Quantum Dynamics as the Foundation of Deductive Theoretical Physics

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Dedicated to my wife Vibhuta
on the occasions of her 62th birthday
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Abstract

There is a fundamental distinction between the two types of physical approaches, the dynamic, strictly causal kind, and the merely statistical kind. The Hilbert space-based formulation of quantum mechanics provides the most prominent example of a statistical order-from-disorder mechanism.

The most striking characteristics of a Hilbert space compared to classical or relativistic geometries are its internal geometry in the form of an inner product and the resulting property of being a closed manifold. Accordingly, a Hilbert space-based formulation of quantum dynamics that provides a statistical order-from-order mechanism must provide the Hilbert space structure with additional tools that continue to guarantee the property of a closed manifold. To ensure that the order-from-order mechanism applies to the laws of classical physics, additional requirements are necessary:

1. *the laws of classical mechanics can be causally described by the "interaction" of quantum-mechanical and quantum-dynamic systems. In particular, the concept of the "Coulomb potential" can ultimately be explained as an approximate model of underlying quantum dynamic models.*
2. *classical phenomena can be traced back to quantum-dynamic causes by means of suitable order-from-order mechanisms and quantum-dynamic fluctuations can be statistically distinguished from quantum-mechanical processes, (ScE).*

The two new concepts of the proposed theory of quantum dynamics are:

1. an extension of the statistical Hilbert space L_2 of quantum mechanics to $H_{-1/2}$
2. two Krein-space representations of the Hilbert space $H_{-1/2}$, which define the concept of a physical ether, consisting of two interconnected ground-state quantum fluctuation and plasma quantum fluctuation models with different superposition and non-superposition states.

The metaphysical prerequisite for both models is a mathematically justifiable "density" assumption regarding this concept of a „physical ether". It includes a concept of dynamic energy that is independent of mechanical energy and independent of the concepts of the theory of motion - that is, in particular, independent of the concepts of space, time, and continuity. The laws of classical mechanics may be interpreted as aproximation solutions in Hilbert scales of underlying variational represention of quantum-mechanical laws governed by the so-called energy method.

At first glance, the extended basis quantum-mechanical Hilbert space framework $H_{-1/2}$ appears to be a cheap magic trick proves to be a valid step toward resolving the problem of a missing physical interpretation of a mathematical "world-point state." The compact embedding of $H_{-1/2}$ into $H_{-\mathrm{n}/2-\varepsilon}$, $\varepsilon > 0$, which is independent of all spatial dimensions, ensures - by virtue of Sobolev's embedding theorem - that continuous fields in classical physics can be generated by quantum-mechanical causes, regardless of the dimension of the underlying Euclidean space. The systems of this extended Hilbert space can thus accommodate the mathematical concept of the Newtonian or Coulomb potential, both of which, for mathematical reasons, only make physical sense in 3D Euclidean space. The concept of phenomenon-specific "point charges" that generate continuous fields throughout 3D Euclidean space can thus be completely replaced. Furthermore, it has been shown that the variation equations of the 3D-NSE initial boundary value problem, based on a "test space" $H_{-1/2}$, avoid existing "blow-up" solutions by means of L_2 test functions. In classical physics, the systems of the extended "test space" $H_{-1/2}$ could therefore be physically interpreted as "dynamic fluid/gas elements."

Content

1. Introduction and structure of the paper 3-5

2. Preliminary remarks by some key figures in the area of field theory 6-18

3. The mathematical toolbox

   a. Compact and symmetric operators, Hilbert scales 19-22
   b. The inner product of energy in the calculus of variations 23-26
   c. The three model problems of PDE in Hilbert spaces 27
   d. The Neumann problem and the Prandtl operator 28
   e. The blow up solutions of the 3D-NSE system 29-30
   f. J. Plemelj's expanded concept of mass and flow 31-33
   g. The radiation problem of the wave equation 34
   h. The global nonlinear stability of the Minkowski space 35
   i. Pseudo-differential operators on R^n 36-37
   j. The quaternion rotation operator 38-41
   k. Potential, angular, and dissipative operators in Krein spaces 42-47
   l. Dehnen's et al. forgotten key regarding a heuristic approach to GRT 48
   m. Kirchhoff's forgotten key regarding the potential of two conductive spheres 49

4. A Hilbert-Krein space based theory of quantum dynamics

   a. A short story about a failed engagement 50
   b. Kant's metaphysical foundations of natural science as a headline 51
   c. Bohm's concept of „Wholeness" and „Implicate Order" 52-53
   d. The concept of dynamic quanta and dynamic fluid particles 54-56
   e. The paradigm shift from inductive to deductive physics 57-58
   f. Two ways to define potential operators in Hilbert-Krein spaces 59-60
   g. The appropriate choice $\alpha = \pm 1/2$ of a Hilbert scale H_α 61
   h. The two quantum fluctuation systems of the Hilbert space $H_{-1/2}$ 62-65

5. A non-exhaustive list of areas of theoretical physics that are affected

   a. Mathematics, natural sciences, and all that 66-71
   b. Maxwell theory and special cases 72-75
   c. Classical physics, relativity, quantum theory, and all that 76-79
   d. Electromagnetism, quantum electrodynamics, and all that 80-81
   e. Atomic nuclei, nucleons, neutrino, ground state of deuteron, and all that 82-88
   f. The real and complex Lorentz groups 89-90
   g. SMEP and all that 91-94
   h. Plasma and all that 95-98
   i. Bose-Einstein condensates, superfluids, superconductivity and Fermi liquids 99
   j. Planck/Schrödinger/Bohm and implicate order dynamics 100-105
   k. Excerpts from relevant articles or books 106-113

6. Perspectives and articles on mathematics, physics, and philosophy 114-223

   Aristotle; Avenarius R.; Barbour J.; Bergson H.; Bethe H. A.; Böhme G.; Bohm D.; Cassirer E.; Capra F.; Chen F. F.; Courant R.; Davidson J.; Dee K.; Dehnen H. et al.; Deleuze G; Deligne P.; Derbyshire J.; Descartes R.; Dirac P. A. M.; Dürr H.-P.; Eco U.; Ehrenhaft F. (and W. Schauberger); Einstein A.; Euler L.; Fermi E.; Feynman R.; Goethe J. W. v.; Gödel K.; Hawking S. W.; Hegel G. W. F.; Heidegger M.; Heisenberg W.; Helmholtz H.; Hildebrandt S; Hübscher A. ; Husserl E.; Kant I.; Kirchhoff G.; Klainerman S.; Kneser A.; Kramers H. A.; Leedskalnin E.; Leibniz G.-W.; Lorentz H. A.; Mach E.; Marx W.; Maupertuis P.; Mijajlovic Z.; Miyamoto K.; Müller O. L.; Nagel Th.; Neuenschwander D. E.; Nietzsche F.; Nussbaumer I.; Penrose R.; Peskin M.; Plaass P.; Planck M.; Plemelj J.; Poluyan P.; Robitaille P.-M.; Rollnik H.; Rovelli C.; Russel R.; Schauberger V.; Schiller F.; Schmicking D. A.; Schopenhauer A.; Schpolski E. W.; Schrödinger E.; Shaw B.; Shu F. H. ; Smolin L.; Spatschek K. H.; Treder H.-J.; Unzicker A.; Vagt C.; Weinberg S.; Weizsäcker C. F. v.; Welzer H.; Weyl H.; Wheeler J. A.; Whitehead A. N.; Wigner E.

7. References and related literature 224-237

1. Introduction and structure of the paper

The mathematical framework of classical physics is provided by calculus, the theory of partial differential equations and integral equations. The expanded mathematical framework in cosmology includes additional concepts from topology, as well as the concepts of metric (Riemann) and linearly normed spaces (Minkowski). The mathematical framework of quantum theory is provided by probability theory and Hilbert space theory (von Neumann). The phenomenon of "gravitation" cannot be described within the framework of the theory of motion, neither in conjunction with Euclidean geometry nor in conjunction with Minkowski geometry. The mathematical solution concept here is "geometrodynamics" (i.e., based on methods of algebraic topology) without metaphysical foundations (PlP). The paradigm of quantum theory for deriving quantum-mechanical equations of motion from classical Lagrangian equations of motion is "quantization of the classical Lagrangian equations of motion." The two mathematical methods for transferring the laws of motion in quantum mechanics to the various gauge theories of quantum dynamics are "regularization" and "renormalization," (DeP).

The key physical concept that comes into play in all of the above theories is that of a "potential." The Newtonian potential and the Coulomb potential are the same mathematical concept. In physics, the Coulomb potential plays a central role not only in classical electrodynamics but also in all plasma theories and nuclear physics, (ChF), (BeH). It may seem both simple and easy to handle from a physicist's perspective, but from a mathematical standpoint, it is by no means so. It describes the so-called fundamental solution of the potential equation for a given "function" - the delta or Dirac "function" - whose "regularity" also depends on the dimension of space R^n.

The concept of transferring the laws of motion from classical physics to relativistic quantum mechanics in a physical vacuum (with the aim of enabling classical, so-called conserved quantities such as charge and current density) is associated with the terms "angular momentum" and "spin-orbit operator." This is an operator that unites two concepts of physical momentum: angular momentum (intrinsic rotation) and linear momentum. The stationary states of the Dirac equation are determined by separating the radial components, thereby yielding two determining equations for calculating the stationary states of the underlying Hamiltonian operator. When calculating the Sommerfeld energy formula for the electron in the hydrogen atom based on an assumed Coulomb potential, these two determining equations yield two power series with different asymptotic behavior for very large and very small radii. In order to predict finite energy values, both power series must converge, which can only be achieved by means of a suitably chosen constant - the Sommerfeld fine-structure constant.

The empirical "measurement method" of quantum mechanics is based on the mathematical methods of probability theory and statistics. The corresponding Hilbert space L_2 is the space of quadratically integrable Sobolev functions. (The integration refers to the Lebesgue integral. These integrals allow for so-called null sets; that is, the Lebesgue integral taken over a null set has the value zero. Such countable sets include, for example, the countably many eigenvalues of a Hamiltonian operator. From a physical perspective, this circumstance leads to the concept of a "wave packet" (which is a divergent integral in the L_2 Hilbert space, since the Hamilton operator is defined only on a subspace of the L_2 Hilbert space). The mathematical solution concept is that of the delta or Dirac distribution. The Sobolev embedding theorem guarantees continuous field solutions of $\Delta u = \delta(x) \in H_{-n/2-\varepsilon}$. Thus, a physical requirement determines the mathematical construct of the Dirac distribution. Multiplication by case-specific "potential constants" then defines the respective "source of causation" for the corresponding physical law. And this brings us full circle to the physical concept of a point charge in Euclidean space, whose mathematical concept is likewise the Dirac "function."

Infinitely small points with finite mass or charge do not exist in nature. Assuming point charges leads to infinite values (divergences) when calculating the interaction of a particle with its own field. At the Planck scale, the concept of continuous spacetime points loses its validity. A point distribution completely breaks down physically at this scale. The proposed model of quantum dynamics ensures continuous fields without having to resort to the concept of a Dirac "function." The related framework of empirical physics, in particular the concept of the "observer," can be described approximately using the methods of quantum mechanics. From a physical perspective, the shared concept of the "observer" corresponds to that of a common, statistically relevant Hamilton operator of both systems ("ground state" and "plasma"), which "exists" independently of the concepts of space, time, and continuity. "De-quantization" toward classical physics enables a subsequent assignment of measurement systems (geometries) appropriately selected for specific phenomena or theories.

Experimentally measured physical constants are interpreted in this model not as fundamental fixed points, but as approximation constants (approximate values) that arise from the reduction of the higher-order Hilbert space.

In quantum mechanics, the Lagrangian formulation depending on certain kinds of fields serves as a bridge to quantum field theory. In quantum field theories, the method of renormalization is used to describe how the dynamics of a quantum system change when we alter the scale (distance, energies) at which we probe it (DeP), p. 553. The fundamental mathematical problem underlying all quantum field theories is "infinite physical quantities (divergences) that arise when calculating different subatomic processes." The conceptual problem in theoretical physics is that each of these different subatomic processes requires its own theory. The basic concept of these renormalization theories is based on the two methods of "regularization" and "dimensional renormalization." Mathematically speaking, the "renormalization method" solves the problem of divergent integrals of all terms in a power series. The method of "dimensional regularization" ensures that the terms of such a series converge, but the series itself remains divergent in every neighborhood of zero - that is, it is undefined for the "physical vacuum region." A renormalization group is intended to explain why decoupling quantum mechanics and quantum dynamics - each with the same minimum number of scales (the "distance") - can work at all, (DeP) p. 560. The theoretical proof of the existence of "dynamic" elementary particles that cannot be directly observed statistically thus becomes a problem in the mathematics of a non-trivial quantized Yang-Mills theory: *"to prove that for any compact simple gauge group F, a non-trivial quantum Yang-Mills theory exists on R^4 and has a mass gap $\Delta > 0$."* https://www.claymath.org/millennium/navier-stokes-equation/.

The basic structure of the proposed theory of quantum dynamics is formed by an extended Hilbert space of the Hilbert space L_2 underlying J. von Neumann's quantum mechanical statistics, (NeJ). The basic requirements for such a theory are as follows

> *Requirement 1:*
>
> *The laws of classical mechanics can be causally described by the "interaction" of quantum-mechanical and quantum-dynamic systems. In particular, the concept of the "Coulomb potential" can ultimately be explained as an approximate model of underlying quantum dynamic models.*
>
> *Requirement 2:*
>
> *classical phenomena can be traced back to quantum-dynamic causes by means of suitable order-from-order mechanisms and quantum-dynamic fluctuations can be statistically distinguished from quantum-mechanical processes, (ScE).*

The proposed two connected basic models for a Hilbert-Krein space-based quantum dynamics in an extended basis quantum mechanical Hilbert space framework $H_{-1/2}$ consists of a physical ground-state model and a physical plasma model. What at first glance appears to be a cheap magic trick proves to be a valid step toward resolving the problem of a missing physical interpretation of a mathematical "world-point state." The metaphysical prerequisite for both mathematical models is a mathematically justifiable "density" assumption regarding this concept of a „physical ether". It includes a concept of dynamic energy that is independent of mechanical energy and independent of the concepts of the theory of motion - that is, in particular, independent of the concepts of space, time, and continuity. The laws of classical mechanics may be interpreted as aproximation solutions in Hilbert scales of underlying variational represention of quantum-mechanical laws governed by the so-called energy method, (VeW). The concept of related complementary variational methods is provided in (ArA).

All chapters of this work are structured to be independent of one another in terms of content. Chapter 4 describes the actual topic of this paper. The titles of the individual sections are intended to provide an overview of the conceptual design elements of the two models, specifically with regard to the preliminary remarks by some key figures in Chapter 2. The authors cited therein were selected from the extensive list of authors in Chapter 6 because they not only pointed out problems with current paradigms related to the topic of this paper at an early stage but also supported these with new conceptual proposals. Chapter 3 provides a toolkit of well-known mathematical methods that might be suitable for linking the proposed basic model with the topics of classical physics, nuclear physics, and the two theories of relativity, which are compiled in Chapter 5, among other places. Chapter 6 comprises a loose compilation of numerous texts from books and

publications on the broader subject matter covered in Chapters 3 and 5, which may not be accessible to everyone.

At this point, at the very latest, the physicist A. Unzicker must be mentioned, whose books have provided the author with motivation and a wealth of inspiration. One might even go so far as to say that one of the aims of this work is to help underpin Unzicker's vision of a "mathematical reality" with a mathematical model. Since the basic mathematical model of the proposed quantum dynamics is free in the choice of a suitable "coordinate system," then, for example, entirely in line with A. Unzicker's "Mathematical Reality," quaternions (with the unit quaternions—which are isomorphic to the group of Lorentz transformations—serving as "coordinate units") could constitute a suitable alternative to the concept of a "space- and time-based quantum world." This would then reduce Pauli's concept of spin matrices and operators in quantum mechanics to what they ultimately are: nothing other than Hamiltonian quaternions. This concept would also be in complete agreement with the work by H. Dehnen and colleagues, titled "A Heuristic Approach to the General Theory of Relativity"—a work also mentioned by A. Unzicker (in his book *Einstein's Lost Key*) and unjustly forgotten:

From the perspective of experimental physics, it is possible to take a heuristic approach to the general theory of relativity while avoiding covariant theory. From an empirical standpoint, it is natural to start from the fact of inertial systems and the Lorentz invariance of the dynamic equations that holds for them (DeH).

From a mathematician's perspective, the "wave-particle duality" is not a duality at all, but simply the observation that, in functional analysis, a linear operator is meaningfully defined within the framework of Hilbert space theory only if, in addition to its mapping rule, it also possesses a suitably defined domain. The domain of the "momentum operator" is compactly embedded in the domain of the "localization operator." Thus, the "wave-particle duality" is nothing more than the expression of a mathematical fact. Whether this fact must be elevated to the status of a physical principle for lack of alternatives is, at the very least, open to doubt. From a mathematical perspective, the obvious task here seems rather to be to place the two physical concepts of "matter" and "field" into a context other than the traditional mechanical "cause-and-effect" relationship, especially since the mechanical effect is inextricably linked to the concept of "energy" and the concept of "absolute time," which, according to the special theory of relativity, are not applicable to a cosmological framework in physics anyway.

2. Preliminary remarks by some key figures in the area of field theory

A. Unzicker

The Mathematical Reality
Mathematics alone must suffice

(UnA2) S. 138: There is overwhelming historical evidence that scientific revolutions have led to simplification, and simplicity has proved to be a fairly good guiding principle for our philosophical considerations so far. But how can simplicity be achieved when describing nature? Ultimately, there is only one way: building a model of reality devoid of any „physical" constants, i.e. justifying them by purely mathematics. If we adhere to a rational philosophy of nature, fundamental constants must be seen as a consequence of our hitherto limited understanding. There is no epistemological reason that forbids describing the world around us in mathematical terms only, without resorting to arbitrary postulates such as „constants of nature". … I am advocating a theory based on pure mathematics for the description of nature. For any real progress, however, c (the speed of light) and h (the Planck constant) must be explained, i.e. manifest themselves as mathematical properties. The entire phenomenology of c and h must turn out to be a necessary characteristic of mathematical objects. And, of course, the properties of these objects, should also shed light on why we perceive them approximately as the 3+1-dimensional world commonly called space and time.

M. Planck

The dynamical and the statistical type of law

(PlM) S. 90: … Physics, too, like the social sciences long before it, has come to recognize the great importance of an approach that is entirely distinct from the purely causal one, and has applied it with ever-increasing success since about the middle of the last century; this is the statistical method, with the development of which the entire recent progress of theoretical physics is most closely connected. Instead of investigating the dynamic laws of an individual process which currently remain entirely obscure without any prospect of tangible success, one first compiles the observations made on a large number of individual processes of a certain type and derives average or mean values from them. For these mean values, certain empirical rules then emerge depending on the particular circumstances of the case, and the rules thus obtained allow us admittedly never with absolute certainty, but nevertheless with a probability that very often amounts practically to certainty, to predict the course of future events in advance not in every detail, but - and this is often what matters most in practical applications - in their average course … At any rate, the situation described above makes it sufficiently clear just how crucial it is to carry out a careful and fundamental distinction between the two types of regularity discussed the dynamic, strictly causal kind, and the merely statistical kind - for understanding the very essence of all scientific knowledge.

A. Einstein

Ether and the theory of relativity

Since, according to our current understanding, even the elementary particles of matter are, in essence, nothing more than condensations of the electromagnetic field, our current worldview recognizes two realities that are conceptually completely distinct from one another, even though they are causally linked: the gravitational ether and the electromagnetic field—or, as one might also call them, space and matter.

Of course, it would be a major step forward if we could succeed in conceiving of the gravitational field and the electromagnetic field together as a unified entity. Only then would the era of theoretical physics, founded by Faraday and Maxwell, come to a satisfactory conclusion. The contrast between ether and matter would then fade, and the entire field of physics would become a coherent system of thought similar to that of geometry, kinematics, and the theory of gravitation as unified by the general theory of relativity. …

In summary, we can say: According to the general theory of relativity, space possesses physical properties; thus, in this sense, an ether does exist. According to the general theory of relativity, a space without an ether is inconceivable; for in such a space, not only would there be no propagation of light, but there would also be no spacetime distances in the physical sense. However, this ether must not be conceived as possessing the property characteristic of ponderable media—namely, consisting of particles that can be tracked over time; the concept of motion must not be applied to it, (EiA5).

E. Schrödinger

Two ways of producing orderlines

„The orderliness encountered in the unfolding of life springs from a different source. It appears that there are two different „mechanisms“ by which orderly events can be produced: the „statistical mechanism“ which produces „order from disorder“ and the new one, producing „order from order“. To the unprejudiced mind the second principle appears to be much simpler, much more plausible. No doubt it is. That is where physicists were so proud to have fallen in with the other one, the „order-from-disorder“ principle, which is actually followed in Nature and which alone conveys an understanding of the great line of natural events, in the first place of their irreversibility. But we cannot expect that the „laws of physics“ derived from it suffice straightaway to explain the behaviour of living matter, whose most striking features are visible based to a large extent on the „order-from-order“ principle. You would not expect two entirely different mechanisms to bring about the same type of law – you would not expect your latch-key to open your neighbour's door as well.

We must therefore not be discouraged by the difficulty of interpreting life by the ordinary laws of physics. For that is just what is to be expected from the knowledge we have gained of the structure of living matter. We must be prepared to find a new type of physical law prevailing in it. Or are we to term it a non-physical, not to say a super-physical law?“

The new principle is not alien to physics

„No, I do not think that. For the new principle that is involved is a genuinely physical one: it is, in my opinion, nothing else that the principle of quantum theory over again. To explain this, we have to go to some length, including a refinement, not to say an amendment, of the assertion previously made, namely, that all physical laws are based on statistics.

This assertion, made again and again, could not fail to arouse contraction. For, indeed, there are phenomena whose conspicuous features are visible based directly on the „order-from-order“ principle and appear to have nothing to do with statistics or molecular disorder.

(ScE) pp. 49-51: „There is, essentially, only one problem in statistical thermodynamics: the distribution of a given amout of energy E over N identical systems. Or perhaps better: to determine the distribution of an assembly of N identical systems over the possible states in which this assembly can find itself, given that the energy of the assembly is a constant E.

„The second attitude makes us think of the n_s „particles present in state α_s“ as a proper vibration (or „hohlraum“ oscillator to use a customary expression) in its n_sth quantum state. (This attitude really corresponds to the so-called second quantization or field quantization). n_s becomes one quantum number … and comes into line with the ordinary view about quantum states and their statistical weight (viz. equal for any two of them). … Bose pointed out (about 1924) that we could, alternative to the „hohlraum“ statistics, speak of photon statistics, but then we had to make it „Bose statistics“. … Since in the Bose case we seem to be faced, mathematically, with a simple oscillator of Planck type, of which n_s is the quantum number, we may ask whether we ought not to adopt for n_s half-odd integer $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \ldots n + \frac{1}{2}, \ldots$ rather than integers. One must think, call that an open dilemma. From the point of view of analogy one would very much prefer to do so. For, the „zero-point energy“ $\frac{1}{2}h\nu$ of a Planck oscillator is not only borne out by directly observation in the case of crystal mattices, it is also immediately linked up with Heisenberg uncertainty relation that one hates to dispense with it. On the other hand, if we adopt it straightaway, we get into serious trouble, especilly on the contemplating changes of the volume (e.g. adiabatic compression of a given volume of black body raditation), because in this process the (infinite) zero-point energy seems to change by infinite amount! So we do not adopt it, and we continue to take for the n_s the integers, beginning with 0.“

The nonlinear Schrödinger equation

(ChF) p. 336: „*This equation has the standard dimensionless form*

$$i\frac{\partial\psi}{\partial t} + p\frac{\partial^2\psi}{\partial x^2} - q|\psi|^2\psi = 0$$

where ψ is the wave amplitude, $i = (-1)^{1/2}$, and p and q are coefficients whose physical significance will be explained shortly. This equation differs from the usual Schrödinger equation

$$ih\frac{\partial\psi}{\partial t} + \frac{h^2}{2m}\frac{\partial^2\psi}{\partial x^2} - V(x,t)\psi = 0$$

in that the potential $V(x,t)$ depends on ψ itself, making plane waves of the non-linear Schrödinger equation in the form. Note however, that V depends only on the magnitude and not on the phase of ψ. This is to be expected, as far as electron plasma waves are concerned, because the nonlinearity comes from the ponderomotive force, which depends on the gradient of the wave intensity.

Plane wave solutions of the (standard dimensionless) equation are modulationally unstable if $pq > 0$; that is, a ripple on the envelope of the wave will tend to grow. For plasma waves, it is easily to see how the ponderomotive force can cause a modulational instability. The ponderomotive force moves both electrons and ions toward the intensity minima, forming a ripple in the plasma density. Plasma waves are trapped in regions of low density."

R. Courant

Empirical evidence and mathematical existence

(HiS) p. 148: „*Empirical evidence can never establish mathematical existence – nor can the mathematician's demand for existence be dismissed by the physicist as useless rigor. Only a mathematical existence proof can ensure that the mathematical description of a physical phenomenon is meaningful*."

W. Heisenberg

Two different types of symmetries or invariances of elementary particles,
electromagnetic charge composed of isospin charge and hypercharge
the latter connected with the concept of strangeness

Definition of elementary particles

(HeW) pp. 1-5: Some fifty year ago only three kinds of „elementary particle" were known. Electron and proton were considered to be the smallest units of matter, the photon or light quantum the smallest unit of energy in radiation. This concept of the „smallest unit" had its origin in the problem of the divisibility of matter. A piece of matter can, by mechanical or chemical means, be decomposed into single atoms. The atom may be separated into its nucleus and the surrounding electron. The atomic nucleus can be split into protons and neutrons.

The only processes in which a splitting of elementary particles could possibly be exspected, are collisions between particles of extremely high energy. … many experiments have led to the result that from such high energy collision between two particles actually many particles may emerge; but the emerging particles are not neccessarily smaller than the colliding particles. On contrary one finds always the same various kinds of particles, whatever the nature of the colliding particles may have been. The phenomena can be described more adequately by saying, that the big kinetic energy of the colliding particles has been transmuted into matter, into the emerging particles („multiple production of particles"). … The different elementary particles may be considered as different forms in which the fundamental substance, matter or energy, can exist. They may be called „smallest units" only in the sense that when they are split, their parts are not smaller units but units of roughly the same size.

Classification of elementary particles

The obvious starting point for a classification is an analysis of the features of the observations and the underlying natural law that in classical physics have been called „laws of conservation"; for instance conservation of energy, momentum, angular momentum, charge, et.. The physicists have learned from the mathematicians that conservation laws are connected with symmetries, with the group structure of the

underlying natural law. If the underlying law is invariant under a certain transformation, the operator performing this transformation may be used for constructing an observable, which stays constant during the processes described by the natural law. The different states of the system may be considered as different representations of the group. These representations can, from a mathematical point of view, be characterized by discrete numbers (in case of compact groups) or continuous parameters (in case of non compact groups), the discrete numbers being called quantum numbers from the phenomenological point of view. The conservation law may be stated by saying that the sum of the quantum numbers or the sum of the continuous parameters of the interacting systems stays constant during the collision process, or, in case of a discrete group, by saying that the product of the quantum numbers (for example, parity) stays constant. All these relations are well known from unrelativistic quantum mechanics; but it is a very important result of all recent observations on elementary particle processes, that in this respect no fundamental differences has been found between elementary particles physics and quantum mechanics. The elementary particles can be classified by quantum numbers, mass-values and lifetimes just like the stationary states in quantum mechanics.

A difference comes in however, by the relevant groups. The most important group in elementary particle physics is undoubtedful the Lorentz group, which replaces the Galilean group of unrelativistic quantum mechanics and produces the conservation laws of energy, momentum and angular momentum. … Another important group is connected with the „iso-spin“ of the particles and with charge symmetry; this group seems to be isomorphic with the group of rotations in threedimensional space or with the group SU_2 (group of unitary and unimodular transformations of two complex variables) and produces the isospin-multiplets which will be discussed in more detail below. The empirical fact that electromagnetic charge is composed of two parts, isospin charge and hypercharge, and that the latter is connected with the concept of strangeness, suggests a connexion between the isospin group and the gauge group producing strangeness. These two groups should therefore probably be combined to the group U_2, (group of unitary transformations of two complex variables), which is just the direct product of SU_2 and one gauge group [(*)]. Finally the observations show the relevance of several discrete groups, space reflection parity P, charge conjugation C, time reversal T and G-conjugation (product of C and isospin reversal).

While the existence of these groups contains the most important information concerning the structure of the underlying natural law, it is somewhat disturbing, that some of these symmetries seems to be only approximately valid. In fact, the existence of multiplets in the spectrum of elementary particles shows already that there are weaker interactions which are less symmetrical. In case of completely symmetry any n-dimensional representation of the group should lead to n identical eigenvalues for the mass. A splitting of these eigenvalues can be produced only by interaction which is less symmetrical. In quantum mechanics it is well known that, for example, an outer electric field destroys the rotational symmetry of the original system and thereby leads to a splitting of the optical lines (Stark effect). The symmetry can be reduced by actions from outside.

On the other hand, the multiplet structure of the optical spectra rests upon the fact that the introduction of the magnetic spin-orbit interaction destroys the independence of the rotational symmetry of the spin and the orbital motion. In this case, it is not an action from outside that breaks the symmetry; by neglecting the magnetic forces, one had, so to speak, artificially produced a higher symmetry (the independence of the rotation of spin and orbit), which is broken afterwards. One learns from these examples that one should distinguish between two different types of symmetry or invariance. Symmetries of the first kind are properties of the underlying law of nature (e.g., invariance under rotations in the quantum mechanics of the individual atom). Symmetries of the second kind are based on approximations that may be more or less justified in the various stationary states of the system in question. In the case of symmetries of the first kind, which can be disturbed by external effects, it must be explained in greater detail what the term “external” can mean in specific terms: the effect can be arbitrarily induced from the outside by the experimenting physicist. In other cases, the effect may come from the world as a whole and cannot be influenced by the physicist. In Einstein’s theory of gravitation, for example, the centrifugal force arises as an external effect caused by the overall state of the world or by masses located very far away. It cannot be arbitrarily altered.

The observation of mass multiplets in the spectrum of elementary particles that are not caused by arbitrary external fields leaves room for two very different interpretations. The symmetries can either be properties of the underlying law of nature - that is, properties of the first kind, which are, however, subsequently broken by the ground state of the world - or they can belong to the second kind; namely, they can be simulated to a

certain extent by approximations. The decision between these two possibilities is, in any case, of extraordinary importance for the formulation of the theory.

(*) In fact, in the standard model the weak and the electromagnetic interactions are unified in what is called *electroweak* theory, where there is a special symmetry related to W^+, W^-, Z^0, and the photon γ, according to the groups $SU(2) \times U(1)$ or, more correctly, $U(2)$, (PeR4) pp. 640/641.

> (PeR4) p. 654: The group might be expressed as $SU(2) \times U(1)/Z_2$, where the $'/Z_2'$ means „factor out by a Z_2 subgroup". However, there is more than one such subgroup, so this notation is not fully explicit. The notation $'U(2)'$ automatically picks out the correct one. (I am grateful to Florence Tsou for this observation.) It seems that the reason that the electroweak symmetry group is not conventionally referred to as $'U(2)'$ is that this does not easily extend to the symmetry of the full standard model, which also incorporates the strong symmetry group $SU(3)$, the full group being a version $SU(3) \times SU(2) \times U(1)/Z_6$.

R. Feynman

QED, The strange theory of light and matter

(FeR2) pp. 4-7: Physics has a history of synthesizing many phenomena into few theories. For instance, in the early days there were phenomena of motion and phenomena of heat; there were phenomena of sound, of light, and gravity. But it was soon discovered that some of these apparently different things were aspects of the same thing. For example, the phenomena of sound could completely understood as the motion of atoms in the air. So sound was no longer considered something addition to motion. It was also discovered that heat phenomena are easily understoodable from the laws of motion. In this way, great globs of physics theory were synthesized into a simplified theory. The theory of gravitation, on the other hand, was not understandable from the laws of motion, and even today it stands isolated from the other theories. Gravitation is, so far, not understandable in terms of other phenomena.

After the synthesis of the phenomena of motion, sound, and heat, there was the discovery of a number of phenomena that we call electrical and magnetic. In 1873 these phenomena were synthesized with the phenomena of light and optics into a single theory by James Clerk Maxwell, who proposed that light is an electromagnetic wave. So at that stage, there were the laws of motion, the laws of electricity and magnetism, and the laws of gravity.

Around 1900 a theory was developed to explain what matter was. It was called the electron theory of matter, and it said that there were little charged particles inside of atoms. This theory evolved gradually to include a heavy nucleus with electrons going around it.

Attemps to understand the motion of the electrons going around the nucleus by using mechnanical laws – analogous to the way Newton used the laws of motion to figure out how the earth went around the sun – were a real failure. All kinds of predictions came out wrong. (incidentally, the theory of relativity, which you all understand to be a great revolution in physics, was also developed at about that time. But compared to this discovery that Newton's law of motion were quite wrong in atoms, the theory of relativity was only a minor modification.) Working out another system to replace Newton's law took a long time because phenomena at the atomic level were quite strange. One had to lose one's common sense in order to preceive what was happening at the atomic level. Finally, in 1926, an „uncommon-sensy" theory was developed to explain the „new type of behavior" of electrons in matter. It looked cockeyed, but in reality is was not: it was called the theory of quantum mechanics. The word „quantum" refers to this peculiar aspect of nature that goes against common sense. It is this aspect that I amgoing to tell you about.

Because the theory of quantum mechanics could explain all chemistry and various properties of substances, it was a tremendous success. But still there was the problem of interaction of light and matter. That is, Maxwell's theory of electricity and magnetism had to be changed to be in accord with new principles of quantum mechanics that had been developed. So a new theory, the quantum theory of the interaction of light and matter, which is called by the horrible name „quantum electrodynamics," was finally developed by a number of physicists in 1929.

But the theory was in trouble. If you calculated something roughly, it would give a reasonable answer. But if you tried to compute it more accurately, you would find that the correction you thought was going to be small

(the next term in a series, for example) was in fact very large – in fact, it was *infinite*! So it turned out you couldn't really compute anything beyond a certain accuracy. …

Using the theory of relativity, Paul Dirac made a relativistic theory of the electron that did not completely tak into account all the effects of the electron's interaction with light. Dirac's theory said that an electron had a magnetic moment – something like the force of a little magnet – that had a strength of exactlty 1 in certain units. Then in 1948 it was discovered in experiments that the actual number was closer to 1.00118 (with the uncertainty of about 3 on the last digit). It was known, of course, that electrons interact with light, so some small correction would be understandable from the new theory of quantum electrodynamics. But when it was calculated, instead of 1.00118 the result was infinity – which is wrong, experimentally!

Well, this problem of how to calculate things in quantum electrodynamics was straightened out by Julian Schwinger, Sin-Itiro-Tomonaga, and myself in about 1948. Schwinger was the first to calculate this correction using a new „shell game"; his theoretical value was around 1.00116, which was close enough to the experimental number to show that we were on the right track. At last, we had a quantum theory of electricity and magnetism with which we could calculate!

R. Feynman

Conservation laws of momentum and angular momentum

(FeR1) pp. 77-80: There are other laws similar to the conservation of energy, in the sense that they are numbers. One of them is the momentum. If you take all the masses of an object, multiply them by the velocities, and add them all together, the sum is the momentum of these particles; and the total amount of momentum is conserved. Energy and momentum are now understood to be very closely related, so I have put them in the same column of our table.

Another example of a conserved quantity is angular momentum. The angular momentum is the area generated per second by objects moving around. For example, if we have a moving object, and we take any centre whateoever, then the speed at which the area swept out by a line from centre to object, increases, multiplied by the mass of the object, and added together for all the objects, is called the angular momentum. And that quantity does not change. So we have conservation of angular momentum. Incidentally, at first sight, if you know too much physics, you might think that the angular momentum is not conserved. Like the energy it appears in different forms.

Although most people think it only appears in motion it does appear in other forms, as I will illustrate. If you have a wire, and move a magnet up into it, increasing the magnetic field through the flux through the wire, there will be an electric current – that is how electric generators work. Imagine that instead of a wire I have a disc, on which there are electric charges analogous to th electrons in the wire. Now I bring a magnet dead centre along the axis from far away, very rapidly up to the disc, so that now there is a flux change. Then, just as in the wire, the charges will start to go around, and if the disc were on a wheel it would be spinning by the time I had brought the magnet up. That does not look like conservation of angular omentum, because when the magnet is away from the disc nothing is turning, and when they are close together it is spinning. We have got turning for nothing, and that is against the rules. „Oh yes," you say, „I know, there must be be some other kind of interaction that makes the magnet spin the opposite way." That is not the case. There is no elecrical force on the magnet tending to twist it the opposite way. The explanation is that angular momentum appears in two forms: one of them is angular momentum of motion, and the other is angular momentum in elecric and magnetic fields. There is angular momentum in the field around the magnet, although it does not appear as motion, and this has the opposite sign to the spin. If we take the opposite case it is even clearer. If we have just the particles, and the magnet, close together, and everything is standing still, I say there is angular momentum in the field, a hidden form of angular momentum which does not appear as actual rotation. When you pull the magnet down and take the instrument apart, then all the fields separate and the angular momentum now appear and the disc will start to spin. The law that makes it spin is the law of induction of electricity. Whether angular momentum comes in units is very difficult for me to answer. …. The angular momentum is always a number of units. It is not the kind of unit, like an electric charge, that you can count. The angular momentum does come in units in the mathematical sense that the number we get in any measurement is a definite integer times a unit. But we cannot interpret this in the same way as with units of electric charge, imaginable units that we can count – one, then another, then another. In the case of angular momentum we cannot imagine them as separate units, but it comes out always as an integer … which is very peculiar.

E. Schrödinger

The Current Situation in Quantum Mechanics
(ScE5) pp. 844-849
translated by DeepL

§ 11 The Abolition of "Entanglement"
The Result Depends on the Will of the Experimenter

We return to the general case of " *entanglement* " without focusing specifically on the particular case of a measurement process, as we did just now. The probability distributions of two particles A and B are supposed to have become entangled through temporary interactions. Now the bodies are to be separated again. Then I can take one of them, say B, and successively supplement my submaximal knowledge of it to a maximum through measurements. I assert: As soon as I succeed for the first time - and not before - first, the entanglement will have just been resolved, and second, through the measurements of B, by utilizing the conditional statements that held, I will have acquired maximal knowledge of A as well. For, first, knowledge of the overall system always remains maximal, because it is in no way compromised by good and accurate measurements. Second: Conditional statements of the form "if A …, then B …" can no longer exist once we have obtained a maximal catalog of B. For this is unconditional, and nothing relating to B can be added to it at all. Third: Conditional statements of the form ("if B … then A …") can be converted into statements about A alone, since all probabilities for B are already known unconditionally. The entanglement is thus completely eliminated, and since knowledge of the entire system has remained maximal, it can only consist in the addition of a maximal catalog for A to the maximal catalog for B.

However, it cannot happen that A becomes maximally known indirectly, through measurements on B, before B is. For then all conclusions work in the reverse direction, i.e., B is as well. The systems become maximally known simultaneously, as claimed. Incidentally, this would also hold true if one did not restrict the measurement to just one of the two systems. But the interesting point is precisely that one *can* restrict it to one of the two; since this achieves the goal.

Which measurements are taken on B and in what order is entirely up to the experimenter's discretion. He does not need to select specific variables in order to exploit conditional statements. He is free to devise a plan that would lead him to maximum knowledge of B, even if he knew nothing at all about B. Nor can it hurt if he carries this plan through to completion. If, after his measurement, he considers whether he has perhaps already reached his goal, it is only to save himself further unnecessary work.

Which A-catalog results indirectly in this way naturally depends on the measurements that occur at B (before the entanglement is completely resolved; not on the later ones, if measurements are continued unnecessarily). Suppose now that I have derived an A-catalog in a specific case. Then I can reflect and consider whether I might have found a *different* one if I had implemented a *different* measurement plan on B. But since I have neither touched system a nor would I have touched it in the hypothetical other case, the statements of the other catalog—whatever they may be—must all be correct *as well*. They must therefore be entirely contained in the first, since the first is maximal. But the second would be maximal as well. Therefore, it must be identical to the first.

Strangely enough, the mathematical structure of the theory by no means automatically satisfies this requirement. Indeed, one can construct examples where the requirement is necessarily violated. It is true that in any given experiment one can actually carry out only *one* sequence of measurements (always on B!), for as soon as that is done, the entanglement is resolved and one learns nothing more about A through further measurements on B.

But there are cases of entanglements in which *two specific* measurement *programs* can be specified for B, each of which 1. must lead to the resolution of the entanglement, 2. must lead to an A-catalog to which the *other cannot* lead at all - whatever measurement values may occur in one case or the other.

The fact is simply that the *two sets* of A-catalogs that can arise in one or the other program are completely distinct and have not a single element in common.

These are particularly extreme cases in which the conclusion is so obvious. In general, one must consider the matter more carefully. If two programs for measurements on B are presented, along with the two series of A-catalogs to which they may lead, then it is by no means sufficient for the two series to have one or a few members in common in order to be able to say: well, then one of these will surely always occur—and thus to regard the requirement as "presumably fulfilled." That is not enough. For *we know* the probability of each measurement on B, considered as a measurement on the entire system, and with many a priori repetitions, each must occur with its intended frequency. The two series of A-catalogs would therefore have to agree, member by member, and moreover, the probabilities in each series would have to be the same. And this is not merely the case for these two programs, but for each of the infinitely many that one can conceive of. There is not the slightest mention of this. The requirement that the A-catalog one obtains should always be the same, regardless of which measurements on B one uses to bring it to light - this requirement is never, ever fulfilled. Let us now discuss a simple "extreme" example.

§ 12 An Example [(1)]

For simplicity's sake, let us consider two systems, each with only *one* degree of freedom. That is, each of them is characterized by *a* coordinate q and a canonically conjugate momentum p. The classical example would be a point mass that can move only along a straight line, or the balls in those toys that small children use to learn arithmetic. p is the product of mass and velocity. For the second system, we denote the determining elements with capital letters Q and P. Whether the two are "strung on the same wire" is something we will not discuss at all in our abstract consideration. But even if they are, it may still be convenient not to calculate q and Q from the same fixed point. The equality $q = Q$ therefore need not imply coincidence. The two systems can nevertheless be entirely separate.

In the cited work, it is shown that an entanglement can exist between these two systems, which *at a certain moment - to which everything that follows refers* - is briefly described by the two equations

$$q = Q\,;\, p = -P.$$

That is to say: *I know that if* a measurement of q on the first system yields a certain value, a Q-measurement performed immediately thereafter on the second system will yield the *same* value, and vice versa; *and I know* that if a p-measurement on the first system yields a certain value, a P-measurement performed immediately thereafter will yield the opposite value, and vice versa. A single measurement of q *or* p *or* Q *or* P breaks the entanglement and reveals as much as possible about *both* systems. A second measurement on the same system now modifies the information about *it*, but reveals nothing more about the other. One cannot, therefore, test both equalities in a single experiment. But one can repeat the experiment a thousand times from scratch; test one or the other equality, as one pleases; and find the one one deigns to test confirmed each time. We assume that this has happened.

If, then, on the thousand-and-first attempt, one feels like dispensing with further tests and instead measuring q on the first system and P on the second, and one finds

$$Q = 4\,;\, P = 7$$

can one then doubt that

$$q = 4\,;\, p = -7$$

would have been a correct prediction for the first system, or

$$Q = 4\,;\, P = 7$$

a correct prediction for the second?

Not fully verifiable in a single experiment - quantum predictions never are - but correct, because whoever had made it would not have been disappointed, whichever half they decided to test.

[(1)] (EiA3) Einstein A., Podolsky B., Rosen N., Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? Physical Review, Vol. 47, 1935

There can be no doubt about this: every measurement is the first on its system. Measurements on separate systems cannot directly influence one another; that would be magic. Nor can they be random numbers, if it is established from a thousand experiments that initial measurements coincide. The experiment catalog $q = 4$; $p = -7$ would, of course, be hypermaximal.

§ 13 Continuation of the example: all possible measurements are uniquely entangled

Now, a *prediction* of this scope is not even possible according to the teachings of quantum mechanics, which we are pursuing here to their ultimate consequences. Many of my friends find this reassuring and explain: what a system *would have* told the experimenter *if* … - has nothing to do with a real measurement and is therefore, from our epistemological standpoint, none of our business.

But let's make the matter crystal clear once again. Let's focus our attention on the system denoted by the lowercase letters p and q; let's call it briefly the "small" system. The situation is as follows. I can pose *one* of two questions to the small system through direct measurement on it: either the one regarding q or the one regarding p. Before I do so, I can, if I wish, have obtained the answer to *one* of these questions through a measurement on the completely separate other system (which we shall regard as an auxiliary apparatus), or I may intend to do so afterward. My small system, like a student in an exam, *cannot possibly know* whether I have done so and for which question, or whether and for which I intend to do so later. From any number of preliminary trials, I know that the student always answers the first question I pose to him correctly. It follows that he *knows* the answer to *both* questions in any case. The fact that answering the first question I choose to ask tires or confuses the student to such an extent that his subsequent answers are worthless does not alter this conclusion in the least. No high school principal, if this situation were to repeat itself with thousands of students of the same background, would judge it differently, no matter how much he might wonder *what* makes all the students so stupid or uncooperative after answering the first question. It would not occur to him that his, the teacher's, consulting a reference book is what prompts the student to give the correct answer in the first place, or even that in cases where the teacher chooses to consult the book only after the student has answered, the student's answer has altered the text in the notebook to the student's advantage.

So my little system has a very specific answer ready for both the q-question and the p-question, in case the one in question is the first one asked directly. This readiness cannot be altered in the slightest by my measuring Q on the auxiliary system (in the illustration: the teacher looking up one of the questions in his notebook but smudging the page where the answer is written with a blot of ink). The quantum mechanic claims that after a Q measurement on the auxiliary system, my small system is assigned a ψ-function in which „q is completely sharp, but p is completely indeterminate." And yet, as already mentioned, not a single iota has changed in the fact that my small system also has a very specific answer ready for the p-question, and indeed the same one as before.

But the matter is even worse. My clever student has a very specific answer ready not only for the q-question and the p-question, but also for a thousand others - and this without my being able to fathom in the least the mnemonic technique by which he accomplishes this. p and q are not the only variables I can measure. Any combination of them, for example,

$$p^2 + q^2$$

corresponds to a very specific measurement according to quantum mechanics. It now turns out that the answer for this, too, can be determined by a measurement on the auxiliary system, namely by measuring $P^2 + Q^2$, and the answers are virtually identical. According to the general rules of quantum mechanics, only one value can result from this sum of squares

$$h, 3h, 5h, 7h, \ldots .$$

The answer that my little system has ready for the $p^2 + q^2$ question (in case this should be the first to be posed to it) must be a number from this series. – The situation is exactly the same with the measurement of $p^2 + a^2p^2$, where$a$ is to be any positive constant. In this case, according to quantum mechanics, the answer must be a number from the following series

$$ah, 3ah, 5ah, 7ah, \ldots$$

For every numerical value of a, one obtains a new question, and for each one, my little system has a number ready from the sequence (formed with the corresponding a).

The astonishing thing is this: these answers cannot possibly be related to one another in the way specified by the formulas! For if q' is the answer provided for the q-question and p' is the answer provided for the p-question, then it is impossible

$$\frac{p'^2+a^2q'^2}{ah} = \text{an odd integer}$$

for certain numerical values of q' and p' and for *any positive number* a. This is not merely an operation with imaginary numbers that cannot actually be determined. Two of the measure numbers can indeed be obtained directly, e.g., q' and p', one by direct measurement and the other by indirect measurement. And then one can convince oneself that the above expression, formed from the measured values q' and p' and an arbitrary a, is not an odd integer.

The lack of insight into the connection between the various answers already given (in the student's "mnemonic technique") is complete; the gap is not filled by some novel algebra of quantum mechanics. The deficiency is all the more puzzling given that, on the other hand, one can prove: entanglement is already uniquely determined by the requirement $q = Q$; $p = -P$.

If we know that the coordinates are equal and the momenta are oppositely equal, then quantum mechanics dictates a *very specific,* one-to-one mapping of *all possible* measurements on the two systems. For *every* measurement on the "small" system, one can obtain the corresponding value through a suitably arranged measurement on the "large" system, and every measurement on the large system simultaneously indicates the result that a certain type of measurement on the small system will yield or has yielded. (Of course, only the initial measurement on each system counts). As soon as we have brought the two systems into a situation where they (in short) coincide in position and momentum, they (in short) also coincide with respect to all other variables.

But we have no idea how the numerical values of all these variables in *one* system are related to one another, even though the system must have a very specific value ready for each one; for we can, if we wish, determine precisely that value on the auxiliary system and then find it consistently confirmed by direct measurement.

Should we now conclude, since we know absolutely nothing about the relationship between the variable values provided in *a* system, that no such relationship exists, and that largely arbitrary combinations can occur? That would mean that such a system of *"one* degree of freedom" would require not merely two numbers for its sufficient description, as classical mechanics held, but far more - perhaps an infinite number. But then it is strange that *two* systems always agree in *all* variables when they agree in two. One would therefore have to assume, secondly, that this is due to our clumsiness; one would have to think that we are practically incapable of bringing two systems into a situation in which they coincide with respect to two variables without, willy-nilly, also bringing about the coincidence for all remaining variables with , even though that would not in itself be necessary. One would have to make these *two* assumptions in order not to perceive the complete lack of insight into the relationship of variable values within a system as a major embarrassment.

§ 14 The Change in Entanglement with Time. Objections to the Special Status of Time

§ 15 Natural principle or mathematical trick?

…

The situation is similar with another system, the electromagnetic field. Its laws are "the embodied theory of relativity"; a non-relativistic treatment is simply impossible. Nevertheless, this field - which, as the classical model of thermal radiation, provided the initial impetus for quantum theory - was the first system to be "quantized." The fact that this could be achieved with simple means is due to the fact that it is a bit easier here, because the photons, the "atoms of light," do not interact directly with one another at all, but only through the mediation of charged particles. Even today, we still do not possess a truly flawless quantum theory of the electromagnetic field. Although one can get quite far with a *structure based on particle systems* following the pattern of non-relativistic theory (Dirac's theory of light [3], (FeE)), one has not quite reached the goal.

D. Bohm

Treatment of quantum fluctuations
A new distinction between implicate and explicate order

Treatment of Quantum fluctuations

(BoD) p. 110-111: Let us begin by assuming some „deterministic" field theory. Its precise characteristics are unimportant for our purposes here. All that is important is to suppose the following properties:

- There is a set of field equations which completely determines the changes of the field with time.

- These equations are sufficiently non-linear to guarantee a significant coupling between all wave components, so that (except perhaps in some approximation) solutions cannot be linearized superposed.

- Even in the „vacuum" the field is so highly excited that the mean field in each region, however small, fluctuates significantly, with a kind of turbulent motion that leads to a high degree of randomness in the fluctuations. This excitation guarantees the discontinuity of the fields in the smalles regions.

- What we usually call „particles" are relatively stable and conserved excitations on top of this vacuum. Such particles will be registered at the large-scale level, where all apparatus is sensitive only to those features that fluctuate rapidly. Thus, the „vacuum" will produce no visible effects at the large-scale level, since its fields will cancel themselves out on the average, and space will be effectively „empty" for every large-scale process (e.g. as a perfect crystal lattice is effectively „empty" for an electron in the lowest band, even though the space is full of atoms).

(BoD1) p. 178: .. accommodation within already existing frameworks of order has generally been considered to be main activity to be emphasized in physics, while the perception of new orders has been thought of as something that happens only occasionally, perhaps in revolutionary periods, during which what is regarded as the normal process of accomodation has broken down.

A new distinction between implicate and explicate order

(BoD1) p. 189: Generally speaking, the laws of physics have thus far referred mainly to the explicate order. Indeed, it may be said that the principle function of Cartesian coordinates is just to give a clear and precise description of explicate order. Now we are proposing that in the formulation of the laws of physics, primary relevance is to be given to implicate order, while the explicate order is to have a secondary kind of significance. Thus, it may be expected that a description in terms of Cartesian coordinates can no longer be given a primary emphasis, and that a new kind of description will indeed have to be developed for discussing the laws of physics.

(BoD1) p. 199: „What we usually call „particles" are relatively stable and conserved excitations on top of this vacuum. Such particles will be registered at the large-scale level, where apparatus is sensitive only to those features of the field that will last a long time, but not to those features that fluctuate rapidly. Thus, the „vacuum" will produce no visible effects at the large-scale level, since its fields will cancel themselves out on the average, and space will be effectively „empty" for an electron in the lowest band, even though the space is full of atoms".

„It is important to emphasize, however, that mathematics and physics are not being regarded here as separate but mutually related structures (so that, for example, one could be said to apply mathematics to physics as paint is applied to wood). Rather, it is being suggested that mathematics and physics are to be considered as aspects of a single undivided whole".

(BoD1) p. 223: It is instructive at this point to contrast the key features of relativistic and quantum theories. As we have seen, relativity theory requires continuity, strict causality and locality. On the other hand, quantum theory requires non-continuity, non-causality (or determinism) and non-locality. So the basic concepts of relativity and quantum theory diretly contradict each other. It is therefore hardly surprising that these two

theories have never been unified in a consistent way. Rather it seems most likely that such a unfication is not actually possible, What very probably needed instead is a qualitatively new theory, from which both relativity and qauntum theory are to be derived as abstractions, approximations and limiting cases. ….

To begin with undivided wholeness means, however, that we must drop the mechanistic order. But this order has beed, for many centuries, basic to ll thinking on physics. As brought out in chapter 5, the mechanistic order is most naturally and directly expressed through the Cartesian grid. .. As we shall see later, even the quatum laws may only be abstractions from still more general laws, of which only some outlines are now vaguely to be seen. So the totality of movement of enfoldment and unfoldment may go immensely beyond what has revealed itself to our observations thus far.

K. Gödel

A new type of cosmological solutions of the gravity field equations

(GöK): All cosmological solutions with non-vanishing density of matter known at present have the common property that, in a certain sense, they contain an „absolute" time coordinate, owing to the fact that there exists a one-parametric system of three-spaces everywhere orthogonal on the world lines of matter. It is easily seen that the non-existence of such a system of three-spaces is equivalent with a rotation of matter relatively to the compass of inertia. In this paper I am proposing a solution (with a cosmological term $\neq 0$) which exhibits such a rotation. This solution, or rather the four-dimensional space S which it defines, has the further properties

(1) S is homogeneous
(2) …. so that any two world lines of matter are equidistant
(3) S has rotational symmetry
(4) … That is, a positive direction of time can consistently be introduced in the whole solution
(5) It is not possible to assign a time coordinate to each space-time point in such a way that the coordinate always increases, if one moves in a positive time-like direction; …
(6) … it is theoretically possible in these worlds to travel into the past, or otherwise influence the past
(7) There exist no three-spaces which are everywhere space-like and intersect each world line of matter in one point
(8) … an absolute time does not exist, even if it is not required to agree in direction with the times of all possible observers (where absolute means: definable without reference to individual objects, such as e.g. a particular galactic system).
(9) Matter everywhere rotates relatively to the compass of inertia with the angular velocity $2\sqrt{\pi\mu\rho}$, where ρ is the mean density of matter and μ Newton's gravitational constant.

H. Weyl

The laws of dynamics are laws of nearby action
cause and effect need to belong to different spheres of existence
while in nature cause and effects lie within the same plane

(WeH) pp. 190-191: The laws of dynamics, since they are laws of nearby action, are a continuous infinitesimal character; they, rather than the forms, are considered as original in physics today. Typical configurations come about, however, when these laws admit of certain discrete solutions of special character, such as static and periodic solutions. … Causal law took the following form: the derivatives with respect to time of the state quantities at a world point are mathematical functions of the state quantities themselves and their spatial derivatives at that point. Consequently, the state of the world at any moment would determine the state at the immediately following moment by means of differential law. Thus only the world's state at a single moment would remain „arbitrary" or „accidental", and from it the world's whole past and future could be computed by integration of the Laplacian world formula. … Furthermore, the causal principle is meant to postulate not merely the existence of functional dependences but also a certain density of their fabric. This is brought out in the customary formulation: „Under the same conditions the same event will repeat itself. Where should the line be drawn between conditions and events happening due these conditions? There is no difficulty here, provided cause and effect belong to different spheres of existence. … But in nature cause and effect lie within the same plane. As a dividing line separating the two one may set down a three-dimensional space-like cross section $t = \text{const}$ through the world, which will dissect it arbitrarily into a „past" half and a „future" half. This leads to the formulation that the content of the past determines by law the content of the future. However this is not the causal law itself but only a special form of it which fits well into field systems. It need not for this reason be unconditionally accepted.

R. Courant

The radiation problems of the wave equation, the Huygens' principle, and spherical waves for arbitrary time-like lines

The radiation problem and the Huygens' principle

For $t > 0$ we seek a solution of the homogenous wave equation $u_{tt} - \Delta u = 0$ which vanishes together with its derivative u_t for $t = 0$ except at the origin of the x-space, and which for $r = \sqrt{x_1^2 + x_2^2 + x_3^2} = 0$, i.e., on the „time-axis", is singular in such a way that

$$^{\circ}u := \lim_{\varepsilon \to 0} \oiint_{O_\varepsilon} \frac{\partial u}{\partial \nu} do = -g(t) \, .$$

Here the integration is at the time t over a sphere O_ε about the origin of the x-space. $\frac{\partial}{\partial \nu}$ denotes the normal of the sphere, do is the surface element, and the radius ε of the sphere is contracted to zero. The function $g(t)$ is the prescibed intensity of radiation as a function of the time, (CoR) p. 695.

Huygens' principle

The value of the solution of Cauchy's problem for the wave equation depends only of the boundary of the domain of dependence in the plane $t = 0$; i.e., it depends only on the initial values of u and its derivatives on the boundary of the base $r = t$ of the characteristic cone, but not on the initial values in the interior of this case. This a special case of the following general rule: Huygens' principle is valid for the wave equation when the number n of the space dimension is odd, but not when n is even, (CoR) p. 690.

The Huygens' principle is valid for the radiation problem with odd space dimensions. The effect, at the point x at the time t, of a disturbance radiating from the origin depends only on the character of this disturbance at a single previous instant of time, the time $t - r$. The disturbance, advancing from the origin with the unit velocity, reaches the point x at just the time t. Impulses at the origin that are sharply defined in time, i.e., represented by functions $g(t)$ which are different from zero only in a short time interval, are thus perceived at any point at the distance r only during a correspondingly short time interval beginning r time units later, (CoR) p. 698.

Families of relatively undistorted spherical waves issuing from a time-like line Λ, spherical waves and the problem of transmitting with perfect fidelity signals in all directions, The Courant conjecture that the four-dimensional physical space-time world of classical physics enjoyes an essential distinction

A family of undistorted progressing waves related to a hyperbolic differential operator L is a family of solution of $L[u] = 0$ depending on an arbitrary function $S(\Phi)$ and having the form $u = S(\Phi(x,t))$, where S is called the wave form and $\Phi(x,t)$ is a fixed phase function of th space variables x, and the time $t = x_0$. A less restricted concept of „relatively undistorted" progressing wave family has the form $u = g(x,t)S(\Phi)$, where $S(\Phi)$ is again arbitrary but also th distortion factor $g(x,t)$.

The problem of transmission of signals is further clarified by the concept of „spherical waves" which generalize the spherical solutions of the three-dimensional wave equation. We confine ourselves to the case of linear differential equations $L[u] = 0$ of second order and consider a time-like line Λ given in the form $\Lambda x_i = \xi(\lambda)$ with a parameter λ. (The time variable is not emphasized here.) With the point $\xi(\lambda)$ as vertex we consider the characteristic conoid or spherical wave front $\Gamma(x,\xi) = 0$.

For a given x, we may determine λ as a function of x from the equation $\Gamma(x,\xi(\lambda)) = 0$; we write $\lambda = \Phi(x)$. The characteristic conoid with vertex $\xi(\Phi(x))$ is given by the equation $\Phi(x) = constant$. A family of relatively undistorted spherical waves issuing from Λ may then be defined as a solution u of the second order differential equation in the form $u(x) = g(x)S(\Phi(x))$ with specific g and arbitrary S.

Little is known about the scope of this concept, which obviously relates spherical waves to the problem of transmitting with perfect fidelity signals in all directions. All we can do here is to formulate a conjecture which will be given some support in article 3:

> *Families of spherical waves for arbitrary time-like lines Λ exist only in the case of two and four variables, and then only if the differential equation is equivalent to the wave equation.*

A proof of this conjecture would show that the four-dimensional physical space-time world of classical physics enjoyes an essential distinction, (CoR) pp. 760-766.

3. The mathematical toolbox

a. Compact and symmetric operators, Hilbert scales

(NiJ), (NIJ1)

The eigenvalue problem for compact symmetric operators

In the following H denotes an (infinite dimensional) real Hilbert space with scalar product $(.,.)$ and the norm $\|\ldots\|$. We will consider mappings $K: H \to H$. Unless otherwise noticed the standard assumptions on K are:

i) K is symmetric, i.e., for all $x, y \in H$ it holds $(x, Ky) = (x, Ky)$

ii) K is compact, i.e., any (infinite) sequence $\{x_n\}$ bounded in H contains a subsequence $\{x_{n'}\}$ such that $\{Kx_{n'}\}$ is convergent

iii) K is injective, i.e., $Kx = 0$ implies $x = 0$.

A first consequence is

Lemma: K is bounded, i.e.

$$\|K\| := \sup_{x \neq 0} \frac{\|Kx\|}{\|x\|} < \infty.$$

Lemma: Let K be bounded, and fulfill condition i) above, but not necessarily the two other conditions ii) and iii). Then $\|K\|$ equals

$$N(K) = \sup_{x \neq 0} \frac{|(x,Kx)|}{\|x\|}.$$

Theorem: There exists a countable sequence $\{\lambda_i, \phi_i\}$ of eigen-elements and eigenvalues $K\phi_i = \lambda_i \phi_i$ with the properties

i) the eigen-elements are pair-wise orthogonal, i.e. $(\phi_i, \phi_k) = \delta_{i,k}$

ii) the eigenvalues tend to zero

iii) for the generalized Fourier sums it holds

$$S_n := \sum_{i=1}^{n} (x, \phi_i)\, \phi_i \to x \quad \text{with } n \to \infty \text{ for all } x \in H$$

iv) the Parseval equation

$$\|x\|^2 = \sum_{i}^{\infty} (x, \phi_i)^2$$

holds for all $x \in H$.

Hilbert Scales

Let H be a (infinite dimensional) Hilbert space with scalar product $(.,.)$ and the norm $\|..\|$. Let A be a linear operator with the properties

A is self-adjoint, positive definite

A^{-1} is compact.

Without loss of generality, possible by multiplying A with a constant, one may assume

$$(x, Ax) \geq \|x\| \quad \text{for all } x \in D(A).$$

Any eigen-element of the compact operator $K = A^{-1}$ is also an eigen-element of A to the eigenvalues being the inverse of the first. Now by replacing $\lambda_i \to \lambda_i^{-1}$ we have that there is a countable sequence $\{\lambda_i, \phi_i\}$ with

$$A\phi_i = \lambda_i \phi_i \ , \ (\phi_i, \phi_k) = \delta_{i,k} \text{ and } \lim_{i\to\infty} \lambda_i$$

and any $x \in H$ is represented by

$$x = \sum_{i=1}^{\infty} (x, \phi_i)\, \phi_i \text{ and } \|x\|^2 = \sum_1^{\infty} (x, \phi_i)^2.$$

Lemma 1: Le $x \in D(A)$, then

$$Ax = \sum_{i=1}^{\infty} \lambda_i (x, \phi_i)\, \phi_i \ , \ \|Ax\|^2 = \sum_{i=1} \lambda_i^2 (x, \phi_i)^2, \ (Ax, Ay) = \sum_{i=1}^{\infty} \lambda_i^2 (x, \phi_i)\, (y, \phi_i).$$

Similarly one can define the spaces H_α with scalar product

$$(x, y)_\alpha = \sum_i^{\infty} \lambda_i^\alpha (x, \phi_i)\, (y, \phi_i) = \sum_i^{\infty} \lambda_i^\alpha x_i y_i \text{ and norm } \|x\|_\alpha^2 = (x, x)_\alpha.$$

The relation to $x \in D(A)$ is given by

$$\|x\|_2^2 = (Ax, Ax)_0 \ , H_2 = D(A).$$

The set $\{H_\alpha | \alpha \geq 0\}$ is called a Hilbert scale. The condition $\alpha \geq 0$ is in the context of this section necessary for the following reasons:

Since the eigen-values λ_i tend to infinity we would have for $\alpha < 0$: $lim\, \lambda_i^\alpha \to 0$. Then there exist sequences $\hat{x} = (x_1, x_2, \dots)$ with

$$\|\hat{x}\|_2^2 < \infty \ , \ \|\hat{x}\|_0^2 = \infty \ .$$

Because of Bessel's inequality there exists no $x \in H$ with $Ix = \hat{x}$. This difficulty could be overcome by duality arguments which we omit here.

Note: The physical model problem of the operator A above is the Laplacian operator on Sobolev spaces.

Note: The Stokes operator associated with space periodicity conditions also possesses a sequence of eigenfunctions w_k, $k \in N$, which form an orthogonal basis of the baseline Hilbert space H, (TeR) p. pp. 8/10,

$$Aw_n = \lambda_n w_n, w_n \in D(A),$$

$$0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \dots, \ \lambda_n \to \infty \text{ for } n \to \infty.$$

There are certain relations between the spaces $\{H_\alpha | \alpha \geq 0\}$ for different indices:

Lemma 2: Let $\alpha < \beta$. Then

$$\|x\|_\alpha \leq \|x\|_\beta$$

and the embedding $H_\beta \to H_\alpha$ is compact.

Lemma 3: Let $\alpha < \beta < \gamma$. Then

$$\|x\|_\beta \leq \|x\|_\alpha^\mu \|x\|_\gamma^\nu \text{ for } x \in H_\gamma$$

with

$$\mu = \frac{\gamma-\beta}{\gamma-\alpha} \text{ and } \nu = \frac{\beta-\alpha}{\gamma-\alpha}.$$

Lemma 4: Let $\alpha < \beta < \gamma$. To any $x \in H_\beta$ and $t > 0$ there is a $y = y_t(x)$ according to

$$\|x-y\|_\alpha \leq t^{\beta-\alpha}\|x\|_\beta$$
$$\|x-y\|_\beta \leq \|x\|_\beta \,,\ \|y\|_\beta \leq \|x\|_\beta$$
$$\|y\|_\gamma \leq t^{-(\gamma-\beta)}\|x\|_\beta \ .$$

Corollary: Let $\alpha < \beta < \gamma$. To any $x \in H_\beta$ and $t > 0$ there is a $y = y_t(x)$ according to

i) $\|x-y\|_\rho \leq t^{\beta-\rho}\|x\|_\beta$ for $\alpha \leq \rho \leq \beta$

ii) $\|y\|_\sigma \leq t^{-(\sigma-\beta)}\|x\|_\beta$ for $\beta \leq \sigma \leq \gamma$.

Remark: Our construction of the Hilbert scale is based on the operator A with the two properties i) and ii). The domain $D(A)$ of A equipped with the norm

$$\|Ax\|^2 = \sum_{i=1} \lambda_i^2 (x, \phi_i)^2$$

turned out to be the space H_2, which is densely and compactly embedded into $H = H_0$. It can be shown that on the contrary to any such pair of Hilbert spaces there is an operator A with the properties i) and ii) such that $D(A) = H_2$ $R(A) = H_0$ and $\|x\|_2 = \|Ax\|$.

Extension and generalizations

For $t > 0$ one may introduce the Hilbert space $H_{(\tau)}$ by an additional inner product resp. norm in the form

$$(x, y)_t = \sum_{i=1} e^{-\sqrt{\lambda_i} t}(x, \phi_i)(y, \phi_i)$$

$$\|x\|^2_{(t)} = (x, x)^2_{(t)} .$$

Now the factor has exponential decay $e^{-\sqrt{\lambda_i} t}$ instead of a polynomial decay in case of λ_i^{α}.

Obviously it holds

$$\|x\|_{(t)} \leq c(\alpha, t)\|x\|_{\alpha} \text{ for } x \in H_{\alpha}$$

with $c(\alpha, t)$ depending only from α and $t > 0$. Thus the (t)-norm is weaker than any α-norm. On the other hand any negative norm index, i.e. $\|x\|_{\alpha}$ with $\alpha < 0$, is bounded by the 0-norm and the newly introduced (t)-norm.

It holds:

Lemma: Let $\alpha > 0$ be fixed. The α-norm of any $x \in H_0$ is bounded by

$$\|x\|^2_{-\alpha} \leq \delta^{2\alpha}\|x\|^2_0 + e^{t/\delta}\|x\|^2_{(t)}$$

with $\delta > 0$ being arbitrary.

Proof: The inequality is a consequence of the following inequality

$$\lambda^{-\alpha} \leq \delta^{2\alpha} + e^{t(\delta^{-1} - \sqrt{\lambda})}, \text{ for any } t, \delta, \alpha > 0 \text{ and } \lambda \geq 1.$$

This holds for the following reasons:

i) if $\lambda^{-1/2} \leq \delta$ then obviously $\lambda^{-\alpha} \leq \delta^{2\alpha}$

ii) in case of $\lambda^{-1/2} \geq \delta$ it holds $e^{t(\delta^{-1} - \sqrt{\lambda})} \geq 1$,

iii) whereas $\lambda^{-\alpha} \leq 1$ is a consequence of $\alpha > 0$ and $\lambda \geq 1$.

The counterpart of the lemma 4 above is

Lemma: Let $t, \delta > 0$ be fixed. To any $x \in H_0$ there is a $y = y_t(x)$ according to

$$\|x - y\| \leq \|x\|$$

$$\|y\|_1 \leq \delta^{-1}\|x\|$$

$$\|x - y\|_{(t)} \leq e^{-t/\delta}\|x\|.$$

b. The inner product of energy in the calculus of variations

(BrK), (VeW)

Let A be a linear operator of the equation $Au = f$ in a given Hilbert space with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|$ with the following properties

i) the operator A is positive, i.e., $(Au, v) > 0 \quad$ for $u \neq 0$

ii) the operator A is symmetric, i.e., $(Au, v) = (u, Av)$

for $u, v \in D(A)$. Within the domain of the operator, an inner product is thus defined in the form

$$a(u, v) \coloneqq (Au, v).$$

The corresponding norm $|||u|||^2 \coloneqq a(u, u)$ allows for an extension of the domain of the inner product $a(u, v)$ from $D(A) \times D(A)$ to $H_A \times H_A$ with $H_A \coloneqq \overline{D(A)}^{|||u|||}$. This yields the following variational equation for the original linear operator equation:

$$u \in H_A \coloneqq \overline{D(A)}^{|||u|||}: \quad a(u, v) \coloneqq (f, v) \text{ for all } v \in H.$$

The norm $|||u|||$ is sometimes also referred to as the energy norm. The corresponding quadratic extremal problem seeks an approximate solution to the above variational equation in a linear subspace of H. If the functional on the right-hand side of the above equation is bounded with respect to the norm $|||u|||$, then the extremal problem has a unique solution.

In quantum mechanics, measurable quantities (observables) are, in the ideal case, described by self-adjoint operators in a Hilbert space. In functional analysis, such an operator is defined only if a domain is assigned to it. The two prototypical operators in quantum mechanics are the momentum and position operators. In the above model, the domain of the momentum operator is the constructed dense subspace $H_A \coloneqq \overline{D(A)}^{|||u|||}) \subset H$. The domain of the position operator is the entire Hilbert space H. It necessarily follows that the two operators are not commutative. The corresponding physical principle of Heisenberg's uncertainty principle thus reflects a mathematical consequence of the mathematical model.

Regarding the linear operator in classical physics we give the following illustrations:

Example 1. $H = L_2(\Omega)$ with Ω a bounded domain in R^3 and the boundary $\partial\Omega$ sufficiently smooth.

$$Au \coloneqq -\Delta u \text{ and } D(A) = \dot{W}_2^2(\Omega), \quad (\dot{W}_2^2(\Omega) \text{ being the intersection of } W_2^2(\Omega) \text{ and } W_0^{1,2}(\Omega)).$$

We introduce a Hilbert scale in the following way: let $\{v_i, \lambda_i\}$ be the orthonormal set of eigen-pairs of A, i.e.

$$-\Delta v_i = \lambda_i v_i \text{ in } \Omega$$

$$v_i = 0 \quad \text{on } \partial\Omega\,.$$

The Hilbert spaces $\{H_\beta | \beta \in R\}$ are spanned by the functions with finite β -norm defined by

$$\|z\|_\beta^2 \coloneqq \sum \lambda_i^\beta z_i^2 \text{ with } z_i \coloneqq (z, v_i).$$

We have the inclusions

$$D(A) \subset H_A = H_1 = W_0^{1,2}(\Omega) \subset L_2\,(\Omega).$$

The operator $A: W_0^{1,2}(\Omega) \to W^{-1.2}(\Omega)$ is called potential operator and $a = \frac{1}{2}\|z\|_1^2$ the potential of a potential function a (ChJ).

Example 2. $H = L_2^{\#}(\Gamma)$ with $\Gamma = S^1(R^2)$, i.e. Γ is the boundary of the unit sphere. Then H is the space of $L_2^{\#}$-integrable periodic functions in R.

$$(Au)(x) \coloneqq \oint k(x-y)u(y)dy \text{ and } D(A) = H$$

with

$$k(y) \coloneqq -ln\left|2sin\frac{y}{2}\right|.$$

With the help of the Fourier coefficients v_i of 2π a -perodic function v defined by

$$v_\nu \coloneqq \frac{1}{2\pi}\oint v(x)e^{-ivx}dx$$

we may introduce for real β the norms

$$\|v\|_\beta^2 \coloneqq \sum|\nu|^{2\beta}v_i^2.$$

The Hilbert spaces $H_\beta = H_\beta(\Gamma)$ are defined similar to example 1. The Fourier coefficients of the convolution Au are

$$(Au)_\nu = \frac{1}{2|\nu|}\cdot u_\nu.$$

This time we have the inclusions $D(A) \subset H_A = H_{-1/2}(\Gamma)$.

Example 3. In the theory of conformal mappings the (circular Hilbert transform) integral operator

$$v = Au$$

$$v(s) \coloneqq \frac{1}{2\pi}\oint \cot(\frac{1}{2}(s-t)u(t)dt$$

plays a central role. It is skew symmetric in $L_2^{\#}(0,2\pi)$. Further A maps the space $\dot{L}_2^{\#}(0,2\pi) \coloneqq L_2^{\#}(0,2\pi)/R$ i.e. (mean-value zero $L_2^{\#}(0,2\pi)$-function on Γ) isometrically onto itself, i.e. $\|Au\| = \|u\|$. The Hilbert inverse relations may be written in the form $A^2 = -I$.

The Hilbert space, (NaS),

$$H \coloneqq \dot{H}_{1/2}^{\#}(S^1)/R$$

is the subspace of $L_2^{\#}(S^1)/R$ comprising real functions of mean-value zero on S^1 which have a half-order derivative also in $L_2^{\#}(S^1)$. The Hilbert space H is isometric to the sequences space

$$l_2^{1/2} \coloneqq \left\{\text{complex sequences u} = u_1, u_2, u_3, \ldots \middle| \sqrt{n}u_n \in l_2 \right\}.$$

The interconnection of the symplectic form

$$S(u,v) \coloneqq \frac{1}{2\pi}\oint udv$$

between the inner product on H and the Hilbert transform operator A is given by

$$S(u,Av) = (u,v)_{1/2} \text{ for all } u,v \in H.$$

We note the relations to Plemelj's extension of the Green formulae, (PlJ). The extended formulae are accompanied by an extended Dirichlet integral $(\nabla u, \nabla v)_0 \cong (u,v)_1 \to (u,v)_{1/2}$, and a related new physical notion of a „mass element" and a „flux through a surface" avoiding the mathematical concept of the normal derivative.

Example 4. Seeking the solution of the Neumann boundary value problem

$$\Delta u = 0 \quad \text{in } R^3 - \Omega$$

$$\frac{\partial u}{\partial n} = f \quad \text{on } S := \partial\Omega.$$

leads to the Prandtl operator, (LiI),

$$(\prod v)(x) := \frac{1}{4\pi} \oiint_{\partial\Omega} v(y) \frac{cos\, \phi_{xy}}{|x-y|^2} dS_y = f(x).$$

The Prandtl operator $\prod : H_r \to H_{r-1}$ is

i) is bounded for $0 \leq r \leq 1$

ii) is Noetherian for $0 < r < 1$.

iii) The solution function of the Neumann problem is represented as double layer potential

$$u(x) := \frac{1}{4\pi} \oiint_{\partial\Omega} v(y) \frac{cos\, \phi_{xy}}{|x-y|^2} dS_y$$

and the exterior Neumann problem admits one and only one generalized solution for $\frac{1}{2} \leq r < 1$.

Example 5. Let P denotes the orthogonal projection operator of $(L_2(\Omega))^3$ (PDO of degree zero) onto the divergence free vector field H_ω consisting of all solenoidal vector functions u, i.e. P denotes the orthogonal projection operator onto the kernel of the divergence operator.

The Stokes operator A is a selfadjoint (Friedrichs extension) operator in $H_A := H_\sigma$ of the non-negative symmetric operator

$$A := -P\Delta \text{ in } D(A) := H_\omega := \{u \in C^2 | divu = 0, u_{n|\partial\Omega} = 0\},$$

and the corresponding Hilbert scale norms are defined by

$$\|u\|_\beta := \|A^{\beta/2} u\|.$$

Putting $B(u) := P(u, \nabla u)$ and assuming $Pu_0 = u_0$ the initial-boundary NSE equations can be representated by

$$\frac{du}{dt} + Au + Bu = Pf\ , u(0) = u_0$$

with

$$Au = Pf \quad \text{in } H_0.$$

As u is divergence free and $u \cdot v$ identically vanishes on $\partial\Omega$ one gets

$$b(u, v, w) := ((u, \nabla)v, w) = \iint_\Omega (u, \nabla)v \cdot w dx = -b(u, w, v)$$

and especially $b(u, v, v) = 0$.

Complementary variational principles and the method of Noble

The method of Noble, (VeW) S. 164, can also be applied to a certain class of nonlinear boundary value problems, (ArA) p. 67 ff.:

Let $(E, \langle , \rangle)$ and $(E', (,))$ be Hilbert spaces and $T: E \to E'$, $T^*: E' \to E$ linear operators fulfilling $(u', Tu) = \langle T^* u', u \rangle$ and let $W: E' x E \to R$ a functional fulfilling

$$T = \frac{\partial W(u',)}{\partial u'} \quad \text{and} \quad T^* = \frac{\partial W(,u)}{\partial u}$$

i.e., the operators T and T^* are derivatives from $W(.,.)$ in the sense of Gateaux, i.e.,

$$lim \frac{F(u+tv)-F(v)}{t} = F_u(v) \text{ for all } v \in E \, .$$

With $a(u,v) = (Tu, Tv)$ and putting $W(u',u) := \frac{1}{2}(u',u') - F(u)$ first, you get $Tu = u'$. This is the condition to which the admissible functions of $J(u) = a(u,u) + 2F(u) \to min$ are subject. Furthermore, as a derivative with respect to u, one obtains $(T^* u', .) = -F_u(.)$. This corresponds to the functional equation

$$(v', T\phi) + F_u(\phi) = 0 \text{ for all } \phi \in U$$

which can also be written in the form

$$(T^* v', \phi) = -F_u(\phi) \text{ for all } \phi \in U$$

where $F(.)$ was assumed to be a convex functional. The key property of the functional $W(.,.)$ is that it is convex with respect to u' and concave with respect to u. This allows us to characterize the solution to the pair of operator equations

$$T = \frac{\partial W(u',)}{\partial u'} \quad \text{and} \quad T^* = \frac{\partial W(,u)}{\partial u}$$

involving the "Hamilton function" $W(.,.)$ as a kind of saddle point. The theory is very general and encompasses not only boundary value problems for differential equations, but also integral equations.

c. The three model problems of PDE in Hilbert spaces

The operator of a linear equation $Au = f$ defined in a Hilbert space is assumed to be positive and symmetric enabling the definition of a norm $|||\quad|||$ on the domain of A. It can be extended to the self-adjoint (Friedrichs) extension of A with domain $H_A := \overline{D(A)}^{|||\quad|||}$. The restrictive assumptions can be exptended to non-linear problems, in case the non-linear term can be interpreted as a compact disturbance operator of such a linear operator A. Then this results into the so-called Garding inequality.

Let (λ_n, φ_n) denote the orthogonal set of eigen-pairs of a linear self-adjoint & positive definite operator A, with A^{-1} compact. Then the Hilbert spaces $\{H_\alpha | \alpha \in R\}$ are spanned by the finite norms

$$\|x\|_\alpha^2 = \sum_1^\infty \lambda_n^\alpha x_n^2 < \infty \,,\; x_n := (x, \varphi_n)$$

accompanied by the inner product $(x,y)_\alpha = \sum_1^\infty \lambda_n^\alpha \, x_n \, y_n$. The extended Hilbert space $H_{\alpha.(\tau)}$ is defined by the inner product resp. norm

$$(x,y)_{\alpha.(\tau)} = \sum_1^\infty \lambda_n^\alpha e^{-\sqrt{\lambda_n}\tau} \, x_n \, y_n, \quad \|x\|_{\alpha.(\tau)}^2 = (x,x)_{\alpha.(\tau)}.$$

We consider the following elliptic, parabolic, and hyperbolic PDE model cases:

(1) elliptic case: the Laplacian (potential) operator
(2) parabolic case: the heat equation operator
(3) hyperbolic case: the d'Alembert wave equation operator

The operators (1) and (2) become well-defined within the H_α (Sobolev function based) framework. This is not the case for the wave equation operator (3) (*). However, it turned out that the extended Hilbert space $H_{(\tau)} := H_{0.(\tau)}$ provides an appropriate domain for the wave equation operator.

ad (1): The Laplacian operator $-\Delta$ is isometric (by construction), i.e.,

$$\|-\Delta u\|_\alpha^2 \cong \|u\|_{\alpha+2}^2 \quad \text{with} \quad \|u\|_\alpha^2 = \sum_1^\infty \lambda_n^\alpha u_n^2 < \infty$$

ad (2): The heat equation operator $H[u](x,t) := \dot{u}(t) - \Delta u(t)$ is isometric, i.e.,

$$|||H[u]|||_\alpha^2 \cong |||u|||_{\alpha+2}^2 \quad \text{with} \quad |||u|||_\alpha^2 := \int_0^T \|u\|_\alpha^2(t)dt, \text{ (NiJ7).}$$

ad (3): A well-defined wave equation operator $D[u](x,t) := \ddot{u}(t) - \Delta u(t)$ equipped with the domain $H_{\alpha.(\tau)}$ is isometric (**), i.e.,

$$\big\|\|D[u]\|\big\|_\alpha^2 \cong \big\|\|u\|\big\|_{\alpha+2}^2 \quad \text{with} \quad \big\|\|u\|\big\|_\alpha^2 := \int_0^\infty \|u\|_{\alpha.\tau}^2 d\tau.$$

(*) The related counter example is given by the function $\Phi(x,t) := e^{-(\frac{1}{2}-(x-t))^2}$, $u(x,t) := t^2\Phi(x,t)$, $f(x,t) := 2\Phi(x,t) - 4t\Phi'(x,t)$; it fulfills the relationships $\dot{\Phi}(x,t) = -\Phi'(x,t)$, $\ddot{\Phi}(x,t) = \Phi''(x,t)$, $\ddot{u}(x,t) - u''(x,t) = f(x,t)$ and $\|u''\|_{L_2(L_2)} \sim \|\Phi''\|_{L_2(L_2)}$, however it holds $\|f\|_{L_2(L_2)} \sim \|\Phi'\|_{L_2(L_2)}$.

(**) We consider the hyperbolic model problem

$$\begin{aligned} \ddot{u} - \Delta u &= f && \text{in } \Omega \times (0,T] \\ u &= 0 && \text{on } \partial\Omega \times (0,T] \\ u(0) = u_0 \;,\; \dot{u}(0) &= u_1 && \text{in } \Omega \;. \end{aligned}$$

Let $\{\lambda_n, \varphi_n\}$ be the orthonormal set of eigen-pairs of $-\Delta$, i.e., $-\Delta\varphi_n = \lambda_n\varphi_n$ and Let $u_n := (u, \varphi_n)$ resp. $n := (f, \varphi_n)$ being the generalized Fourier coefficient. Then it holds $\ddot{u}_n(t) + \lambda_n u_n(t) = f_n(t)$ and $u_n(0) = \dot{u}_n(0) = 0$ with the solution

$$u_n(t) = \tfrac{1}{\sqrt{\lambda_n}} \int_0^t \sin(\sqrt{\lambda_n}(t-\tau)\, f_n(\tau)d\tau \;.$$

Lemma:
$$\int_0^T \|u\|_{k+2,(t)}^2 dt \le c \int_0^T \|f\|_{k,(t)}^2 dt \;.$$

Proof: It holds for $\tau \le t$
$$\begin{aligned} \int_0^T \|u\|_{k+2,(t)}^2 dt = \sum \lambda_n^{k+2} \int_0^T e^{-\sqrt{\lambda_n}t} u_n^2(t)dt &\le \sum \lambda_n^{k+2} \int_0^T e^{-\sqrt{\lambda_n}t} \Big[\tfrac{1}{\sqrt{\lambda_n}} \int_0^t \sin(\sqrt{\lambda_n}(t-\tau)\, f_n(\tau)d\tau\Big]^2 dt \\ &\le \sum \lambda_n^{k+1} \int_0^T e^{-\sqrt{\lambda_n}t} (\int_0^t \sin(\sqrt{\lambda_n}(t-\tau)d\tau) \Big[\int_0^t \sin(\sqrt{\lambda_n}(t-\tau)d\tau\, f_n^2(\tau)d\tau\Big] dt \\ &\le \sum \lambda_n^{k+1/2} \int_0^T e^{-\sqrt{\lambda_n}t} \Big[\int_0^t f_n^2(\tau)d\tau\Big] dt \le \sum \lambda_n^{k+1/2} \int_0^T e^{-\sqrt{\lambda_n}t} \Big[\int_0^t f_n^2(\tau)d\tau\Big] dt \;. \end{aligned}$$

Exchanging the order of integration gives

$$\int_0^T \int_0^t e^{-\sqrt{\lambda_n}t} f_n^2(\tau)d\tau dt = \int_0^T \int_t^T e^{-\sqrt{\lambda_n}t} f_n^2(\tau)dt d\tau = \int_0^T f_n^2(\tau)dt \Big[\int_t^T e^{-\sqrt{\lambda_n}t}\, d\tau\Big] \le \tfrac{1}{\sqrt{\lambda_n}} \int_0^T f_n^2(\tau)dt \;.$$

d. The Neumann problem and the Prandtl operator

For a closed connected surface $S \subset R^3$ one can seek the solution of the Neumann boundary value problem

$$\begin{aligned} \Delta u &= 0 && \text{in } R^3 - S \\ \frac{\partial u}{\partial n} &= f && \text{on } S \end{aligned}$$

as the double layer potential, (see (LiI) chapter 4, "*Neumann Problem and Integral Equations with Double Layer Potential*"),

$$u(x) := \frac{1}{4\pi} \oiint_S v(y) \frac{\cos \phi_{xy}}{|x-y|^2} dS_y$$

whereby the unknown function $v(y)$ is obtained by the equation

$$\overline{P}[v](x) := \frac{1}{4\pi} \oiint_S v(y) \frac{\cos \phi_{xy}}{|x-y|^2} dS_y = f(x).$$

Here ϕ_{xy} is the angle between the vector $|x-y|$ and the normal n_y to the surface S at the point y, and $v(y)$ is the density of the double layer potential. The operator $\overline{P}$ is called the Prandtl operator. It has the following properties (LiI) pp. 105, 108, 109, 111, 115:

Theorem:

i) There is a representation $\overline{P} = A + K$ with

$$(Av)(x) := \frac{1}{4\pi} \oiint_S \frac{v(y)}{|x-y|^3} dS_y \quad \text{and} \quad (Kv)(x) := \frac{1}{4\pi} \oiint_S k(x,y) v(y)\, dS_y$$

whereby $\left|k(x,y) dS_y\right| \leq \frac{c}{|x-y|}$

ii) The Prandtl operator $\overline{P} : H_r \to \hat{H}_{r-1}$ is bounded for $0 \leq r \leq 1$, and for $0 < r < 1$ it is Noetherian, i.e. it has a right regularizer R with $R\overline{P} = RL + RN$; the operator RN is compact in H_r , the operator R is bounded from $\hat{H}_{r-1}$ to H_r, the operator N is bounded from H_r to H_0, and the operators NR and LR are a compact operators in $\hat{H}_{r-1}$ (*)

iii) For $v \in H_r$, $r \geq 1/2$, the function

$$u(x) := \frac{1}{4\pi} \oiint_S v(y) \frac{\cos \phi_{xy}}{|x-y|^2} dS_y$$

is an element of $H_1(R^3 - S)$

iv) For $1/2 \leq r < 1$ the exterior Neumann problem admits one and only on generalized solution.

Note: The Neumann problem for the pressure field $p(\vec{x}, t)$ of the Navier-Stokes equations is given by

$$\begin{aligned} \Delta p &= \rho(\vec{v} \cdot \nabla \vec{v} - \vec{f}) && \text{in } \Omega \\ \frac{\partial p}{\partial n} &= -\left[\mu \Delta \vec{v} - \rho \vec{v}_1 \cdot \nabla \vec{v} - \vec{f}\right] \cdot \vec{n} && \text{at } \partial\Omega \end{aligned}$$

where $\vec{n}$ denotes the outward unit normal to the domain Ω. It follows that the prescription of the pressure at the bounding walls or at the initial time independently of $\vec{v}$, could be incompatible with the initial and boundary conditions of the NSE PDE system, and therefore, could render the problem ill-posed (GaG), (HeJ).

(*) for the definition of $\hat{H}_{r-1}$ see (LiI) pp. 95, 108

(**) for a closed connected surface $S \subset R^n$ it holds $\hat{H}_{r-1} = H_{r-1}$

e. The blow up solutions of the 3D-NSE system

The content of this section is basically taken from (GiY). With c we denote numeric constants which may have different values at different places.

Let P be the orthogonal projection operator of $(L_2(\Omega))^n$ onto the divergence free vector field H_ω consisting of all solenoidal vector functions u, i.e. the operator is an orthogonal projection onto the kernel of the divergence operator. It is a Pseudo-Differential operator (PDO) of degree zero [(EsG]). The Stokes operator A is a selfadjoint operator in H_σ, being the Friedrichs extension of the non-negative symmetric operator $-P\Delta$ in H_ω defined for all $u \in C^2$ with $divu = 0$ and $u_{n|\partial\Omega} = 0$. The Stokes operator enables the definition of a related Hilbert scale ($\alpha \in R$) with corresponding norm

$$\|u\|_\alpha := \left\|A^{\alpha/2}u\right\| \ .$$

Throughout this paper, if not explicitly mentioned, we assume $p = 2$ and $n = 3$ for $(L_p(\Omega))^n$.

Using the Stokes operator and its related Hilbert scale framework the Navier-Stokes equations can be represented as an evolution equation in H_0. Since $P(gradp) = 0$ one gets

$$Au = Pf \ \text{ in } H_0 \ .$$

Putting $B(u) := P(u, grad)u)$ and assuming $Pu_0 = u_0$ the NSE initial-boundary equation is given by

$$(*) \qquad \frac{du}{dt} + Au + Bu = Pf \ , u(0) = u_0.$$

As u is divergence free and $u \cdot v$ identically vanishes on $\partial\Omega$ one gets

$$b(u, v, w) := ((u, grad)v, w) = \iint_\Omega (u, grad)v \cdot w dx = -b(u, w, v)$$

and especially $b(u, v, v) = 0$.

The linear homogeneous part of (*) enables the definition of a semigroup generated by the Stokes operator which leads to the representations

$$u(t) = e^{-tu_0} + \int_0^t e^{-(t-s)A}Pf(s)ds + \int_0^t e^{-(t-s)A}Bu(s)ds.$$

Multiply u to both sides of the homogeneous equation of (*) and integrate over Ω one gets

$$(\dot{u}, u) + (Au, u) + (Bu, u) = 0 \ , u(x, 0) = u_0.$$

This yields

$$\frac{d}{dt}\|u\|^2 + 2\left\|A^{1/2}u\right\|^2 = 0$$

respectively integrating over $[0, t]$

$$\|u(t)\|_0^2 + 2\int_0^t \|u\|_1^2(\tau)d\tau = \|u_0\|_0^2 \ .$$

A priori estimates for higher space derivatives of u are derived by multiplying the homogeneous equation (*) with $A^{2\sigma}u$ leading to

$$(\frac{du}{dt}, A^{2\sigma}u) + (Au, A^{2\sigma}u) + (Bu, A^{2\sigma}u) = 0$$

resp.

$$\frac{1}{2}\frac{d}{dt}\|A^{\sigma}u\|^2 + \left\|A^{\sigma+1/2}u\right\| + (A^{\sigma-1/2}Bu, A^{\sigma+1/2}u) = 0\,.$$

Estimates of $A^{\sigma-1/2}Bu$ depend on the space dimension n, which is the root cause for the "still open NSE questions" for the space dimension $n = 3$. The essential a priori estimates are e.g. given in (GiY)

$$(**) \qquad \frac{1}{2}\frac{d}{dt}\|u\|_1^2 + \|u\|_2^2 \cong \frac{1}{2}\frac{d}{dt}\left\|A^{1/2}u\right\|^2 + \|Au\|^2 \le c\begin{cases}\|u\|^2\left\|A^{1/2}u\right\|^4 \cong \|u\|_0^2\|u\|_1^4 & n=2\\ \left\|A^{1/2}u\right\|^6 \cong \|u\|_1^6 & n=3\end{cases}.$$

Putting

$$y(t) := \begin{cases}\|u\|_{2\sigma}^2 & n=2 \quad 0<\sigma<1/2\\ \|u\|_1^2 & n=3\end{cases}$$

one gets

$$y'(t) \le c\cdot\begin{cases}\|u\|_1^2\cdot y(t) & n=2\\ \|u\|_1^6\cdot y^3(t) & n=3\end{cases}$$

For $n = 2$ this leads to a global boundedness estimate in the form

$$z'(t) \le c\cdot\|u\|_1^2\cdot z(t) \quad \text{resp.} \quad z(t) \le z(0)\cdot e^{c\int_0^t\|u\|_1^2(s)ds}.$$

The special case $\sigma = 1/2$ lead also to a global estimate, based on an argument to be found e.g. in (TeR) 3.1.

For $n = 3$ every positive solution of $y'(t) = cy^3(t)$ blows up, i.e. there is no global estimate by this method.

Note: For $-1 \le \alpha \le 1$ the operator $A^{\alpha} := \int_0^{\infty}\lambda^{\alpha}dE_{\lambda}$ with domain

$$D(A^{\alpha}) := \left\{u \in L_{\sigma}^2(\Omega\,;\ \|A^{\alpha}u\|_2^2 = \int_0^{\infty}\lambda^{2\alpha}d\|E_{\lambda}u\|_2^2\right\}$$

is a positive, self-adjoint operator, (SoH) p. 133.

Lemma (SoH) p. 141): Let $n \ge 2$, $0 \le \alpha < \frac{n}{4}$, $2 \le q < \infty$ such that $2\alpha + \frac{n}{q} = \frac{n}{2}$ then $D(A^{\alpha}) \subset L_{\mathrm{q}}(R^n)^n$.

f. J. Plemelj's expanded concept of mass and flow
in complete analogy with the Newton potential

(PlJ) pp. 9-12, 15, 17, 20-22
translated by DeepL
original german text see below

§ 4 On an Extension of the Validity of Green's Theorems

Green's theorems are subject to certain restrictive conditions that prove to be quite problematic in some analyses. The first of these is the assumption of continuity and the unambiguous determinability of the normal direction $\frac{\partial V}{\partial n}$, which is what gives the theorem its clear meaning. *The derivation is based on the assumption that the potential and its first derivatives are finite and continuous up to the boundary, where there is a continuous transition to continuous values at the boundary*. However, a closer - and, incidentally, quite simple - examination of the derivation of Green's formulas shows that they hold true even when the normal direction undergoes a discontinuity at individual points, provided that the first derivatives of the potentials do not exceed finite values anywhere in the vicinity of such points. However, in the formulas, only those first derivatives appear that correspond to the direction of the normal at the boundary. It now seems reasonable to assume that the existence and continuity of this single derivative, together with the continuity of the potential itself at the boundary, is sufficient for the validity of Green's formulas. In fact, the validity of these formulas can be easily seen if one makes the very restrictive assumption regarding the shape of the boundary that, in addition to the continuous normal, there is also a circle of curvature present everywhere; while this is not bound by the condition of continuity, the radius of curvature is everywhere a finite quantity $\lambda < a$ different from zero. Since two adjacent normals meet only at the center of curvature, the set of points whose distance from the boundary (in the normal) is everywhere the same will form a parallel curve that does not intersect itself anywhere, and whose points are uniquely associated with the points of the given boundary curve such that corresponding points share a common normal, (PlP) p. 9.

§ 5 The Concepts of Mass and Flow

According to equation

$$\int \frac{\partial U}{\partial n}(s) = 0, \tag{6}$$

when the integral is extended over the boundary of a closed field in which the potential is continuous both inside and on the boundary, and the normal derivatives also have a continuous profile, However, if one attempts to extend Equation (6) to infinite fields and to such functions that, at sufficiently large distances, have a form

$$U = m log\left(\frac{1}{r}\right) + u,$$

with regular u at infinity, then the integral $\int \frac{\partial U}{\partial n}(s)$, taken only over the boundary lying in the finite region, is no longer zero. One can now, as above, consider only that part of the region of regularity that lies within a very large circle of radius and then examine the contribution to the integral $\int \frac{\partial U}{\partial n}(s)$ resulting from integration over the large circle. In the limit $lim\, R = \infty$, however, one obtains (in the case of the Newton potential, would stand 4π in place of 2π). Thus, for infinite fields you have

$$m = -\frac{1}{2\pi}\int \frac{\partial U}{\partial n}(s)ds, \tag{11}$$

where the integration must now be restricted to the boundary lying within the finite region.

If the boundary consists of several separate curve segments, it is appropriate to designate that part of the integral (11) corresponding to a self-contained segment of the boundary as the mass on that curve segment. Thus, the total mass is obtained as the sum of the masses on the individual closed curve segments. For a potential that is regular at infinity, the sum of the masses is zero, but they need not vanish individually. The vanishing of the mass on each individual closed curve segment is a very remarkable property, which is why such potentials play a prominent role in potential theory.

From the integral $\int \frac{\partial U}{\partial n}(s)$ along a non-closed curve, a property of great importance follows from equation (6). Namely, the integral depends only on the endpoints and not on the specific form of the integrating curve connecting them, in such a way that all integrals corresponding to integrating paths that emerge from one another through continuous deformation within the region of regularity are equal to one another. Thus, if p and q are two points in the region of regularity and they are connected by any curve (which has tangents), then $\int_p^q \frac{\partial U}{\partial n}(s)ds$ (where p and q do not belong to the boundary) is well-defined and has a value that does not depend on the specific shape of the curve. This can be seen by drawing a closed curve through p and q applying equation (6) to it (the full analogy holds for the Newtonian potential).

The integral between the points p and q is

$$\overline{U}(q) = -\int_p^q \frac{\partial U}{\partial n}(s)ds,$$

because it is independent of the curve, a well-defined function of the limits p and q and shall be denoted by $\overline{U}(q)$ in its dependence on q. This function is continuous for a continuous change of the upper limit q. If one now moves the point q from the interior of the domain of regularity toward a boundary point σ, the integral value will change continuously. Now it may happen that, despite the nonexistence of a specific value for the derivatives q, the integral $\int_p^q \frac{\partial U}{\partial n}(s)ds$ nevertheless tends toward a very specific limit value, which would then be denoted by $\overline{U}(\sigma) = -\int_p^\sigma \frac{\partial U}{\partial n}d\sigma$. The same applies when the second point is moved toward a boundary point σ_0. This explains the meaning of the integral

$$\overline{U}(\sigma) = -\int_{\sigma_0}^{\sigma} \frac{\partial U}{\partial n}ds$$

between two boundary points even when it cannot be formed along the boundary due to the nonexistence of $\frac{dU}{dn}$. If the integral $\overline{U}(\sigma)$ undergoes a continuous change as σ is continuously moved, then infinitesimal segments $d\sigma$ of the boundary will also correspond to an infinitesimal change in $\overline{U}(\sigma)$, which we can denote by $d\overline{U}(q)$. The connection with the ordinary derivatives is now easy to establish. Namely, if the quantity $\overline{U}(\sigma)$ is differentiable along the boundary, i.e., if $\frac{d\overline{U}(\sigma)}{d\sigma}$ exists, then this differential quotient is equal to $-\frac{dU}{dn}$. We thus have in the new concept a substitute for the ordinary derivatives, but one of far greater power. For $d\overline{U}(\sigma)$ can indeed make good sense even if no such sense exists for $\frac{d\overline{U}(\sigma)}{d\sigma}$. *This is a generalization, just as the extension of differentiable functions to continuous ones is*. The greater significance of the new concept is not hard to see, for now one is independent of both the existence of the normals and the derivatives at the boundary (even the assumption of the rectifiability of the boundary curve is irrelevant). Thus, for example, a generalization of the task of specifying normal derivatives of a potential at the boundary will consist in prescribing the corresponding value $d\overline{U}(\sigma)$ to each element $d\sigma$ of the boundary.

In its physical application to the flow of electricity, this new concept has already been given the name "*intensity of the current*" flowing through this section of the boundary. I shall therefore, for lack of a term free from such application, use the designation "*current*" or "*flow*" on the boundary section, (PlJ), pp. 10–12.

§ 7 The Location of Extreme Values

The equation

$$U(p) = \frac{1}{2\pi}\int U(s)\cdot\frac{\partial log\frac{1}{r_{ps}}}{\partial n_s}ds - \frac{1}{2\pi}\int log\frac{1}{r_{ps}}\cdot\frac{\partial U}{\partial n_s}ds$$

allows us to easily express the value of a potential at the center of a circle in terms of the peripheral values of the potential. This yields the central value

$$U_o = \frac{1}{2\pi r}\int U(s)\cdot ds$$

This incorporates Gauss's theorem:

The value of a potential at the center of a circle is the average of the peripheral values, (PlP) p. 15.

§ 8, The Potentials of the Single and Double Layers

Until now, it has been customary to assume the form

$$V(p) = \int log\frac{1}{r_{ps}}\mu'(s)ds$$

for the *potential of the single layer*, whereby $\mu'(s)$ was then called the *mass density* of the distribution. *However, such an assumption proves to be such a far-reaching restriction that it deprives the potential of most of its effectiveness.* For more in-depth investigations, the potential V proves to be usable only in the form

$$V(p) = \int log\frac{1}{r_{ps}} \cdot d\mu_s \,.$$

The infinitesimal quantity $d\mu_s$ can then be called the *mass element*, but one can speak of mass densities only if the differential quotient $\frac{d\mu_s}{ds} = \mu'(s)$ exists, (PlJ) p. 17.

§ 9, The Mass of the Potentials V and W and the Behavior at Infinity

The behavior of the potentials V and W at infinity was already discussed in § 6. If one sets

$$m = \int d\mu_s,$$

by extending the integration over the entire boundary, then for all large distances R the potential V has a series expansion of the form

$$V = mlog\frac{1}{R} + A\frac{x}{R^2} + B\frac{y}{R^2} + \cdots,$$

analogously, for the potential W of the double layer, one has a series expansion

$$W = A\frac{x}{R^2} + B\frac{y}{R^2} + \cdots.$$

The omitted terms are of higher order in $\frac{x}{R^2}$ and $\frac{y}{R^2}$. From this consideration it follows that the potential V is generally not regular at infinity, whereas the potential W is regular at infinity and has the value zero there. If, in the potentials V and W, the integration extends over several self-contained curves, then the potentials can be represented as the sum of potentials arising from the integration over each individual curve. From the fact that the integral $\int \frac{\partial U}{\partial n_s} ds$ over every curve that encloses a region regular with respect to the potential U is zero, it is easy to conclude that the mass of V on a closed curve segment over which the integral $V(p) = \int log\frac{1}{r_{ps}} \cdot d\mu_s$ is to be understood is equal to that part of the integral $\int d\mu_s$ which results from integration over this closed curve segment. Our definition of mass thus agrees with the classical one, according to which is indeed designated as mass. *An analogous consideration regarding the potential shows that the mass of the double-layer potential is zero on each individual closed curve segment of the boundary.* This excellent property of the potentials becomes very prominent in the boundary value problems of potential theory as soon as we deal with multiple connected regions, but has so far been insufficiently emphasized, (PlP) pp. 20.

§ 10, The Boundary Values of the Potentials V and W

1. *If the potential of the single layer $V(p) = \int log\frac{1}{r_{ps}} \cdot d\mu_s$ lies below a specified limit in any vicinity of a boundary point, then it is also continuous and passes continuously through the boundary.*

2. *The potential of the double layer undergoes discontinuities at a point on the boundary curve where the distribution is continuous, (PlP) pp. 20–22.*

g. The radiation problem of the wave equation

The following information is taken from the book *Methods of Mathematics and Physics* by R. Courant p. 682. We will limit ourselves to the four-dimensional Euclidean space-time continuum. If the aforementioned hypothesis - which has so far remained unproven - were to prove true, the existence of spherical waves would imply a characterization of this space. For the sake of simplicity, we will limit ourselves to the following two model cases, without limiting the generality of the result.

Huygens' principle asserts: *The value of the solution of Cauchy's problem for the wave equation depends only of the boundary of the domain of dependence in the plane* $t = 0$*; i.e., it depends only on the initial values of* u *and its derivatives on the boundary of the base* $r = t$ *of the characteristic cone, but not on the initial values in the interior of this case*. This a special case of the following general rule: *Huygens' principle is valid for the wave equation when the number* n *of the space dimension is odd, but not when* n *is even*, (CoR) p. 690.

T**he radiation problem**: For $t > 0$ we seek a solution of the homogenous wave equation $u_{tt} - \Delta u = 0$ which vanishes together with its derivative u_t for $t = 0$ except at the origin of the x-space, and which for $r = \sqrt{x_1^2 + x_2^2 + x_3^2} = 0$, i.e., on the „time-axis", is singular in such a way that

$$^{\circ}u := \lim_{\varepsilon \to 0} \oiint_{O_\varepsilon} \frac{\partial u}{\partial \nu} do = -g(t) \, .$$

Here the integration is at the time t over a sphere O_ε about the origin of the x-space. $\frac{\partial}{\partial \nu}$ denotes the normal of the sphere, do is the surface element, and the radius ε of the sphere is contracted to zero. The function $g(t)$ is the prescibed intensity of radiation as a function of the time, (CoR) p. 695.

The Huygens' principle is valid for the radiation problem with odd space dimensions. The effect, at the point x at the time t, of a disturbance radiating from the origin depends only on the character of this disturbance at a single previous instant of time, the time $t - r$. The disturbance, advancing from the origin with the unit velocity, reaches the point x at just the time t. Impulses at the origin that are sharply defined in time, i.e., represented by functions $g(t)$ which are different from zero only in a short time interval, are thus perceived at any point at the distance r only during a correspondingly short time interval beginning r time units later, (CoR) p. 698.

Spherical Waves. The problem of transmission of signals is further clarified by the concept of „spherical waves" which generalize the spherical solutions of the three-dimensional wave equation. We confine ourselves to the case of linear differential equations $L[u] = 0$ of second order and consider a time-like line Λ given in the form $\Lambda x_i = \xi(\lambda)$ with a parameter λ. (The time variable is not emphasized here.) With the point $\xi(\lambda)$ as vertex we consider the characteristic conoid or spherical wave front $\Gamma(x, \xi) = 0$.

For a given x, we may determine λ as a function of x from the equation $\Gamma(x, \xi(\lambda)) = 0$; we write $\lambda = \Phi(x)$. The characteristic conoid with vertex $\xi(\Phi(x))$ is given by the equation $\Phi(x) = constant$. A familiy of reatively undistorted spherical waves issuing from Λ may then be defined as a solution of the second order differential equation in the form

$$u(x) = g(x) S(\Phi(x))$$

with specific g and arbitrary S.

Little is known about the scope of this concept, which obviously relates spherical waves to the problem of transmitting with perfect fidelity signals in all directions. All we can do here is to formulate a conjecture which will be given some support in article 3:

> *Families of spherical waves for arbitrary time-like lines* Λ *exist only in the case of two and four variables, and then only if the differential equation is equivalent to the wave equation.*

A proof of this conjecture would show that the four-dimensional physical space-time world of classical physics enjoyes an essential distinction, (CoR) p. 763/764.

Note: The S^1 and S^3 are the only spheres with a "continuous" group structure.

h. The global nonlinear stability of the Minkowski space

The aim of this book („The Global Nonlinear Stabiltiy of the Minkowski Space“) is to provide a proof of the nonlinear gravitational stability of the Minkowski space-time. More precisely, our work accomplishes the following goals:

1. It provides a constructive proof of global, nontrivial solutions to the Einstein-Vacuum equations, which look, in the large scale, like the Minkowski space-time. In particular, these solutions are free of black holes and singularities.

2. It provides a detailed description of the sense in which these solutions are close to the Minkowski space-time in all directions and gives a rigorous derivation of the laws of gravitational radiation proposed by Bondi. It also describes our new results concerning the behavior of the gravitational field at null infinity.

3. It obtains these solutions as dynamic developments of all initial data sets, which are close, in a precise manner, to the initial data set of the Minkowski space-time, and thus it establishes the global dynamic stability of the matter.

4. Though our results are established only for developments of initial data sets which are uniformly close to the trivial one, they are in fact valid in the complement of the domain of influence of a sufficiently large compact subset of the initial manifold of any „strongly asymtotically flat“ intial data set, (ChD) p. 1.

(ChD) pp. 1, 10-13: „Einstein's field equations is about an unified theory of space-time and gravitations; the space-time (M, g) is the unknown, where M denotes a 4-dimensional manifold; one has to find an Einstein metric g, fulfilling the Einstein field equations. This is basically the equality $G = T$, whereby G denotes the Einstein tensor and T denotes the energy momentum tensor (e.g. the Maxwell equations). The Einstein-Vacuum equations (in the absense of matter, i.e. $T = 0$) are given by $R = 0$, whereby R denotes the Ricci tensor. Its simplest solution is the Minkowski space-time with its canonical coordinate system. Apart from Minkowski space-time it is not known, if there are any smooth, geodesically complete solution, which becomes flat at the infinity on any given spacelike direction. The main difficulties one encounters in the proof for the Cauchy Einstein-Vacuum equations with given initial data are:

- the problem of coordinates
- the strongly nonlinear hyperbolic features of the Einstein equations.

The problem of coordinates comes along with the concept of manifolds. To write the equations in a meaningful way, one seems forced to introduce coordinates. Such coordinates seem to be necessary even to allow the formulation of well-posed Cauchy problems and a proof of a local in time existence result. Nevertheless, as the particular case of wave coordinates illustrates, the coordinates may lead, in the large, to problems of their own.“

The main goal of this work („The Evolution Problem in General Relativity“) is to revisit the proof of the global stability of Minkowski space by D. Christodoulou and S. Klainerman. We provide a new self-contained proof of the main part of that result, which concerns the full solution of the radiation problem in vacuum, for arbitrarily asymptotically flat initial data sets. This can be interpreted as a proof of the global stability of the external region of Schwarzschild spacetime.

The proof, which is a significant modification of the arguments in (ChD), is based on a double null foliation of spacetime instead of the mixed null-maximal foliation used in equations and leads to important technical simplifications.

i. Pseudo-differential operators on R^n

The Pseudo-Differential operator (p.o.) with symbol $|\xi|^{-\alpha}$ has the form, (EsG) p. 39

$$(\Lambda^{-\alpha}u)(x)=\frac{2^{\alpha}\Gamma\left(\frac{\alpha+n}{2}\right)}{\pi^{\frac{n}{2}}\Gamma\left(-\frac{\alpha}{2}\right)}p.v.\int_{-\infty}^{\infty}\frac{u(y)}{|x-y|^{\alpha+i\beta+n}}dy\ ,\ 0<\alpha<n.$$

In particular, when $\alpha=1$, we get

$$(\Lambda^{-1}u)(x)=-\frac{\Gamma\left(\frac{n-1}{2}\right)}{2\pi^{\frac{n+1}{2}}}p.v.\int_{-\infty}^{\infty}\frac{u(y)}{|x-y|^{n-1}}dy\ ,\ n\geq 2\ .$$

The Newtonian potential p.o. for $n\geq 3$

For $\alpha=-2$ and $n\geq 3$, the function

$$(\Lambda^{-2}u)(x)=-\frac{\Gamma\left(\frac{n-2}{2}\right)}{4\pi^{\frac{n}{2}}}p.v.\int_{-\infty}^{\infty}\frac{u(y)}{|x-y|^{n-2}}dy=-(\Delta\Lambda^{-1})u(x)\ \ ,n\geq 3\ .$$

is called a *Newtonian potential*.

The Riesz transform p.o. for $n\geq 3$

Exampe 3.3 in (EsG) p. 39: Let k be a positve integer. Then $\Lambda^{2k}(\xi)=|\xi|^{2k}$ and hence $\Lambda^{2k}u=(-\Delta)^k u$, where$\Delta$ is the Laplace operator. If $\alpha>0$ is not even, then there exists and even integer $2m>0$ such that If $\alpha=2m+\gamma$ is where $-2<\gamma<0$. Consequently,

$$(\Lambda^{\alpha}u)(x)=(\Lambda^{2m+\gamma}u)(x)=(-\Delta)^m\frac{2^{\gamma}\Gamma\left(\frac{n+\gamma}{2}\right)}{\pi^{\frac{n}{2}}\Gamma\left(-\frac{\gamma}{2}\right)}p.v.\int_{-\infty}^{\infty}\frac{u(y)}{|x-y|^{n+\gamma}}dy\ ,\ n\geq 2,$$

Note: In particular, the p.o. $\Lambda=-(\Delta\Lambda^{-1})$ has the form, (EsG) p. 39

$$(\Lambda u)(x)=-(\Delta\Lambda^{-1})u(x)=-\Delta\frac{\Gamma\left(\frac{n-1}{2}\right)}{2\pi^{\frac{n+1}{2}}}p.v.\int_{-\infty}^{\infty}\frac{u(y)}{|x-y|^{n-1}}dy=-\frac{\Gamma\left(\frac{n-1}{2}\right)}{2\pi^{\frac{n+1}{2}}}p.v.\int_{-\infty}^{\infty}\frac{\Delta_y u(y)}{|x-y|^{n-1}}dy\ .$$

The Riesz p.o. with symbol with symbol $\sigma_k=\frac{\xi_k}{|\xi|}$ and $\oint_{S^{n-1}}\sigma_k ds_\sigma$ has the form, (EsG) p. 44

$$(R_k u)(x)=-\mathrm{i}\frac{\Gamma\left(\frac{n+1}{2}\right)}{\pi^{\frac{n+1}{2}}}p.v.\int_{-\infty}^{\infty}\frac{x_k-y_k}{|x-y|^{n+1}}u(y)dy\ .$$

Note: The Riesz transformations are the n-dimensional generalizations of the 1-dimensional Hilbert transformation. They arise when study the Neumann problem in upper half-plane. The Riesz transform commutes with translations and homothesis, having nice properties relative to rotation, (PeB), (StE) (*).

(*) If $j\neq j$ then R_jR_k is a singular convolution operator. On the other hand, it holds $R_j^2=-(1/n)I+A_j$ where A_j is a convolution operator. The following identities are valid

$$\|R_j\|=1\ ,R_j^*=-R_j\ \ ,\ \sum R_j^2=-I\ \ ,\ \ \sum\|R_j u\|^2=\|u\|^2\ ,u\in L_2\ .$$

The "rotation property" plays a key role in the context of the rotation group $SO(n)$: Let $m:=m(x):=(m_1(x),\dots m_n(x))$ be the vector of the Mikhlin multipliers of the Riesz operators and$\rho=\rho_{ik}\in SO(n)$, then

$$\begin{aligned}m(\rho(x))=\rho(m(x)),\text{ whereby }m_j\big(\rho(x)\big)&=\sum\rho_{jk}m_k(x)\\&=c_n\int_{S^{n-1}}(\tfrac{\pi i}{2}sign(x\rho^{-1}(y))+log\left|\tfrac{1}{x\rho^{-1}(y)}\right|)\tfrac{y}{|y|}d\sigma(y)\\&=c_n\int_{S^{n-1}}(\tfrac{\pi i}{2}sign(xy)+log\left|\tfrac{1}{xy}\right|)\tfrac{y}{|y|}d\sigma(y)\ .\end{aligned}$$

The special Calderón-Zygmund p.o. Λ with symbol $|\xi|$ (*)

The special Calderón-Zygmund operator Λ with symbol $|\xi| = \sum_{k=1}^{n} \xi_k \frac{\xi_k}{|\xi|}$ has the form, (EsG) p. 44

$$(\Lambda u)(x) = (\sum_{k=1}^{n} R_k D_k u)(x) = -\frac{\Gamma\left(\frac{n-1}{2}\right)}{2\pi^{\frac{n+1}{2}}} p.v. \int_{-\infty}^{\infty} \frac{\Delta_y u(y)}{|x-y|^{n-1}} dy \, .$$

$$= -\frac{\Gamma\left(\frac{n-1}{2}\right)}{2\pi^{\frac{n+1}{2}}} p.v. \int_{-\infty}^{\infty} \frac{\Delta_y u(y)}{|x-y|^{n-1}} dy = -(\Delta\Lambda^{-1})u(x) \, .$$

Note: The special Calderón-Zygmund p.o. are convolution operators $T(f) = S * F$ with a distribution S defined by symbols $m(\omega) \in C^{\infty}(R^n - \{0\})$ with the following properties, (MeY)

i) $m(\mu\omega) = m(\omega), \mu > 0$

ii) the mean of $m(\omega)$ on the unit sphere is zero

iii) it holds $m(\omega) = \frac{\omega_j}{|\omega|}$.

Note: The Calderón-Zygmund operators Λ with symbol $|\nu|$ and its inverse operator Λ^{-1} may be represented in the following forms, (EsG) 3.15, 3.17, 3.35, (LiI) p. 58 ff.,

$$(\Lambda u)(x) = (\sum_{k=1}^{n} (R_k D_k u)(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\pi^{\frac{n+1}{2}}} \sum_{k=1}^{n} p.v. \int_{-\infty}^{\infty} \sum_{k=1}^{n} \frac{x_k - y_k}{|x-y|^{n+1}} \frac{\partial u(y)}{\partial y_k} dy$$

$$= -\frac{\Gamma\left(\frac{n-1}{2}\right)}{2\pi^{\frac{n+1}{2}}} p.v. \int_{-\infty}^{\infty} \frac{\Delta_y u(y)}{|x-y|^{n-1}} dy = -(\Delta\Lambda^{-1})u(x)$$

$$(\Lambda^{-1} u)(x) = \frac{\Gamma\left(\frac{n-1}{2}\right)}{2\pi^{\frac{n+1}{2}}} p.v. \int_{-\infty}^{\infty} \frac{u(y)}{|x-y|^{n-1}} dy \, , n \geq 2 \, .$$

Note: The function spaces on R^n are related to Sobolev function spaces and continuous field spaces on bounded domains $\Omega \subset R^n$ with Lipschitz boundaries by the Sobolev and the Rellich embedding theorems:

i. $H_k(\Omega) \subset C^0(\bar{\Omega})$, $k > n/2$

ii. $H_{k'}(\Omega) \subset H_k(\Omega)$, $k < k'$.

Note: For space dimension $n \geq 2$ the Calderón-Zygmund operator in the form $-i\Lambda = -\mathrm{i} \sum_{k=1}^{n} R_k D_k u$ serves as an alternative to the Schrödinger operator $-i\nabla$. For space dimension $n = 1$ the Riesz operators reduces to the Hilbert transform operator, i.e., the expression takes the form $P = -iH\left[\frac{d}{dx}\right]$, (BrK1), (MeY) p. 5.

Note: In the context of the Navier-Stokes Equations we mention the Leray-Hopf projector P onto the space of divergence-free L_2-funcions given by

$$P := Id - R \otimes R =: Id - Q = Id - \frac{D \otimes D}{D^2} Id - \Delta^{-1}(\nabla \times \nabla) \, .$$

It is well defined due to the Helmholtz-Weyl decomposition of the L_2 space into the space of divergence-free function and the space of gradients of functions in $W^{1,2}$.

j. The quaternion rotation operator

(KuJ) pp. 123-134

The text in this section is merely a summary of some characteristics from (KuJ) relevant to this paper; the passages quoted directly are written in italics:

Notations and Definitions*: Quaternions are denoted by lower case letters, say p or q or r. Bold faced letters denote ordinary vectors in the three-dimensional space, namely R^3. Vectors in three dimension space may be written as triple of real numbers (that is, scalars), so that we write this orthogonal basis as*

$$\boldsymbol{i} = (1,0,0),\ \boldsymbol{j} = (0,1,0),\ \boldsymbol{k} = (0,0,1).$$

Now a quaternion may be regarded as a 4-tuple of real numbers, that is, as an element of R^4. In this case we would write $q = (q_0, q_1, q_2, q_3)$, where q_0, q_1, q_2 and q_3 are simple real numbers or scalars. We shall adopt an alternative way of representating a quaternion. First, we define scalar part to be some real number or scalar, say q_0. Then we define a vector part, say $\boldsymbol{q}$, which is an ordinary vector in R^3, $\boldsymbol{q} = \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3$, and the quaternion as the sum $q = q_0 + \boldsymbol{q} = q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3$. The scalars q_0, q_1, q_2, q_3 are called the components of the quaternion. The complex conjugate resp. the related norm of the quaternion $q = q_0 + \boldsymbol{q} = q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3$ is given by the quaternion $q^* = q_0 - \boldsymbol{q} = q_0 - \boldsymbol{i}q_1 - \boldsymbol{j}q_2 - \boldsymbol{k}q_3$ resp. $N(q) = \sqrt{q^*q}$.

Note*: A vector $\boldsymbol{v} \in R^3$ can simply be treated as though it were a quaternion $q \in R^4$ whose real part is zero. Such a quaternion is called a pure quaternion.*

Note*: One cannot expect our quaternion rotation operator to consist simply of a single quaternion.*

Note: *In terms of a given input vector $\boldsymbol{v}$ we have two possible triple-product quaternion operators, defined by*

$$\boldsymbol{w}_1 = q\boldsymbol{v}q^* \qquad (5.5)$$

$$\boldsymbol{w}_2 = q^*\boldsymbol{v}q \qquad (5.6)$$

The algebraic action of Equation 5.5 is illustrated in Figure 5.2. In summary, to this point we have found a way to handle vectors in quaternion space, and we have found two quaternion operators which take vectors into vectors. So, now we ask

What geometric interpretation can we give these operators?

The quest for geometric interpretations for the triple quaternion products in Equations 5.5 and 5.6 leads us to ask the next question:

Is there some way to associate an angle with a quaternion, analogous to the way we earlier associated an angle with a rotation matrix?

If so, perhaps we can also associate an angle with the above two quaternion operators. The answer to our question is Yes, and in our consideration we discover an interesting and helpful fact.

From this point on, the quaternion used to define the operator will always be a unit or normalized quaternion, that is a quaternion wirh norm 1. … If $q = q_0 + \boldsymbol{q}$ has the norm 1 then $q_0^2 + |\boldsymbol{q}|^2 = 1$. For any angle θ we know that $cos^2\theta + sin^2\theta = 1$, i.e, there must be some angle θ such that $cos^2\theta = q_0^2$ and $sin^2\theta = |\boldsymbol{q}|^2$. This angle can be defined uniquely if we place the proper restriction on its domain. In general, we will ask that θ satisfy the restriction $-\pi < \theta \leq \pi$. In this way we have an angle, namely the angle θ defined above, associated with the quaternion q. We will find it convenient to write the unit quaternion $q = q_0 + \boldsymbol{q}$ in terms of this angle. Suppose we define a unit vector $\boldsymbol{u}$, which represents the direction of $\boldsymbol{q}$ by writing $\boldsymbol{u} = \frac{\boldsymbol{q}}{|\boldsymbol{q}|} = \frac{\boldsymbol{q}}{sin\theta}$, then we may write the unit quaternion q in terms of the angle θ and the unit vector $\boldsymbol{u}$ in R^3 as

$$q = q_0 + \boldsymbol{q} = cos\theta + \boldsymbol{u}sin\theta.$$

Note, that for a quaternion q expressed in this form, substituting $-\theta$ for θ, (whatever geometric meaning the angle θ might have) we get the complex conjugate of q. That is,

$$cos(-\theta) + \boldsymbol{u}\, sin(-\theta) = cos(\theta) - \boldsymbol{u}\, sin(\theta) = q^*.$$

A special quaternion product

Suppose we have two unit quaternions, say p and q, with both having the same (direction of **q)** *vector in $\boldsymbol{u} \in R^3$. We associate an angle with the quaternion and an angle α with the quaternion β. Then we may write*

$$p = \cos(\alpha) + \boldsymbol{u}sin(\alpha),\ q = \cos(\beta) + \boldsymbol{u}sin(\beta).$$

The quaternion product rule gives

$$r = pq = \big(\cos(\alpha) + \boldsymbol{u}sin(\alpha)\big)\big(\cos(\beta) + \boldsymbol{u}sin(\beta)\big) = \cos(\alpha+\beta) + \boldsymbol{u}sin(\alpha+\beta) = \cos(\gamma) + \boldsymbol{u}sin(\gamma).$$

This is an interesting result. It says, if we multiply the quaternions, p and q, each having the same vector, $\boldsymbol{u}$, then the product is a quaternion

$$r = \cos(\gamma) + \boldsymbol{u}sin(\gamma).$$

also having the same vector, $\boldsymbol{u}$.

In this example we will associate the angle $\theta = \frac{\pi}{6}$, ($cos\left(\frac{\pi}{6}\right) = \frac{1}{2}$, $sin\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$), with the quaternion q, while retaining the basis vector $\boldsymbol{k}$ as the vector of q. Then we write the quaternion q in the form $q = cos(\theta) + \boldsymbol{k}sing\theta$ in the form $q = q_0 + \boldsymbol{q} = cos\theta + \boldsymbol{k}sin\theta = \frac{1}{2}\big[\sqrt{3} + \boldsymbol{k}\big]$. We apply the quaternion operator of Equation 5.5 to the basis vector $\boldsymbol{v} = 1\boldsymbol{i} + 0q_2 + 0\boldsymbol{k}$. Again it not difficult to verify that the following computations are correct. We have

$$w = q\boldsymbol{v}q^* = (\tfrac{\sqrt{3}}{2} + \tfrac{1}{2}\boldsymbol{k})(0 + \boldsymbol{i})(\tfrac{\sqrt{3}}{2} - \tfrac{1}{2}\boldsymbol{k}) = \left(\tfrac{\sqrt{3}}{2}\boldsymbol{i} + \tfrac{1}{2}\boldsymbol{j}\right)\left(\tfrac{\sqrt{3}}{2} - \tfrac{1}{2}\boldsymbol{k}\right) = \tfrac{1}{2}\big[\boldsymbol{i} + \sqrt{3}\boldsymbol{j}\big].$$

The result is again a pure quaternion, as should be the case. The vector in R^3 which corresponds to this pure quaternion is $\boldsymbol{w} = \frac{1}{2}\big[\boldsymbol{i} + \sqrt{3}\boldsymbol{j}\big]$. Notice again that $\boldsymbol{w}$ is a unit vector, and that the vector $\boldsymbol{w}$ lies in the second quadrant. We ask

> *Geometically, how are we to view the vector-frame action of this quaternion operator $q\boldsymbol{v}q^*$?*

In answering this question, an important consideration is how one views the resultant final relationship between the input vector $\boldsymbol{v}$, the output vector $\boldsymbol{w}$ and the coordinate frame with the standard orthogonal basis $\{\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}\}$. The answer is conditional; one may adopt either of the following two distinctly different perspectives.

***First Perspective**:*

The first perspective is that of an observer seated on and fixed with respect to the coordinate frame $\{\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}\}$. To her it appears that the quaternion operator, $q\boldsymbol{i}q^*$, has rotated the vector $\boldsymbol{v}$, about $\boldsymbol{k}$, as an axis through an angle $2\theta = \frac{\pi}{3}$, that is, in the counter-clockwise direction. From this perspective it is convenient to think of the coordinate frame being fixed, while the vector is rotated. This is sometimes called a point rotation.

***Second Perspective**:*

The second perspective is that of an observer seated on and fixed with respect to the vector $\boldsymbol{v}$. To him it appears that this same quaternion operator, $q\boldsymbol{v}q^*$, has rotated the coordinate frame $\{\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}\}$, about $\boldsymbol{k}$ as an axis, through an angle, $-\frac{\pi}{3}$, that is, in the clockwise direction. From this second perspective we think of the vector $\boldsymbol{v}$ as being fixed, while the coordinate frame is rotated. This is sometimes called a frame rotation.

***Choice of Perspective**:*

Which of these two perspectives is to be preferred in a given application is usually a subject matter – a decision made by the practitioner. In many of the applications which we consider in later chapters, we will use the operator qvq^*, and interpret it geometrically to be a frame rotation. Incidentally, note that the signs of the angles in these two perspectives are reversed if the operator is qvq^*.

> ***Theorem 5.1**: For any unit quaternion*
>
> $$q = q_0 + \boldsymbol{q} = q_0 + \boldsymbol{i}q_1 + \boldsymbol{j}q_2 + \boldsymbol{k}q_3 = cos(\theta) + \boldsymbol{u}sin(\theta)$$
>
> *and for any vector $\boldsymbol{v} \in R^3$ the action of the operator*
>
> $$\boldsymbol{w} = L_q(\boldsymbol{v}) := q\boldsymbol{v}q^*$$
>
> *on $\boldsymbol{v}$ may be interpreted geometrically as a rotation of the vector $\boldsymbol{v}$ through an angle 2θ about $\boldsymbol{q}$ as the axis of rotation.*

Note, in Figure 5.4., that the vector $\boldsymbol{v}$ and its image $\boldsymbol{w}$ may be viewed as generators of the right-circular cone whose axis is the quaternion vector $\boldsymbol{q}$ and whose circular base in this instance contains the vectors, $\boldsymbol{n}$ and $\boldsymbol{m}$. Thus, the vector $\boldsymbol{v}$ and its image $\boldsymbol{w}$ are related by the rotation described in the above theorem.

The various vectors in Figure 5.4 implicitly suggest the existence of some reference coordinate frame. From *afar*, God sees the *frame* within which all of this is happening. But we *local creatures*, are a much too loosely coupled part of it all. As observer: we must more closely relate to the *fixed axis* of rotation, the generators $\boldsymbol{v}$ and $\boldsymbol{w}$, the *fixed point*, and/or the *frame*.

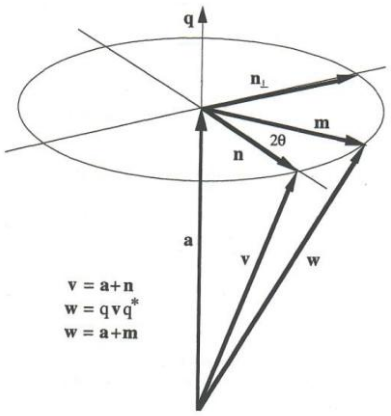


Figure 5.4: Rotation Operator Geometry

So the observer somehow finds a seat fixed in the implicit frame. What she sees happening, under the action of the rotation operator qvq^*, is that the vector $\boldsymbol{v}$ goes to $\boldsymbol{w}$. That is, $\boldsymbol{v}$ is rotated positively through an angle 2θ about an axis whose direction is defined by $\boldsymbol{q}$. This is exactly the situation described earlier called First Perspective.

On the other hand, if I go find myself a seat on the vector $\boldsymbol{v}$, then, for the same operator qvq^*, what I see happening is that the coordinate frame rotates in a negative direction through the angle 2θ about this same axis which is defined by the quaternion vector $\boldsymbol{q}$. This is what we earlier called Second Perspective.

Up to this point we have used the operator of Equation 5.5. We could just as well have used the quaternion operator $L_{q^*}(\boldsymbol{v}) := q^*\boldsymbol{v}q$ of Equation 5.6. Keeping in mind our remarks on the observer's perspective, made above, we then would have proved the following theorem:

***Theorem 5.2**: For any unit quaternion*

$$q = q_0 + \boldsymbol{q} = cos(\theta) + \boldsymbol{u}sin(\theta)$$

and for any vector $\boldsymbol{v} \in R^3$ the action of the operator

$$\boldsymbol{w} = L_{q^*}(\boldsymbol{v}) := q^*\boldsymbol{v}q$$

may be interpreted geometrically as

- *a rotation of the coordinate frame with respect to the vector $\boldsymbol{v}$ through an angle 2θ about $\boldsymbol{q}$ as the axis, or,*
- *an opposite rotation of the vector $\boldsymbol{v}$ with respect to the coordinate frame through an angle 2θ about $\boldsymbol{q}$ as the axis.*

It is important to note just how closely related these two theorems are: If in Theorem 5.1 we change the sign of the angle and therefore the direction of rotation, then Theorem 5.1 becomes Theorem 5.2, and vice versa. This tends to confirm that if the angle may be chosen from $-\pi < \theta \leq \pi$ then we could get by with only one Theorem – and carefully choose the proper persepective.

Even though we stated earlier that with the appropriate choice of the angle θ (specifically whether θ is defined to be positive or defined to be negative) the two opertors of Equations 5.5 and 5.6 may be in fact equivalent. However, it will still be important in many applications to maintain the distinction between the results of these two theorems. Hence from this point on we shall uniformly interpret the quaternion rotation operator of Equation 5.5 geometrically as a point rotation with respect to the frame, while we interpret the quaternion rotation operator of Equation 5.6 as a coordinate frame rotation with respect to the point or vector space. The importance of this disctinction is found only in the need to decide which of these two geometric interpretations is appropriate in a given application.

We again emphasize that the choice of which of these two quaternion operators to use in a given application is actually arbitrary, and so the choice should be made on the basis of what seems natural an convenient. We can hardly emphasize too much the fact that the important consideration is the relative relationship between the vectors and the coordinate frame. It is most important to realize that in a specific application we begin with an initial relative relationship between the vectors and the coordinate frame, and that these quaternion rotation operators then simply modify this relative relationship. Which rotations one uses to get to the desired final relationship, that is, whether one uses point rotations (frame fixed) or frame rotations (points fixed), is not the significant matter; these two perspectives are identical except for the sign on the rotation angle. Having said this, however, we must also say that in a given application the choice must be made explicitly and carefully.

k. Potential, angular, and dissipative operators in Krein spaces

The potential operator $\boldsymbol{Wx \coloneqq P_1x - P_2x}$
(VaM) pp. 55, 86, 88, 90-91

The term "potential operator" comes up in the topic „Variational Methods for the Study of Nonlinear Operators, (VaM).

Definition 5.2 (VaM) p. 55: We say that the operator $\boldsymbol{F}$, from a Banach space E into the conjugate space E^*, is potential on some set $H \subset E$, if there exists a functional f such that $gradf(x) = \boldsymbol{F}(x)$ for every $x \in H$.

A Krein space is decomposed into an orthogonal sum of two spaces H_1 and H_2 ($H = H_1 \otimes H_2$) associated with two projection operators P_1 and P_2 onto H_1 and H_2. From the functional

$$\varphi(x) := \big((x)\big)^2 = \|P_1x\|^2 - \|P_2x\|^2$$

we have

$$\|P_1x\|^2 = \tfrac{1}{2}\|x\|^2 + \tfrac{1}{2}\big((x)\big)^2 \; ; \; \|P_2x\|^2 = \tfrac{1}{2}\|x\|^2 - \tfrac{1}{2}\big((x)\big)^2.$$

According to (10.7), (VaM) p. 86, the gradient of $\varphi(x) := \big((x)\big)^2$ is given by

$$grad\big((x)\big)^2 = 2P_1x - 2P_2x.$$

The operator $J = P^+ - P^-$ is the *fundamental symmetry* belonging to the fundamenal decomposition $H = H_1 \otimes H_2$, (BoJ) p. 52. In other words, the potential operator W on all of the space H of the form

$$Wx \coloneqq \tfrac{1}{2}grad\big((x)\big)^2 = P_1x - P_2x$$

corresponds to the symmetry operator J of a Krein space.

Hyperbolids, hyperbolic regions, conical regions
(VaM) pp. 90-93

The manifold $\big((x)\big) = c > 0$ represents a hyperboloid in the Hilbert space H. The points of the hyperbolic region V_c resp. the points of the conical region V_0 satisfy the conditions

$$\big((x)\big) = \sqrt{\|P_1x\|^2 - \|P_2x\|^2} \geq c > 0 \quad , \quad \big((x)\big) = \sqrt{\|P_1x\|^2 - \|P_2x\|^2} \geq 0.$$

Evidently $V_c \subset V_0$. We remark that if x is an interior point of the conical region V_0, i.e.,

$$\big((x)\big) = \sqrt{\|P_1x\|^2 - \|P_2x\|^2} = \alpha > 0,$$

then those points of the ray $tx, t \in [0,\infty)$ for which $t \geq c/\alpha$ belong to the hyperbolic region $\mathrm{V_c}$, and those for which $0 \leq \mathrm{t} < \mathrm{c}/\alpha$ do not belong to $\mathrm{V_c}$. If x is not an element of $\mathrm{V_0}$, then the ray $tx, t \in [0,\infty)$ does not have any point in common with $\mathrm{V_c}$. Thus, every interior ray of the conical region $\mathrm{V_0}$ intersects the hyperbolid $\big((x)\big) = c > 0$ in a single point. We denote by K the boundary of the conical region V_0. The manifold K is defined by the condition $\big((x)\big) = 0$. If we look at the unit sphere S^1 ($\|x\|^2 = 1$), then those points of S^1 for which $\|P_1x\| = \|P_2x\|$ belong to $V_0 - K$. Those rays tx ($x \in S^1$) for which $\|P_1x\|^2 > \|P_2x\|^2$ intersect the hyperboloid $\big((x)\big) = x > 0$ at a point whose distance from θ is given by

$$t = \frac{c}{\sqrt{\|P_1x\|^2 - \|P_2x\|^2}}.$$

From this we see that $t \to \infty$ if $(\|P_1x\|^2 - \|P_2x\|^2) \to \infty$, i.e., the manifold K is an asymptotic conical manifold for the hyperboloid $\big((x)\big) = c > 0$.

Hyperboloids generated by operators

Let be a self-adjoint operator B defined on all of the Hilbert space H. The operator B induces a decomposition of H into a direct sum of the subspaces H_1 and $H_1^{\perp}$ and thereby generates a hyperboloid

$$\big((x)\big) = \sqrt{\|P_1x\|^2 - \|P_2x\|^2} = c > 0,$$

where P_2 is the projection onto $H_1^{\perp}$. In case where the positive part of the spectrum of B lie in an interval $[m, b]$, where $m > 0$, then the inequality

$$\|Bx\| \geq \frac{m}{\sqrt{2}}\sqrt{\big((x)\big)^2 + \|x\|^2} \geq \frac{m}{\sqrt{2}}\sqrt{c^2 + \|x\|^2}$$

holds for every x in the hyperbolic region V_c

$$\big((x)\big) = \sqrt{\|P_1x\|^2 - \|P_2x\|^2} \geq c > 0,$$

as well as in the conical region V_0, i.e., when $c = 0$.

Angular and dissipative operators

(BoJ) pp. 52-55

Let $\underline{P}^{++} \coloneqq \{x \in H; (x,x) > 0 \; or \; x = 0\}$ and $\underline{P}^{+} \coloneqq \{x \in H; (x,x) > 0 \; or \; x = 0\}$; if the inner product on H is indefinite on H, then none of the sets $\underline{P}^{+}, \underline{P}^{++}, \underline{P}^{-}, \underline{P}^{--}$ is a subspace, (BoJ) p. 6.

Let B be a decomposable, non-degerated inner product space. Consider the fundamental projectors belonging to a fundamental decomposition

$$(*) \quad H = H^{+} \otimes H^{-}, H^{+} \subset \underline{P}^{++}, H^{-} \subset \underline{P}^{-} \text{ of } H.$$

We set $J = P^{+} - P^{-}$, and say that J is the *fundamental symmetry* belonging to the fundamenal decomposition (*).

The term „symmetry" for the linear operator J is justified by the fact that for every $x \in H$ the vectors x and Jx lie symmetrically with respect to the subspace H^{+}. Namely, if

$$(**) \qquad x = x^{+} + x^{-}, \quad x^{+} \in H^{+}, x^{-} \in H^{-}$$

then

$$(***) \qquad Jx = x^{+} - x^{-}, \quad x^{+} \in H^{+}, x^{-} \in H^{-}.$$

Since the system (**), (***) can be solved for x^{+} and x^{-}, the fundamental symmetry J uniquely specifies the fundamental decomposition to which it belongs. This entitles us to speak of „fundamental decomposition belonging to J". Making use of relation (**), (***) one easily verifies the following.

Lemma 11.1: A fundamental symmetry J is completely invertible, and we have $J = J^{-1}$. Moreover, J is symmetric as well as isometric.

….

By the aid of J-norms an extremly useful description of semidefinite subspaces can be given. For this purpose we need one more definition.

Let L be a subspace of the decomposable, non-degerated inner product space H. Consider the fundamental decomposition

$$H = H^{+} \otimes H^{-}, H^{+} \subset \underline{P}^{++}, H^{-} \subset \underline{P}^{--}$$

and the corresponding fundamental projectors P^{+}, P^{-}. Suppose that $P^{+}|_{L}$ is invertible. Then every $x \in L$ is uniquely specified by its projection $P^{+}x$, so that the formula

$$K^{+}P^{+}x = P^{-}x, x \in L$$

defines an linear operator $K^{+} = K^{+}(L)$ from H^{+} to H^{-} with domain $D(K^{+}) = P^{+}(L)$ and range $R(K^{+}) = P^{-}(L)$. We say, K^{+} is the *angular operator* of the subspace L with respect to H^{+}. Similarly, if $P^{-}|_{L}$ is invertible, then the angular operator $K^{-} = K^{-}(L)$ of L with respect to H^{-} can be defined by the relation

$$K^{-}P^{-}x = P^{+}x, x \in L \;\; ;$$

we have $D(K^{-}) = P^{-}(L)$, $R(K^{-}) = P^{+}(L)$.

Theorem 11.6: Suppose that $H^{+} \otimes H^{-}, H^{+} \subset \underline{P}^{++}, H^{-} \subset \underline{P}^{--}$. Let L be a subspace of H. If K^{+} (K^{-}) is the angular operator of L with respect to H^{+} (H^{-}) then

$$(1) \quad L = \{x^{+} + K^{+}x^{+} \colon x^{+} \in L_1\}$$

$$(2) \quad L = \{x^{-} + K^{-}x^{-} \colon x^{-} \in L_2\}$$

where L_1 (L_2) is a subspace of H^{+} (H^{-}). Conversely, if L can be written in the form (1) (resp. (2), where L_1 (L_2) is a subspace H^{+} (H^{-}) and K^{+} (K^{-}) is a linear operator from L_1 to H^{-} (from L_2 to H^{+}), then K^{+} (K^{-}) is the angular operator from L with respect to H^{+} (H^{-}).

Theorem 11.7: Suppose that $H^+ \otimes H^-$, $H^+ \subset \underline{P}^{++}$, $H^- \subset \underline{P}^{--}$and denote the corresponding fundamental symmetry by J. A subspace $L \subset H$ is positive if and only if the angular operator K^+ of H with respect to H^+ exists and satisfies the condititon

$$\|K^+x^+\|_J \leq \|x^+\|_J\,,\ x^+ \in D(K^+).$$

In particular, positive definite subspaces are characterized by the property

$$\|K^+x^+\|_J < \|x^+\|_J\,,\ x^+ \in D(K^+), x^+ \neq 0$$

and neutral subspaces by

$$\|K^+x^+\|_J = \|x^+\|_J\,,\ x^+ \in D(K^+)\,.$$

For negative subspaces similar statements, involving K^- instead of K^+, are valid.

A Reducing of the Invariant Semidefinite Subspace Problem
for Krein Noncontraction to such a Problem for Krein Isometry
Choroszavin S. A., (ChS)

The tradititional question of the theory of linear operators is the question of the existence of a (nontrivial) invariant subspace. In the case of indefinite inner product spaces it is of interest a special kind of subspaces:

Definition: A subspace $L \subset H$ is said to be positive (negative) iff $\langle x, x\rangle \geq 0$, (≤ 0) for any $x \in L$.

Definition: A subspace $L \subset H$ is said to be *maximal positive (negative)* iff it is positive (negative) and maximal through such of subspaces in the sence of the set theory.

Theorem: In order that the subspace $L \subset H$ should be maximal positive, a necessary and sufficient condition is that there should exist a linear $K : H^+ \to H^-$ such that

$$\|K\| \leq 1 \text{ and } L = \{x^+ + K^+x^+ : x^+ \in H^+\}.$$

Such an operator K is said to be angular operator of L and it is unique (by fixed L).

Note: The structure of closed positive subspace L is analogous: $L = \{x^+ + K^+x^+ : x^+ \in L^+\}$ where L^+ is a closed linear subspace of $H^+$and $K^+ : L^+ \to H^-$ is linear with $\|K\| \leq 1$.

Definition: A linear bounded $T : H \to H$ is said to be J-noncontraction or $\langle\,,\rangle$-noncontraction or Krein noncontraction, iff

$$\langle Tx\,, Tx\,\rangle \geq \langle x\,, x\,\rangle,\ x \in H.$$

We are interested in the existence of maximal positive invariant subspaces for Krein noncontraction:

Theorem 1: If every J-isometry has nontrivial positive invariant subspace then every J-noncontraction has such a subspace.

Theorem 2: If every J-binoncontractive J-isometry has maximal positive invariant subspace then every J-noncontraction has such a subspace.

An Example of J-Unitary Operator
Solving a Problem, Stated by M.G. Krein
Choroszavin S. A., (ChS1)

This paper describes the construction of a J-unitary operator that possesses certain spectral and invariant properties, building on previous work and focusing on linear operators in separable Hilbert spaces.

Definition: Given an operator T and a number c, we write

$$S(T,c) \coloneqq \{x \in H | \exists M \geq 0 \forall N \geq 0 : \|T^N x\| \leq M c^N\}, r(T) \coloneqq \text{spectral radius of } T .$$

Remark 1: Given a linear bounded operator T and a T-invariant subspace L such that $r(T|L_1) \leq c$ then

$$L \subset S(T, c+\varepsilon).$$

Remark 2: $$S(T_1 \otimes T_2, c) = S(T_1, c) + S(T_2, c)$$

Theorem 1: Given a number $c \geq 1$, there exists a J-unitary operator $\hat{V}$, such that

- $r(\hat{V}) = r(\hat{V}^{-1}) = c$
- die Mengen $S(\hat{V}), c) = S(\hat{V}^{-1}, c) = S(\hat{V}^{*-1}, c) = S(\hat{V}^{*}, c) = \{0\}$
- there exist maximal strictly positive and strictly negative $\hat{V}^{\pm}$-invariant subspaces
- L_+, L_- such that they are mutually -orthogonal and $L_+ + L_-$ is dense in the space
- if L_1 is non-zero $\hat{V}$-invariant subspace, $\hat{V}L_1 \subset L_1$, then $r(\hat{V}|L_1) = r(\hat{V})$
- if L_2 is non-zero $\hat{V}^{-1}$-invariant subspace, $\hat{V}^{-1}L_2 \subset L_2$, then $r(\hat{V}^{-1}|L_2) = r(\hat{V}^{-1})$.

Models of Z-Orbits of Unitary in Indefinite Inner Product Spaces Operators
Construction of Z-Orbits in Spaces with an Indeterminate Scalar Product
Choroszavin S. A., (ChS2)

This document deals with the construction and modeling of Z-orbits of unitary operators in indefinite inner product spaces, in particular in Krein spaces, using bilateral weighted shifts. The text describes how to define a (pre)Hilbert scalar product structure on a linear space H_0 equipped with an injective operator U_0 and a vector $x_0 \in H_0$, such that U_0 becomes unitary, even in the case of an indeterminate scalar product.

Alternating Pairs
(BoJ) pp. 114-115

An ordered pair $\{L_1, L_2\}$ of subspaces of the Krein space H will be called *an alternating pair* provided L_1 is positive, L_2 is negative, and $L_1 \perp L_2$. If, in addition, L_1 is maximal positive and L_2 is maximal negative, we say that $\{L_1, L_2\}$ is an alternating maximal pair.

By an *alternating extension* of the alternating pair $\{L_1, L_2\}$ we mean an alternating pair $\{L'_1, L'_2\}$ such that $L_1 \subset L'_1, L_2 \subset L'_2$.

Theorem 9.1: Every alternating pair in the Krein space H can be extended to an alternating maximal pair.

Dissipative Operators in Hilbert Space
(BoJ) pp. 116-118

The purpose of this section is to give an idea of how one applies alternating pairs.

Definition: A linear operator A with $D(A)$ in a Hilbert space H is called *dissipative* if, for all $\lambda > 0$ and for all $x \in D(A)$, the following holds: $\|(\lambda I - A)x\| > \lambda\|x\|$, or, equivalently, if $\forall x \in D(A)$ there exists a continuous duality pair (x^*, x) such that the following holds: $Re(Ax, x^*) \leq 0$ with $\|x^*\| = \|x\|$.

Note: A dissipative operator (e.g., the Laplace operator) is the infinitesimal generator of a so-called strongly continuous contractive semigroup. The operator reduces the energy or norm in a system. It describes the temporal variation of a temperature field or corresponds to friction. It causes the total length (norm) of vectors in Hilbert spaces to shrink. The heat conduction (operator) semigroup is a so-called -semigroup (that is, a strongly continuous semigroup). It is also called the Brownian semigroup because of its connection to Brownian motion. The name heat conduction group reflects its close connection to the heat conduction equation, (WeD) pp. 388, 411.

Theorem VII.4.16 (Lumer-Phillips Theorem): Let A be a linear operator on a Banach space X. The operator A generates a contraction semigroup if and only if A is dissipative and $\lambda_0 - A$ is surjective for some $\lambda_0 > 0$, (WeD) p. 406.

Let H_0 be a Hilbert space with inner product $[\circ, \circ]_0$ and norm $\|\circ\|_0$. Let T_1, T_2 be dissipative operators in H_0; by this we mean that T_1, T_2 are linear operators with dense domains satisfying

$$[T_1x_1, x_1]_0 + [x_1, T_1x_1]_0 \leq 0, x_1 \in D(T_1)$$
$$[T_2x_2, x_2]_0 + [x_2, T_2x_2]_0 \leq 0, x_2 \in D(T_2).$$

Suppose also that

$$[T_1x_1, x_2]_0 = [x_1, T_2x_2]_0, x_1 \in D(T_1), x_2 \in D(T_2).$$

The question is whether there exist operators $T_1 \subset T_1^{(0)}$, $T_2 \subset T_2^{(0)}$ in H_0 which are *maximal dissipative* (i.e., dissipative operators having no dissipative proper extension) and satisfy the analogue of

$$\left[T_1^{(0)}\, x_1, x_2\right]_0 = \left[x_1, T_2^{(0)}\, x_2\right]_0, x_1 \in D(T_1^{(0)}), x_2 \in D(T_2^{(0)}).$$

We are led to the following result. If $\left\{L_1^{(0)}, L_2^{(0)}\right\}$ is an alternating maximal pair extending the graphs $\{x_1, -T_1x_1\}$, $\{x_2, T_2x_2\}$, $x_1 \in D(T_1)$, $x_2 \in D(T_2)$, then the operators $T_1^{(0)}$, $T_2^{(0)}$ defined by the relations $\left\{x_1, -T_1^{(0)}x_1\right\} = L_1^{(0)}$, $\left\{x_2, T_2^{(0)}x_2\right\} = L_2^{(0)}$, yield a solution of our problem, and every solution can be obtained in this way. Thus the existence of a solution is guaranteed by Theorem 9.1.

I. Dehnen et al.'s overlooked key to a heuristic approach to GRT

This chapter refers to the work, (DeH) Dehnen H., Hönl H., Westphal K., *A Heuristic Approach to General Relativity*, Annalen der Physik, Vol. 461, No. 7-8, (1960), 370-406

The excerpts from the above publication that are essential to this work are reproduced in translated form in Chapter 6. A. Unzicker's books have numerous points of reference not only with regard to the above publication but also in many ways to the present work. The title of this section refers to one of the books of A. Unzicker, titled "Einstein's Lost Key: How We Overlooked the Best Idea of the 20th Century" (UnA1). This "lost key" is the concept of a "variable speed of light."

The key statements relevant to this work are briefly summarized below. The conclusion essential to this work concerns the relationship between general relativity and quantum theory: *the space-time continuum is a continuum approximation of interactions between elementary particles under the reinstated Mach principle*. This also brings Dirac's "large number hypothesis" back into focus: it relates the ratio of the size of a proton to the size of the universe to the ratio of electric and gravitational forces (UnA1).

The key points of the work by Dehnen et al. can be briefly summarized with the following keywords:

- the special theory of relativity, which explains a variable speed of light (in which "light" and "mass" influence each other), accounts for all phenomena of general relativity
- when one moves to "large" speeds comparable to the speed of light, general relativity only becomes of decisive importance in regions of "cosmic" scale, whose dimensions are no longer small relative to the mean radius of curvature of the world. With the question of the nature of the world on a grand scale, however, Mach's principle is also restored to its former prominence.
- Among the great variety of exact solutions to the field equations of general relativity, there are only a few that come into consideration for describing real conditions, both in terms of empirical facts and in terms of the a priori requirements to be placed on a theory
- If the physical requirement for covariant field equations is dropped, the Mach principle regains validity as a "regulatory" principle (in Kant's sense) in the role of a selection principle, serving to sift out a few models from the large number of possible ones as physically viable.

On the Relationship Between General Relativity and Quantum Theory
The space-time continuum as a continuum approximation
of interactions between elementary particles

The concept of a "vacuum" presupposes that of inertial frames of reference in both gravitational theories and quantum field theory. It therefore makes good physical sense to ask about the origin of inertial systems and the Lorentz group. This question is not to be asked within quantum field theory (it is, so to speak, "bracketed out" there); however, it must be asked in principle in order to account for cosmological theories of gravity. The action quantum$h$ is entirely independent of the gravitational field, i.e., it is not influenced even by arbitrarily strong gravitational fields. This suggests that the scope of the laws of the theory of relativity and that of quantum theory are far apart; or, more precisely: that the aspect under which the physical world appears in general relativity is fundamentally different from under which it appears in quantum theory. The space-time continuum of the theory of relativity likely has the physical significance of a continuum approximation of interactions between bodies (elementary particles), which are governed by the laws of quantum theory. This approximation will be all the more accurate the more material bodies are involved in its construction. The space-time continuum would thus be merely the "stage" (res extensa) on which the actual events of the world - quantum events - unfold.

m. Kirchhoff's forgotten key $\psi(x)$

The function $\beta(x)$ is first mentioned in the context of physical applications by G. Kirchhoff, (KiG). This paper addresses the problem first formulated by *Poisson* finding the distribution of electric charge on two conductive spheres such that the potential of the charge distributed on one sphere is given for all points on the line of symmetry. For the series formulated by Poisson, Kirchhoff derives a closed-form expression that bears a resemblance to series found in the theory of elliptic functions. The formula derived by Kirchhoff

$$f(x) = h\frac{b}{1+b}\frac{1}{1-x}\left[\psi\left(\frac{b}{1+b}\frac{x}{1-x}\right) - \psi\left(\frac{b}{1+b}\frac{1}{1-x} - 1\right)\right]$$

shows the electric potential on two conductive spheres for points outside the central line. The letter b denotes the radius of the second sphere, and the letter h denotes the potential of all the free electricity in the first sphere.

Kirchhoff's solution formula is formulated from the perspective of the second sphere; it therefore also applies from the perspective of the first sphere, with a corresponding change in the choice of parameters.

The proposed Hilbert space-based theory of quantum dynamics provides the mathematical foundation of quantum mechanics. Empirical measurements in quantum mechanics are not a passive reading of a specific value based on one or more units of measurement, but rather an active process. Mathematically, quantum mechanical measurements are probabilistic. In a separable Hilbert space, a particle (element) is represented as a superposition of various basis states, which are formed from the eigenpairs of a measurement operator with specific properties. From the perspective of empirical physics, the most fundamental level of a measurement is thus based on probability theory. No matter how appropriately modified, Kirchhoff's formula for the distribution of electric charge on two conductive spheres cannot therefore play a direct role within the framework of quantum dynamics. Kirchhoff's model is thus in no way directly applicable - in any form - within the framework of quantum theory.

However, a key feature of quantum dynamics is the possibility of a quantum pair with its own dynamic energy. Based purely on analogical considerations, Kirchhoff's model therefore lends itself as a prototype for an approximate model of classical physics. The relationships between quantum-mechanical and quantum-dynamic quantities that must be taken into account could be reflected in the conditions $Re(1 + b \pm a) > 0$ of the following formulas:

Lemma (GrI) 3.541, 8.366: For parameters that satisfy the inequality $Re(1 + b \pm a) > 0$, it holds

i) $$\int_0^\infty e^{-x}\frac{sinh(ax)}{sinh(bx)}dx = \frac{1}{2b}\left[\psi\left(\frac{1}{2}+\frac{1+a}{2b}\right) - \psi\left(\frac{1}{2}+\frac{1-a}{2b}\right)\right]$$

ii) $$\int_0^\infty e^{-x}\frac{sinh(ax)}{sinh(bx)} - \frac{sinh(-ax)}{sinh(bx)}dx = 2\int_0^\infty e^{-x}\frac{sinh(ax)}{sinh(bx)}dx = \frac{1}{b}\left[\psi\left(\frac{1}{2}+\frac{1+a}{2b}\right) - \psi\left(\frac{1}{2}+\frac{1-a}{2b}\right)\right]$$

iii) $$\psi\left(\frac{1}{2}\right) = -\gamma - 2ln2 \approx -1.963\ 510\ 026\ \ldots\ .$$

4. The Hilbert space-based theory of quantum dynamics

a. A short story of the failed engagement of quantum mechanics and relativity

The story is about the engaged couple $E = mc^2$ (Einstein) and $E = \pm\sqrt{E_0^2 - h^2c^2\nabla^2}$ (Schrödinger), whom Dirac wanted to marry.

Experiments in particle accelerators confirm an increase in mass at high kinetic energies. The end products of nuclear reactions are lighter than their initial nuclei, with the release of high kinetic energy. The kinetic energy, mass, velocity, and momentum of the particles can be linked together with the physical concept of momentum $p := mv$ in the formula $E_{kin}^2 = \frac{m}{2}v^2 = \frac{p^2}{2m}$. Accordingly, Schrödinger's quantum mechanical operator representation (without taking into account Planck's "action" constant h) resulted in the form

$$E_{kin.QM}^2 = \frac{P^2}{2m} \text{ with } P := -i\nabla .$$

In classical mechanics, localized states are the so-called mass points. In classical field theory, the localized states are the wave packets. To make both localized states mathematically comparable, the Dirac "function" is required. This is not a function in the strict sense, but a distribution whose "regularity" depends, incidentally, on the dimension of the space. This mathematically ensures that the resulting mathematical fields are continuous ($\delta \in H_{-n/2-\varepsilon}$). The corresponding physical concepts are called wave packets and group velocity.

Quantum mechanics is a statistical theory in the mathematical sense. Its "world" states are elements of the Hilbert space L_2. The symmetric potential operator $-\Delta$ (with domain H_2) plays an essential role in both classical potential theory and Schrödinger's quantum mechanics. The theory of the calculus of variations provides a *self-adjoint* extension for this potential operator $-\Delta$ with domain H_1, (VeW). Accordingly, there is the following equivalent representation of the potential equation with reduced domain $u \in H_1$:

$$(\nabla u, \nabla v)_0 = (u, v)_1 = (f, v) \quad \forall v \in H_1 .$$

The invariant energy value is provided free of charge in the form of the standard $E_{kin} = \|u\|_1 = \|\nabla u\|_0$. For localized wave packets, the "Heisenberg uncertainty" between the localization and energy operators naturally persists due to the required different domains of definition H_0 and H_1 .

Einstein's formula $E = mc^2$ is a proposed mathematical description of a relativistic classical mass point m_0, which is expressed by the power series

$$\frac{1}{\sqrt{1-x}} = \sum_{n=0}^{\infty} \frac{(2n)!}{2^{2n}(n!)^2} x^n = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \cdots$$

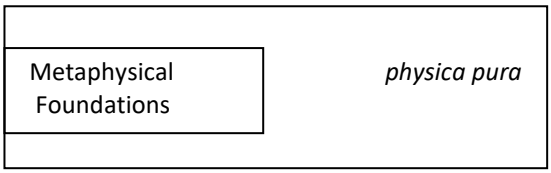
is given. For $x = \frac{v^2}{c^2}$ and $(1 - \frac{v^2}{c^2})^{-1/2} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$, in a first approximation (!) and the relativistic "mass increase formula" resulting from the Lorentz transformation $m = m_0(1 - \frac{v^2}{c^2})^{-1/2}$, this yields Einstein's approximation formula for total energy: it is composed of the kinetic (!) energy of the "rest mass" m_0, multiplied by the square of the speed of light of a massless photon, and the classical kinetic energy in the form $E^2 = E_0^2 + E_{kin}^2 = m_0c^2 + E_{kin}^2$. From this, Dirac's relativistic version of the Schrödinger equation is derived in the form $E^2 = E_0^2 + P^2c^2$. And this form of the total energy of the quantum system is the actual (mathematical) reason why the (physical) engagement had to fall apart when Dirac attempted to reconcile Schrödinger's quantum mechanics with Einstein's special theory of relativity: Instead of continuing to work with the by definition positive energy norms of the calculus of variations, one resorts again to the "merely" symmetric potential operator $-\Delta$ to thereby derive the following relation $E^2 = E_0^2 - c^2\Delta$. It is in this way that the mathematically necessary "negative energies" first come into play, which run completely counter to the physical concept of energy (the fundamental concept of invariance in physics par excellence). Moreover, *in considerations involving the energy of nuclei the mass M is important. According to Einstein's relation, the energy equivalent of a change in mass ΔM is $\Delta E = \Delta Mc^2$. Such changes in mass occur when protons and neutrons are changed from one configuration to another in which they are bound more or less strongly*, (BeH) p. 2.

b. Kant's metaphysical foundations of natural science as a headline

The „Pure Part“ Within The Doctrine of Nature
(PlP) p. 314-315

„The pure part has a structure that can be indicated as follows:

Pure Part (*physica generalis*)



Here only the actual core, i.e., the metaphysics of material nature itself, is of finite scope, while the pure part (the physica pura) is in principle infinite.

Thus the pure part contains nothing but pure apodeictic knowledge, whose objective reality is also guaranteed in the mathematical part by the metaphysics that serves as its basis – sometimes that does not hold for all concepts of pure mathematics. For „*all mathematical concepts are not in themselves knowledge; except insofar as one presupposes that there are things that allow being presented to us only in accord with the form of that pure sensible intuition*“. This presupposition, however, is given to mathematics in this application in the physica generalis precisely by the core of the pure part, i.e., the Metaphysical Foundation, and this, as already noted, is the proper content of the assertion, „that the art of measurement (Meßkunst, i.e., applied mathematics) and philosophy are two completely different things, although in natural science they do indeed shake hands with each other“.

Kant make the application of mathematics into the criterion for how much „science proper“ is contained in a special doctrine of nature. From this, one sees that the „application of mathematics“ is not thought of as restricted to the pure part but rather by means of the pure part obviously extends to the entire empirical, but apodeictic (proper) science.“

c. Bohm's concept of „Wholeness" and „Implicate Order"

An attempt at a brief summary of Bohm's book
"Wholeness and the Implicate Order"

Order and measurement in classical physics are characterized by Cartesian coordinates and the assumption of a universal and absolute temporal order that is independent of space, as well as the assumption of the absoluteness of this (Euclidean) system of order and measurement. The general characteristic of this classical structure is the ability to analyze (decompose) everything into separate parts (either small rigid bodies or an idealization of particles without spatial extent).

The laws of physics in the context of the mechanics of motion describe the relationships between the motions of all parts, with each law relating the motion of each part to the arrangement of all other parts. This law is deterministic and causal in nature. Every external disturbance can be regarded as a cause that produces a describable effect, which is then, in principle, transmitted to every part of the system.

Bohm's holistic approach aims to unify quantum theory with the theory of general relativity. Building on relativistic quantum mechanics, Bohm's conceptually described path consists in reconfiguring the organizational structure of classical field theories. The concept of order is not understood here as merely a regular arrangement of objects or a uniform sequence of events. Instead, the desired overall order is "implicate" in every segment of space and time. Bohm's assumption is that, at a deeper subquantum level, there are additional hidden variables that more precisely determine the fluctuations in the results of individual quantum-mechanical measurements.

Bohm's conceptual proposal for restructuring the existing framework of relativistic quantum theory focuses on its four main aspects, which are

- the indivisibility of the action quantum
- the wave-particle duality of matter
- the properties of matter as statistical possibilities
- non-causal correlations (the Einstein–Podolsky–Rosen paradox).

Bohm's section on the "Treatment of Quantum Fluctuations" begins with the following sentences, (BoD) pp. 110–111:

"Let us begin by assuming some 'deterministic' field theory. Its precise characteristics are unimportant for our purposes here. All that is important is to suppose the following properties:

1. *There is a set of field equations that completely determines the changes in the field over time.*
2. *These equations are sufficiently nonlinear to guarantee significant coupling between all wave components, so that (except perhaps in some approximation) solutions cannot be linearized and superimposed.*
3. *Even in the "vacuum," the field is so highly excited that the mean field in each region, however small, fluctuates significantly, with a kind of turbulent motion that leads to a high degree of randomness in the fluctuations.* This excitation guarantees the discontinuity of the fields in the smallest regions.
4. *What we usually call "particles" are relatively stable and conserved excitations superimposed on this vacuum. Such particles will be detected at the large-scale level, where all instruments are sensitive only to those features that fluctuate rapidly. Thus, the "vacuum" produces no visible effects at the large-scale level, since its fields cancel each other out on average, and space is effectively "empty" for every large-scale process (e.g., just as a perfect crystal lattice is effectively "empty" for an electron in the lowest band, even though the space is filled with atoms).*

It is evident that there would be no way to solve such a set of field equations directly. The only possibility would be to try to deal with some kind of average *field equations. In general, we might hope that a set of such average quantities would, at least to some approximation,* determine themselves *independently of the infinitely complex fluctuations within the associated region of space. To the extent that this " " occurs, we could obtain approximate field laws associated with a certain scale, but these laws cannot be exact because the nonlinearity*

of the equations means that fields will necessarily be coupled in some way to the internal fluctuations that have been neglected. As a result, the mean fields will also fluctuate randomly around their average behavior. .. To sum up, we are reorganizing the non-countable infinity of field variables, and we are treating explicitly only some countable sets of these reorganized coordinates."

Bohm's concept can be summarized roughly as follows:

To unify quantum theory with the theory of gravitation, a division of the uncountably many field variables of classical Lagrangian field equations into two separate implicit and explicit structures is proposed. The infinitely uncountable number of field variables is divided into an infinitely countable number of so-called explicit field variables and, in turn, an infinitely uncountable number of implicit (so-called hidden) field variables. The explicit variables provide the information for empirical physics, while the hidden variables "unfold" indirectly in the sense of Property 3 above. Consequently, Bohm therefore asserts

> "*It is important to emphasize that mathematics and physics are not regarded here as separate but mutually related structures. Rather, it is suggested that mathematics and physics are to be considered as aspects of a single undivided* whole," (BoD1) p. 199.

A philosophical aside: *In light of Kant's postulate regarding the metaphysical foundations of theoretical physics—which are not limited to purely mathematical concepts—Bohm's statement means nothing other than that theoretical physics becomes a descriptive science and thus no longer belongs to the natural sciences. By the same token, the "law without law at the basis of physics" in geometrodynamics (the principle that "the boundary of the boundary of a manifold is zero," (CiI), p. 49) can then also be elevated to a mathematical a priori principle of physics.*

The question of how the "unfolding process" of quantum fluctuations in Euclidean space can be explained from the perspective of theoretical physics thus becomes a question for mathematics. Since the cardinality of the irrational numbers is the same as that of the entire set of real numbers, however, the answer is easy to formulate. With regard to quantum mechanics, this means that the set of countably infinite "measurement values" of a quantum measurement operator—however it may be defined—constitutes a null set with respect to the Lebesgue integral; that is, from a probabilistic (and thus quantum-mechanical) perspective, they cannot play a physical role.

A digression into mathematical logic: *However, the question of the cardinality of the real numbers—in comparison to the identical cardinality of the natural numbers and the rational numbers—is a central topic in set theory and mathematical logic and serves as a bridge to metamathematical questions such as the consistency of mathematics. The countable "Gödel numbers" play a central role in the proof of Gödel's completeness theorem, which forms a bridge between the formal concept of proof and the structure-based concept of inference.*

As a preview of the following sections: *Bohm's guiding principle of a "divided overall structure into an explicitly measurable structure and an underlying implicit (hidden) structure" cannot be mathematically "realized" using methods of analysis, but it can certainly be realized in the structure of a Hilbert space scale. This is because these Hilbert spaces are, by definition, separable—that is, they are based on a countable set of eigenpairs. In the terminology of quantum theory, one would say that they are based on the underlying physical assumption of a countable number of measured values. Mathematically, this means that every embedded Hilbert space is complemented by an additional (closed) subspace of the next larger Hilbert space. This subspace is not a Hilbert space and has no separable structure. However, this embedding is compact and thus offers the opportunity, within the framework of Hilbert space theory, to construct compact operators that, in turn, can form discrete (i.e., countable) eigenpairs. Expressed in physical terms, this means that "if one suitably extends the domain of a 'measurement operator' to a larger Hilbert space, the original operator can be supplemented by an additive compact operator that can be interpreted as a 'perturbation operator' of the countable set of its measured values."*

d. The concept of dynamic quanta and dynamic fluid particles

The "elementary entities of physics" can be summarized in the three major complexes, (WeC2) p. 147.

"Stable" matter:	proton, neutron, electron
"Electromagnetism":	Electromagnetic field
"Gravitation":	Gravitational field.

The two new physical concepts—dynamic quanta and dynamic fluid or dynamic gas particles—can be viewed, in light of Kant's *Metaphysical Foundations of a Theoretical Physics*, as the metaphysical foundation of a theoretical physics intended to replace the current "elementary entities of physics." Such a re-foundation then entails the requirement that the laws arising from it either remain consistent with existing laws or, within the framework of a new or appropriately modified theory, do not contradict known empirical findings.

From a purely conceptual standpoint alone, any form of dynamic quanta or dynamic gas or liquid particles must be embedded in a "finer" structure than the existing particle types. A mathematical "refinement" resulting from a refinement of the "real numbers" (the "particle type" of analysis and classical physics, but also of Riemannian geometry) is evident from the outset.

However, following the principle of simplicity, a Hilbert space framework—which also serves as the mathematical foundation of quantum mechanics—is well-suited for the "refinement" of structures.

The empirical "measurement method" of quantum mechanics is based on the mathematical methods of probability theory and statistics. The corresponding Hilbert spaceL_2 is the space of quadratically integrable Sobolev functions. (The integration refers to the Lebesgue integral. These integrals allow for so-called null sets; that is, the Lebesgue integral taken over a null set has the value zero. Such countable sets include, for example, the countably many eigenvalues of a Hamiltonian operator. From a physical perspective, this circumstance leads to the concept of a "wave packet" (which is a divergent integral in the L_2 Hilbert space, since the Hamilton operator is defined only on a subspace of the L_2 Hilbert space). The mathematical solution concept is that of the delta or Dirac distribution. The Sobolev embedding theorem guarantees continuous field solutions of $\Delta u = \delta(x) \in H_{-n/2-\varepsilon}$. Thus, a physical requirement determines the mathematical construct of the Dirac distribution. Multiplication by case-specific "potential constants" then defines the respective "source of causation" for the corresponding physical law. And this brings us full circle to the physical concept of a point charge in Euclidean space, whose mathematical concept is likewise the Dirac "function."

This leads to the following guiding principle for the construction of dynamic quanta and dynamic gas or liquid particles within the framework of a Hilbert space environment:

A refinement of quantum-mechanical systems must, from a metrological standpoint, make it possible to formulate a connection between quantum-mechanical and quantum-dynamic laws.

The conceptual framework of this work is based on an a priori, mathematically justifiable, deductively constructed "quantum world" that underlies the a posteriori (statistically relevant) quantum mechanical world. This "dynamic" quantum world (another term for it would be "the world of quantum fluctuations that 'interacts' with a mechanical quantum world") possesses its own energy type, which can also be mathematically justified by the self-adjoint counterpart of the self-adjoint mechanical potential operator, namely the self-adjoint J-symmetric operator of a Krein space. By design, it is also ensured that the "quantum fluctuation," with its own a priori energy type (in the sense of Kant's "metaphysical foundation of theoretical physics") and corresponding definite and indefinite norms, continues to allow for the familiar statistical analysis using the tools of probability theory. The expansion of the possibilities of current statistics occurs in such a way that the domain of a *self-adjoint* potential (Laplace) operator is further extended from the Hilbert space H_1 to the Hilbert space $H_{1/2} = H_1 \otimes H_1^{\perp}$. What at first glance appears to be a cheap magic trick proves to be a valid step toward resolving the problem of a missing physical interpretation of a mathematical "world-point state."

With the above extension, the statistical test function space is extended from H_0 to the Hilbert space $H_{-1/2} = H_0 \otimes H_0^{\perp}$. This enables test functions that render the Dirac function mathematically superfluous as a mechanical "world state point." The above variational equation thus follows for given distribution functions $f \in H_{-1/2}$ in the form, (BrK),

$$u \in H_{1/2}\colon\ (\nabla u, \nabla v)_{-1/2} = (u, v)_{1/2} = (f, v)_{-1/2}\ \ ,\forall v \in H_{1/2}\ .$$

The distribution function $f \in H_{-1/2}$ can certainly be interpreted here as an alternative Newtonian gravitational potential or as an alternative Coulomb electric potential, subject to fewer mathematical restrictions and offering improved physical capabilities.

Two notes regarding the following sections: The full Hilbert-scale H_α can be extended to the Hilbert space $H_{(\tau)}$ defined by the inner product or norm

$$(x, y)_{(\tau)} = \sum_1^\infty e^{-\sqrt{\lambda_i}\tau}\, x_n\, y_n,\ \ \|x\|_{(\tau)}^2 = (x, x)_{(\tau)}$$

If the symbol x' is interpreted as "momentum" and the parameter τ as some form of "time" or "frequency," the norm of $H_{(\tau)}$ is linked to the energy Hilbert space $H_{1/2} = H_1 \otimes H_1^{\perp}$ by the energy equation of a vibrating string governed by the norm of $H_{(\tau)}$ in the form

$$\|x\|_{1/2}^2 = \tfrac{1}{2}\int_0^\infty \left[\|x'\|_{(\tau)}^2 + \|\dot{x}\|_{(\tau)}^2\right] d\tau\ .$$

Mathematically, due to its compact embedding in $H_{-1/2}$, the closed subspace $H_0^{\perp}$, there exists a compact operator that can be interpreted as a compact perturbation of the H_1 -coercive Laplacian operator, where the "energetic" coerciveness condition can be applied:

> *A variational representation of an operator in the form $B = A + K$, where A is a H_α -coercive operator with a compact disturbance K , satisfies a coerciveness (Garding-type inequality) condition of the form*
>
> $$(Bu, v) \geq c \cdot \|u\|_\alpha \|v\|_\alpha - (Ku, v) \text{ or } (Bu, v) \geq c_1 \cdot \|u\|_\alpha^2 - c_2 \cdot \|u\|_\beta^2$$
>
> $H_\beta \subset H_\alpha$ *is compactly embedded.* For related arguments concerning the Boltzmann-Landau equations we refer to (LiP), (LiP1), see also (AzA).

The concept of the Hilbert space extension $H_{-1/2} = H_0 \otimes H_0^{\perp}$ can also be applied to the Schrödinger operator (BrK1). Comparable physical "phenomena" appear in classical physics as turbulence in the form of nonlinear functions or differential operators. In the above context, it is therefore natural to refer to the nonlinear Schrödinger equation. This is characterized by a potential function V that depends on the magnitude of the wave amplitude ψ but not on its phase. "*This is to be expected, as far as electron plasma waves are concerned, because the nonlinearity arises from the ponderomotive force, which depends on the gradient of the wave* intensity," (ChF) p. 336.

The proof of concept for the dynamic fluid-particle-based variational form of the 3D nonlinear and non-stationary Navier-Stokes equations system is provided on the following page.

A bounded -$H_{1/2}$ -energy norm estimate for the 3D-NSE problem

Navier-Stokes Equation - Clay Mathematics Institute

"*This is the equation that governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: Do solutions exist, and are they unique? Why seek a proof? Because a proof provides not only certainty but also understanding.*

Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier-Stokes equations. Although these equations were written down in the 19th century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory that will unlock the secrets hidden in the Navier-Stokes equations."

For space dimensions $n = 3$, there is no global boundedness estimate for the variational representation of the homogeneous 3D-NSE system in the form

$$u \in H_1: \ (\dot{u}, v)_{-1/2} + (Au, v)_{-1/2} + (Bu, v)_{-1/2} = 0 \text{ for all} v \in H_{-1/2}$$
$$(u(0), v)_{-1/2} = (u_0, v)_{-1/2} \text{ for all} v \in H_{-1/2} .$$

For $-1 \le \alpha \le 1$, the Stokes operator A can be generalized to the operators $A^\alpha := \int_0^\infty \lambda^\alpha dE_\lambda$ with domain

$$D(A^\alpha) := \{u \in L^2_\sigma(\Omega\,;\ \|A^\alpha u\|_2^2 = \int_0^\infty \lambda^{2\alpha} d\|E_\lambda u\|_2^2\}$$

which are positive and self-adjoint, (SoH) p. 133. It turned out that the nonlinear energy term of the 3D nonlinear, nonstationary NSE system is bounded with respect to the norm $\|u\|_{1/2}$ as a simple consequence of the Sobolevskii inequality (GiY):

Substituting $B(u) := P(u, grad)u)$ into the NSE and assuming $Pu_0 = u_0$, the NSE initial-boundary equation is given by

$$\frac{du}{dt} + Au + Bu = Pf \ , u(0) = u_0 .$$

By formally multiplying the homogeneous equation by $A^\alpha v$ with appropriately selected α, this can be expressed in the variational form

$$u \in H_{1/2}: \ (\dot{u}, v)_{-1/2} + (Au, v)_{-1/2} + (Bu, v)_{-1/2} = 0 \text{ for all } v \in H_{-1/2}$$
$$(u(0), v)_{-1/2} = (u_0, v)_{-1/2} \text{ for all } v \in H_{-1/2} .$$

Setting $v = u$ yields

$$\frac{1}{2}\frac{d}{dt}\|u\|^2_{-1/2} + \|u\|^2_{1/2} \le \left|(Bu, u)_{-1/2}\right| \le \|u\|_{-1/2}\|Bu\|_{-1/2} \cong \|u\|_{-1/2}\left\|A^{-1/4}Bu\right\|_0.$$

The last term can be estimated using the Sobolevskii estimates in the form, (*)

$$\left\|A^{-1/4}Bu\right\|_0 \le c\|u\|_1^2.$$

Setting $y(t) := \|u\|^2_{-1/2}$, we obtain $y'(t) \le c \cdot \|u\|_1^2 \cdot y^{1/2}(t)$, and therefore, by Gronweall's lemma

$$\|u(t)\|_{-1/2} \le \|u(0)\|_{-1/2} + \int_0^t \|u\|_1^2(s)ds \le c\{\|u_0\|_{-1/2} + \|u_0\|_0^2\},$$

which guarantees global boundedness by the a priori energy estimate provided that $u_0 \in H_0$.

(*) Lemma 3.2 (GiY): For $0 \le \delta < 1/2 + n \cdot (1 - 1/p)/2$, it holds that $\left|A^{-\delta}P(u, grad)v\right|_p \le M \cdot \left|A^\theta u\right|_p \cdot |A^\rho u|_p$ with a constant $M := M(\delta, \theta, \rho, p)$ if $\delta + \theta + \rho \ge n/2p + 1/2$, $\theta, \rho > 0$, $\theta + \rho > 1/2$. Setting $p := 2$, $\delta := 1/4$, $\theta := \rho := 1/2$, and satisfying $\theta + \rho \ge \frac{1}{4}(n + 1) = 1$, it follows

$$\left\|A^{-\delta}P(u, grad)u\right\| \le c\left\|A^\theta u\right\| \cdot \|A^\rho u\| = c\|u\|_{2\theta} \cdot \|u\|_{2\rho} = c\|u\|_1^2$$

or

$$\frac{1}{2}\frac{d}{dt}\|u\|^2_{-1/2} + \|u\|^2_{1/2} \le \left|(Bu, u)_{-1/2}\right| \le c \cdot \|u\|_{-1/2}\|u\|_1^2.$$

e. The paradigm shift from inductive to deductive physics

Mathematics is a descriptive science. It constructs logical structures based on a few propositions, the axioms. Physics is, by its very nature, an inductive science, although an inductively constructed structure can ultimately be "recast" into a deductive form. In the words of C. F. Weizäcker, the terms "deduction" and "induction" evoke the image of a pyramid that either rests on a single point or culminates in a single point. Physics thus allows for a dual structure

- the phenomenological structure of empirical physics
  in which knowledge is gained through systematic observation, measurement, and experimentation

- the concrete structure of theoretical physics
  in which knowledge is gained through the development of mathematical models and logical concepts.

Over the course of physics' development, a multitude of phenomena have been "recast" into a few theories. Acoustic phenomena could be explained by the motion of atoms in the air. Phenomena in thermodynamics could likewise be described by the laws of motion. Phenomena of light and optics could be described by the laws of electromagnetism (Maxwell's field theory). The formulation of the laws of quantum mechanics and "cosmos mechanics" could not be integrated into classical mechanics. They required new physical concepts and mathematical models. Today's formulations of the laws of the various quantum dynamics theories and of the two "cosmic mechanics" and "cosmic dynamics" theories—the special and general theories of relativity—represent the latest stage of this development.

The gain in understanding achieved by integrating existing theories into a single framework thus came to an end with the emergence of quantum mechanics and the general theory of relativity. Neither their underlying mathematical models nor their underlying physical concepts can be described within a single mathematical model based on a common set of fundamental physical concepts.

From an empirical standpoint, inquiring into the origin of inertial systems and the Lorentz group makes good physical sense (DeH). In quantum mechanics, the concept of a vacuum already presupposes the concept of inertial systems. Bohm's approach to describing laws of motion using classical Lagrangian functions and quantum fluctuations in the vacuum involves a distinction between an "explicate order" and an "implicate order" of the underlying transfinite set of field variables. From a mathematical standpoint, the countable rational numbers represent, in a sense, the empirically observable physical vacuum, while the uncountable remainder (in the language of analysis, the irrational numbers) represents the metaphysical mathematical remainder. Consequently, Bohm therefore asserts

> "*It is important to emphasize that mathematics and physics are not regarded here as separate but mutually related structures. Rather, it is suggested that mathematics and physics are to be considered as aspects of a single undivided* whole," (BoD1) p. 199.

The actual problem from the perspective of empirical physics therefore persists even in Bohm's approach, since the cardinality of the irrational numbers—to stay with the metaphor—is the same as the cardinality of the entire set of real numbers ($(2^{\aleph})$. However, with regard to the "measurement method" of quantum mechanics using methods of probability theory, this means that in Bohm's approach, the set of countably infinite "measurement values" of a Hamilton operator—however it may be defined—forms a null set; that is, from a quantum-mechanical perspective, they play no role.

The concept of transferring the laws of motion from classical physics to relativistic quantum mechanics in a physical vacuum (with the aim of enabling classical, so-called conserved quantities such as charge and current density) is associated with the terms "angular momentum" and "spin-orbit operator." This is an operator that unites two concepts of physical momentum: angular momentum (intrinsic rotation) and linear momentum. The stationary states of the Dirac equation are determined by separating the radial components, thereby yielding two determining equations for calculating the stationary states of the underlying Hamiltonian operator. When calculating the Sommerfeld energy formula for the electron in the hydrogen atom based on an assumed Coulomb potential, these two determining equations yield two power series with different asymptotic behavior for very large and very small radii. In order to predict finite energy values, both power series must converge, which can only be achieved by means of a suitably chosen constant—the Sommerfeld fine-structure constant.

The two mathematical methods for transferring the laws of kinematics from quantum mechanics to the various theories of quantum dynamics are “regularization” and “renormalization.” Renormalization in gauge theories is the mathematical procedure used in quantum field theories to “eliminate” infinite physical quantities (divergences) that arise when calculating subatomic processes. Different processes require different renormalization groups; this is the methodological reason for the current three decoupled quantum field theories. A renormalization group is intended to explain why decoupling quantum mechanics and quantum dynamics—each with the same minimum number of scales (the “distance”)—can work at all. Renormalization theory works in two steps: 1) regularization; 2) renormalization, (DeP) p. 560:

> *In quantum theory, we consider Lagrangians that depend on certain types of fields. We are interested in correlation functions of the form* $\langle T(\phi_1(x_1),..\phi_N(x_N))\rangle$ *, where* T *denotes time ordering. It is usually easier to compute the Fourier transform of this function. Feynman rules allow us to do this perturbatively, i.e., to obtain the answer in the form of a power series in coupling constants. This answer presents two problems*
>
> - *every term of the series is given by a divergent integral*
> - *even if all terms are convergent, the series diverges in any neighborhood of zero.*
>
> *Renormalization theory helps to overcome the first problem. The second problem concerns the (difficult) question of whether the theory actually exists nonperturbatively. If it does, this problem can be solved by treating the series expansion not as a Taylor series but as an asymptotic expansion of a certain function.*
>
> *Renormalization theory works in two steps: 1) regularization; 2) renormalization.*
>
> Regularization *involves introducing some kind of cutoff, which makes the integral convergent. After regularization, the integral is convergent but depends on the regularization parameter* Λ *. If* Λ *tends to infinity (i.e., when the cutoff is removed), the integral tends to infinity. This problem is resolved by the second step—renormalization. Specifically, we find a way to change the parameters in the Lagrangian in such a* Λ *-dependent way that when* Λ *and the parameters in the Lagrangian are varied simultaneously—with* $\Lambda \to \infty$ *—the integral converges to a finite limit. This limit is then taken to be the value of the integral.*
>
> Dimensional regularization *preserves all symmetries, but is only valid in the perturbation expansion. The idea behind dimensional regularization is to compute the integral corresponding to each Feynman graph in the “space” of dimension D, where D is a generic complex number.*

Mathematically speaking, the “renormalization method” solves the problem of divergent integrals of all terms in a power series. The method of “dimensional regularization” produces convergent terms in this power series, but the series itself remains divergent in every neighborhood of zero - that is, it is undefined for the “physical vacuum region.” Thus, both in the case of the Sommerfeld energy formula and within the framework of gauge theories, mathematical convergence conditions lead to the definition of physical “constant of nature” values or require *a priori* conditions for a phenomenon-specific physical vacuum. In any case, the calculated solutions are mathematical approximations of an empirically observable physical “reality.”

The concept of classical Lagrangian functions is incompatible with both theories of relativity. There are good reasons to rule out extending general relativity in such a way that it can incorporate quantum theory (DeH). Furthermore, from the fact that the action quantum is entirely independent of the gravitational field (i.e., it is not influenced even by arbitrarily strong gravitational fields), it can be concluded that the scopes of application of both theories are far apart. Thus, there is no hope of deriving the laws of quantum theory from those of the theory of relativity. Rather, the space-time continuum is likely to have the physical significance of a continuum approximation of interactions between bodies (elementary particles). From the perspective of experimental physics, a heuristic approach to general relativity is possible without resorting to covariant theory. From an empirical standpoint, it is natural to start from the fact of inertial systems and the Lorentz invariance of the dynamic equations of motion (DeH) that applies to them.

In summary, it can be concluded that a physical space in which the concept of motion is not applied is indispensable as the foundation for a theory of quantum dynamics. Such a theory must then, among other things, include the mathematical description of a physical vacuum, and the concept of motion of particles that can be tracked over time must be derivable from it. To this end, it is necessary that the fundamental concepts of such a quantum theory of dynamics - from which quantum-mechanical motions, and subsequently classical and “cosmic” motions, can be derived - be independent of the respective “spacetime geometries” required in each case. Viewed from this perspective as well, the concept of a Hilbert space with its inherent geometric structure (made possible by the concept of an inner product) thus presents itself as a suitable modeling framework. Given the close connection between the Hilbert space and the Krein-Hilbert space - which is endowed with both a definite norm (and thus a metric) and an indefinite norm (metric) - and considering that the development of Krein space theory was spurred by the work of Dirac and Pauli in the field of quantum field theory, the circle closes on Heisenberg’s proposal for a mathematical formalism for a unified field theory (HeW).

f. Two ways to define potential operators in Hilbert-Krein spaces

The term "potential operator" comes up in the topic „Variational Methods for the Study of Nonlinear Operators, (VaM):

Definition 5.2 (VaM) p. 55: We say that the operator $\boldsymbol{F}$, from a Banach space E into the conjugate space E^*, is potential on some set $H \subset E$, if there exists a functional f such that $gradf(x) = \boldsymbol{F}(x)$ for every $x \in H$.

The concept of a potential operator in Hilbert spaces

Let H be a (infinite dimensional) Hilbert space with scalar product $(.,.)$ and the norm $\|..\|$. Let A be a linear operator with the properties

i) A is self-adjoint, positive definite

ii) A^{-1} is compact.

Without loss of generality, possible by multiplying A with a constant, one may assume

$$(x, Ax) \geq \|x\| \qquad \text{for all } x \in D(A).$$

The Fredholm-Riesz-Schauer theory states that there are at most a countable number of eigenvalues λ_n of A^{-1} with $\lambda_n \neq \lambda_m$, $n \neq m$, $|\lambda_n| \geq \lambda > 0$, and corresponding eigenelements x_n. Any eigen-element of the compact operator $K = A^{-1}$ is also an eigen-element of A to the eigenvalues being the inverse of the first. Now by replacing $\lambda_n \to \lambda_n^{-1}$ we have that there is a countable sequence $\{\lambda_n, \varphi_n\}$ with

$$A\varphi_n = \lambda_n \varphi_n \ , \ \ (\varphi_n, \varphi_m) = \delta_{n,m} \ \text{ and } \ \lim_{n\to\infty} \lambda_n \to \infty$$

and any $x \in H$ is represented by

$$x = \sum_{i=1}^{\infty} (x, \varphi_n)\, \varphi_n \ \text{ and } \ \ \|x\|^2 := \sum_1^{\infty} x_n^2, \ x_n := (x, \varphi_n).$$

The x_n are called generalized Fourier coefficients. Based on this basic concept of a linear operator with properties i) and ii), further Hilbert spaces $\{H_\alpha | \alpha \in R\}$ can be defined by the corresponding convergence condition of their underlying norms

$$\|x\|_\alpha^2 = \sum_1^{\infty} \lambda_n^\alpha x_n^2 < \infty \ , \ x_n := (x, \varphi_n)$$

accompanied by the inner product $(x, y)_\alpha = \sum_1^{\infty} \lambda_n^\alpha \, x_n \, y_n$. A characteristic feature of these norms is their polynomial convergence behavior with respect to the exponent α. There are certain relations between the spaces $\{H_\alpha | \alpha \geq 0\}$ for different indices:

i) Let $\alpha < \beta$. Then $\|x\|_\alpha \leq \|x\|_\beta$ and the embedding $H_\beta \subset H_\alpha$ is compact

ii) Let $\alpha < \beta < \gamma$. Then $\|x\|_\beta \leq \|x\|_\alpha^\mu \|x\|_\gamma^\nu$ for $x \in H_\gamma$ with $\mu = \frac{\gamma-\beta}{\gamma-\alpha}$ and $\nu = \frac{\beta-\alpha}{\gamma-\alpha}$

iii) Let $\alpha < \beta < \gamma$. To any $x \in H_\beta$ and $t > 0$ there is a $y = y_t(x)$ according to

$$\|x - y\|_\alpha \leq t^{\beta-\alpha} \|x\|_\beta$$
$$\|x - y\|_\beta \leq \|x\|_\beta \ , \ \|y\|_\beta \leq \|x\|_\beta$$
$$\|y\|_\gamma \leq t^{-(\gamma-\beta)} \|x\|_\beta$$

iv) Let $\alpha < \beta < \gamma$. To any $x \in H_\beta$ and $t > 0$ there is a $y = y_t(x)$ according to

$$\|x - y\|_\rho \leq t^{\beta-\rho} \|x\|_\beta \quad \text{for } \ \alpha \leq \rho \leq \beta \ , \ \|y\|_\sigma \leq t^{-(\sigma-\beta)} \|x\|_\beta \qquad \text{for } \ \beta \leq \sigma \leq \gamma \ .$$

For $\tau > 0$ one may introduce the Hilbert space $H_{(\tau)}$ by an additional inner product resp. norm in the form

$$(x,y)_{(\tau)} = \sum_{n=1}^{\infty} e^{-\sqrt{\lambda_n}\tau}(x,\varphi_n)(y,\varphi_n)$$

$$\|x\|_{(\tau)}^2 = (x,x)_{(\tau)}^2 .$$

Now the factor has exponential decay $e^{-\sqrt{\lambda_n}\tau}$ instead of a polynomial decay in case of λ_n^{α}.

Obviously it holds

$$\|x\|_{(t)} \le c(\alpha,\tau)\|x\|_{\alpha} \text{ for } x \in H_{\alpha}$$

with $c(\alpha,\tau)$ depending only from α and $\tau > 0$. Thus the (τ)-norm is weaker than any α-norm. On the other hand any negative norm index, i.e. $\|x\|_{\alpha}$ with $\alpha < 0$, is bounded by the 0-norm and the newly introduced (τ)-norm. For $\alpha > 0$ be fixed and $\delta > 0$ being arbitrary the α-norm of any $x \in H_0$ is bounded by

$$\|x\|_{-\alpha}^2 \le \delta^{2\alpha}\|x\|_0^2 + e^{\tau/\delta}\|x\|_{(t)}^2, \quad \|x\|_{-1/2}^2 \le \delta\|x\|_0^2 + e^{\frac{\tau}{\delta}}\|x\|_{(\tau)}^2 .$$

For $\tau, \delta > 0$ be fixed the counterpart of the properties i)-iv) above is that for any $x \in H_0$ there is a $y = y_{\tau}(x)$ according to

i) $\|x - y\| \le \|x\|$

ii) $\|y\|_1 \le \delta^{-1}\|x\|$

iii) $\|x - y\|_{(\tau)} \le e^{-\tau/\delta}\|x\|.$

The concept of a potential operator in Krein spaces

A Krein space is decomposed into an orthogonal sum of two spaces H_1 and H_2 ($H = H_1 \otimes H_2$) associated with two projection operators P_1 and P_2 onto H_1 and H_2. From the functional

$$\varphi(x) := \big((x)\big)^2 = \|P_1 x\|^2 - \|P_2 x\|^2$$

we have

$$\|P_1 x\|^2 = \tfrac{1}{2}\|x\|^2 + \tfrac{1}{2}\big((x)\big)^2 ; \ \|P_2 x\|^2 = \tfrac{1}{2}\|x\|^2 - \tfrac{1}{2}\big((x)\big)^2.$$

According to (10.7), (VaM) p. 86, the gradient of $\varphi(x) := \big((x)\big)^2$ is given by

$$grad\big((x)\big)^2 = 2P_1 x - 2P_2 x.$$

The operator $J = P^+ - P^-$ is the *fundamental symmetry* belonging to the fundamenal decomposition $H = H_1 \otimes H_2$, (BoJ) p. 52. In other words, the potential operator W on all of the space H of the form

$$Wx := \tfrac{1}{2} grad\big((x)\big)^2 = P_1 x - P_2 x$$

corresponds to the symmetry operator J of a Krein space.

g. The appropriate choice $\alpha = \pm 1/2$ of a Hilbert scale H_α

The appropriate choice of a Hilbert scale H_α with respect to the requirements 1 & 2 follows from the relationship between all Hilbert scale norms $\|x\|_\alpha^2 = \sum_1^\infty \lambda_n^\alpha x_n^2 < \infty$, $x_n := (x, \varphi_n)$ and a further extended special Hilbert scale norm

$$\|x\|_{(\tau)}^2 = \sum_1^\infty e^{-\sqrt{\lambda_n}\tau} x_n^2 < \infty, \tau > 0 \, .$$

The norm $\|x\|_{(\tau)}^2$ is weaken (exponential decay with respect to the eigenvalues λ_n) than all norms $\|x\|_\alpha^2$ (polynominal decay with respect to the eigenvalues λ_n). For corresponding approximation properties we refer to the appendix. The two types of norms are related to each other by the identity

$$(*) \quad \|x\|_{\alpha-1/2}^2 = \int_0^\infty \|x\|_{\alpha.(\tau)}^2 d\tau \, .$$

That means that the extension from $\alpha = 0$ to $\alpha = -1/2$ is consistent with the property (*). It proves to be sufficient in that, from the standpoint of empirical physics, it guarantees the existence of continuous fields as a result of the embedding $H_{-1/2} \subset H_{-n/2-\epsilon}$. (*) Furthermore, the identity (*) allows us to define a "physical instant" $t > 0$ in the following way

$$\text{(A)} \quad \|x\|_{-1/2}^2(t) := \sum_1^\infty \frac{1}{\sqrt{\lambda_n}} x_n^2(t) \text{ with } x_n^2(t) := x_n^2 \int_t^\infty e^{-\sqrt{\lambda_n}\tau} \, .$$

Note: In the extended Hilbert space framework the variational Poisson equation for a given *distribution* functions $f \in H_{-1/2}$ is given in the form (see also (BrK1)

$$u \in H_{1/2}: \ (\nabla u, \nabla v)_{-1/2} = (u, v)_{1/2} = (f, v)_{-1/2} \ \ \forall v \in H_{1/2} \, .$$

The distribution function $f \in H_{-1/2}$ can certainly be interpreted here as an alternative Newtonian gravitational potential or as an alternative Coulomb electric potential, subject to fewer mathematical restrictions and offering improved physical capabilities.

Note: If the symbol x' is interpreted as "momentum" and the parameter τ is interpreted as some form of "time" or "frequency" of quantum fluctuations the norm of $H_{(\tau)}$ is linked to the energy Hilbert space $H_{1/2} = H_1 \otimes H_1^\perp$ by the energy equation of a „vibrating string" governed by the norm of $H_{(\tau)}$ in the form

$$\|x\|_{1/2}^2 = \frac{1}{2} \int_0^\infty \left[\|x'\|_{(\tau)}^2 + \|\dot{x}\|_{(\tau)}^2 \right] d\tau \, .$$

Note: The Hlbert space $H_{(\tau)}$ provides the appropriate framework to allow well-defined hyperbolic initial-boundary vaule problems.

Note: The concept of the Hilbert space extension $H_{-1/2} = H_0 \otimes H_0^\perp$ can also be applied to define an alternative Schrödinger operator (BrK1). Comparable physical "phenomena" appear in classical physics as turbulence in the form of nonlinear functions or differential operators. In the above context, it is therefore natural to refer to the nonlinear Schrödinger equation. This is characterized by a potential function V that depends on the magnitude of the wave amplitude ψ but not on its phase. "*This is to be expected, as far as electron plasma waves are concerned, because the nonlinearity arises from the ponderomotive force, which depends on the gradient of the wave* intensity," (ChF) p. 336.

Note: Mathematically, due to its compact embedding in $H_{-1/2}$, the closed subspace $H_0^\perp$, there exists a compact operator that can be interpreted as a compact perturbation of the H_1 -coercive Laplacian operator, where the "energetic" coerciveness condition can be applied:

> *A variational representation of an operator in the form $B = A + K$, where A is a H_α -coercive operator with a compact disturbance K, satisfies a coerciveness (Garding-type inequality) condition of the form*
>
> $$(Bu, v) \geq c \cdot \|u\|_\alpha \|v\|_\alpha - (Ku, v) \text{ or } (Bu, v) \geq c_1 \cdot \|u\|_\alpha^2 - c_2 \cdot \|u\|_\beta^2$$
>
> *where $H_\beta \subset H_\alpha$ is compactly embedded.* For related arguments concerning the Boltzmann-Landau equations we refer to (LiP), (LiP1), see also ((AzA).

Note: In the appendix a bounded-$H_{1/2}$ -energy norm estimate for the 3D-NSE problem is provided. If the systems of the underlying extended „test" space $H_{-1/2}$ are interpreted as dynamic fluid particles this means that such a concept avoids „blow up" solutions of the 3D Navier-Stokes initial boundary value problem. The $H_{-1/2}$-systems may be interpreted as dynamic fluid/gas particles.

Note: In (NaS) there is an interesting relationship of the $H_{1/2}$ space to „The Teichmüller theory and the universal period mapping via quantum calculus and the $H_{1/2}$ space on the unit circle".

(*) The embedding $H_{-1/2} \subset H_{-n/2-\epsilon}$, which is independent of all spatial dimensions, ensures - by virtue of Sobolev's embedding theorem - that continuous fields in classical physics can be generated by quantum-mechanical causes, regardless of the dimension of the underlying Euclidean space. The systems of this extended Hilbert space can thus accommodate the mathematical concept of the Newtonian or Coulomb potential, both of which, for mathematical reasons, only make physical sense in 3D Euclidean space. The concept of phenomenon-specific "point charges" that generate continuous fields throughout 3D Euclidean space can thus be completely replaced.

h. The two quantum fluctuation systems of the Hilbert space $H_{-1/2}$

The concept of a „physical ether"

The quantum-mechanical basis Hilbert spaces $H_{-1/2}$ can be characterized by the norms of a "mechanical" Hamilton operator A with a countable number of eigenpair solutions. The two quantum fluctuation systems it contains are the connected "ground state" and "plasma" quantum fluctuation systems. Together, these two systems define the "physical ether," (EiA). They differ from $H_{-1/2}$ systems in that they have different indefinite norms (i.e., different invariants) and an additional structural element in the form of an extra (dynamic) potential operator. The methodological difference between the ground-state model and the plasma model is the additional *a priori* metaphysical assumption of a plasma density equal to twice the *a priori* metaphysical vacuum density (PlP).

The physical ether is endowed with a mathematically justifiable "density" $\frac{1}{4n-1}$ to define appropriate related quantum numbers:

> *The mathematical concept behind the construction of appropriate "implicate fluctuation" values is based on the different Schnirelmann densities of odd and even numbers. For odd natural numbers, the Schnirelmann density is ½; for even numbers, it is zero. The fundamental mathematical quantity that determines the respective quantum numbers is a density distribution of the odd and even numbers in the following form:* $\frac{2n-1}{4n-1}+\frac{2n}{4n-1}=1$ *. Accordingly, the distribution ratio* $\frac{1}{4n-1}$ *forms the basis for defining the "implicate fluctuation" values used to construct the dynamic quantum state or quantum energy norms.*

The physical properties of the"physical ether" are:

1. There is a concept of dynamic energy independent of the concept of mechanical energy, defined independently of the concepts of kinematics

2. The "physical ether" possesses the characteristics of a "ground state" of possible ponderable matter and a "plasma state" of possible non-ponderable matter [(*)]. This physical ether is independent of the concepts of kinematics - that is, in particular, independent of the concepts of space, time, and continuity. The characteristic property of a plasma state is the "potential" of two plasma systems that superimpose on every quantum-mechanical system.

3. The conceptual framework of empirical physics for physical measurements - in particular, the concept of the "observer" - can be described approximately using the methods of quantum mechanics and the methods of approximation theory in Hilbert scales. From a physical perspective, the concept of the "observer" corresponds to that of a common, statistically relevant Hamilton operator of the ground-state and plasma systems. "De-quantization" with respect to the downstream concepts of classical physics is achieved by means of a downstream assignment of suitably chosen metric systems (geometries) to the respectively compactly embedded approximation Hilbert spaces of the underlying basis Hilbert space.

[(*)] The requirements that a physical ether with the attributes "ground state" and "plasma" must meet in the basic model are based on Einstein's descriptive qualities of a physical ether, summarized in the requirement that a physical ether must exist without concepts from the mechanics of motion, (EiA) and the conceptual tension between the two groups of terms "matter" and "electromagnetism."
The fact that "plasma" should also be considered part of the concept of a physical ether follows from the fact that about 95% of the universe consists of the phenomenon known as the 'vacuum.' The same proportion applies to the empty space between a proton and an electron. The remaining 5% of the universe's vacuum consists roughly of 5% matter, 25% dark matter, and 70% dark energy. Nearly all (about 99%) of the 5% matter in the universe is in a 'plasma state.' A presumed sophisticated physical concept of "dark matter" "explains" the phenomenon of spiral shapes in the universe. A presumed sophisticated physical concept of "dark energy" explains the phenomenon of the cosmic microwave background radiation (LaM).

The mathematics of the ground state model and the plasma model

The basis quantum-mechanical Hilbert space framework is the Hilbert space $H_{-1/2} = H_0 \otimes H_0^{\perp}$ defined by the finite norms

$$\|x\|^2_{-1/2} = \int_0^{\infty} \|x\|^2_{(\tau)} d\tau = \sum_{n=1}^{\infty} \frac{1}{\sqrt{\lambda_n}} x_n \varphi_n \,, \quad x_n := (x, \varphi_n).$$

The quantum numbers of the two interconnected quantum fluctuation systems are based on the *a priori* density assumptions of a physical ether σ_n. The quantum numbers of the ground-state model correspond to this physical density σ_n of the ether.

	Ground-state model	Plasma model
Quantum fluctuation numbers	$\sigma_n := \frac{1}{4n-1}$ $\Gamma_n^{\pm} := \frac{1}{\sqrt{\lambda_n}} \pm \frac{\sigma_n}{2\sqrt{\lambda_n}} > 0$ $\Gamma_n^{+} + \Gamma_n^{-} = \frac{2}{\sqrt{\lambda_n}}$ $\Gamma_n^{+} - \Gamma_n^{-} = \frac{\sigma_n}{\sqrt{\lambda_n}}$	$2\sigma_n := \frac{2}{4n-1}$ $\Pi_n^{\pm} := \frac{1}{\sqrt{\lambda_n}} \pm \frac{2\sigma_n}{2\sqrt{\lambda_n}} = \frac{1}{\sqrt{\lambda_n}}(1 \pm \sigma_n) > 0$ $\Pi_n^{+} + \Pi_n^{-} = \frac{2}{\sqrt{\lambda_n}}$ $\Pi_n^{+} - \Pi_n^{-} = \frac{2\sigma_n}{\sqrt{\lambda_n}}$
	$x^+ := \frac{1}{2}\sum_{n=1}^{\infty} \Sigma_n^{+} x_n \varphi_n$ $x^- := \frac{1}{2}\sum_{n=1}^{\infty} \Sigma_n^{-} x_n \varphi_n$ $x = x^+ + x^- = \sum_{n=1}^{\infty} \frac{1}{\sqrt{\lambda_n}} x_n \varphi_n$	$x^+ := \frac{1}{2}\sum_{n=1}^{\infty} \Pi_n^{+} x_n \varphi_n$ $x^- := \frac{1}{2}\sum_{n=1}^{\infty} \Pi_n^{-} x_n \varphi_n$ $x = x^+ + x^- = \sum_{n=1}^{\infty} \frac{1}{\sqrt{\lambda_n}} x_n \varphi_n$
Indefinite norms functionals	$\|\|x^+\|\|^2_{\sigma} := \frac{1}{2}\sum_1^{\infty} \Sigma_n^{+} x_n^2$ $\|\|x^-\|\|^2_{\sigma} := \frac{1}{2}\sum_1^{\infty} \Sigma_n^{-} x_n^2$ $\varphi_{\sigma}(x) := \|\|x^+\|\|^2_{\sigma} - \|\|x^-\|\|^2_{\sigma}$	$\|\|x^+\|\|^2_{2\sigma} := \|x\|^2_{-1/2} + \|\|x\|\|^2_{2\sigma}$ $\|\|x^-\|\|^2_{2\sigma} = \|x\|^2_{-1/2} - \|\|x\|\|^2_{2\sigma}$
Norms	$\|\|x\|\|^2_{\sigma} = \sum_{n=1}^{\infty} \frac{\sigma_n}{\sqrt{\lambda_n}} x_n^2$ $\|\|x\|\|^2_{\sigma,\perp} := \sum_{n=1}^{\infty} \frac{1-\sigma_n}{\sqrt{\lambda_n}} x_n^2$	
	$\|\|x\|\|^2_{\sigma} = (W_{\sigma} x, x) = \sum_{n=1}^{\infty} \frac{\sigma_n}{\sqrt{\lambda_n}} x_n^2$ with	$\|\|x\|\|^2_{2\sigma} := 2(W_{\sigma} x, x) = 2\|\|x\|\|^2_{\sigma}$ with
	$W_{\sigma} x := \frac{1}{2} grad \varphi_{\sigma}(x)$ $= x^+ - x^- = \sum_{n=1}^{\infty} \frac{\sigma_n}{\sqrt{\lambda_n}} x_n \varphi_n$	$W_{2\sigma} x := \frac{1}{2} grad \varphi_{2\sigma}(x)$ $= x - x^- = \sum_{n=1}^{\infty} \frac{1-\sigma_n}{\sqrt{\lambda_n}} x_n \varphi_n$
Norm estimates	$\|\|x\|\|^2_{\sigma} \le \frac{1}{3}\|x\|^2_{-1/2} < \frac{1}{2}\|x\|^2_{-1/2} <$ $< \frac{2}{3}\|x\|^2_{-1/2} \le \|\|x\|\|^2_{\sigma,\perp}$	$\frac{1}{3}\|x\|^2_{-1/2} \le \|\|x^-\|\|^2_{2\sigma}$ $\|\|x^-\|\|^2_{2\sigma} < \|x\|^2_{-1/2} - \|\|x\|\|^2_{\sigma}$

The characteristic feature of the ground-state system is that it allows for three non-superimposable states, which can be characterized by the quantum spectrum range $\left[\frac{1}{3}, \frac{2}{3}\right]$.

The characteristic feature of the plasma system is that it allows only a dynamic quantum-antiquantum plasma pair. It is characterized by the fact that it does not allow for any non-superimposed quantum-mechanical states. From a mathematical perspective, it is interesting that the additional *a priori* physical fundamental condition of a plasma state at the quantum level simplifies probability-theory-based quantum mechanics (ponderable matter) and, at the same time, this plasma state can serve as the basis for continuous fields in the sense of a Maxwell-Mie field theory, (WeH) p. 171 ff., (WEH1) p. 206 ff..

Outlook

The following descriptions are deliberately phrased in the subjunctive, as they represent the speculative thoughts of a mathematician who, even during his studies, was exposed only to the concepts of economics.

The "vacuum model" and the "plasma" model, as well as their relationship to one another, have been fully described from a mathematical perspective. To make sense of this relationship from a physical perspective as well, the obvious next steps could be based on Weizäcker's grouping of the "elementary entities of physics" into the three complexes, (WeC2) p. 147

"Stable" matter:	proton, neutron, electron
"Electromagnetism":	Electromagnetic field
"Gravitation":	Gravitational field.

According to the current understanding of an atom (with its two components, the nucleus and the electron shell), the proton is stable within the atom; electrons and positrons are stable in the electron shell. The "ground state model," with its three distinct superposition states, could be considered as an alternative model.

In the absence of a conceptual foundation for all existing (phenomenon-specific) plasma theories, the plasma model is certainly worthy of being tested as a candidate for a new theory of plasma dynamics. From the perspective of empirical physics, it could thus provide a suitable theoretical model for the Landau damping phenomenon, and thereby resolve the conceptual problem posed by the two currently incompatible models - one based on the concept of continuous, collision-free fields (Coulomb potential-based) and the other on mechanical collision fields (Landau-Boltzmann collision operator-based) - each of which is underpinned by different physical "forces."

Mie theory is an attempt to arrive at a field theory of matter without overstepping the scope of known electromagnetic state variables - that is, an attempt to derive matter from the field (WeH1). The two-component model of the plasma model could thus also provide the appropriate mathematical framework for an extended Maxwell-Mie theory. From an empirical perspective, the two-component plasma model would also be suitable in light of the Ehrenhaft phenomenon of photophoresis and Ehrenhaft's interpretation of this phenomenon:

> "*The light induces electric and magnetic charges (poles) upon the particles if they are illuminated by concentrated light predominantly of shorter wavelengths*," (EhF).

The ether-based model is independent of spacetime geometries; that is, the assignment of the geometries indispensable to empirical physics occurs only in the context of constructing approximate solutions using the approximation methods of the calculus of variations in Hilbert spaces. This means, among other things, that a "reengineering" of Maxwell's equations could open up a range of new possibilities for bringing together various concepts of theoretical physics (geometries, unit quaternions that are isomorphic to the Lorentz group, the complex Lorentz transformation group for two-component systems, the hidden symmetry of the Coulomb problem, etc.) into a single integrated model. This could be linked to a new role for the so-called "constant of nature," which could be interpreted as metrologically relevant "approximation constants" of "deeper-lying quantum-dynamic processes." In this context, special reference should be made to the book by A. Unzicker titled "The Mathematical Reality." With regard to the relationship between quantum theory and general relativity and their classical test - which can also be explained by a variable speed of light - Unzicker's book titled "Einstein's Lost Key" should also be mentioned here, and particular attention should be drawn to the article by H. Dehnen and colleagues cited in this book in connection with Mach's principle, (DeH).

The concept of massless light quanta that "transmit" signals and energy, Newton's concept of a theory of light refraction, and Lorentz's interpretation of the transformation equations of the same name as effects on moving objects caused by their motion through the ether (SuL, p. 61) cannot be explained within the framework of a theory of motion. A new, overarching research question could therefore arise with regard to a theory of light that does not yet exist.

The following list is intended merely to highlight additional possibilities:

- the concept of “positronium” at the time of the “Big Flash,” when the electron and the positron had equal mass and orbited each other “on equal footing” (UnA2), p. 96
- the only two known superfluids, the list of ideal gases, the atomic number in the periodic table, which roughly corresponds to the rounded relative atomic mass
- Fermi gas and the shell model in nuclear physics
- $\beta^{\pm}$ disintegration
- Plemelj’s concept of a “flux” on the surface without the need for the concept of a normal vector
- P.-M. Robitaille’s papers (RoP1)–(RoP4); see also (UnA4)
- Photosynthesis:$CO_2 + H_2O + light \rightarrow CH_2O\ (theoretical\ carbohydrate) + O_2$
- Molecular „Vitalism“, (KiM).

Note: The physical *a priori* ether model - or the “interaction” of mechanical and dynamic quantum states - allows for a remarkable comparison with Deleuze’s philosophy *Difference and Repetition*. In (PlM) and (PlM1), various variations on Deleuze’s conceptual framework are formulated, elucidating what he actually means by *Difference and Repetition*.

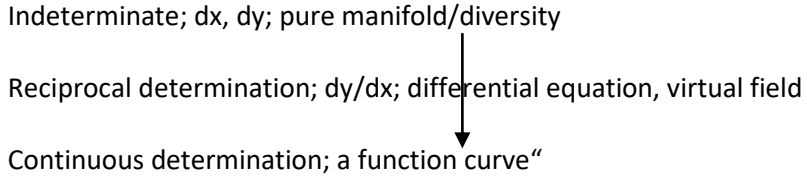
The undifferentiated abyss and *difference* as *THE* determination

Quote, (PlM) I, p. 426 ff.: “The difference between formless (undifferentiated) abyss/chaos and form must be something that lies outside of representation. Difference is therefore the process by which forms emerge and come into being from the formless. This emergence cannot be derived from the structure of the already completed form. … Thus, beneath the ordered level of representation—the settled distribution—there is not immediately chaos, the undifferentiated abyss. In between lies a dynamic layer that cannot be fully represented. … This layer is highly dynamic. It repeatedly absorbs elements from the chaos and the undifferentiated abyss, thereby “renewing” itself and revitalizing itself. As a result, orders within the settled distribution can also be destroyed. ….

Difference is the arrow pointing downward: from the undifferentiated abyss/chaos through semi-chaos/chaaosmos to order. Our “stable” orders then emerge from semi-chaos.

Repetition is the renewal, the repetition of the process “*Difference as* THE *Determination*”: The world is not created out of absolute chaos once at the beginning of time, after which everything is ordered and proceeds according to a grand equation of the world. Rather, the dynamic, sub-representative continues to operate. ….

Difference and Repetition as a Conceptual Framework in Mathematics/Physics (see also (DeG) p. 220)



5. Additional notes

a. Mathematics, natural sciences, and all that

Note (R. Penrose's „Road To Reality"): R. Penrose's „Road To Reality" gives a complete guide to the physical laws of the universe on the basis of current physical paradigms accompanied by supporting mathematical tools.

Note (current paradigm in physics): The physical models in different physical areas are decoupled and differently scaled according to their different levels of granularity (e.g., the „SMEP"-layer, the „thermodynamics" layer, the „relativity" layer). Conceptually speaking, reducing the number of scales requires new „nature constants".

Note (A. Unzicker's "Mathematical Reality"): A. Unzicker's "Mathematical Reality" could be interpreted as a kind of re-engineering approach of current physical paradigms justified by a critical analysis from a physicists perspective of the current usage of the „nature constant" concept. The aspiration of "Mathematical Reality" is,

> „to form a consistent picture of reality by observing nature from the cosmos to elementary particles", (UnA2).

Remark (the mathematical framework for physical laws): The mathematical framework of the proposed physical modelling framework are built on functional analysis and on number theory. The central branches from functional analysis are the theory of Krein spaces enabling hermitian operators in spaces with an indefinite metric, and approximation theory in Hilbert scales enabling by their compactly embeddedness properties. Related physical requirements to those branches first appeared in papers from Dirac, Pauli, and Heisenberg. The Krein space based mathematical concepts of "potential", "potential operators", "maximal definite subspaces", "maximal dissipative operators", "hyperboloids generated by operators" etc., are accompanied by corresponding mathematical constants; those constants are supposed to provide mathematically justified "physical potential barriers" between physical-statistical worlds and an overall mathematical reality". The essential concept behind the (vacuum, plasma, electromagnetic) quanta pair number systems is based on number theory.

Note (different number of scales): In classical mechanics one deals with the three scales, „distance", „time", and „mass"; in non-relativistic quantum theory and classical relativity one deals with two scales, „distance", and „time"; in relativistic quantum theory one deals with only one scale, the „distance", (DeP) p. 551.

Note: About 95% of the universe is about the phenomenon „vacuum". The same proportion applies to the emptyness between a proton and an electron. The remaining 5% of universe's vacuum consists roughly of 5% matter, of 25% sophisticated „*dark matter*", and of 70% sophisticated „*dark energy*". Nearly all (about 99%) of the 5% matter in the universe is in "plasma state". A presumed physical concept of „*dark matter*" „explains" the phenomenon of the spiral shapes in the universe. A presumed physical concept of „*dark energy*" explains the phenomenon of the cosmic microwave background.

Note (the mass gap problem of the classical Yang-Mills theory): The Maxwell fields can carry energy from one place to another. The classical Yang-Mills theory is a generalization of the Maxwell theory of electromagnetism where the invented *chromo*-electromagnetic field also carries charges for low energy scales. As a classical field theory it has solutions which travel at the speed of light so that its quantum version should describe massless particles (gluons). However, the postulated phenomenon of color confinement permits only bound states of gluons, forming massive particles. This is the mass gap. The proposed Maxwell-Mie quanta energy field model makes the Yang-Mills theory (which is anyway restricted to low energy scales) obsolete.

Note: The quantum theory gets primacy regarding the classical theory with its most perfect design, the general relativity theory. Therefore, the laws of the metric field, which are in principle independent from the laws of the quantum theory, have no absolute validity. The regularity of the metric field – indeed in a statistical way – would be tied with elementary particle interaction, like it is furthermore „located" in the sense of the Mach principle, (DEH).

Note: The Mach principle is a cosmological principle; as there are multiple cosmological models, it becomes also a selection principle to select the few physical relevant cosmological models. Therefore, in the sense of Kant, it it not a „constitutive" principle (like the general co-variance of the field equations), but a „regulative" principle, (DeH).

Note:The most advanced mathematics of "galactic dynamics" is about collisionsless Boltzmann and Poisson equations accompanied by the probability of a given star to be found in unit phase-space volume near the phase-space position $(\boldsymbol{x},\boldsymbol{v})$, (BiJ) p. 555.

Note: The Planck action constant is independent from any weak or strong gravitation field. It therefore somehow mirrors the fundamental difference of physical macro and micro world, (DeH).

Note (the Maxwell and the Einstein equations): In the Maxwell equations „charges tell the electromagnetic fields how to vary". In the Einstein's field equations „space-time geometry tells mass-energy how to move" and „mass-energy tells space-time geometry how to curve".
The Einstein operator is given by $G = R_{ik} - R\frac{g_{ik}}{2}$ with the corresponding gravity field equations $G = -\kappa T_{ik}$ and the corresponding motion equations

$$\frac{d}{d\tau}\left(g_{\mu,\nu}\frac{dx^{\mu}}{d\tau}\right) = \frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial x^{\nu}}\frac{\partial x^{\alpha}}{\partial \tau}\frac{\partial x^{\beta}}{\partial \tau}$$

for the path $x^{\mu} = x^{\mu}(t)$ of a particle.

The change from the Newton model is about a change from the Newton potential equation $-\Delta\Phi = -4\pi k\rho$ (applying the Dirac (delta) function on the right side of the PDE) to the Einstein equation $G = -\kappa T_{ik}$, going along with a change from the motion equations from

$$\frac{d^2\vec{x}}{dt^2} = -grad\Phi \quad \rightarrow \quad \frac{d}{d\tau}\left(g_{\mu,\nu}\frac{dx^{\mu}}{d\tau}\right) = \frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial x^{\nu}}\frac{\partial x^{\alpha}}{\partial \tau}\frac{\partial x^{\beta}}{\partial \tau}.$$

Instead of one potential equation there are now 10 equations with 10 potentials Φ_{ik}; instead of a linear operator, there is now a non-linear operator. The gravity potential is no longer the sum of single gravitation potentials. The matter is described by the energy-momentum tensor T_{ik}, reflecting the principles of energy and momentum conservation. The matter generates the space-time structure, particles move along of geodesics and the potentials Φ_{ik} are functions of the energy-momentum tensor T_{ik} ($\Phi_{ik} = f(T_{ik})$).

Note (the Cosmological Microwave Background Radiation and Big Bang models): The CMBR provides us with the most important evidence supporting the Big Bang model. Big Bang models are on the basis of general relativity and follow from a number of assumptions, (LaM) p. 7:

- homogeneity of space applies. Thus it is assumed that all points of space are equivalent and the properties associated with each point are the same
- isotropy of space applies. This means that there is no privileged direction in space
- the matter in the universe can be described very simple in terms of what is called a perfect fluid. In this case its properties are completely given by ist density ρ and its pressure p
- the laws of physics are the same everywhere.

Note (Water, Hydrogen Bonding, and the Microwave Background): *In this work, the properties of the water are briefly revisited. Though liquid water has a fleeting structure, it displays an astonishingly stable network of hydrogen bonds. …. This simple analysis reveals that the oceans have a physical mechanism at their disposal, which is capable of generating the microwave background*", (RoP2).

Note: R. Penrose: How special was the Big Bang?

> „in order to produce an universe resembling the one in which we live, the Creator would have to aim for an absurdly tiny volume of phase space of possible universes – about $1/10^{10^{123}}$ of the entire volume, for the situation under consideration", (PeR) p. 444.

Note: (galactic kinematics, cosmic time, Hubble law, and ordinary differential equations): The kinematics of an universe observed to be homogeneous and isotrop on large scales are describes by the Hubble parameter $H(t)$ and a scale factor $a(t)$ depending by a cosmic time parameter t:

> "Consider the triangle defined by three nearby fundamental observers. As the universe evolves, the triangle may change in size, but cannot change in shape or orientation – in the contrary case, it would define a preferred direction, therby violating the isotropy assumption. Thus, if $r_{ij}(t)$ is the length of the side joining oberserver i and j at cosmic time t, we must have $r_{ij}(t) = r_{ij}(t_0)a(t)$, where $a(t)$ is independent of i and j. Since this argument holds for all fundamental observers, the distance between any two of them must have the form $r(t) = r(t_0)a(t)$, where the scale factor is a universal function, which may normalize so that $a(t_0) = 1$ at the present cosmic time t_0. The relative velocity of the two observers is
>
> $$v(t) = \frac{dr}{dt} = r(t_0)\dot{a}(t) = r(t)\frac{\dot{a}(t)}{a(t)} = r(t)H(t),$$
>
> where $H(t)$, is the Hubble parameter. At the present time, $H(t_0) = H_0$ is the Hubble constant. The Hubble law $v = H_0 r$ is a consequence of homogeneity and isotropy resp. in a homogeneous, isotropic universe the Hubble law remains true at all times, but the Hubble constant varies with cosmic time", (BiJ1) p. 38.

Note (non-relativistic resp. relativistic gravitational instability of the universe): The two magic tricks to analyse the (non-relativistic resp. the relativistic) gravitational instability of the universe is based on a simple continuity equation of fluid elements in the form

$$\frac{\partial \rho}{\partial t} + 3H(t)\rho + \nabla \cdot (\rho \vec{v}) = 0$$

in combination with a related fluid-particle Lagrangian. Taking into account gravitational and pressure forces influencing those fluids (after some linearization) the main non-relativistic equation becomes the form

$$\frac{\partial \boldsymbol{v}}{\partial t} + 2H(t)\vec{v} = -\frac{1}{a^2}\left(\frac{1}{\rho_0}\nabla p_1 + \nabla \Phi_1\right),$$

while the (by special relativity modified) "relativistic" equation becomes the form

$$\frac{\partial \psi}{\partial t} + 2H(t)\psi = -\frac{1}{a^2}\left(\frac{p_1}{\rho_0 + \rho_0/c^2} + \Phi_1\right),$$

where the density ρ of the (non-relativistic) Poisson equation is replaced by the "relativistic" density in the form $\rho + \frac{3p}{c^2}$, (BiJ1) pp. 722/723.

Note (mechanical & dynamical energy types): Based on appropriate properties provided by a Krein-Hilbert space framework there are positive, increasing potential differences between the three 2-component dynamic field types starting from the vacuum energy fields up to the electromagnetic energy fields. The quanta type specific laws are governed by the principle of conservation of total energy, defined by the sum of two „complementary" mechanical & dynamical energies of the related considered physical system.

Note: The probably most fundamental mathematical theorem in physics is E. Noether's theorem. It effects a huge class of conservation laws governing symmetries of space, time, and „internal" variables. Noether's theorem relates conservation to invariance, and thus to symmetry. This theorem provides the mathematical foundation of the whole quantum mechanics. However, the conservation of electric charge emerges from a more abstract symmetry called „gauge invariance".

Note: (renormalization group equation and symmetry break down): The behavior of a physical system depends on a scale (of energies, distances, momenta, etc.) at which the behavior is studied. The change of behavior when the scale is changed, is described by the renormalization group equation. In quantum field theory, the dependence of the behavior on the scale is often expressed mathematically by the fact that in order to regularize (i.e., render finite) Feynman diagram integrals one must introduce auxiliary scales, cutoffs, etc. The effect of these choices on the physics is encoded into the renormalization group equation. The "case" if there is no related (G-invariant) realisation in a quantum theory (i.e., if there is a difference between realizations and vacuum states) then one says that the symmetry is broken, (DeP1) p. 1125.

Note: The conservation principles of energy, linear momentum, angular momentum, and electric charge are amoung the most fundamental principles of physics. … The notion „conservation“ as in „conservation of energy“ is not the same as „invariant“. They are related, …, but they are not synonymous. The momentum or energy of a system of particles may be conserved but not necessarily invariant, (NeD) pp. 1, 4.

Remark: The mathematical notion for the invariant quantities in the conservation laws of mechanics and electrodynamics is called „functional“. In mathematics, „functionals“ are a central concept in Hilbert space theory playing a key role in variational methods for the study of nonlinear (potential) operators, (ChJ), (VaM). At the same time, the L_2- Hilbert space is a well established mathematical framework for thermostatistics and quantum mechanics.

Note: (zero point energy and symmetry break down): Physics is scale dependent and decoupling. The down (complexity) causality thinking results into a degrease of the number of scales, while the number of «nature constants» increases. The effect of the required auxiliary scales, cutoffs, etc. on the physics is encoded into the renormalization group equation. The "case" if there is no related (G-invariant) renormalization realisation (example ground state energy) is called "symmetry break down", (DeP1) p. 1119 ff.

Remark: „*Lorentz succeeded in reducing all electromagnetic happenings to Maxwell's equations for free space*“, (EiA5). The proposed model provides an inner product of a „free space“ electrino-positrino energetical quanta pair Hilbert space framework w/o any space-time-momentum conceptual notions.

Remark: The Lorentz transformation in special relativity is a simple type of rotation in hyperbolic space. We note that the characteristics of hyperbolic PDE is about their „time-symmetry“. We further note that the hyperbolic wave operator equipped with a $H_{(\tau)}$-based domain is stronly hyperbolic operator. This property is the counterpart of the related strongly elliptic potential operator equipped with a H_{α}-based domain.

Note: The Lorentz transformation group and related components, (StR): A Lorentz transformation is a linear transformation mapping space-time onto space-time preserving the Lorentz-invariant scalar product of two four-vectors $x \coloneqq (x^0, \vec{x}), y \coloneqq (y^0, \vec{y})$ with $\vec{x} \coloneqq (x^1, x^2, x^3), \vec{y} \coloneqq (y^1, y^2, y^3)$ given by $x \cdot y \coloneqq x^0 y^0 - \vec{x}\vec{y}$. Two Lorentz transformations can be connected to one another by a continuous curve of Lorentz transformations. Therefore, the Lorentz transformations form a group, the Lorentz group. The Lorentz group has four components, each of which is connected in the sense that any one point can be connected to any other, but no Lorentz transformation in one component can be connected to another in another component.

> „The full group of Lorentz transformations is the group of transformations that leaves the Minkowski metric invariant. Here is why. Parity (mirroring of all three spatial axes) is the Lorentz transformation. But in the space of all possible Lorentz transformations there is no continuous path that starts out at the Identity, and so are the pure Lorentz boosts, but one cannot reach Parity by pure boosts or pure rotations or combinations of the two.) So the real Lorentz group splits up into at least two disconnected components: the Lorentz transformations that one can reach via continuous path from the Identity (the „restricted“ Lorentz transformations), and the Lorentz transformations that one can reach via continuous path from Parity. And there is another split, namely the split between the Lorentz transformations that include Time Reversal and the ones that do not. So the Lorentz group has at least disconnected components. In fact it has exactly four disconnected components. …. The classical real Klein Gordon field is a real scalar field whose field values are invariant under the restricted Lorentz transformations. The restricted Lorentz transformations are the ones that are continuously connected to the Identity. They include spatial rotations and Lorentz boosts. They include neither P nor T nor PT. The law of evolution on the Klein Gordon field, the Klein Gordon equation is invariant under the restricted Lorentz tranformations“, (CaC) p. 636.

The four „connection“ possibilities are characterized by four different $\det(.) = \pm 1$ conditions containing the four different Lorentz transformations

(1) "1" (3) time inversion $I_t = T$

(2) space inversion $I_s = P$ (4) space-time inversion.

There are three related sub-groups of the Lorentz group, (A) the orthochronous Lorentz group (containing „1“ and the space inversion); (B) the proper Lorentz group (containing the „1“ and the space-time inversion; it is associated to the group of $2x2$ complex matrices of determinant one, which is denoted by $SL(2, C)$), which is important in describing the tranformation properties of spinors), and (C) the orthochorous Lorentz group (containing the space inversion and the time inversion).

The Lorentz transformation in special relativity is modelled by the restricted Lorentz group, the group of $2x2$ complex matrices of determinant one, $SL(2,C)$. It is isomophic to the symmetry group $SU(2) \cong SL(2,C)$, containing as elements the complex-valued rotations, which can be written as a complex-valued matrix of type

$$\begin{pmatrix} a+ib & c+id \\ -c+id & a-ib \end{pmatrix}$$ with determinant one.

Note: For the relations of the Lorentz group in the context of the SRT and the GRT to Minkowskian quaternions, belonging to the wider class of complex quaternions (biquaternions), we refer to (GiP).

Note: In (LeS1) new real linear quaternions are introduced to obtain a quaternionic version of the Lorentz group without the use of complexified quaternions) and a quaternionic metric tensor is defined, overcoming difficulties concerning the appropriate transformations on the $3+1$ space-time.

Note: For quaternionic analysis and elliptic boundary value problems we refer to (GuK).

Note: The spin of an elementary particle is its eigen-rotation with exactly two rotation axes, one parallel and one anti-parallel axis to a magnetic field. This is the 2×2 complex number scheme, where every „normal“ rotation is contained twice. Consequently, an electron has a charge only half of the Planck's quantum of action. For a quaternionic equation representation of the motion of a particle with an electric charge in a electromagnetic field manifesting the relativistic covariance of classical electromagnetism we refer to (GiP). In (ArI) a quaternionic unification of electromagnetism and hydrodynamics is provided.

Note: In (HuM) the concept of the unit quaternion is applied to enable statistical analysis for rotations in 3D electron cryo-microscopy. Concepts of distance and geodesic between spatial rotations is introduced and developed to enable comparisons and interpolations between rotations. Statistical methods for performing sampling and numerical analysis in the rotational space are introduced and developed. A description for the molecular symmetry and the corresponding method of space division for asymmetric units are developed based on the unit quaternion.

Note (is the quaternion rotation operator): The quaternions provide an appropriate field to address the „translation-rotation“ (linear and angular rotation) „permutation“ requirement. The perhaps primary application of quaternions is the quaternion rotation operator. This is a special quaternion triple-product (unit quaternions and rotating imaginary vector) competing with the conventional (Euler) matrix rotation operator. The quaternion rotation operator can be interpreted as a frame or a point-set rotation, (KuJ). Its outstanding advantages compared to the Euler geometry are

- the axes of rotation and angles of rotation are independent from the underlying coordinate system and directly readable
- there is no need to to take care about the sequencing of the rotary axes.

Note: In (SaM) a generalized quaternionic quantum wave equation formulation is used to construct general plane waves enabling corresponding generalized Klein Gordon and Helmholtz equations.

Note: MacFarlane (MaF) introduced the set of hyperbolic quaternions. The hyperbolic quaternions are not commutative like real quaternions. But the set of hyperbolic quaternions contains zero divisors.

Note: In (LeS) the isomorphism between unitary quaternions and space time rotations is extended to Lorentz boosts. From the transformation properties of two-component spinors a quaternionic representation for the space-time algebra is derived. Additionally, a quaternionic bi-dimensional version of the Dirac equation is derived.

Note: In (KrR) variable orthonormal sets, so-called moving frames, to generalize the notion „differentiability“ and „complex-analyticity“ by a hypercomplex (quaternionic) differential form calculus.

Note: In (BoI) a continuous wavelet transform is built on the upper sheet of the 2-hyperboloid H_+^2.

Note (StR): The complex Lorentz group $L(C)$ has just two connected components, $L_+(C)$ and $L_-(C)$. Additionally, the transformations 1 and -1, which are disconnected in the real Lorentz group L, are connected in the complex Lorentz group. Just as the restricted Lorentz group is associated with $SL(2,C)$ the complex Lorentz group is associated with $SL(2,C) \otimes SL(2,C) \cong SU(2) \otimes SU(2)$. The latter group is the set of all pairs of $2x2$ matrices of determinant one with the multiplication law

$$\{A_1, B_1\}\{A_2, B_2\} = \{A_1A_2, B_1B_2\}.$$

> **In summary**: While two (real) Lorentz transformations need to be connected to one another by an appropriately defined continuous curve of Lorentz transformations (the Lie group concept), there are two pairs of components of the complex Lorentz group, which are both already connected by definition accompanied by a related multiplication law.

Note (mathematics and physics): Most laws of physics are derived by a statistical mechanisms (thermo-statistics accompanied by the concept of entropy), which E. Schrödinger called „order-from-(atomic) disorder" mechanism. As a consequence, the physics (with the claim to be the foundation of chemistry) is not able to provide any fundamental law derived by an „order-from-order" mechanism as common „law" with chemistry and biology.

Note (physics and chemistry): The current understanding of the relationship of physics and chemistry may be briefly sketched by H. Weyl's statement that

> „the valence bonds are an abbreviated symbol for the actual quantum-physical forces acting between the atoms, which themselves are complex dynamical system", (WeH) p. 266.

Note (dead and living matter): The contrast of dead and living matter may be briefly sketched by H. Weyl's statement that

> „One of the profoundest enigmas of nature is the contrast of dead and living matter. …. Incidentally, the gap between organic and inorganic matter has been bridged to a certain extent by the discovery of virusses. Virusses are submicroscopic entities that behave like dead inert matter unless placed in certain living cells. …. Many virusses have the structure typical of inorganic matter; they are crystals", (WeH) p. 276.

Remark (mathematics and consciousness): The electrinos and the positrinos may be interpreted as binary quanta information carriers enabling a link to information and consciousness theory. With the proposed conception of non-mechanical binary quanta information carriers the synapses (neuronal net) model is no longer restricted to mechanical signals with velocities limited by the speed of light, enabling other kinds of potential differences between biological synapses governed by dynamical energy quanta.

Remark (mathematics and philosophy): There is an analogy to Leibniz's conception of (otherworldly) monades and their role defining a preestablished (mechanical) harmony.

> „The classical philosopher of a dynamic world presentation is Leibniz. … For him the real of movement does not lie in a pure change of the location, but in a moving force „La substance est un etre capable d'action – une force primitive – overspatial, immaterial. … The last element is the dynamic point, from which the force erupts as an otherworldly power, an indecomposable strechless unit: the monade", (WeH2) p. 51
>
> „And so we can conclusively state the relationship of the least action principle to Kant's Critique of Judgement in the following form: the principle of least action in its most modern generalization is a maxim of the reflective judgement", (KnA) p. 55.

Note: The group $SO(4)$ is no simple Lie group. The complex Lorentz group $S^3 \times S^3 \cong SU(2) \times SU(2)$ (the hidden symmetry group of the Coloumb problem and the related Rydberg spectrum) is isomorphic to the two isomorphic normal subgroups of the group $SO(4)$. Beside the group S^3 (the unit quaternions of the quaternion algebra $|\boldsymbol{H}$) it contains isomorphic normal subgroups $G \coloneqq \psi(S^3 \times e)$, $G' \coloneqq \psi(e \times S^3)$, where ψ denotes the surjective orthogonal mapping $\psi(a,b) : |\boldsymbol{H} \rightarrow |\boldsymbol{H}, x \rightarrow axb$, (EbH) For more details about the underlying simple quaternion rotation operator see (KuJ) p.127.

b. Maxwell theory and special cases

Note: Electric Current and Its Magnetic Field
„When a metallic wire is connected between two oppositely charged spheres, the electric charges cancel each other out completely, and the entire system is uncharged. This process has been given the name "electric current." While electricity is flowing, i.e., while the electric current exists, several effects are observed, some in the surrounding space and some within the current path itself. The latter heats up, and a magnetic needle located in the surrounding space is deflected: the current produces heat and generates a magnetic field in the surrounding space. … How the potential differences arise within the element cannot be understood without delving into the molecular constitutions of the element's constituents; Maxwell's theory leaves this aside: it accepts the existence of these potential differences without asking where they come from", (ScS) S. 51/53.

Note: The magnetic field of a linear current
„In a plane perpendicular to the current-carrying conductor, the field lines are closed curves, namely circles concentric with the axis of the current. .. The magnetic field has no "longitudinal" component, i.e., no component parallel to the current-carrying conductor. … The loop voltage for a closed curve encircling the current is nonzero. … If the surrounding space is a vacuum, we can characterize the magnetic field as a source-free vortex field, since the field lines are all closed curves. The formula div(B)=0 implies that there is neither true nor free magnetism—which are, after all, identical in a vacuum," (ScS) S. 54/55.

Note: *„Maxwell's equations determine the electric field when the distribution of electric charges and currents is known. However, the laws governing the behavior of currents and charges are unknown to us. We do know that electric forces consist of elementary particles (electrons, positive nuclei), but we do not understand this from a theoretical standpoint. We do not know the energetic factors that cause the arrangement of electricity into particles of a certain size and charge, and all attempts to complete the theory in this regard have so far failed. We therefore know, if we may at all base ourselves on Maxwell's equations, the energy tensor for the electric fields only outside the elementary particles.*

It is true that attempts have been made to remedy this deficiency by conceiving of the elementary electric particles as genuine singularities. In my view, however, this amounts to abandoning any real understanding of the structure of matter. It seems to me far better to admit our current inability than to settle for a pseudo-solution," (EiA6) S. 52.

Note: The Maxwell (field) theory of electrodynamics plays an important role in quantum theory, as well as in the relativity theory. The Maxwell fields can carry energy from one place to another. It describes the electricity dynamics of an a priori existing charged elementary particle (called electron) in an idealized semiconductor world governed by an electric and a magnetic field. The induced electric (current) force is modelled by the sum of an electrical conductor line current and a so-called displacement current. The latter one is a cross-section line reduced 1st order approximation of a virtual electrical insulator field shriveled up to an „insulator line current" accompanied by the notions of „time" and „distance".

Note (KiA): In the Maxwell equations the charge density ρ of the medium „generates" the electric displacement field $\vec{D}$ by Gauss' Electric Law ($div_x(\vec{D}) = \rho$), and the current density $\vec{J}$ „generates" the magnetic field $\vec{M}$ by Ampere's Law ($\frac{\partial \vec{D}}{\partial t} - curl_x \vec{M} = -\vec{J}$). In case of stationary fields the Faraday's Law of Induction reduces to $curl_x \vec{E} = 0$. In case of a homogeneous medium if there exists a potential function u Gauss' Electric Law yields the Poisson equation $-\varepsilon \Delta u = \rho$, where ε denotes the permittivity constant. Thus, the electrostatics potential in a homogeneous medium (e.g., in vacuum) is described by the basic elliptic partial differential equation (PDE) $-\Delta u = \frac{\rho}{\varepsilon}$, (KiA). The same concept applies for Newton's gravitational scalar potential defined by $-\Delta u = 4\pi G\rho$, (G gravitational constant, ρ mass density).

Note: Mathematical spoken, the regularity of the given volume charge density resp. the gravitational mass density in form of a given Dirac distribution function $constant \cdot \delta(x)$ in R^3, $\delta(x) \in H_{-3/2-\varepsilon}$, $\varepsilon > 0$, ensures continuous Coulomb resp. Newton potential function solution $u \in C^0$.

Note: The operator concerned with the time-harmonic Maxwell equation and the radiation problem is the D'Alembert operator related to the wave equation. The electrodynamic in the special relativity theory is described by the four-vector formalism of the space-time given by the equation $\vec{A} = \frac{4\pi}{c}\vec{j}$, with the four-vector potential $\vec{A}$, where its curvature determines the electric and magnetic field forces, and $\vec{j}$ denotes the four-current-density. The solution of time-harmonic Maxwell equations in a vacuum leads to the Helmholtz equation. The fundamental solution of the Helmholtz equation at the origin is given by spherical wave fronts. The time-dependent magnetic field has the form of the Hertz dipole centered at the origin, (KiA) p. 1.

Note: (KiA) p. 1 ff.: Electromagnetic wave propagation is described by four particular equations, the Maxwell equations, which relate five vector fields $\vec{E}$ (electric field), $\vec{D}$ (electric displacement), $\vec{H}$ (magnetic field), $\vec{B}$ (magnetic flux density), $\vec{I}$ (current density), and the scalar field ρ (charge current). ρ and $\vec{I}$ can be interpreted as macroscopic mean values of the free charge and current densities in the medium. In differential form the Maxwell equations read as follows:

$$\frac{\partial \vec{B}}{\partial t} + curl_x \vec{E} = 0 \qquad \text{(Faraday's Law of Induction)}$$

$$\frac{\partial \vec{D}}{\partial t} - curl_x \vec{H} = -\vec{I} \qquad \text{(Ampere's Law)}$$

$$div_x \vec{D} = \rho \qquad \text{(Gauss' Electric Law)}$$

$$div_x \vec{B} = 0 \qquad \text{(Gauss' Magnetic Law) .}$$

In domains where the equations are satisfied one derives from the identity $div\ curl_x \vec{H} = 0$ the well-known *Equation of Continuity*

$$\frac{\partial \rho}{\partial t} = div \frac{\partial \vec{D}}{\partial t} = div\left(curl\vec{H} - \vec{I}\right) = -div\vec{I}\,.$$

The Constitutive Equations: In the general setting the Maxwell equations are not yet complete, Obviously, there are more unknowns than equations. The *Constitutive Equations* couple them:

$$\vec{D} = \vec{D}(\vec{E}, \vec{H}) \ , \vec{B} = \vec{B}(\vec{E}, \vec{H}).$$

The electric properties of the material, which give these relationships are complicated. In general, they depend not only on the molecular character but also on macroscopic quantities as density and temperature of the material. Also, there are time-dependent dependencies as, e.g., the hysteresis effect, i.e. the fields at time t depend also on the past.

As a first approximation one starts with representations of the form

$$\vec{D} = \vec{E} + 4\pi\vec{P}, \vec{B} = \vec{H} - 4\pi\vec{M}$$

where $\vec{P}$ denotes the electric polarization vector and $\vec{M}$ the magnetization of the material. These can be interpreted as mean values of microscopic effects in the material. Analoguously, ρ and $\vec{I}$ are macroscopic mean values of the free charge and current densities in the medium.

If we ignore ferro-electric and ferro-magnetic media and if the fields are relatively small, one can model the dependencies by linear equations of the form

$$\vec{D} = \epsilon\vec{E}, \vec{B} = \mu\vec{H}$$

with the two matrix-valued functions, the *dielectric tensor* ϵ: $R^3 \to R^{3x3}$ and the *permeability tensor* μ: $R^3 \to R^{3x3}$. In this case the medium is called linear.

The special case of an *isotropic medium* means that polarization and magnetization do not depend on the directions. Otherwise the medium is called *anisotrop*. In the isotropic case dielectricity and permeability can be modeled as just real valued functions, and one have

$$\vec{D} = \epsilon\vec{E},\ \vec{B} = \mu\vec{H}$$

with real valued functions $\epsilon, \mu : R^3 \to R$.

In the simplest case these functions ϵ and μ are constant and we call such a medium *homogeneous*. It is the case, e.g., in vacuum.

We indicated already that also ρ and $\vec{I}$ can depend on the material and the fields. Therefore, we need a further relation. In conducting media the electric field induces a current. In a linear approximation this is described by Ohm's Law:

$$\vec{I} = \sigma\vec{E} + \vec{I}_e$$

where $\vec{I}_e$ is the external current density. For isotropic media the function $\sigma : R^3 \to R$ is called the *conductivity*. If $\sigma = 0$, then the material is called *dielectric*. In vacuum, we have $\sigma = 0$ and $\epsilon = \epsilon_0 \approx 8.854 \cdot 10^{-12} \frac{AS}{Vm}$, $\mu = \mu_0 \approx 4\pi \cdot 10^{-7} \frac{Vs}{Am}$. In anisotroc media, also the function σ is matrix valued.

The special vacuum case: Vacuum is a homogeneous, dielectric medium with $\epsilon = \epsilon_0$, $\mu = \mu_0$, and $\sigma = 0$, and no charge distributions and no external currents; that is, $\rho = 0$ and $\vec{I}_e$. The Faraday's Law of induction takes the form

$$\mu_0 \frac{\partial \vec{H}}{\partial t} + curl_x \vec{E} = 0\ .$$

Assuming sufficiently smooth functions a differentiation with respect to time t and an application of Ampere's Law yields

$$\epsilon_0\mu_0 \frac{\partial^2 \vec{H}}{\partial t^2} + curl\ curl\ \vec{H} = 0.$$

The term $c = 1/\sqrt{\epsilon_0\mu_0}$ has the dimension of a velocity and is called the *speed of light*.

The special Electro- and Magnetonstatics case: Next we consider the Maxwell system in the case of stationary fields; that is, the five vector fields $\vec{E}$ (electric field), $\vec{D}$ (electric displacement), $\vec{H}$ (magnetic field), $\vec{B}$ (magnetic flux density), $\vec{I}$ (current density), and the scalar field ρ (charge current) are constant with respect to time. For the electric field $\vec{E}$ this situation in a region Ω is called *electrostatics*. The law of induction reduces to the differential equation

$$curl\ \vec{E} = 0 \text{ in } \Omega.$$

Therefore, if Ω is simply connected, there exists a potential $u : \Omega \to R$ with $\vec{E} = -\nabla u$ in Ω. In a homogeneous medium Gauss' Electric Law yields the Poisson equation

$$\rho = div\ \vec{D} = -div\left(\epsilon_0\vec{E}\right) = -\epsilon_0\Delta u$$

For the potential u. Thus, the electrostatics is described by the basic elliptic patial differential equation $-\Delta u = \rho/\epsilon_0$. Mathematically, we are led to the field of *potential theory*.

In magnetostatics one considers $\vec{H}$ being constant in time. For the magnetic field the situation is different because by Ampere's law we have $curl\vec{H} = \vec{I}$. Thus in general $curl\vec{H}$ does not vanish. However, according to Gauss' magnetic law we have

$$div\vec{B} = 0.$$

From this identity we conclude the existence of a vector potential $\vec{A}: R^3 \to R^3$ with $\vec{B} = -curl\vec{A}$ in D. Substituting his into Ampere's Law yields (for homogeneous media Ω) after multiplication with μ_0 the equation

$$-\mu_0\vec{I} = cur\ curl\ \vec{A} = \nabla div\vec{A} - \Delta\vec{A}.$$

Since $curl\nabla = 0$ we can add gradients ∇u to $\vec{A}$ without changing $\vec{B}$. We will see later that we can choose u such that the resulting potential $\vec{A}$ satisfies $div\vec{A} = 0$. This choice of normailzation is called *Coulomb gauge*. With this normalization we get the Poisson equation

$$-\Delta\vec{A} = -\mu_0\vec{I}$$

also in magnetostatics. We note that in this case the Laplacian operator is vector valued and has to be taken componentwise.

Note: Considering wave phenomena the most important situation are the special *Time-Harmonic Fields*. For further details we refer to (KiA).

Note: (UnA2) p. 152: Quote from R. Feynman:

> *„The solution is about a qualitative justification for the fact, that (these) two phenomena, c and h, occur in nature". …. the proposed strategy in (UnA1) leads to the 4-dimensional unit sphere S^3, with its underlying „natural" quaternions numbers. … „The quaternionic multiplication of a spatiotemporal derivative with electromagnetic potential is given by*
>
> $$\left(\frac{\partial}{\partial t}, \vec{\nabla}\right) \times \left(\varphi, \vec{A}\right) = \frac{\partial\varphi}{\partial t} - \vec{\nabla}\cdot\vec{A}\frac{\partial\vec{A}}{\partial t} + \vec{\nabla}\varphi + \vec{\nabla}\times\vec{A}\,.$$
>
> *The last two terms precisely match the known expressions for the electric and magnetic fields $\vec{E}$ and $\vec{B}$"*, Richard Feynman.

Note: (FeR) pp. 200-201: *„Take Hamilton's: the physicists threw away most of this very powerful mathematical system, and kept the part – the mathematically trivial part – that became vector analysis. But when the whole power of quaternions was needed, for quantum mechanics, Pauli re-invented the system in a new form. Now, you can look back and say that Pauli's spin matrices and operators were nothing but Hamilton's quaternions"*, Richard Feynman.

c. Classical physics, relativity, quantum theory, and all that

Barbour, Dicke, Dirac, Einstein, Higgs, Lorentz, Mach, Newton, Plemelj, Prandtl …

Note (Newtonian theory, (PeR4) p. 431): The quantities Newtonian energy, momentum, and angular momentum have a well-defined meaning in Newtonian theory. Their vital importance is that they are conserved – for a system not acted upon by external forces – in the sense that the total energy, momentum, and angular momentum are constant in time. The energy of a system may be considered to be composed of two parts, namely the kinetic energy (i.e. the energy of motion) and the potential energy (the energy stored in the forces between particles). The kinetic energy of a (structureless) particle, in the Newtonian theory, is given by the expression $E_{kin} = \frac{1}{2}mv^2$, where m is the mass of the particle and v is the speed. To obtain the entire kinetic energy, we simply add the kinetic energies of all the individual particles (… we may refer to their energy as heat energy). To obtain the total potential energy, we need to know something of the detailed nature of all the forces involved. Neither the total kinetic energy nor the total potential energy need be individually conserved, but the total is.

The momentum $\vec{p}$ of a particle is a vector quantity, given by the expression $\vec{p} = m\boldsymbol{v}$, where $\boldsymbol{v}$ is the vector describing the velocity. To get the entire momentum, one takes the vector sum of all individual momenta. This total quantity is also conserved in time.

For the Newtonian theory it holds the Galilean relativity. How do our conservation laws manage to survive when neither the energy nor the momentum is left unchanged as we move from one inertial frame to another? … It turns out that conservation of energy and momentum in the first frame goes over to conservation of energy and momentum in the second frame provided we take into account that mass is also conserved.

In Newtonian mechanics there are also other conserved quantities

- The angular momentum
- For a single particle $N = t\vec{p} - m\vec{x}$**.**

Note (Newtonian dynamics): The Newtonian dynamics is governed by the gravitational (Newton) potential at a „point“ in space equipped with a mass m. The reference point, where the potential is zero, is by convention infinitely far away from any mass, resulting in a negative potential at any finite distance. The field of gravity potentials is called the gravitational field. If the field is nearly independent of position the gravitational accelleration g (the standard gravity on the surface of the earth) can be considered constant. In that case, the difference in potential energy from one height to another is, to a good approximation, linear to the difference in height: $\Delta U \approx mg\Delta h$.

Note (Einstein's formula $E = mc^2$): According to Einstein's formula $E = mc^2$ mass and energy are two sides of the same coin; in simple words, there is no mass creation out of energy and the other way around, there is only mass into energy conversion and vice versa; consequently, the distinction between bright matter/energy and dark matter/energy is either nonsense or defines a new kind of energy, which is different from the current two physical-mechanical energy concepts as defined by Leibniz ($E_{kin} = \frac{1}{2}mv^2$ of a moving point in space) and Newton ($\Delta E_{pot} \approx mg\Delta h$; the difference in potential energy from one height to another of two points in space accompanied by the gravitational (Newton) potential at a „point“ in space equipped with a mass m).

Note: (relativistic energy, momentum, and angular momentum, (PeR4) p. 434): Similar as space and time become united in relativity to become the single entity „spacetime“, the momentum and energy become unitied. There is the energy-momentum 4-vector, whose spatial components are $(p^1, p^2, p^3) = c^2\vec{p}$, and whose time-component p^0 measures not only the total energy but also, equivalently, the total mass m of the system according to $p^0 = E = mc^2$, qhich incorporates Einstein's famous mass-energy relation.

Note (relativity and quantum theory): Relativity theory applies to macroscopic bodies, such as stars. Quantum theory has its roots in the microscopic world. The two theories operate with different mathematical concepts - the four dimensional Riemann space and the infinite dimensional Hilbert space, respectively. Therefore, from a mathematical perspective the two theories could not be united, that is, there exists no mathematical formulation to which both of these theories are approximations, while *„all physicists believe that a union of the two theories is inherently possible and that they shall find it*“, (WiE).

Note (Einstein's field equations and the Einstein-Hilbert action functional): The GRT is the probably most prominent example of a theory, which can derived from two conceptually different design processes; it can be expressed in two different ways: Einstein's field equations and the Einstein-Hilbert action functional.

Note (SRT and GRT): The special relativity theory is about the gravitational dynamics in the universe, where each of the affected single „elementary particle" type is modelled as an element of the Minkowski space-time continuum; mathematically speaking, this is a Banach space equipped with an indefinite inner product. The general relativity theory is about the gravitational dynamics in the universe, where each of the affected single „elementary particle" type is modelled as an element of a four-dimensional Riemannian (space-time) *manifold* continuum; therefore, the GRT is a *field on field* theory.

> General relativity is the discovery that spacetime and the gravitational field are the same entity. What we call „spacetime" is itself a physical object, in many respects similar to the electromagnetic field. We can say that GR is the discovery that there is no spacetime at all. What Newton called „space", and Minkowski called „spacetime", is unmasked: it is nothing but a dynamic object – the gravitational field – in a regime in which we neglect its dynamics. …., the universe is not made up of fields on spacetime; it is made up of fields on fields, (RoC).

Physically speaking, the Riemannian *manifold* continuum governs the gravitational movements of all affected mechanical matter/energies in the universe (replacing the Newton potential), while at the same time, those movements influence the curvature („geometry") of the Riemannian manifold. In simple words, physical-mechanical effects (actors on the stage) influence the mathematical framework (the geometry of the stage), while at the same time, the (stage) framework determines the actions of the actors. Needless to say, that in such an actor-stage dynamical world there is no room and opportunity for naturalists to observe the show on stage.

> A purely „geometrodynamics" proclaims a *law without law at the basis of physics*, where it is possible to derive the dynamical equations for matter and fields from the extremely simple but central identity of algebraic topology: the principle that the *boundary of the boundary of a manifold* is zero, (CiL) p. 49.

Note: *Einstein's "general relativity" or ""geometric theory of gravitation" or "geometrodynamics", has two central ideas:*

> *(1) Space-time geometry "tells" mass-energy how to move; and*
> *(2) mass-energy "tells" spacetime geometry how to curve.*

ad (1): We have just seen that the way spacetime tells mass-energy how to move, is automatically obtained from the Einstein field equations by using the identity of Riemannian geometry, known as the Bianchi identity, which tells us that the covariant divergence of the Einstein tensor is zero.

In other words, Einstein geometrodynamics has the important and beautiful property that the equations of motion are a direct mathematical consequence of the Bianchi identities

ad (2): According to an idea of extreme simplicity of the laws at the foundations of physics, what one of us has called „the principle of austerity" or „law without law at the basis of physics", in geometrodynamics it is possible to derive the dynamical equations for matter and fields from the extremely simple but central identity of algebraic topology: the principle that the boundary of the boundary of a manifold is zero", (CiI) p. 49.

Note: The prize being paid for a physical "purely geometrodynamics" interpretation is, (TrH1),

- giving up the fundamental principle of nature, the least action principle
- requiring so-called Einstein spaces
  - gravitation models without sources
  - not identical with SRT-Minkowski space equipped with an indefinite inner product.

Note: Each Hilbert space is a Banach space; each Banach space is a metric space; each metric space is a topological space. However, only the Hilbert space has a geometric structure enabled by the inner product.

Note („Einstein's lost key, „a variable speed of light", (UnA1)): This idea is in line with thoughts and models from Schrödinger, Mach, Dicke, Sciama. Dicke's related theory is in agreement with all known four classical tests of the GRT (light defection, gravitational shift, radar echo delay, perihelion advance of the planet Mercury); although this theory means a huge simplification compared to the GRT, it's the GRT-manifolds-on-manifolds theory were all cosmology theories are refering to.

Note (the four classical tests of the GRT): In (DeH) the four classical tests of the GRT, (1) light defection, (2) gravitational redshift, (3) radar echo delay, (4) the perihelion advance of the planet Mercury, are explained all with variable speed of light, the essential concept of Dicke's theory and Einstein's formula about the „effect of gravitational field" on clocks.

> Einstein's formula says the speed of light near the sun c differs from „normal" speed c_0 only minutely, by a factor litte smaller than 1 that contains both the gravitational potential $\Phi = -\frac{GM}{r}$ (M mass of the sun, r distance from the sun) and the speed of light, c= $c_0(1+\frac{\Phi}{c^2})$, (UnA1) pp. 77, 142.

Note (symmetry and permanent elementary particles): According to the "Big-Bang Theory" in the early universe pressures and temperature prevented the permanent establishment of elementary particles. None of the invented elementary particles of the SMEP were able to form stable objects until the universe had cooled beyond the so-called „supergravity phase". „Symmetry" is thought of as an overall governing concept already existing during the chaos and flux of the early universe, before and during virtual particles are created and destroyed until today. This „symmetry" concept is accompanied by the concept of a „time symmetric, mirror-like quality to every interaction in the early universe". Physical conservation laws governed by this „symmetry principle" limit the possible interactions between particles. Imaginary processes that violate conservation laws are forbidden. So the „existence of symmetry" provides the source of order to the early universe. Technically speaking, the „symmetry" „modelling assumption" of whatever is required to explain E. Schrödinger's order-from-order mechanisms governing regular courses of events in natural sciences. The proposed physical modelling framework is purely based on a mathematical fundamental building block, which is governed by the fundamentally different Snirel'man densities of odd and even integers is.

Note (Dirac's (one system based) radiation theory of an electron): The two-component Maxwell-Mie system provides the concept of a single convection electromagnetic current. This puts the spot on Dirac's (one system based) radiation theory of an electron accompanied by three energy attributes of an „electron" one quantum system, the „*mechanical energy*" of the quantum system, the „*radiation energy*" of the quantum system, and a small remaining „*coupling energy*" between the mechanical and the radiation energy of the quantum system.

Note: Big Bang models follow from a number of (rather simplifying) mathematical assumptions, e.g. (1) homogeneity of space, (2) isotropy of space, (3) *matter can be described as perfect fluid*, (4) laws of physics are the same everywhere, (LaM) p. 7. The most advanced mathematical modelling framework for galactic dynamics is about „*equilibria of collisionless systems*", ((BiJ). Nearly all of the unicerse's matter is „*in plasma state*". However, the most advanced plasma dynamics models are about statistical theories, (e.g. accompanied by the Fokker-Planck equation and the Vlasov equation) or Magnetohydrodynamics assuming a macroscopic hydrodynamic bahavior of the plasma gas, (CaF). In both cases, the fundamental defining property of a plasma gas is neglected, that there are approximately the same (arbitrarily small or large) numbers of positive and negative charge carries.

Note: According to the "Big Bang Theory" in the early universe pressures and temperature prevented the permanent establishment of elementary particles. None of the invented elementary particles of the SMEP were able to form stable objects until the universe had cooled beyond the so-called "supergravity phase". "*At the end of the famous first three minutes after the Big Bang the universe was made up of mainly light, neutrinos and anti-neutrinos*", (WeS).

Note: The "*universe formation*" of plasma matter is governed by the "*Landau damping*" effect, which therefore becomes a characteristic of "*plasma quanta dynamics*".

Note: Einstein's mass-energy conservation law $E = mc^2$ and the definition of temperature in the form $\frac{1}{T} = k \cdot \frac{1}{W}\frac{dW}{dE}$ are only valid for mechanical energy Hilbert space governed laws., e.g. (FiW), (VoH).

Note: On the macroscopic level potential is simply energy per mass:

> *„The enigmatic formula $G \approx c^2 \frac{R_U}{M_U}$, M_U, R_U total mass resp. the visible radius of the universe, can be interpreted as the equivalence (in this superfical view) of the kinetic and potential energy of the universe. … Whereas the relation $G \approx c^2 \frac{R_U}{M_U}$ as such is only numerical, Schrödinger went one step further and realized that the concept of the gravitational potential was concealed in the*

formula. Potential is simply energy per mass, for which Newton had derived an expression in his theory of gravitation: $\varphi = -\frac{GM}{r}$*, when a mass is at a distance* r *from the sun with mass* M*,"* (UnA1) p. 117, see also (UnA2) p. 69.

Note: The article (DeH) explains all known tests of the (GRT) theory with variable speed of light with the following conclusion:

> *„Es wäre demnach konsequent, den Gültigkeitsbereich der allgemeinen Relativitätstheorie grundsätzlich auf das makroskopische Verhalten der Körper einzuschränken und darauf zu verzichten, die Raum-Zeit-Struktur der allgemeinen Relativitätstheorie bis in die Dimensionen der Elementarteilchen und Atome fortzusetzen*".

Note: Dirac's „Large Number Hypothesis" links the size and mass of the universe with the ratio of the two forces at work when a „proton" and an „electron" in a hydrogen atom orbit one another, (UnA1) p. 152.

Note: *„It is my conviction that general relativity is deeply Machian in a sense that unfortuntely Einstein never managed to pinpoint accurately and that preciscly this very Machian nature of general relativity is the main cause of the difficulties that stand in the way of its quantization*", (BaJ) p. 571.

Note: Dirac's observations leading to his large numbers hypothesis, ((UnA) p. 255) is about the coincidence of the two relations $\frac{M_U}{m_p} \approx \frac{R_U^2}{r_p^2} \approx [10^{39}]^2 = 10^{78}$ and $\frac{F_e}{F_g} = \frac{3 \cdot 1836 \cdot M_U \cdot r_p}{2 \cdot 137 \cdot R_U \cdot m_p} \approx 2{,}3 \cdot 10^{39}$; ($r_p \approx 0{,}84 \cdot 10^{-15} m$ resp. $m_p \approx 1{,}6726 \cdot 10^{-27} kg$ denote the radius resp. the mass of the proton, and F_e resp. F_g denote the electric resp. the gravitational force). It establishes the connection between cosmology and elementary partices formulated in the form $\frac{F_e}{F_g} \approx \frac{R_U}{r_p} = \tau$, (‚epoch'), (UnA2) pp. 73/79/88.

Note (DiP2): *„By measuring along the time-axis with respect to which the matter in the neighbourhood of the point is at rest, we get an absolute measure of time* (of the visible universe horizon), *called the epoch*"; *… only the difference of two epochs can enter into laws of nature …*

Note: *„The theory of Maxwell and Lorentz cannot hold for the interior of the electron; therefore, from the point of view of ordinary theory of electrons we must treat the electron as something given* a priori*, as a foreign body in the field. A more general theory of electrodynamics has been proposed by Mie, by which it seems possible to derive the matter from the field",* (WeH1) pp. 206-208.

Note: *„The current understanding of the relationship of physics and chemistry may be briefly sketched by the statement that*

> „the valence bonds are an abbreviated symbol for the actual quantum-physical forces acting between the atoms, which themselves are complex dynamical system", (WeH) p. 266.

Note: *„The current understanding of the relationship of physics and biology may be briefly sketched by the statement that*

> „One of the profoundest enigmas of nature is the contrast of dead and living matter. …. Incidentally, the gap between organic and inorganic matter has been bridged to a certain extent by the discovery of virusses. Virusses are submiroscopic entities that behave like dead inert matter unless placed in certain living cells. …. Many virusses have the structure typical of inorganic matter; they are crystals", (WeH) p. 276.

d. Electromagnetism, quantum electrodynamics, and all that

Note: Half of the four Maxwell equations,

$$div(\vec{B}) = 0\ ,\ rot(\vec{E}) + \frac{\partial}{\partial t}\vec{B} = 0\ ,$$

are „just" a mathematical consequence of the definition of the magnetic field $\vec{B}$. They are derived via a differentiating process, applying the div- resp. the rot-operator to the definition of the magnetic field $\vec{B} := rot\vec{A}$, whereby $\vec{A}$ denotes an arbitrary (differentiable) vector field. In other words, there are no magnetic charges foreseen telling the fields, how to vary, (SuL).

The other half of the Maxwell equations,

$$div(\vec{E}) = \rho, rot(\vec{B}) - \frac{\partial}{\partial t}\vec{E} = \vec{j}\ ,$$

are the consequences of a more specifically defined vector field $\vec{A}$. In this case there is an underlying scalar field of $\vec{A}$ regarding the time variable, reflecting the space-time geometry structure. It enables the definition of an electric field $\vec{E}$ given by, (SuL)

$$\vec{E} := -\frac{\partial \vec{A}}{\partial t} - grad(A_0).$$

In other words, only electric charges tell the electro-magnetic fields, how to vary. Reversely, there is only the Lorentz force

$$\vec{F} = e(\vec{v} \times \vec{B}),$$

where „the magnetic field tells the electrons, how to move". From a physical modelling perspective, this „imbalance" challenge has been overcome by the concept of „displacement current".

Note: The Maxwell equations unify the behaviour of electric fields, magnetic fields, and even the light; they are the first of the relativistic field equations, (PeR4) p. 441. The vanishing divergence of the charge-current vector provides the equation of conservation of electric charge in spacetime. The reason that it is referred to as a „conservation equation" comes from the mathematical theorem of exterior calculus accompanied by an integration over a closed 3-surface Q in a Minkowski space, (PeR4) p. 446.

Note (the energy tensor of the electromagnetic fields): The energy tensor of the electromagnetic fields is only known outside of the electrons.

Note: Maxwell's equations determine the electromagnetic field, when the distribution of electric charges and currents is known. However, the laws which govern the currents and charges are not known:

> „We do know, indeed, that electricity consists of elementary particles (electrons, positive nuclei), but from a theoretical point of view we cannot comprehend this. We do not know the energy factors which determine the distribution of electricity in particles of definite size and charge, and all attempts to complete the theory in this direction have failed. If then we can build upon Maxwell's equations at all, the energy tensor of the electromagnetic field is known only outside the charged particles. In these regions, outside of charged particles, the only regions in which we can believe that we have the complete expression for the energy tensor, we have $\frac{\partial T_{ij}}{\partial x_j} = 0$." (EiA4).

Note (electro-magnetohydrodynamics): MHD is concerned with the motion of electrically conducting fluids in the presence of electric or magnetic fields. In MHD one does not consider velocity distributions. It is about notions like number density, flow velocity and pressure. The MHD equations are derived from continuum theory of non-polar fluids with three kinds of balance laws:

- conservation of mass/energy
- balance of angular momentum (Maxwell equations)
- balance of linear momentum.

Note (F. Ehrenhaft's photophoresis): Classical theoretical physics does not known about magnetism, (RoH). F. Ehrenhaft's discovery of the „photophoresis" phenomenon discovery is still neglected, (EhF) p. 243.

> Ehrenhaft's „photophoresis" is about light inducing not only electric but also magnetic charges (poles) upon the particles if they are illuminated by concentrated light preponderantly shorter wave lengths.

The proposed electromagnetic dynamical quanta field pair provides an appropriate model for F. Ehrenhaft's discovery.

Remark: In the proposed framework Maxwell's „line current" (of a conductor) and the related sophisticated (time- and initial-value depending isolator) „displacement current" (both restricted to the cross section area of the imaginary „semi-conductor line") are replaced by truly „mechanical energy" based electricity and magnetism fields governed by an overall conservation of total (mechanical and dynamical) energy.

Note: Mie's theory is about an electric pressure field counterbalancing the electricity field E of the Maxwell equation.

> In the statical case Mie's equation states that $\mathrm{E} - \mathrm{grad}(\Phi) = 0$ that is, the electric force E is counterbalanced in the ether by an „electrical pressure" Φ, (WeH1) p. 206 ff.

Note: The Mie-Theory was proposed by G. Mie to modify the Maxwell equations to solve the underlying problem of matter by explaining why the field possesses a granular structure and why the knots of energy remain intact in spite of the back-and-fourth flux of energy and momentum", (WeH) p. 171. The underlying problem of matter of the Maxwell equations is, that they cannot hold the interior of the electron. (WeH1) p. 206 ff..

Note: The Mie Theory is basically about a new physical concept of "*cohesive electric pressure*". It overcomes current modelling challenges of the Maxwell equations concerning the electromagnetic currents. The Maxwell-Mie model of the *„electromagnetic quanta dynamics*" may be interpreted as "*cohesive electric & magnetic pressures*". It is in line with Leedskalnin's claim, that magnetic and electric current is (basically) the same. It delivers an explaning of Ehrenhaft's discovery of the photophoresis phenomenon and his related observations that the movement of „light particles" in a field (in combination with an occuring *centripetal force*) do not run in straight lines, but run in paths in extremely regular forms, sizes and orbital frequencies, (EhF), (EhF1), (LeE). Ehrenhaft's observation is also in line with Schauberger's "screw movement and implosion theory", that Nature tries to prevent straight movement prefering planetary resp. cycloidal movements, (AlO) p. 222, (LaS) S. 226.

Note: In the Maxwell theory and the related SMEP the spin of an elementary particle is its eigen-rotation with exactly two rotation axles, one parallel and one anti-parallel axis to a magnetic field. This is the 2×2 complex number scheme $SL(2, C) \cong SU(2)$, where every „normal" rotation is contained twice. Consequently, an electron has a charge only half of the Planck's quantum of action. It is applied in describing the transformation properties of spinors.

Note: The experimental observations of the spectra of atoms and their decomposition into magnetic and electric fields showed a decomposition of spectral lines or of electron beams into an even number of components, while the angular momentum multiplets were only composed by an odd number of multiplets with the numbers $2l + 1$, (RoH) p. 217.

Note (quantum electrodynamics, or the theory of the Lamb shift): Whereas Newton's theory of gravitation still had obvious connections with experience, experience entered the formulation of matrix mechanics only in the refined or sublimated form of Heisenberg's prescriptions. The quantum theory of the Lamb shift, as conceived by Bethe and established by Schwinger, is a purely mathematical theory and the only direct contribution of experiment was to show the existence of a measurable effect. The agreement with calculation is better than one part in a thousand, (WiE).

Note (gauge bosons; field quanta): Gauge bosons arise spontaneously without external influence and you can freely select certain parameters locally without anything changing of the related interaction.

e. Atomic nuclei, nucleons, neutrino, ground state of deuteron, and all that

Basic facts about nuclei

(BeH) pp. 1-6

Each atomic nucleus has a charge Ze, a mass M, and a mass number A. Ze is an integral multiple of the charge e of the proton. M is very close to an integral multiple of the proton mass. The integer A which gives the multiple closest to M is the mass number.

The nuclear charge Z determines all the chemical properties associated with an element. It has values from $Z = 0$ (neutron) to $Z = 101$ for observed nuclei. Some of these do not occur in nature: $Z = 0, 43, 61, 85, 87$ (87 occurs in very small abundance as a member of a branch of the radioactive family of Ac), 93 through 101.

The mass number A ranges from $A = 1$ (proton or neutron) to $A = 255$. Nearly every mass number in this range is found in nature. There are good reasons why notable exceptions $A = 5$ and $A = 8$ are not stable long enough to be observed even in the laboratory. The mass numbers of form $4n + 1$ beyond 209 (Bi) are not found in nature, but many of them have been produced in the laboratory. These nuclei belong to a radioactive series which does not contain any long-lived members and, therefore, could not have survived on earth.

Isotopes.
Nuclei of the same Z, but different A are called isotopes. On the average there are about three stable isotopes for each Z. To distinguish isotopes A is usually written as a right superscript, and for convenience Z is sometimes written as a left superscript. To illustrate: Si^{28} and Si^{30} are stable isotopes of Si. In addition to the stable isotopes, most elements also possess radioactive isotopes; e.g., Si has the known isotopes Si^{27} and Si^{31}. Of these, Si^{27} is β^+-radioactive (having too little mass for its charge) and decay with a half-life of 4 seconds to Al^{27} and a positron

$$Si^{27} = \beta^+ + Al^{27}$$

Si^{31} (having too little charge for its mass) decays with a half-life of 170 minutes to P^{31} and an electron

$$Si^{31} = \beta^- + P^{31}$$

Isobars.
For a given A, there may well be several possible values of Z (isobars). There are many instances of stable isobaric pairs, e.g., ${}^{16}S^{36}$ and ${}^{18}A^{36}$, or ${}^{44}Ru^{104}$ and ${}^{46}Pd^{104}$, and some stable isobaric triples, e.g., ${}^{40}Zr^{96}$, ${}^{42}Mo^{96}$ and ${}^{44}Ru^{96}$, as well as numerous radioactive isobars.

Regularities.
There are several stricing regularities in a table of stable nuclei. Nuclei of even Z are much more numerous than those of odd Z. Nuclei of even A are more numrious than those of odd A; the exceptions are ${}^{1}H^{2}$, ${}^{3}Li^{6}$, ${}^{5}B^{10}$, and ${}^{7}N^{14}$. (There are also ${}^{19}K^{40}$, and ${}^{71}Lu^{176}$, but these are not properly stable, being β-radioactive with very long lifetimes). The fact that nuclei with odd Z cannot have even A with the listed exceptions is what makes stable nuclei with even Z more numerous than those with odd Z, for a nucleus with even Z may have A either odd or even. Für odd A, there are apparently no preference between even Z and odd Z.

Table IA (Sample of Isotope Statistics) illustrating all three rules.

Z	Number of Stable Isotopes	Number of Odd A	Number of even A
48	8	2	6
49	2	2	0
50	10	3	7
51	2	2	0

Energy.
In considerations involving the energy of nuclei the mass M is important. According to Einstein's relation, the energy equivalent of a change in mass ΔM is

$$\Delta E = \Delta M c^2.$$

Such changes in mass occur when protons and neutrons are changed from one configuration to another in which they are bound more or less strongly.

Modern mass spectrographic techniques permit the determination of M to better than 1 part in 10^6 which just makes possible the determination of the decrease in the atomic weight of a heavy atom due to the binding of the electrons in the field of the nucleus.

For example, from the atomic weight scale based on O^{16}

$$M(O^{16}) = 16.00000$$

Nuclear reaction data give $M\left({}^1H^1\right) = 1.008142$, $M\left({}^0n^1\right) = 1.008982$. On the superposition that the O^{16} nucleus is made up of 8 protons and 8 neutrons, the binding energy is

$$8M\left({}^1H^1\right) + 8M\left({}^0n^1\right) - 16.00000 = 0.13699 \text{ mass units.}$$

It is to be noted that the masses of the neutral atoms O^{16} and ${}^1H^1$ are used here. The justification for this is the masses of 8 electrons of the O^{16} are canceled in the calculation by the masses of the 8 electrons of the hydrogen. (The change in the mass of the 8 electrons, due to their stronger binding around the O^{16} nucleus, is beyond the experimental error in the mass determinations.) Two quantities sometimes useful in describing the binding energy of the nuclei are:

$$\text{Mass defect} = \Delta = A - M(A) \text{ ; Mass excess} = -\Delta \text{ ; Packing fraction} = f = -\Delta/A$$

Consider now a nuclear reaction

$${}^3Li^7 + {}^1H^1 \rightarrow {}^2He^4 + {}^2He^4$$

Both the mass number and charge balance. In addition, mass-energy conservation must hold. The balance sheet is as follows:

Initial mass:		
$M({}^3Li^7)$	7 mass units	+ 16.97 Mev
$M({}^1H^1$	1 mass unit	+ 7.58 Mev
Total	8 mass units	24.55 Mev
Final mass:		
$2\ {}^2He^4$	8 mass units	+2 x 3.61 Mev
Mass decrease=energy decrease		17.33 Mev

We have used the mass defect in energy units, from the Appendix.

The relation $E = Mc^2$ gives the conservation factor from the mass units to electron volts of energy

$$1 \text{ millimass unit} = 0.93114 \text{ Mev}$$

The mass decrease here is then $= 0.01862$ mass units. If the Li and H had little velocity, the α-particles would fly off nearly opposite directions, each carrying 8.67-Mev kinetic energy. Systematic observations of reactions such as this have verified the Einstein relation very accurately over a great range of nuclear phenomena and are one of the strongest bulkwarks of the special theory of relativity. In all nuclear reactions involving heavy particles only, energy has been found to be strictly conserved.

Stability.
For a nucleus to be stable it must have a mass which is less than the combined masses of any pair of nuclei made by subdividing it. For example, ${}^3Li^7$ is stable against the subdivision

$$ {}^3Li^7 \rightarrow {}^2He^4 + {}^1He^3 $$

Because $M({}^3Li^7) = 7.01882$ and $M({}^2He^4 + {}^1He^3) = 4.00387 + 3.01700 = 7.02087$. ${}^2He^5$ is unstable because the decomposition

$$ {}^2He^5 \rightarrow He^4 + n^0 $$

is energetical possible. The mass of He^5 can be found by studying the reaction

$$ Li^7 + H^2 \rightarrow He^4 + He^5. $$

Knowing the masses $M(Li^7)$, $M(H^2)$, and $M(He^4)$, and measuring the kinetic energy and momentum of Li^7, H^2, and He^4, we can determin the mass of He^5. It is 5.0137 mass units. (This is 0.9 millimass unit greater than $M(He^4) + M(n^0)$. (There is the possiblity that the measured mass of He^5 might not be for the ground state, but in all known nuclear reactions involving heavy particles, whenever a reaction yields an excited state, it also yields the ground state. Since the experiment gives a unique mass, it is presumed to correspond to the ground state.) Li^7 is unstable to the decomposition $Li^5 \rightarrow {}^2He^4 + He^1$ and Be^8 to the decomposition $Be^8 \rightarrow He^4 + He^4$. This explains the absence of nuclei of mass numbers 5 and 8 which was mentioned above.

Fundamental particles in nuclei.
The word nucleon is often used to refer to either a proton or a neutron. Present idea are that a nucleus is composed of nucleons: Z protons and $A - Z$ neutrons. This replaces older conceptions of a nucleus made up of protons and electrons. Thus the binding energy of any nucleus will be

$$ \text{Binding Energy} = (Z)M({}^1H^1) + (A - Z)({}^0n^1) - M(A, Z). $$

It is fairly certain now that the nucleons within nuclear matter are in a state made rather different from their free condition by the proximity of other nucleons. The exchange of transient mesons, for example, is a typical picture of the mode of interaction of nearby nucleons. Such processes cannot affect over-all energy relationships if initial and final states are precisely defined. The binding energy is just the energy difference between the assembled nucleus and a collection of stationary, well-separated nucleons having Z protons and $A - Z$ neutrons.

β-Disintegration

(BeH) p. 25

Negative β-disintegration consists in the conversation of a neutron into a proton and an electron. Since all three particles are assumed to have spin(1/2) and Fermi statistics, this reaction will not conserve spin and statistics unless it is assumed that an additional particle of spin(1/2) and Fermi statistics is emitted. To conserve charge this particle must be neutral. It is also clear that its mass must be small, and it is therefore called *neutrino*.

The emitted β-particles are found to have a continuous distribution of energies, up to a certain maximum E_0, rather than a single energy. The neutrino is therefore also needed to conserve energy; it is assumed to take the remaining energy, $E_0 - E$, where E is the electron energy. This hypothesis is strongly supported by the fact that the maximum electron energy is found within experiment error to be equal to the energy available for the reaction, as determined from mass data. This shows also that the neutrino mass must be assumed neglegible. Experimental data directly supporting this assertion may be found, for instance, in the β-disintegration of H^3 into He^3 with low and well-measured maximum electron kinetic energy of only 18.5 ± 0.5 kev. Careful measurements have been made the threshold energy of the reaction $H^3(p, n)H^3$. The incident proton energy for threshold was measured and corrected to center-of-mass coordinates by the factor ¾.

The mass-energy

$$763.7 \pm 1\ kev + \ H^3 + \ H^1 = H^3 + n$$

or

$$He^3 - H^3 = 763.7 - (n - H^1) kev = -19.3 \pm 1.3\ kev \ \text{(measured)}$$

where the neutron-proton mass difference is obtained from other reactions not involving neutrinos and from mass-spectrographic data and is $783 \pm 1.5\ kev$.

The energy available for β-emmision is just the mass difference $H^3 - He^3$, so that

$$E_{available} = +19.3 \pm 1.3\ kev.$$

This checks with the observed maximum electron energy, and the energy balance proves directly that the neutrino mass must be less than 1 or 2 kev, a range fixed by the uncertainties of measurement. By use of the formalism of β-spectrum, an even closer limit on the neutrino mass can be obtained; viz., it is certainly less than 1 kev.

Qantitative Theory of Nuclear Forces
(BeH) p. 33

The theory of nuclei is to be contrasted with the theory of atoms. In the latter the principle force between the constituent particles, electrons and nuclei, was known when the theory got under way, and the problem was to find the proper mechanics to describe the motion of those particles under the given force; quantum mechanics is the answer of this problem. In nuclei, there are a good reasons to believe that quantum mechanics is correct (the success of the Gamow theory of α-particles decay is one example), but the forces are unknown.

Non-Additivity of the Nucleon Moment
(BeH) p. 39

The whole discussions of magnetic moments has assumed that the moments measured for the free nucleons are entirely unchanged for the nucleons interacting in the deuteron. Relativistic effects of nucleon motion and the picture of the possible transient presence of other particles with different charges and magnetic polarization densities makes perfect additivity and absence of deviations from the free nucleon values seem rather unrealistic assumptions. Clear evidence for other contributions to the magnetic moment comes from the very accurate measurements on the heavier nuclei beyond the deuteron, H^3 and He^3.

Ground State of the Deuteron
(BeH) p. 42

The most important experimental basis for the theory of the deuteron is its binding energy. This can be obtained by using the photodisintegration of deuterons,

$$H^3 + h\nu \ \rightarrow H^1 + \ {}^0n^1$$

This reaction takes place when $h\nu$ is greater than the binding energy of the deuteron; the difference between $h\nu$ and the binding energy appears as kinetic energy of the neutron and the proton. Because the momentum of the γ-ray is so small, the momenta of proton and neutron are very nearly equivalent and opposite, and, since their masses are almost exactly equal, they share the excess energy, $h\nu$ minus binding energy, very nearly equally. The energy E of the proton can be determined by measuring the total ionization it produces or by measuring its range. The binding energy is then $h\nu - 2E$.

Capture of Neutrons by Protons
(BeH) p. 81

This is the process inverse to photodisintegration. The cross section for capture can be obtained from that for photodisintegration by statistical considerations such as those that follows.

Saturation of Nuclear Forces
(BeH) pp. 108-109

The binding energy and volume of nuclei are proportional to A, the mass number. This is not in accord with a law of force which gives equal interactions between all pairs of particles in the nucleus, for there are then $\frac{A(a-1)}{2}$ might be expected, if not a higher power of A due to increased packing with more interaction. Instead, the nucleus binding energy seems similar to the integral energies of bulk matter, in which 2 pounds has twice as much energy and volume as 1 pound.

To accout for this phenomeon of „saturation of nuclear forces," in which one particle apparently interacts with only a limited number of others, various hypothesis have been made, and various other assumptions about the nature of the forces can be shown to be impossible.

Among the impossible assumptions is that which has been used in this book so far, namely, an ordinary potential, attractive at all distances and independent of the angular momentum, because it is easily shown that such a potential does not give saturation. This is so even if the Coulomb repulsion of the protons is taken into account. The proof can be carried out with various degrees of exactness, using the variational method. This method is based on the Schrödinger variational problem which states that the quantity

$$\Omega = \int \psi H \psi d\tau \,/ \int \psi^2 d\tau$$

is a minimum when ψ is the correct eigenfunction of the most lowest eigenvalue E_0 of H, and the minimum value of Ω is E_0. Thus, if the assumed Hamiltonian operator representing the interaction of the particles in a given nucleus is sandwiched between any arbitrary ψ in this expression for Ω, the value of Ω must be greater (i.e., less negative) than the correct energy of that nucleus. The simplest ψ's are plane waves inside a box representing the nucleus. If the size of the box is adjusted to give as low an Ω as possible, this size comes out about equal to the range of nuclear forces, which is clearly much too small. Further, it gives a potential energy proportional to A^2, and a kinetic energy proportional to $A^{5/2}$. The size of the coefficients of these powers is such that the potential energy dominates for $A > 50$; for $A = 238$ the binding energy is greater than 238 mass units. This is convincing evidence tha the ordinary potential will not work, and this is true independently of the shape of the potential (square well, exponential, Gaussian, etc.).

The Shell model
(BeH) p. 162

The Fermi gas model is the simplest form of a wide class of models for many particle systems. These models may be called single-particle models, for they treat the interacting system as a collection of independent single particle systems, each of which moves in a potential which is in fact determined by the average motion of all other particles. Atoms are treated best by this model (Hartree model), at least when somewhat extended to take care of the Pauli principle. For the atomic electrons, interacting through a long-range smooth Coulomb force, and with a well-defined center of force, the nucleus, to account for most of the potential, this model is immediately attractive and works as well one might guess. But for the short-range nuclear forces, with no particular center of force, the model appears hard to justiy. Nevertheless, because of its simplicity the Hartree model was tried from early days of nuclear physics. But only in 1949 (M. Mayer, H. Jensen, et al.) was the special scheme hit upon which makes the independent particle model so successful that with its aid we can predict most of the properties of the first few states of nearly every individuel nuclear species.

The atomic shell model effects show up most obviously in the closed-shell structure of the noble gases, whose high stability is apparent. Like-wise for nuclei, though perhaps not so markedly, the special stability of particular numbers of neutrons, protons, and both, is easy to see and has been pointed out since 1934 (Elsässer). The stability of the α-particle and of O^{16} can be seen by a glancé at their binding energies compared to those of the neighboring nuclei, and similar, less quantitative arguments point to the special stability of nuclei for which either N or Z (or indeed both) have values $2, 8, 20, 28, 50, 82$ and 126. These are jocularly called „magic" numbers, and their effects can be found by careful examination in the wide variety of nuclear properties.

β-Disintegration
(BeH) p. 217

Fermi introduced a new interaction between the nucleon and the two light particles, electron and neutrino. His interaction was chosen in analogy with the interaction between charges and electromagnetic fields in quantum electrodynamics. The heavy particles are to act as sources and sinks of the light particles.

Electron capture
(BeH) p. 230

In this type of β-transition an atomic orbital electron is absorbed by a nuclear proton, changing the proton into a neutron, and resulting in the emission of a neutrino. The neutrino alone carries off the full energy of the transition and is therefore monoenergetic. Examples of electron capture are given in Chapter III, and an experiment which measures the recoil momentum given to the nucleus by the outgoing neutrino is described in Chapter VI.

Neutrino and Antineutrino
(BeH) p. 231

The reactions of β-transition may be summarized as shown:

β^-decay $\quad n^* \rightarrow \beta^- + p^* + \nu_1 \quad$ (a)

β^+decay $\quad n^* \rightarrow \beta^+ + n^* + \nu_2 \quad$ (b)

Electron capture $p^* + \beta^- \rightarrow n^* + \nu_2 \quad$ (b‘)

where the asterik denotes that the nucleon may not be free, but bound into a nucleus. Reaction (b) and (b‘) are fully equivalent, and as we have seen may be in fact compete for the decay of one and the same nuclear state. There is evidently a particular relationship between the role of the electron and that of the positron in these two reactions; the emission of a positron in reaction (b) is fully equivalent to the absorption of a negative electron in reaction (b‘). The electron and positron differ, of course, in other respects; their electric charge and magnetic moments are equal in magnitude but opposite in sign. Two spin(1/2) particles so related are said to be particle and the corresponding antiparticle: the Dirac theory predicts that such a pair will exist for every type of spin(1/2) elementary particle. The creation of such a pair by γ-rays is well known of course for electrons and positrons.

If the spin(1/2) particle, the neutrino, has this property, then it too will have a counterpart, the antineutrino. In that case, the neutral particle ν_1, emitted in ordinary β^--decay (which we will call the neutrino) can be absorbed in the reaction:

$$p^* + \nu_1 \rightarrow n^* + \beta^+.$$

Since this is exactly the reaction obtained from reaction (b) by considering the absorption of a neutrino ν_1, to be equivalent to the emission of an antineutrino, ν_2. But a neutrion could *not* be absorbed to induce a β^-decay; for that, the reaction must read instead

$$n^* + \nu_2 \rightarrow \beta^- + p^*$$

obtained from reaction (a) in the analogous way; an incident *anti-neutrino* is required.

Some further notes

Note: The α-radiation consists of α particles, i.e. ${}_2^4H$ nuclei. The first characterization of a neutron (with mass number 1 and order number 0) was given by J. Chadwick in the form

$$ {}_4^9Be + {}_2^4H \rightarrow {}_6^{12}C + {}_0^1n. $$

Note: An α-decay is only possible for nuclei with mass numbers > 208 and order numbers > 82. Two examples of α-decays are given by

$$ {}_{84}^{210}Po \rightarrow {}_{82}^{206}Pb + {}_2^4H, \quad {}_{90}^{228}Th \rightarrow [{}_{88}^{224}Ra]^* + {}_2^4H. $$

Note: The γ-radiation triggers no change of mass and order numbers. It is accompanied by a line spectrum occuring when electrons change between the several energy layers of the electron shell. Two examples of γ-decays are given by

$$ [{}_{52}^{125}Te]^* \rightarrow {}_{52}^{125}Te + \gamma, \quad [{}_{88}^{224}Ra]^* \rightarrow {}_{88}^{224}Ra + \gamma. $$

Note: The $\beta^- \cong {}_{-1}^0e$-decay is an electron emission process. The ${}_{-1}^0e$ contains no nucleon. A neutron can change into a proton when it emits an electron (the proton with the nucleus is denoted by ${}_1^1p$; the free proton is denoted by ${}_1^1He$). If a β^- particle is emitted the number of neutrons in the nucleus is reduced by 1 and the number of protons is increased by 1, i.e., the total number of nucleons keeps unchanged, and the ratio between neutron and protons decreases.

Note: An $\beta^+ \cong {}_1^0e$-decay is an electron capturing process resp. a positron emission process (also the ${}_1^0e$ contains no nucleon). It only happens in case of instable nuclei with a small neutron/proton ratio, which do not occur in nature. The emission of a positron changes a proton into a neutron, i.e. the number of neutrons in the nucleus increases by 1 and the number of protons decreases by 1.

Note (Artificially created β^+-decay and *neutrinos*): Additionally a nuclide with a large number of protons accompanied by an appropriate ratio of neutrons and protons can also emit positrons. This process is accompanied by the radiation energy of a neutrino and x-ray.

Note: The radioactive nuclide ${}_6^{14}C$ permanently arises in the atmosphere from cosmic neutron radiation and nitrogen (and Wilson's „cloud chamber" observation) are modelled by

$$ {}_7^{14}N + {}_0^1n \rightarrow {}_6^{14}C + {}_1^1H, \qquad ({}_7^{14}N + {}_2^4He \rightarrow {}_8^{17}O + {}_1^1H), $$

while the ${}_6^{14}C$ will be oxidized in the atmosphere to CO_2 and vanishes by β^- decay according to

$$ {}_6^{14}C \rightarrow {}_7^{14}N + {}_{-1}^0e\,. $$

f. The real and complex Lorentz groups

(StR)

The Lorentz transformation in special relativity is a simple type of rotation in hyperbolic space.

The Lorentz group L has four components, each of which is connected in the sense that any point can be connected to any other, but no Lorentz transformation in one component can be connected to another in another component. One of this components is the restricted Lorentz group, which is the group of $2x2$ complex matrices of determinant one, $SL(2,C)$. It is isomophic to the symmetry group $SU(2) \cong SL(2,C)$, containing as elements the complex-valued rotations, which can be written as a complex-valued matrix of type

$$\begin{pmatrix} a+ib & c+id \\ -c+id & a-ib \end{pmatrix} \text{ with determinant one.}$$

It is important in describing the transformation properties of spinors. In SMEP the group $SU(2) \cong SL(2,C)$ describes the weak force interaction with 3 bosons W^+, W^-, Z.

Another group associated with the Lorentz group L is the complex Lorentz group $L(C)$, (*). It has just two connected components, $L_+(C)$ and $L_-(C)$. The transformations 1 and -1, which are disconnected in L are connected in $L(C)$. In other words, the complex Lorentz transformation connects

- the two components containing the 1-transformation and space-time inversion
- the two components containing the space inversion and the time inversion.

Just as the restricted Lorentz group is associated with $SL(2,C)$, the complex Lorentz group is associated with $SL(2,C) \otimes SL(2,C) \cong SU(2) \otimes SU(2)$. There is also a two-to-one homomorphism from $SL(2,C) \times SL(2,C)$ onto $L(+,C)$.

The spin of an elementary particle is its eigen-rotation with exactly two rotation axes, one parallel and one anti-parallel axis to a magnetic field. This is the 2×2 complex number scheme, where every „normal“ rotation is contained twice. Consequently, an electron has a charge only half of the Planck's quantum of action.

(*) The complex Lorentz group is composed of all complex matrices satisfying

$$\Lambda^{\kappa}{}_{\mu}\Lambda_{\kappa\nu} = g_{\mu\nu} \text{ or } \Lambda^T G \Lambda = G, \qquad (1\text{-}5).$$

It has just two connected components, $L_+(C)$ and $L_-(C)$ according to the sign of det(Λ). The transformations 1 and -1, which are disconnected in L are connected in $L(C)$. In other words, the complex Lorentz transformation connects

- the two components containing the 1-transformation and space-time inversion, i.e. the pair
$$\{\det(\Lambda) = +1\,, \det(\Lambda^0{}_0 = +1)\}, \{\det(\Lambda) = +1\,, \det(\Lambda^0{}_0 = -1)\},$$
- the two components containing the space inversion and the time inversion, i.e. the pair
$$\{\det(\Lambda) = -1\,, \det(\Lambda^0{}_0 = +1)\}, \{\det(\Lambda) = -1\,, \det(\Lambda^0{}_0 = -1)\}.$$

Summary:

While two (real) Lorentz transformations need to be connected to one another by an appropriately defined continuous curve of Lorentz transformations, there are two pairs of components of the complex Lorentz transform, which are both already connected by definition.

Just as the restricted Lorentz group is associated with $SL(2,C)$, the complex Lorentz group is associated with $SL(2,C) \otimes SL(2,C)$. The latter group is the set of all pairs of $2x2$ matrices of determinants one with the multiplication law

$$\{A_1, B_1\} \cdot \{A_2, B_2\} = \{A_1 A_2, B_1 B_2\}.$$

Is is easy to see that only matrix pairs which yield a given $\Lambda(\mathrm{A},\mathrm{B})$ are $(\pm\mathrm{A}, \pm\mathrm{B})$. In particular,

$$\Lambda(-1,1) = \Lambda(1,-1) = -1.$$

The corresponding complex Poincare group admits complex translation but also the multiplication law

$$\{a_1, \Lambda_1\} \cdot \{a_2, \Lambda_2\} = \{a_1 + \Lambda_1 a_2, \Lambda_1 \Lambda_2\}.$$

It has two components $\mathrm{P}_{\pm}(C)$, which are distinguished by $\det(\Lambda)$ and a corresponding inhomogeneous group to $SL(2,C)$.

Note: The S^1 and S^3 are the only spheres with a "continuous" group structure.

Note: The complex Lorentz transformation $S^3 \times S^3 \cong SU(2) \times SU(2)$ is the set of all pairs of 2×2 matrices with determinant one, which are connected by a multiplication law. It is the hidden symmetry group of the Coulomb problem, (RoH) p. 163.

Epimorphisms $\boldsymbol{S^3 \to SO(3)}$, $\boldsymbol{S^3 \times S^3 \to SO(4)}$

The group S^3 denotes the unit quaternions of the quaternion algebra $|\boldsymbol{H})$. With every $a \in S^3$ and with every $(a, b) \in S^3 \times S^3$ we may associate the orthogonal mappings

$$\varphi(a) : Im|\boldsymbol{H} \to Im|\boldsymbol{H}\,, x \to axar{a} \quad , \quad \psi(a,b) : |\boldsymbol{H} \to |\boldsymbol{H}\,, x \to ax\bar{b}$$

We consider the mappings $\varphi : S^3 \to \mathrm{O}(|\boldsymbol{H})$, $\psi : S^3 \times S^3 \to \mathrm{O}(|\boldsymbol{H})$. Those mappings are group homomorphisms. The kernel groups each have two elements: $kernel(\varphi) = \{\pm e\}$, $kernel(\psi) = \{\pm(e, e)\}$. The image groups satisfy $\varphi(S^3) = SO(Im|\boldsymbol{H})$, $\psi(S^3 \times S^3) = SO(|\boldsymbol{H})$. The sets $G := \psi(S^3 \times e)$, $G' := \psi(e \times S^3)$ are normal subgroups of the group $SO(4)$, which are isomorphic to the group S^3 under the isomorphisms $a \to \psi(a, e)$, $b \to \psi(e, b)$ respectively, EbH) p. 216-217.

Note: S^3 is isomorphic to $SU(2)$, (EbH) p. 212. We see from the foregoing that there are natural group epimorphisms $S^3 \to SO(3)$, $S^3 \times S^3 \to SO(4)$, whose kernels each have 2 elements. As S^3 is isomorphic to $SU(2)$ there are also correspondingly epimorphisms $SU(2) \to SO(3)$, $SU(2) \times SU(2) \to SO(4)$, with kernels of 2 elements.

g. SMEP and all that

Note: In quantum theory, decoupling of scales is not at all obvious. Indeed, because of the uncertainty principle, we have to work at all scales at once. The renormalization group explains why decoupling survives in quantum theory, (DeP) p. 554

Note: In classical mechanics, there are 3 basis units of measurement (distance D, time T, mass M), and all others can be expressed through them. Thus, in classical mechanics we deal with three scales. In nonrelativistic quantum theory and in classical relativity there remains only two of them, as in the first case we can express M through T and D using the Planck constant, and in the second T can be expressed via D using the speed of light. Thus, in relativistic quantum theory we only have one scale – the scale of distances. Equivalently, we can use the inverse scale – the scale of momenta. Thus we have:

SMALL distances, times = LARGE momenta, energies, masses", (DeP) p. 554.

Note (Dirac's (quantum) single system model): Dirac's (electron) single system model is basically about an elementary particle accompanied by three energy type attribute values and two particle type values. The three energy type attribute values describe the the energy of the atom, the electromagnetic energy of the radiation field, and the (small) coupling energy of the atom and the radiation field:

> „Dirac's theory of radiation is based on a very simple idea; instead of considering an atom and the radiation field with which it interacts as two distinct systems, he treats them as a single system whose energy is the sum of three terms: one representing the energy of the atom, a second representating the electromagnetic energy of the radiation field, and a small term representing the coupling energy of the atom and the radiation field", (FeE).

The two particle type attribute values distinguish between spin(0) and spin(1/2) elementary particles (the spin(1/2) hypothesis).

> „Identical particles obey either Fermi statistics or Bose statistics; … Electrons obey Fermi statistics. To determine the statistics of nuclei, we shall investigate how an exchange of identical nuclei will affect the sign of the wave function for a molecule", (BeH) p. 20.

Note (determining nuclear spin): „*Each nucleus has an intrinsic angular momentum which interacts with angular momenta of electrons or other nuclei. It is measured in units of the Planck constant and, according to quantum mechanics, can take only integral or half-integral values. Three methods of determining nuclear spin are*", (BeH) p. 19:

- Hyperfine structure of spectra
- Zeeman spectra
- Band spectra.

Note: „*In the most simple case of a 1D Coulomb potential box the solutions of the Dirac model can be interpreted as scattering or binding of particles or anti-particles*", (WaA) p. 185.

Note (interaction effects between electro-spin and nuclear spin): Dirac's relativistic quantum mechanics is only concerned with the main part between the interaction effects between electro-spin and nuclear spin governed by the Coulomb potential. The „Lamb shift" phenomenon is interpreted as the radiation correction term of this approximation, i.e., from a modelling perspective the Lamb shift phenomenon is interpreted as a consequence of interaction between the electron and fluctuations of a quantized radiation field.

Note: „*All in all, there are many indications that electrons, including their strange spin behavior, are described more simple by* $S^3 \cong SU(2)$. *In any case, despite the elegant representation Dirac had developed, it cannot be claimed that this sheds light on the reason for the existence of spin*", (UnA2) p. 183.

Note (scattering processes): „*Scattering processes are an important theoretical tool to explore microscopic interaction effects. The interpretation of the considered experiments resulted into the large number of propagated elementary particles of the SMEP, because on the short range energy level there was the need for two additional „strong and weak" EP interaction interpretations. The current supposition is that there are three related quantum field theories, the QED, the QCD, and the QFD*", (WaA) p. 189.

Note: The Standard Model of Elementary Particles (SMEP) is concered with gauges theory and variational principles. The gauge invariance is the main principle in current SMEP theory. Each of the three observed or assumed „forces", the weak & strong forces of particle interactions, and the electromagnetic interactions are related to a specific gauge group. Conceptually, the SMEP starts with a set of fermions, e.g. the electron in quantum electrodynamics. If a theory is invariant under transformations by a symmetry group one obtains a conservation law and quantum numbers. Gauge symmetries are local symmetries that act differently at each space-time point. They automatically determine the interaction between particles by introducing bosons that mediate the interaction. $U(1)$, the complex unit circle numbers, describes the electromagnetic interaction with one boson (Einstein's photon) and one quantum number (charge Q). The group $SU(2)$ of complex, unitary 2×2 matrices with determinant 1 describes the weak force interaction with three bosons W^+, W^-, Z, while the group $SU(3)$ of complex, unitary $3x3$ matrices describes the strong force interaction with eight bosons (gluons).

Note (Yang-Mills theory and the mass gap): The classical Yang-Mills theory is the generalization of the Maxwell theory of electromagnetism where the chromo-electromagnetic field itself carries charges. As a classical field theory it has solutions which travel at the speed of light so that its quantum version should describe massless particles (gluons). However, the postulated phenomenon of color confinement permits only bound states of gluons, forming massive particles. This is the mass gap.

Note: In SMEP the group $SU(2) \cong SL(2, C)$ describes the weak force interaction with 3 bosons W^+, W^-, Z, while the charged particles W^+, W^- have resemblance to positrons and electrons, and the neutral Z particle corresponds to the photon, (UnA3) S. 191. It describes the „how" of the β-decay process. This is the (about 15 minutes) decay of a neutron into a proton, an electron, and an antineutrino. Unfortunately, this (weak interaction process) theory does not say anything about the „why" accompanied by related physical laws.

Note (β-decay): *„Nobody knows to this day, why this process occurs and takes only 15 minutes. The „how-process" described by the symmetry group $SU(2)$ is based on the idea that there is a physical substance called nucleon with two states, called „neutron" and „proton" (the two „spin-states" of a nucleon), and where the root cause of their „folding over/flipping" is called „weak interaction" (which is not a „force" in a true sense of this word)*", (UnA3) p. 189.

Note: A mathematical curiousity in the electroweak theory:

> „In the standard model the weak and the electromagnetic interactions are unified in what is called electroweak theory, where there is a special symmetry related to $\mathrm{W}^+, \mathrm{W}^-, \mathrm{Z}^0$, and the photon γ, according to the groups $\mathrm{SU}(2) \times \mathrm{U}(1)$ or, more correctly, $\mathrm{U}(2)$. The group might be expressed as $\mathrm{SU}(2) \times \mathrm{U}(1)/\mathrm{Z}_2$, where the $'/\mathrm{Z}_2'$ means „factor out by a Z_2 subgroup". However, there is more than one such subgroup, so this notation is not fully explicit. The notation $'\mathrm{U}(2)'$ automatically picks out the correct one. (I am grateful to Florence Tsou for this observation.) It seems that the reason that the electroweak symmetry group is not conventionally referred to as $'\mathrm{U}(2)'$ is that this does not easily extend to the symmetry of the full standard model, which also incorporates the strong symmetry group $\mathrm{SU}(3)$, the full group being a version $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)/\mathrm{Z}_6$", (PeR4) p. 641, 654.

Note Higgs P. W., Spontaneous Symmetry Breakdown without Massless Bosons, (HiP)

> *Abstract*: We examine a simple relativistic theory of two scalar fields, first discussed by Goldstone, in which as a result of spontaneous breakdown of $U(1)$ symmetry one of the scalar bosons is massless, in conformity with the Goldstone theorem. When the symmetry group of the Lagrangian is extended from global to local $U(1)$ transformations by the introduction of coupling with a vector gauge field, the Goldstone boson becomes the longitudinal state of a massive vector boson whose transverse states are the quanta of the transverse gauge field. A perturbative treatment of the model is developed in which the major features of these phenomena are present in zero order. Transition amplitudes for decay and scattering processes are evaluated in lowest order, and it is shown that they may be obtained more directly from an equivalent Lagrangian in which the original symmetry is no longer manifest. When the system is coupled to other systems in a $U(1)$ invariant Lagrangian, the other systems display an induced symmetry breakdown, associated with a partially conserved current which interacts with itself via the massive vector boson.

Note (The Higgs mechanism): „*The Higgs mechanism of spontaneous symmetry breakdown allows gauge fields to acquire mass. In spite of these refinements, the basic fact remains that the existence of gauge fields is a consequence of the existence of gauge-invariant action densities for particle fields*", (BlD) xi. It builds on an extended from global to local U(1) transformations symmetry group of the underlying Lagrangian. It explains the mass of the gauge W- and Z- (weak interaction) bosons of the weak "nuclear-force".

Note (Yang-Mills type forces and the Higgs mechanism):
"*It is fine that the gauge field of electromagnetism has zero mass because there the force is mediated by photons, which are massless. However, Yang-Mills type forces must arise from the exchange of massive particles because of the observed short range of these forces. The Higgs mechanism helps in two ways. First, gauge fields can acquire mass by the symmetry breaking. Second, the undesirable Goldstone bosons (which arise in the symmetry-breaking process) can be usually gauged away*", (BlD) 10.3.

Note: The underyling Lie-groups of the SMEP and the Teichmüller theory are related to the several unit spheres in the following form

- the 1-dimensional unit sphere S^1 in R^2 corresponds to the Lie group $U(1)$. The related number grid is built by the Eisenstein numbers
- the compactification of the field of complex numbers C, the Riemann sphere, is homeomorphic to S^2. It plays a key role in the Teichmueller theory. We note the relationship of the Teichmüller space with the fractional Hilbert space $H_{1/2}$, (NaS)
- the 3-dimensional S^3 unit sphere is isomorphic to $SU(2)$
- the S^1 and S^3 are the only spheres with a "continuous" group structure, (EbH) 7.2. The groups S^1 and S^3 have parameter representations, (EbH) 3.5.4 (2'), 7.3.2 (3). The spheres S^0, S^1, S^3, S^7 are the only parallelizable spheres.

Note: The complex Lorentz group associated with $SL(2,C) \otimes SL(2,C) \cong SU(2) \otimes SU(2)$ plays a key role in the proof of the PCT theorem, where PCT stands for P = space inversion; T = time inversion; C = charge conjugation. This theorem is one of the rarely theorem, which is mathematically proven like the Noether theorem.

Note (the hidden symmetry of the Coulomb problem): The Coulomb problem has the symmetry group $SU(2) \otimes SU(2)$, (RoH) p. 172.

Note (solid state physics, phonon): „*The energy of a lattice vibration is quantized. The quantum of energy is called a phonon in analogy with the photon of the electromagnetic wave. ... The energy of an elastic mode of angular frequency ω and the related zero point energy of the mode are equivalent to a quantum harmonic oscillator frequency, the energy eigenvalues in the form $\left(n + \frac{1}{2}\right) \frac{h}{2\pi}\omega$ and $\frac{1}{2} \frac{h}{2\pi}\omega$. ... A phonon of wavevector K will interact with particles such as photons, neutrons, and electrons as if it had a momentum $\frac{h}{2\pi}K$. However, a phonon does not carry physical momentum*", (KiC) p. 99.

Note (free-electron theory and an infinite resistance of insulators): Insulators show a specific resistance to electricity which may be 10^{26} times greater than that of metals, which is a phenomenon never properly understood on the basis of the "real theory,":

> „The success of Bohr's early and pioneering ideas on the atom was always a rather narrow one and the same applies to Ptolemy's epicycles. Our present vantage point gives an accurate description of all phenomena which these more primitive theories can describe. The same is not true any longer of the so-called free-electron theory, which gives a marvelously accurate picture of many, if not most, properties of metals, semiconductors, and insulators. In particular, it explains the fact, never properly understood on the basis of the "real theory," that insulators show a specific resistance to electricity which may be 10^{26} times greater than that of metals. In fact, there is no experimental evidence to show that the resistance is not infinite under the conditions under which the free-electron theory would lead us to expect an infinite resistance. Nevertheless, we are convinced that the free-electron theory is a crude approximation which should be replaced, in the description of all phenomena concerning solids, by a more accurate picture. If viewed from our real vantage point, the situation presented by the free-electron theory is irritating but is not likely to forebode any inconsistencies which are unsurmountable for us. The free-electron theory raises doubts as to how much we should trust numerical agreement between theory and experiment as evidence for the correctness of the theory. We are used to such doubts", (WiE).

Note: „*In solid state physics there is the „free electron model of metal" providing insight into the heat capacity, thermal conductivity, electrical conductivity, magnetic susceptibility, and electrodynamics of metal. But the model fails to hepl us with other large questions: te occurence of positive values of the Hall coefficient; the relation of conduction electrons in the metal to the valence electron of free atoms; and many transport properties, particularly magnetotransport. We need to have a less naive theory, and fortunately it turns out that almost any simple attempt to improve upon the free electron model is enormmously profitable*", (KiC) p. 163.

Note: The colorful ornamentation of quantum electrodynamics

„In former times, classical physics upheld the picture that it was the electric and gravitational fields in space that caused the accelerations of charged particles. Quantum electrodynamics completely abandons this idea in favor of the exchange of borrowed photons. Feynman's theory worked so well that particle physicists decided to use it as a blueprint for all other interactions. Though the old wave-particle quantum theory of Bohr, Heisenberg, and Schrödinger should be a caveat against describing everything with particles, the idea entered the back door and seized hold of modern physics.

But unlike quantum electrodynamics, the results of its extension to nuclear physics, called quantum chromodynamics, are anything but precise [(*)]. It is therefore utter speculation that imposing the concept of quantum electrodynamics on atomic nuclei is the right way to go. Nevertheless, theorists almost exclusively walk on this we—trodden path", (UnA) p. 146

[(*)] The magnetic moment of an electron (its inherent magnetism) and the so-called Lamb shift in the spectral lines of a hydrogen atom.

Note: Big simplicity at the big flash

„The hydrogen atom would then be similar to an object now called positrinium, consisting of an electron and its antiparticle positron that orbit each other. The definition of the fine structure constant implies that $\frac{1}{\alpha} \sim 137$ *is the ratio of speed of light* c *to the electron's velocity on the innermost orbit of the hydrogen atom*", (UnA2) p. 96.

Note: *„All in all, there are many indications that electrons, including their strange spin behavior, are described more simple by* S^3*. In any case, despite the elegant representation Dirac had developed, it cannot be claimed that this sheds light on the reason for the existence of spin*[(*)]", (UnA2) p. 183.

[(*)] The spin matrices introduced by Pauli 1927 are also isomorphic to the unit quaternions and the simplest non-abelian Lie-group $SU(2)$

h. Plasma and all that

Note: Nearly all of the matter in the universe consists of "plasma". Similar to the notion „elementary particle“, there is no unique mathematical-physical definition of the notion „plasma particle“. The key differentiator between plasma to neutral gas or neutral fluid is the fact that its electrically positively and negatively charged kinematical particles are strongly influenced by electric and magnetic fields, while neutral gas is not.

Note: The high level plasma definition is about a neutral gas composed by many electrical charged (and also neutral) particles, those behavior is primarily determined by their collective degrees of freedom, (SpK).

Note: Plasma is an ionized gas consisting of approximately equal numbers of positively charged ions and negatively charged electrons. The number of neutral particles (atomes or molecules) is irrelevant for the definition of a plasma. The number of positively and negatively charged particles per considered volume element may be arbitrarily small oder arbitrarily large, but both numbers need to be approximately identical (in order to have no internal macroscopic electrostatic fields, (BiJ) p. 46. The interactions of positively charged ions and negatively charged electrons are determined by long-range electrical forces.

Note: Conceptually, „plasma particles“ need to fulfill the following two pre-requisites, (CaF) p. 1:

> (1) there must be electromagnetic interactions between charged particles
> (2) the number of positively and negatively charged particles per considered volume element may be arbitrarily small oder arbitrarily large, but both numbers need to be approximately identical. The number of neutral particles (atomes or molecules) is irrelevant for the definition of a plasma.

Note: Plasma physics is about classical statistical fluid mechanics and classical fluid dynamics. The underlying related mathematical models are grouped by different physical application areas resp. chosen mathematical tools accompanied by correspondingly defined different types of „plasma matter gases“ („hot“, „medium“, „cold“), e.g., there are

- neutral and plasma gas models, (BiJ), (ChF), (DeR)
- radiation fluid hydrodynamics, (MiD)
- gas dynamics and radiation hydrodynamics in astrophysics (ShF)
- magnetodynamics in plasma physics (CaF)
- flow radiation and vortices in superfluids (AnJ)
- condensation energy in the Ginzburg-Landau model (AnJ)
- magnetism in condensed matter, (BlS).

Note: The number of neutral particles (atomes or molecules) is irrelevant for the definition of a plasma. The number of positively and negatively charged particles per considered volume element may be arbitrarily small oder arbitrarily large, but both numbers need to be approximately identical (in order to have no internal macroscopic electrostatic fields, (BiJ) p. 46.

> A cycloton radiation occurs in magnetized plasmas, due to the magnetic centripetal acceleration of the charged particles as they spiral about magnetic fields, (BiJ) p. 6. The condition for a low-density plasma is that the average time between collisions is much more greater than the cyclotron period. „Cold“ plasma is accompanied by the (Non-Maxwellian-Boltzmann) electron velocity distribution under equilibrium conditions and at rest, given by $F_0(v) = n_0\delta(v_x)\delta(v_y)\delta(v_z)$, (BiJ) p. 492. A plasma is sometimes referred to as being „hot“, if it is nearly fully ionized. Examples of fully ionized plasma are the solar wind (interplanetery medium), stellar interiors (the sun's core), and fusion plasmas (plasma-universe.com).

Note (Debye shielding, sphere, double layer potential): A fundamental characteristic of the behavior of a plasma is its ability to shield out electric potentials that are applied to it.

Note: The Debye length is an important physical parameter for the description of a plasma. It provides a measure of the distance over which the influence of the electric field of an individual charged particle (or of a surface at some nonzero potential) is felt by the other charged particles inside the plasma. The charged particles arrange themselves in such a way as to effectively shield any electrostatic fields within a distance of the order of the Debye length. … It is convenient to define a Debey sphere as a sphere inside the plasma of radius equal to the Debye length, (BiJ) p. 8.

Note: The Debye shielding length is a characteristic of typical plasma properties. It is derived from the Poisson equation for the electrostatic (Coulomb) potential of the related Debye ball.

Note: The mathematical tool to distinguish between unperturbed cold and hot plasma is about the Debye length and Debye sphere (DeR). The corresponding interaction (Coulomb) potential of the non-linear Landau damping model is based on the (Poisson) potential equation with corresponding boundary conditions.

Note (the „hot" vs. „cold" plasma modelling case): The „hot (collisionsfree) plasma" corresponds to purely „dynamical plasma". The „cold plasma" corrresponds to „dynamical electromagnetics". The „medium heat plasma" modelling case is the given by the related weight factors of the case specific ratio between the affected two quanta pairs.

Note (Plasma dynamics): Plasma is that state of matter in which the atoms or molecules are found in an ionized state. The interactions of electrons and ions are determined by long-range electrical forces. The many forms of collective motion in a plasma are the result of coupling the charged-particle motion to the electromagnetic field. Therefore, the electromagnetic field which accompanies the particle motion is also a random nonreproducible quantity in a turbulent plasma. Measurements have shown that the fields excited in a plasma during the development of turbulence do in fact have a random nature, (TsV) p. 4.

The Landau damping phenomenon

> *„Landau damping is a characteristic of collisionless plasmas, but it may also have application in other fields. For instance, in the kinetic treatment of galaxy formation, stars can be considered as atoms of a plasma interaction via gravitational rather then electromagnetic forces*", (ChF) p. 245.

The Landau damping phenomenon is a wave damping without energy dissipation by elementary particle collisions, i.e., it is about the possibility of resonance between the wave phase velocity and the velocity of individual electrons; it is governed by the Coulomb potential.

> (DeR) p. 94: „The Landau damping phenomenon is complementary to the properties of electro-magnetic forces, which weaken themselves spontaneously over time w/o increase of entropy or friction. Landau damping involves a flow of energy between single particles on the one hand side, and collective excitations of plasma on the other side".

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„Landau damping models are applied to model the capability of stars to organize themselves in a stable arrangement as resonances in an inhomogeneous medium producing wave absorption (in space rather than in time) (ShF). If stars are considered as atoms of a plasma interacting via gravitational forces rather than electromagnetic forces (as a model for kinetic treatment of galaxy formation), instabilities of the gas of stars can cause spiral arms to form, but this process is limited by Landau damping", (ChF) p. 245.

Note (linear & nonlinear Landau damping phenomenon): Current mathematical models in plasma physics distinguish between linear and nonlinear Landau damping terms (while in both cases the energetic root cause of the Landau damping phenomenon is based on the Coulomb potential), indicating that this phenomenon arises from two different physical effects.

> (ChF) p. 248-249: „There are actually two kinds of Landau damping: linear Landau damping, and nonlinear Landau damping. Both kinds are independent of dissipative collisional mechanisms. If a particle is caught in the potential well of a wave, the phenomenon is called „trapping". Particles can indeed gain or lose energy in trapping. However, trapping does not lie within the purview of the linear theory. …. Trapping is not in the linear theory. When a wave grows to a larger amplitude, collisonless damping with trapping occur. One then finds that the wave does not decay monotonically; rather the amplitutes fluctuates during the decay as the trapped particles bounce back and forth in the potential wells. This is nonlinear Landau damping. .. Since the linear Landau damping is derived from a linear theory, … the nonlinear Landau damping must arise from a different physical effect. The question is: Can untrapped electrons moving close to the phase velocity of the wave exchange energy with the wave?"

The Vlasov-Poisson-Boltzmann (VPB) system

The continuity equation of ideal magneto-hydrodynamics is given by, (DeR) (4.1)

$$\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho \vec{v}) = 0$$

with $\rho = \rho(\vec{x}, t)$ denoting the mass density of the fluid and $\boldsymbol{v}$ denoting the bulk velocity of the macroscopic motion of the fluid. For a corresponding microscopic kinetic description of plasma fluids $\rho(\vec{x}, t)$ is replaced by a function $f(\vec{x}, \vec{v}, t)$. This function is the number density of particles whose position lies within the small volume element d^3x at the position x, and whose velocity lies within the velocity space element $d^3\vec{v}$ at $\vec{v}$, at the time t, (DeR) 5.1. The fundamental equation which $f(\vec{x}, \vec{v}, t)$ has to satisfy is the (kinetical) Boltzmann equation, (ChF) p. 230,

$$\frac{\partial}{\partial t}f + v \cdot \nabla_x f + \frac{F}{m} \cdot \frac{\partial f}{\partial v} = (\frac{\partial f}{\partial t})_\epsilon .$$

Here F is the force acting on the particles, and $(\frac{\partial f}{\partial t})_\epsilon$ is the time rate of change of f due to collisions. The meaning of the Boltzmann equation become clear if one remembers that f is a function of seven independent variables. Therefore, the total derivative of f with time is given by

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt} + \frac{\partial f}{\partial v_x}\frac{dv_x}{dt} + \frac{\partial f}{\partial v_x}\frac{dv_y}{dt} + \frac{\partial f}{\partial v_x}\frac{dv_z}{dt} .$$

From the Newton's third law $F = m\frac{dv}{dt}$ it follows

$$\frac{\partial f}{\partial v_x}\frac{dv_x}{dt} + \frac{\partial f}{\partial v_x}\frac{dv_y}{dt} + \frac{\partial f}{\partial v_x}\frac{dv_z}{dt} = \frac{F}{m} \cdot \frac{\partial f}{\partial v} .$$

Together with $v \cdot \nabla_x f = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$ one gets

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v \cdot \nabla_x f + \frac{F}{m} \cdot \frac{\partial f}{\partial v} .$$

As $\frac{df}{dt}$ is the convective derivative in the phase space the Boltzmann equation simply says that $\frac{df}{dt}$ is zero unless there are collisions.

In sufficiently hot plasma the current paradigm is that collisions ($^{\pm}$particles interaction) can be neglected. If futhermore the force $F = m\frac{dv}{dt}$ (Newton's third law), is entirely electromagnetic the Vlasov equations takes the special form, (ChF) p. 233,

$$\frac{\partial}{\partial t}f + \vec{v} \cdot \nabla_x f + \frac{q}{m}(\vec{E} + \vec{\boldsymbol{v}} \times \vec{B}) \cdot \frac{\partial}{\partial v} f = 0 .$$

Vlasov's mathematical argument against the Landau equation (leading to this equation) was, that "the Landau model of pair collisions is formally not applicable to Coulomb interaction due to the divergence of the kinetic terms". Because of its comparative simplicity, this is the equation most commonly studied in kinetic plasma theory.

Based on the perturbation split $f(\vec{x}, \vec{v}, t) = f_0(\vec{v}) + f_1(\vec{x}, \vec{v}, t)$ the first order Vlasov equation for electrons is given by, (ChF) 7.4,

$$\frac{\partial}{\partial t}f_1 + \vec{v} \cdot \nabla_x f_1 - \frac{e}{m}\vec{E}_1 \frac{\partial}{\partial \boldsymbol{v}} f_0 = 0 .$$

If f_0 is a Maxwellian the corresponding dispersion relation (in a weak sense) is given by

$$1 + \frac{\omega_p^2}{k^2}\ {}_xH\left[\frac{\partial}{\partial v_x}\tilde{f}_0\right]\left(\frac{\omega}{k}\right) = 0 .$$

The counterpart of the critical term of the linearized Vlasov equation ($(\nabla W * \rho) \cdot \nabla_v f^0$) in the Vlasov equation is given by the non-linear term $F[f] \cdot \nabla_v f$, whereby

$$F[f](t,x) \coloneqq -\iint \nabla W(x-y) f(t,y,w) dw dy.$$

Because of the corresponding Vlasov-Poisson model

$$F = -\nabla W, -\Delta_x W = \rho, \;\; W = \frac{1}{4\pi|\vec{x}|} *_{\vec{x}} \rho, \quad \rho(\vec{x},t) = \int_{R^n} f(\vec{x},\vec{v},t) d\vec{v}$$

the combination of both systems is called the Vlasov-Poisson-Boltzmann (VPB) system. The extension of the VPB system, where the Vlasov force F (or self-consistent force, or mean force …) is replaced by the Lorentz force determined by the electro-magnetic field created by the particles themselves is described in (LiP).

The non-linear collision operator of the Landau equation

The one-component plasma model of the non-linear collision operator of the Landau equation is given by

$$Q(f,f) = \frac{\partial}{\partial v_i}\left\{\int_{R^N} a_{ij}(v-w)\left[f(w)\frac{\partial f(v)}{\partial v_j} - f(v)\frac{\partial f(w)}{\partial w_j}\right] dw\right\}$$

with

$$a_{ij}(z) \coloneqq \frac{1}{|z|}\left\{\delta_{ij} - \frac{z_i z_j}{|z|^2}\right\} \coloneqq \frac{1}{|z|} P(z) = \frac{1}{|z|}[Id - \bar{Q}](z) \text{ and } \bar{Q}(z) \coloneqq (R_i R_j)_{1\leq i,j\leq N}.$$

Here $P(z)$ resp. R_i denote the Leray-Hopf resp. Riesz operators; the symbol function $a(z)$ is symmetric, non-negative and even in z; f denotes an unknown function corresponding at each time t to the density of particle at the point x with velocity v. Therefore, the Leray-Hopf (pseudo differentia) operator with the symbol $b_{ij}(z) = za_{ij}(z) = \delta_{ij} - \frac{z_i z_j}{|z|^2}$ may be interpreted as a kind of linearized Landau operator. It is of order zero.
Mathematically speaking, the Leray-Hopf operator may be interpreted as (mechanical collision) compact disturbance operator of a (dynamical) potential (energy) operator accompanied by H_α $\alpha \in [0,1]$, scale domains.

Note: Vlasov's mathematical argument against the Landau equation and Landau's reply: Vlasov's mathematical argument against the Landau equation (leading to the Vlasov equation) was, that "this model of pair collisions is formally (!) not applicable to Coulomb interaction due to the divergence of the kinetic terms". Vlasov's formula for the plasma dielectric for the longitudinal oscillators is based on the integral, ((ShF) p. 392)

$$W\left(\frac{\omega}{k}\right) = -\int_{-\infty}^{\infty} \frac{F_0{}'(v)dv}{\frac{\omega}{k} - v} \;.$$

As Landau pointed out, this model overlooks the important physical phenomenon of electrons travelling with exactly the same material speed $v_\varphi = \frac{\omega}{k}$ and the wave speed v.

i. Bose-Einstein condensates, superfluids, superconductivity and Fermi liquids

Note (AnJ) p. 1: Superconductivity, superfluitity, and Bose-Einstein condensation are among the most fascinating phenomena in nature. Their strange and often surprising properties are direct consequences of quantum mechanics. This is why they only occur at low temperature, and it is very difficult (but hopefully not impossible!) to find a room temperatur superconductor. But, while most other quantum effects only appear in matter on the atomic or subatomic scale, superfluids and superconductors show the effects of quantum mechanics acting on the bulk properties of matter on a large scale. In essence they are macroscopic quantum phenomena.

Note (AnJ) p. 2: Bose-Einstein condensates

In 1924 the Indian physicist S. N. Bose wrote to Einstein describing a new method to derive the Planck black-body radiation formula. At that time Einstein was already world-famous and had just won the Nobel prize for his quantum mechanical explanation of the photoelectric effect. Bose was a relatively unknown scientist working in Dacca (now Bagladesh), and his earlier letters to European journals had been ignored. But Einstein was impressed by the novel ideas in Bose's letter, and helped him to publish the results. The new idea was to treat the electromagnetic waves of the black-body as a gas of *identical particles*. For the first time, this showed that the mysterious light quanta, introduced by Planck in 1900 and used by Einstein in his 1905 explanation of the photo-electric effect, could actually be thought of as particles of light, that is, what we now call photons. Einstein soon saw that the same method could be used not only for light, but also for an ideal gas of particles with mass. This was the first proper quantum mechanical generalization of the standard classical theory of the ideal gas developed by Boltzmann, Maxwell, and Gibbs. We know now that there are two distinct quantum ideal gases, corresponding to either Bose-Einstein of Fermi-Dirac statistics. The method of counting quantum states introduced by Bose and Einstein applies to *boson* particles, such as photons or ^4He atoms.

Note (AnJ) p. 21: The are only two superfluids: ^4He and ^3He

There are only two superfluids which can be studied in the laboratory: ^4He and ^3He. Unlike all other substances they are unique because they remain in the liquid state even down to absolute zero in temperature. The combination of the light nuclear masses and the relatively weak van der Waals interactions between the particles means that at low pressure they do not freeze into a crystalline solid. All other elements and compounds eventually freeze into solid phases at low temperatures.

Despite the identical electronic properties of the ^4He and ^3He atoms, they have completely different properties at low temperatures. This is not due to the difference in nuclear mass, but arises because ^4He is a spin zero boson, while ^3He is a spin ½ fermion (due to the odd number of spin ½ constituents in the nucleus). As we shall see below, although they both form superfluid phases, the origin and physical properties of these superfluids is completely different. Superfluidity in ^4He occurs below $2.17\ K$, and was first discovered and studied in the 1930s. On the other hand, the superfluitity of ^3He only occurs below about $2\ mK$, three orders of magnitute lower in temperature than for ^4He. Another surprising difference is that ^3He has more than one distinct superfluid phase. Two of the superfluid transitions in ^3He were first observed by Osheroff, Richardson, and Lee in 1972, and they received the 1996 Nobel prize in physics for this work.

Note: (ClR) p. 332: *„After the discovery of the spin density fluctuations in Fermi liquids there has been considerable interest in deviations from the Landau theory of ^3He. Spin fluctuations have* two *effects: (1) they change the single-particle excitation spectrum and (2) act as a collective resonance of the system, both leading to $T^3 ln \frac{T}{\theta}$ contributions to the specific heat of ^3He for examples.*

An adequate starting point for the thermodynamics of a Fermi liquid with Fermi and Bose-like excitations would be the propagator and vertex renormalized representation of the thermodynamical potential Y as given for example by Bloch. The elementary excitation spectrum enters through the singularities of the two and four point functions. Unfortunately this functional has a very complicated analytical structure."

j. Planck's, Schrödinger's & Bohm's concepts of a dynamic implicate quantum dynamics governed by an order-from-order mechnanism

This section provides additional information from the protagonists mentioned above, in relation to the introductory remarks at the beginning of this paper.

M. Planck
(PlM)

"At the very least, it would be entirely wrong to seek a fundamental difference, for example, in the fact that in the realm of natural science, regularity is everywhere absolute and the course of phenomena is a necessary one that admits of no exceptions, whereas in the realm of the mind, the pursuit of causal connections is at times always guided through a degree of arbitrariness and chance.

How can one derive real laws at all from the observation of processes whose course, both as a whole and in detail, remains for the time being entirely subject to blind chance?

Physics, too, has come to recognize the great significance of an approach entirely different from the purely causal one; this is the statistical method, with the development of which the entire recent progress of theoretical physics is most closely connected. Instead of investigating the dynamic laws of an individual process which currently remain entirely obscure without any prospect of tangible success, one first compiles only the observations made on a large number of individual processes of a certain type and derives average or mean values from them. … For practical physics, this has indeed become indispensable.

After all, the situation described above makes it sufficiently clear just how crucial it is to carry out a careful and fundamental distinction between the two types of regularity discussed the dynamic, strictly causal, and the merely statistical for understanding the very essence of any scientific knowledge."

E. Schrödinger
(ScE1), pp. 77-78

Summarizing the physical situation

… the point to emphasize again and again is that to the physicist the state of affairs is not only not plausible but most common exciting, because it is unprecedented. Contrary to the common belief, the regular course of events, governed by the laws of physics, is never the consequence of one well-ordered configuration of atoms – not unless that configuration of atoms repeats itself a great number of times, either as in the period crystal or as in a liquid or in a gas composed of a great number of identical molecules.

Even when the chemist handles a very complicated molecule in vitro he always faced with an enormous of like molecules. To them his laws apply. He might tell us, for example, that one minute after he has started some particular reaction half of the molecules will have reacted, and after a second minute three-quarters of them will have done so. But whether any particular molecule, supposing you could follow its course, will be among those which have reacted or among those which are still untouched, he could not preduct. That is a matter of pure chance.

D. Bohm

This chapter is an attempt to provide a concise summary of Bohm's vision of a new holistic framework for physical classical, quantum, and relativity theories. In his book *Wholeness and the Implicate Order*, (BoD1), D. Bohm posits the two complementary concepts of an explicit and an implicit order, which together constitute undivided wholeness.

Bohm's proposal for creating a holistic structure encompassing all physical theories consists of a further development of the various underlying organizational structures and scales. To this end, he takes the indeterministic features of quantum theory as his starting point. In a certain sense, these reflect the actual behavior of matter in the atomic and nuclear realms.

Classical physics presupposes a specific order and a corresponding measure, characterized by the use of certain Cartesian coordinates and the assumption of a universal and absolute temporal order that is independent of space. Within this system of order and measurement, certain structures are possible; they are essentially based on the assumption of a quasi-rigid body as the basic element.

The laws of classical physics express the relationships between the motions of all parts, whereby a law relates the motion of each part to the arrangement of all parts. This law is deterministic in form, since the only determining factors of the system are the initial position and the velocities of all its parts. It is also causal, since every external disturbance can be regarded as a cause that produces a describable effect, which is then, in principle, transmitted to every part of the system.

A general feature of the emergence of new orders in physics consists in the tendency to regard certain fundamental concepts of order as permanent and unchanging. In this context, it was the task of physics to accommodate new observations by making adjustments within these fundamental concepts of order in order to account for the new facts.

In classical physics, the "fact" (Latin "facere," "that which has been made") was "made" from the standpoint of the order of planetary orbits, which are determined by measurements of space and time. In the theory of relativity, the "fact" was "made" from the standpoint of Riemannian geometry as well as that concept of measure which requires notions such as "curvature of space." In quantum theory, the "fact" was "made" from the standpoint of the order of energy levels, quantum numbers, symmetry groups, etc., as well as appropriate measures (e.g., scattering cross-sections, charges, particle masses).

Fact and theory appear as different aspects of a single whole, in which analysis into individual interacting parts is irrelevant. This means that we should not always forcefully attempt to adapt the theory to such facts that correspond to the respective generally accepted concepts of order, but that we should, when necessary, also be prepared to consider changes to our own conceptions of the "fact" itself, since these might be necessary for the assimilation of such "facts" into new theoretical concepts of order.

A unity of observation, measurement setup, and theoretical understanding requires considering a fact of new order. An example of the connection between measurement setup and theory is the lens, which enables an analysis of a system and its parts. Relativity theory and quantum theory require an undivided whole. The concept proposed by Bohm is that of the hologram, an instrument that "describes the whole."

Physical laws primarily refer to the order of an undivided whole in a substantive description, similar to that shown by a hologram, rather than an order of analyzing this content into separate parts, as exemplified by the lens.

The new concept of order as an undivided whole must not be understood exclusively as a regular arrangement of objects or as a uniform sequence of events. Instead, the overall order is implicitly (folded into) and contained within every segment of space and time. The word "implicit" is derived from the Latin verb "implicare," which means "to wrap in, to link." With this, we could explore the idea that every segment, in a certain sense, carries within itself the "wrapped" or "folded" overall structure.

Treatment of Quantum fluctuations
(BoD) p. 110-111

Let us begin by assuming some „deterministic“ field theory. Its precise characteristics are unimportant for our purposes here. All that is important is to suppose the following properties:

- There is a set of field equations which completely determines the changes of the field with time.

- These equations are sufficiently non-linear to guarantee a significant coupling between all wave components, so that (except perhaps in some approximation) solutions cannot be linearized superposed.

- Even in the „vacuum“ the field is so highly excited that the mean field in each region, however small, fluctuates significantly, with a kind of turbulent motion that leads to a high degree of randomness in the fluctuations. This excitation guarantees the discontinuity of the fields in the smalles regions.

- What we usually call „particles“ are relatively stable and conserved excitations on top of this vacuum. Such particles will be registered at the large-scale level, where all apparatus is sensitive only to those features that fluctuate rapidly. Thus, the „vacuum“ will produce no visible effects at the large-scale level, since its fields will cancel themselves out on the average, and space will be effectively „empty“ for every large-scale process (e.g. as a perfect crystal lattice is effectively „empty“ for an electron in the lowest band, even though the space is full of atoms).

….
The spatial average of a field coordinate is related to its canonically conjugate field momentum. Each of these field coordinates fluctuates randomly. This means that its instantaneous time derivative is infinite (as is also the case with the Brownian motion of a particle). Consequently, there is no exact method for defining such an instantaneous time derivative. If, instead, one considers average changes in the field over a small time interval, this yields an average value of the field momentum in the form

$$\overline{\pi_k} = a(\frac{\Delta\phi_k}{\Delta t})$$

where a is the constant of proportionality. If the field fluctuates in a random way, then by the very definitions of randomness, the region over which it fluctuates during the time Δt is given by

$$(\overline{\delta\phi_k})^2 = b\Delta t \text{ or } \left|\delta\phi_k\right| = \sqrt{b}\sqrt{\Delta t},$$

where b is another constant of proportionality, associated with the mean magnitude of the random fluctuation of the field.

Of course, the precise manner of fluctuation of the field is determined by the infinity of deeper field variables not taken into account, but in this context of the level in question, nothing determines this precise behaviour. In other words, $\delta\phi_k$ represents the maximum possible degree of determination of ϕ_k within the level of field quantities averaged over similar intervals of time.

From the definition of $\overline{\pi_k}$, we see that π_k will also fluctuate at random over the range

$$\delta\pi_k \cdot \delta\phi_k = ab.$$

Thus the product of the maximum degree of determination of π_k and that of ϕ_k is a constant, ab, independent of the time interval, Δt, (BoD1) pp. 113-115.
It is immediately clear that the above result shows a strong analogy to Heisenberg's principle, $\delta p \cdot \delta q > h$. The constant ab plays the role that Planck's constant, h, plays in Heisenberg's principle. The universality of h therefore implies the universality of ab.

It is easy to suggest a theory having the characteristics described above. Thus, that the „zero-point“ field fluctuations were in some kind of statistical equilibrium correspondingly to an extremely high temperature, T. The mean fluctuation in the energy per degree of freedom would, according to the equi-partition theorem, be

of the order $k \cdot T$, but the mean energy is also proportional to the mean of $(\frac{\delta\phi}{\delta t})^2$ (as happens for example in a collection of harmonic oscillators). Thus, we write

$$a\overline{(\frac{\delta\phi}{\delta t})^2} = kT = \frac{a}{b^2}\overline{(\pi)^2}$$

where k is Boltzmann's constant and a is a suitable constant of proportionality.

In the next step Bohm shows how quantization, i.e, the indivisiblity of the quantum of action, fits into his notions concerning a sub-quantum-mechanical level. He developed a theory involving a non-denumerable infinity of field variables that *has room* for the quantization of action according to the usual rules of quantization of the quantum theory and suggested a more definite theory, which give possible physical reasons explaining why action is quantized by the rules above, and which shows possible limitations on the domain of validity of these rules. Thereby is the main task to show that the constants of the motion (which are, for harmonic oscillators, proportional to the energy of a large-scale collective degree of freedom) are quantized by the rule $J = nh$, where n is an integer, and h is Planck's constant. Such a demonstration will constitute an explanation of the wave-particle duality, since the collective degrees of freedom are already known to be wavelike motions, with harmonically oscillating amplitutes. In general, these waves will take the form of fairly localized packets, and if these pakets have discrete and well-defined quantities of energy, momentum, and other properties, they will at higher level, reproduce all the essential characteristics of particles. Yet they will have inner wavelike motions which will reveal themselvse only under condititons in which there exist systems that can respond significantly to these finer details, (BoD1) pp. 117-133.

Classical physics implies a certain basic descriptive order and measure. This may be characterized as the use of certain Cartesian coordinates and by the notion of universal and absolute order of time, independent of that of space. This further implies the absolute character of what may be called Euclidian order and measure (i.e., that characteristic of Euclidian geometry). The laws of physics express the reason in the movement of each part of the configuration of all the other parts. This law is deterministic in form, in that the only contingent features of the system are the initial positions and velocities of all its parts. It is also *causal*, in that any external disturbances can be treated as a *cause*, which produces a specifiable effect that can in principle be propagated to every part of the system, (BoD1) p. 153.

One of the first real breaks in classical notions of order and measure came with the theory of relativity. It is significant to point out here the root of the theory of relativity was probably in a question that Einstein asked himself when he was fifteen years old: „What would happen if one were to move at the speed of light and look in a mirror?" Evidently, one would see nothing because the light from one's face would never reach the mirror. This led Einstein to feel that light is somehow basically different from other forms of motion.

From a more modern vantage point, we can emphasize this difference yet more by considering the atomic structure of the matter out which we are constitute. If we went faster than light, then, as a simple calculation shows, the electromagnetic fields that hold our atoms together would be left behind us (as the waves produced by an aeroplane are left behind it when it goes faster than sound). As a result, our atoms would disperse, and we would fall apart. So it would make no sense to suppose that we could go faster than light, (BoD1) p. 155.

Relativity introduces new notions concerning the order and measure of time. These are no longer absolute, as was the case in Newtonian theory. Rather, they are now relative to the speed of a coordinate frame. This relativity of time is one of the radically new features of Einstein's theory, (BoD1) p. 156.

The quantum theory implies a much more radical change in notions of order and measure than even relativity did. To understand this change, one has to consider four new features of primary significance introduced by this theory, (BoD1) pp. 162-164

- Indivisibility of the quantum of action
- Wave-particle duality of properties of matter
- Properties of matter as statistically revealed potentialities
- Non-causal correlations (the paradox of Einstein, Podolsky and Rosen).

A general feature of the emergence of new orders has been a tendency to regard certain basic notions of order as permanent and unchangeable. … Before the advent of quantum theory and the theory of relativity, the accommodation within already existing frameworks of order has generally been considered to be the main activity to be emphasized in physics, (BoD1) p. 177.

The undivided wholeness of modes of observation, instrumentation and theoretical understanding implies the need to consider *a new order of fact*, i.e, the fact about the way in which modes of theoretical understanding and of observation and instrumentation are related to each other.

An example of the very close relationship between instrumentation and theory can be seen by considering the *lens*, which was indeed one of the key features behind the development of modern scientific thought. The essential feature of a lens is, that it forms an *image* in which a given point P in the object corresponds (in a higher degree of approximation) to a point Q in the image. However, as we have seen relativity and quantum theory imply undivided wholeness, in which analysis into distinct and well-defined parts is no longer relevant, (BoD1) pp. 182-183.

What is being suggested here is that the consideration of the difference between lens and hologram can play a significant part in the perception of a new order that is relevant for physical laws. As Galileo noted the distinction between a various medium and a vacuum and saw that physical law should refer primarily to the order of motion of an object in a vacuum, so we might now note the distinction between a lens and a hologram and consider the possibility that physical law should refer primarily to an order of undivided wholeness of the content of a description similar to that indicated by the hologram rather than to an order of analysis of such content into separate parts indicated by a lens, (BoD1) p. 186.

All this calls attention to the relevance of a new distinction between implicate and explicate order. Generally speaking, the laws of physics have thus far refered mainly to the explicate order. Indeed, it may be said that the principle function of Cartesian coordinates is just to give a clear and precise description of explicate order. Now, we are proposing that in the formulation of the laws of physics, primary relevance is to be given to the implicate order, while explicate order is to have a secondary kind of significance. Thus it may be expected that a description in terms of Cartesian coordinates can no longer be given a primary emphasis, and that a new kind of description will indeed have to be developed for discussing the laws of physics, (BoD1) p. 190.

It is important to emphasize, that mathematics and physics are not being regarded here as separate but mutually related structures. Rather, it is being suggested that mathematics and physics are to be considered as aspects of a single undivided whole, (BoD1) p. 199.

Of course, in the quantum theory, the algebraic terms are interpreted as standing for „physical observables“, to which they correspond. However, in the approach that is being suggested here, such terms are not to be regarded as standing for anything in particular. Rather, they are to be considered as extensions of the general language. A single algebraic symbol is thus similar to a word, in the sense that its implicit meaning comes out fully only in the way in which the language as a whole is used.

This approach is indeed used in a great deal of modern mathematics, especially in number theory. Thus, one can start with what are called undefinable symbols. Rather, only relationships and operations in which these symbols take part are relevant, (BoD1) p. 206.

Though the unified field theory was not successful in this attempt to provide an ultimate mechanistic basis for physics in term of the field concept, it nevertheless did show in a concrete way how consistency with the theory of relativity may be achieved by deriving the particle concept as an abstraction from an unbroken and undivided totality of existence. Thus, it helped to strenghten the challenge posed by relativity theory to the prevailing mechanistic order.

The quantum theory presents, however, a much more serious challenge to this mechanistic order, going far beyond that provided by the theory of relativity. As seen in chapter 5, the key features of the quantum theory that challenge mechanism are:

1. Movement is in general discontinuous, in the sense that action is constituted of indivisible quanta (implying also that an electron, for example, can go from one state to another, without passing through any states in between).

2. Entitities, such as an electron, can show different properties (e.g., particle-like, wavelike, or something in between), depending on the environmental context within which they exist and are subject to observation.

3. Two entities, such as electrons, which initially combine to form a molecule and then separate, show a peculiar non-local relationship, which can best be described as a non-causal connection of elements that are far apart (as demonstrated in the experiment of Einstein, Podolsky and Rosen).

The three key features of the quantum theory given do, however, clearly show the inadequacy of mechanistic notions. Thus, if all actions are in the form of discrete quanta, the interactions between different entities (e.g., electrons) constitute a single structure of indivisible links, so that the entire universe has to be thought of as unbroken whole.

To begin with undivided wholeness means, however, that we must drop the mechanistic order. But this order has been, for many centuries, basic to all thinking on physics. As brought out in chapter 5, the mechanistic order is the most naturally and directly expressed through the Cartesian grid, (BoD1) p. 221-223.

Up till now we have contrasted implicate and explicate orders, treating them as separate and distinct, but as suggested in chapter 6, the explicate order can be regarded as a particular or distinguished case of a more general set of implicate orders from which latter it can be derived. What distinguishes the explicate order is that what is thus derived is a set recurrent and relatively stable elements that are outside of each other. This set of elements (e.g., fields and particles) then provides the explanation of that domain of experience in which the mechanistic order yields an adequate treatment, (BoD1) p. 226.

(Such) enfoldment and unfoldment in the implicate order may evidently provide a new model of, for example, an electron, which is quite different from that provided by current mechanistic notion of a particle that exists at each moment only in a small region of space and that changes its position continuously with time. What is essential to this new model is that the electron is instead to be understood through a total set of unfolded ensembles, which are generally not localized in space, (BoD1) p. 323-233.

From our consideration of how the general structure of matter can be understood in terms of the implicate order, we now come to certain new notions of cosmology that are implicit in what is being done here.
In this connection it may be said that space, which has so much energy, is full rather than empty, (BoD1) p. 240, 242.

As the plant is formed, maintained and dissolved by the exchange of matter and energy with its environment, at which point can we say that there is a sharp distinction between what is alive and what is not? Clearly, a molecule of carbon dioxid that crosses a cell boundary into a leaf does not suddenly „come alife“ nor does a molecule of oxygen suddenly „die“ when it is relased to the atmoshere. Rather, life itself has to be regarded as belonging in some sense to a totality, including plant and environment. It may indeed be said that life is enfolded in the totality and that, even when it is not manifest, it is somehow „implicit“ in what we generally call a situation in which there is no life, (BoD1) p. 246.

k. Excerpts from relevant articles or books

D. Bohm
Wholeness and the implicate (and explicate) order in physical law

„What we usually call „particles" are relatively stable and conserved excitations on top of this vacuum. Such particles will be registered at the large-scale level, where apparatus is sensitive only to those features of the field that will last a long time, but not to those features that fluctuate rapidly. Thus, the „vacuum" will produce no visible effects at the large-scale level, since its fields will cancel themselves out on the average, and space will be effectively „empty" for an electron in the lowest band, even though the space is full of atoms", (BoD1) p. 111

„What is being suggested here is that the considerations of the difference between lens and hologram can play a significant part in the perception of a new order that is relevant for physical law. … the word „implicit" means „to fold inward", (BoD1) p. 186

„It is important to emphasize, however, that mathematics and physics are not being regarded here as separate but mutually related structures (so that, for example, one could be said to apply mathematics to physics as paint is applied to wood). Rather, it is being suggested that mathematics and physics are to be considered as aspects of a single undivided whole", (BoD1) p. 199

„Explicate order arises primarily as a certain aspect of snese of perception and of experience with the content of such sense perception", (BoD1) p. 200.

R. Courant

„Empirical evidence can never establish mathematical existence – nor can the mathematician's demand for existence be dismissed by the physicist as useless rigor. Only a mathematical existence proof can ensure that the mathematical description of a physical phenomenon is meaningful", (HiS) p. 148

H. Dehnen et al.

"*Soll das Prinzip der fiktiven Veränderung physikalischer Größen (insbesondere auch bei den universellen Naturkonstanten) generell durchführbar sein, so muß es sich auf beliebige elementare Wechselwirkungen ausdehnen lassen. Auf diese Weise kann auch verständlich gemacht werden, daß man die räumliche Ausdehnung und Zerfallswahrscheinlichkeiten der Atomkerne grundsätzlich ebensogut zur Längen- und Zeitmessung benutzen kann wie die Eigenschaften der Elektronenhülle der Atome, beispielsweise eine „Cäsium-Uhr" und eine „Ammoniak-Uhr" gleichermaßen für die Zeitmessung im Gravitationsfeld geeignet sind. …*

Prinzipiell sind in (statistischen) Gravitationsfeldern nur Effekte nachweisbar, in welche Differenzen des Newtonschen Potentials an verschiedenen Raumstellen eingehen. …

Zusammenfassend können wir also sagen, daß sich die gesamte Perihelbewegung folgendermaßen zusammensetzt: Die Massenveränderlichkeit im Gravitationsfeld liefert bereits den vollen Betrag derselben, die Massenveränderlichkeit im Sinne der speziellen Relativitätstheorie ein weiteres Drittel, während die Korrektur der Newtonschen Gravitationskraft dem Betrage nach ebenfalls ein Drittel liefert, jedoch die Periheldrehung um diesen Betrag verkleinert. …

Mach hat die Vermutung ausgesprochen, daß das Zusammenfallen eines „dynamischen" und anderseits eines rein „kinetisch" definierten Bezugssystems nicht zufällig sein könne. …

Mit der Frage nach der Beschaffenheit der Welt im Großen wird aber auch das Machsche Prinzip wieder in seine alten Rechte eingesetzt. …

Es wäre demnach konsequent, den Gültigkeitsbereich der allgemeinen Relativitätstheorie grundsätzlich auf das makroskopische Verhalten der Körper einzuschränken und darauf zu verzichten, die Raum-Zeit-Struktur der allgemeinen Relativitätstheorie bis in die Dimensionen der Elementarteilchen und Atome fortzusetzen. Diese Anschauung wird gerade durch das Machsche Prinzip nahegelegt: denn nach diesem können Raum und Zeit nur

als denkbare Wechselwirkungen zwischen Körpern und Ereignissen einen Sinn haben, nicht aber als absolute, physikalisch wirksame Realitäten aufgefaßt werden. Daher dürfte das Raum-Zeit-Kontinuum der Relativitätstheorie die physikalische Bedeutung einer Kontinuumsapproximation von Wechselwirkungen zwischen Körpern (Elementarteilchen) besitzen, welche von den Gesetzen der Quantentheorie beherrscht werden. Diese Approximation wird umso genauer sein, je mehr materielle Körper an ihrem Aufbau beteiligt sind. Das Raum-Zeit-Kontinuum wäre demnach nur der „Schauplatz" (res extensa), auf dem sich das eigentliche Geschehen der Welt, das Quantengeschehen, abspielt", (DeH), cited in (UnA1) p. 142 with the statement:

„yet the article does no less than explain all known tests of the theory with variable speed of light!"

F. Ehrenhaft

„Light beams must have electric stationary components in the direction of the wave front normal, and that consequently there must be stationary electric potential differences between different points along the beam ; and that there must be also a stationary magnetic field in the beam of light with potential differences. Hence, the light beam must have a magnetizing effect, and the charge of a magnet should be changed by light", (EhF1).

A. Einstein

"Nach unserer bisherigen Erfahrung sind wir nämlich zum Vertrauen berechtigt, daß die Natur die Realisierung des mathematisch denkbar Einfachsten ist", (EiA) S.130
" A theoretical construction is unlikely to be true, unless it is logically very simple" (UnA) p. 11
"In a reasonable theory, there are no numbers which can be only determined empirically", (UnA) p. 217

The meaning of relativity: „Maxwell's equations determine the electromagnetic field when the distribution of electric charges and currents is known. But we do not know the laws which govern the currents and charges. We do know, indeed, that electricity consists of elementary particles (electrons, positive nuclei), but from a theoretical point of view we cannot comprehend this. We do not know the energy factors which determine the distribution of electricity in particles of definite size and charge, and all attempts to complete the theory in this direction have failed. If then we can build upon Maxwell's equations at all, the energy tensor of the electromagnetic field is known only outside the charged particles", (EiA4) p. 24.

Ether and the theory of relativity: „Lorentz succeeded in reducing all electromagnetic happenings to Maxwell's equations for free space.

As to the mechanical nature of the Lorentzian ether, it may be said of it, in a somewhat playful spirit, that immobility is the only mechanical property of which it has not been deprived by H. A. Lorentz. It may be added that the whole change in the conception of the ether which the special theory of relativity brought about, consisted in taking away from the ether its last mechanical quality, namely, its immobility. … Generalizing we must say this: -- There may be supposed to be extended physical objects to which the idea of motion cannot be applied. They may not be thought of as consisting of particles which allow themselves to be separately tracked through time. In Minkowski's idiom this is expressed as follows: -- Not every extended conformation in the four-dimensional world can be regarded as composed of world-threads. The special theory of relativity forbids us to assume the ether to consist of particles obserbale through time, but the hypothesis of ether in itself is not in conflict with the special theory of relativity. Only we must be our guard against ascribing a state of motion to the ether.", (EiA5).

E. Fermi

Quantum Theory for Radiation: „Dirac's theory of radiation is based on a very simple idea; instead of considering an atom and the radiation field with which it interacts as two distinct systems, he treats them as a single system whose energy is the sum of three terms: one representing the energy of the atom, a second representating the electromagnetic energy of the radiation field, and a small term representing the coupling energy of the atom and the radiation field.

If we neglect this last term, the atom and the field could not affect each other in any way; that is, no radiation energy could be either emitted or absorbed by the atom. A very simple example will explain these relations. Let

us consider a pendulum which corresponds to the atom, and an oscillating string in the neighborhood of the pendulum which represents the radiation field. If there is no connection between the pendulum and the string, the two systems vibrate quite independently of each other; the energy is in this case simply the sum of the energy of the pendulum and the energy of the string with no interaction term. To obtain a mechanical representation of this term, let us tie the mass M of the pendulum to a point A of the string by means of a very thin and elastic thread a. The effect of this thread is to perturb slightly the motion of the string and of the pendulum. Let us suppose for instance that at the time $t = 0$, the string is in vibration and the pendulum is at rest. Through the elastic thread a the oscillating string transmits to the pendulum very slight forces having the same periods as the vibrations of the string. If these periods are different from the period of the pendulum, the amplitude of its vibrations remains always exceedingly small; but if a period of the string is equal to the period of the pendulum, there is resonance and the amplitude of vibration of the pendulum becomes considerable after a certain time. This process corresponds to the absorption of radiation by the atom. If we suppose, on the contrary, that at the time $t = 0$ the pendulum is oscillating and the string is at rest, the inverse phenomenon occurs. The forces transmitted through the elastic thread from the pendulum to the string put the string in vibration; but only the harmonics of the string, whose frequencies are very near the frequency of the pendulum reach a considerable amplitude. This process corresponds to the emission of radiation by the atom", (FeE).

R. Feynman

„Somebody makes up a theory: The proton is unstable. They make a calculation and find that there would be no protons in the universe any more! So they fiddle around with their numbers, putting a higher mass into the new particle, and after much more effort they predict that the proton will decay at a rate slightly less than the last measured rate of the proton has shown not to decay at. When a new experiment comes along and measures the proton more carefully, the theories adjust themselves to squeeze out from the pressure", (UnA) p. 162.

M. Heidegger

The Age of the World Picture: "modern physics is called mathematical because, in a remarkable way, it makes use of a quite specific mathematics. But it can proceed mathematically in this way only because, in a deeper sense, it is already itself mathematical", (HeM).

W. Heisenberg

Introduction to the Unified Field Theory of Elementary Particles: "The mathematical formalism contains some unconventional features which formerly have rendered its understanding somewhat difficult: the indefinite metric in Hilbert space and the degeneracy of the ground state. But in recent years the indefinite metric has been studied in connexion with the Bleuler-Gupta version of quantum electrodynamics and with the Lee-model, the degeneracy of the ground state plays an important part in modern solid state physics", (HeW) vi.

Sir J. Jeans
The universe around us

A substance which consists solely of atoms of a single kind is described as an element, while one which contains more than one kind of atom is described as a compound.
…
Analysis of all known terrestrial substances has, so far, revealed only 92 essential different kinds of atoms. And even of these 92, the majority are exceedingly rare, most common substances being formed out of the combinations of only about 14 different atoms, say hydrogen (H), carbon (C), nitrogen (N), oxygen (O), sodium (Na), magnesium (Mg), Aluminium (Al), silicon (Si), phosphorus (P), sulphur (S), chlorine (Cl), potassium (K), calcium (Ca), and iron (Fe). … In this way, the whole earth, with its endless diversity of substances, is found to be a building built of standard bricks – the atoms. And of these only a few types, about 14, occur at all abundantly in the structure, the others appearing but rarely.

We shall see below (pp. 164, 165) that the various kinds of atoms occur in much the same relative proportions in the stars as on earth. Thus twelve of the fourteen elements which are abundant on earth are abundant also in the stars. This is not surprising if we consider that the earth probably came into being as a condensation of the gases in the atmoshere of one particular star – namely, the sun (p. 245). Hydrogen and helium are less abundant on earth than in stellar atmospheres, but there is a reason for this also. When the earth was still a

diffuse ball of hot gas, ist graviational power would not be adequate to hold down the rapidly moving atoms of theses substances (p. 212) so that these would rapidly diffuse away and be lost to the earth for ever. Thus little helium remains on earth, while hydrogen is found only in combination with other atoms of other substances, (JeJ) pp. 110-111.

From a study of the spectrum of a star we can tell what chemical substances are present in its atmosphere. A query means that the estimate is uncertain, and a double query that it is very uncertain, while a blank means that no specific evidence of the presence of either the element or its compounds has been found in the sun, (JeJ) pp. 163-164.

Element	*Relative no. of atoms*		*Element*	*Relative no. of atoms*		*Element*	*Relative no. of atoms*
Hydrogen	*1000000000*		*Sodium*	*500000*		*Scandium*	*130*
Helium	*30000000??*		*Magnesium*	*600000*		*Titanium*	*5000*
Lithium	*3*		*Aluminium*	*80000*		*Vanadium*	*3000*
Beryllium	*2?*		*Silicon*	*1000000*		*Chromium*	*16000*
Boron	*3000*		*Phosphorus*	*300?*		*Manganese*	*25000*
Carbon	*1000000*		*Sulphur*	*16000?*		*Iron*	*500000*
Nitrogen	*3000000?*		*Chlorine*	*-*		*Cobalt*	*13000*
Oxygen	*30000000*		*Argon*	*-*		*Nickel*	*30000*
Flurine	*30000?*		*Potassium*	*200000?*		*Copper*	*3000*
Neon	*-*		*Calcium*	*160000*		*Zine*	*2500*

A query means that the estimate is uncertain, and a double query that it is very uncertain, while a blank means that no specific evidence of the presence of either the element or ist compounds has been found in the sun

D. E. Neuenschwander

„There is no continuous infinitesimal transformation for charge conjugation. No states exist that carry charge values in a continuum from the -e electric charge of an electron to the +e of the positron, or between the $I_z = \pm 1/2$ *isospin eigenvalues. How do we define invariance for discrete symmetries?“* (NeD) 9.1.

C. S. Peirce

„Aus dieser ersten und in einer Hinsicht einzigen Regel der Logik, daß man, um zu lernen, den Wunsch haben muß zu lernen, und sich dabei nicht mit dem zufrieden geben darf, was man schon zu denken geneigt ist, ergibt sich ein Folgesatz, der an sich schon verdient, auf jede Mauer in der Stadt der Philosophie zu stehen: Behindere nicht den Gang der Forschung*“,* (PaH) S. 9.

„.. es gibt drei universale Kategorien. Da alle drei ständig gegenwärtig sind, ist es unmöglich, eine reine Idee irgendeiner von ihnen zu bilden, die absolut von den anderen unterschieden ist. Ja, selbst so etwas wie ihre ausreichend klare Unterscheidung kann nur das Ergebnis langen und angestrengten Forschens sein. Sie können mit Erstheit, Zweitheit und Drittheit bezeichnet werden. Erstheit ist das, was so ist, wie es eindeutig und ohne jede Beziehung auf etwas anderes ist. Zweitheit ist das, was so ist, wie es ist, weil eine zweite Identität so ist, wie sie ist, ohne Beziehung auf etwas Drittes. Drittheit ist das, dessen Sein darin besteht, eine Zweitheit hervorzubringen. Es gibt keine Viertheit, die nicht bloß aus Drittheit bestehen würde“, (PaH) S. 31.

R. Penrose

The Emperors's New Mind
Why Quantum Gravity?

„What is there that is new to be learnt, concerning brains or minds, from what we have seen in the last chapter? Though we may have glimpsed some of the all-embracing physical principles underlying the directionality of our perceived „flow of time“, we seem, so far, to have gained no insights into the question of why we perceive time to flow, or, indeed, why we perceive at all. In my opinion, much more radical ideas are needed. My presentation so far has not been particularly radical, though I have sometimes provided a different emphasis from what is

usual. We have made our acquaintance with the second law of thermodynamics, and I have attempted to persuade the reader that the origin of this law – presented to us by Nature in the particular from that she has indeed chosen – can be traced to an enormous geometrical constraint on the big bang origin of the universe: the Weyl curvature hypothesis. Some cosmologists might prefer to characterize this initial constraint somewhat differently, but such a restriction on the initial singularity is indeed necessary. The deductions that I am about to draw from this hypothesis will be considerably less conventional than is the hypothesis itself. I claim that we shall need a change in the very framework of the quantum theory!

…

I shall argue very differently. For I am not concerned so much with the effects that quantum mechanics might have on our theory (Einstein's general relativity) of the structure of space-time, but with the reverse: namely the effects that Finstein's space-time theory might have on the very structure of quantum mechanics", (PeR) pp. 450/451.

P. Plaass

Kant's Theory of Natural Science: „(Therefore) the principles of natural science cannot derive from mathematics alone since it does not in itself deal with existence but only with the possibility of things. While mathematics is also crucial for the a priori foundation of physics, it is not sufficient; only metaphysics deals with the question of existence. Therefore, a metaphysical foundation of natural science that simultaneously includes the basis for the application of mathematics to nature is necessary to assure that mathematics can be applied to what belongs to the existence of natural things – and not just their possibility. The mathematizability of nature (which Galileo and Newton simply presupposed in order to ground physics mathematically) is itself dependent on a metaphysics of nature. If the mathematizability of nature is simply hypothesized and left unexamined, the result is to fall back onto uncritical metaphysical assumptions that fail to deal with the underlying problems and hence leave one's position open to the kind of skeptical undermining disclosed by Hume", (PlP) p. 92

P.-M. Robitaille

Fourty lines of evidence that the solar body is comprised of condensed matter
The Sun on trial, Liquid metallic hydrogen as a solar building block

„Forty lines of evidence will be presented that the solar body is comprised of, and surrounded by, condensed matter. These 'proofs' can be divided into seven broad categories: 1) Planckian, 2) spectroscopic, 3) structural, 4) dynamic, 5) helioseismic, 6) elemental, and 7) earthly. Collectively, these lines of evidence provide a systematic challenge to the gaseous models of the Sun and expose the many hurdles faced by modern approaches. Observational astronomy and laboratory physics have remained unable to properly justify claims that the solar body must be gaseous. At the same time, clear signs of condensed matter interspersed with gaseous plasma in the chromosphere and corona have been regrettably dismissed. As such, it is hoped that this exposition will serve as an invitation to consider condensed matter, especially metallic hydrogen, when pondering the phase of the sun", (RoP).

Blackbody radiation and the loss of universality,
Implications for Planck's formulation and Boltzmann's constant

„Through the reevaluation of Kirchhoff's law Planck's blackbody equation loses its universal significance and becomes restricted to perfect absorbers. Consequently, the proper application of Planck's radiation law involves the study of solid opaque objects, typically made from graphite, soot, and carbon black. The extension of this equation to other materials may yield apparent temperatures, which do not have any physical meaning relative to the usual temperature scales. Real temperatures are exclusively obtained from objects which are known solids, or which are enclosed within, or in equilibrium with, a perfect absorber. For this reason, the currently accepted temperature of the microwave background must be viewed as an apparent temperature. Rectifying this situation, while respecting real temperatures, involves a reexamination of Boltzman's constant. In so doing, the latter is deprived of its universal nature and, in fact, acts as a temperature dependent variable. In its revised form, Planck's equation becomes temperature insensitive near 300 K, when applied to the microwave background", (RoP1).

Water, Hydrogen Bonding, and the Microwave Background

„In this work, the properties of the water are briefly revisited. Though liquid water has a fleeting structure, it displays an astonishingly stable network of hydrogen bonds. Thus, even as a liquid, water possesses a local

lattice with short range order. The presence of hydroxyl ($O-H$) and hydrogen ($H\cdots OH_2$) bonds within water, indicate that it can simultaneously maintain two separate energy systems. These can be viewed as two very different temperatures. The analysis presented uses results from vibrational spectroscopy, extracting the force constant for the hydrogen bonded dimer. By idealizing this species as a simple diatomic structure, it is shown that hydrogen bonds within water should be able to produce thermal spectra in the far infrared and microwave regions of the electromagnetic spectrum. This simple analysis reveals that the oceans have a physical mechanism at their disposal, which is capable of generating the microwave background", (RoP2).

C. Rovelli

„The thermal time hypothesis: In Nature, there is no preferred physical time variable t. There are no equilibrium states ρ_0 preferred a priori. Rather, all variables are equivalent: we can find the system in an arbitrary state ρ; if the system is in a state ρ, then a preferred variable is singled out by the state of the system. This variable is what we call time. …. In other words, it is the statistical state that determines which variable is physical time, and not any a priori hypothetical „flow" that drives the system to a preferred statistical state", (RoC) p. 143.

B. Russell

„"Substance", in a word, is a metaphysical mistake, due to transference to the world-structure of the structure of sentences composed of a subject and a predicate", (RuB1) p. 212

„Hume had proved that the law of causality is not analytic, and had inferred that we could not be certain of its truth. Kant accepted the view that it is synthetic, but nevertheless maintained that it is known a priori. He maintained that arithmetic and geometry are synthetic, but are likewise a priori. He was thus led to formulate his problem in these terms:
How are synthetic judgements a priori possible? The answer to this question, with its consequences, constitutes the main theme of The Critique of Pure Reason.

Space and time, Kant says, are not concepts; they are forms of „intuition". (The German word is „Anschauung", which means literally „looking at" or „view". The word „intuition", though the accepted translation, is not altogether a satisfactory one)", (RuB1) p. 680.

E. Schrödinger
Two ways of producing orderlines

„The orderliness encountered in the unfolding of life springs from a different source. It appears that there are two different „mechanisms" by which orderly events can be produced: the „statistical mechanism" which produces „order from disorder" and the new one, producing „order from order". To the unprejudiced mind the second principle appears to be much simpler, much more plausible. No doubt it is. That is where physicists were so proud to have fallen in with the other one, the „order-from-disorder" principle, which is actually followed in Nature and which alone conveys an understanding of the great line of natural events, in the first place of their irreversibility. But we cannot expect that the „laws of physics" derived from it suffice straightaway to explain the behaviour of living matter, whose most striking features are visible based to a large extent on the „order-from-order" principle. You would not expect two entirely different mechanisms to bring about the same type of law – you would not expect your latch-key to open your neighbour's door as well", (ScE1) p. 80

The principle of objectivation

"Science aims at nothing but making true and adequate statements about its object. The scientist only imposes two things, namely truth and sincerity, imposes them upon himself and upon other scientists. In the present case the object is science itself, as it has developed and has become and at present is, not as it ought to be or ought to develop in future", (ScE1) p. 117

Form, not substance, the fundamental concept

„The new idea is that what is permanent in these ultimate particles or small aggregates is their shape and organization. The habit of everyday language deceives us and seems to require, whenever we hear the word „shape" or „form" pronounced, that it must be the shape or form of something, that a material substratum is

required to take on a shape. Scientifically this habit goes back to Aristotle, his causa materialis and causa formalis. But when you come to the ultimate particles constituting matter, there seems to be no point in thinking of them again consisting of some material. They are, as it were, pure shape, nothing but shape; what turns up again and again in successive observations is this shape, not an individual speck of material." (ScE3) p. 125.

L. Smolin

The Trouble with Physics

"The fact that there are that many freely specifiable constants in what is supposed to be a fundamental theory is a tremendous embarrassment", (SmL1) p. 13, (UnA) p. 11

Problem 1 (problem of quantum gravity): Combine general relativity and quantum theory into a single theory that can claim to be the complete theory of nature

Problem 2 (foundational problems of quantum mechanics): Resolve the problems in the foundations of quantum mechanics, either by making sense of the theory as it stands or by inventing a new theory that does make sense

Problem 3 (the unification of particles and forces): Determine whether or not the various particles and forces can be unified in a theory that explains them all as manifestations of a single, fundamental entity

Problem 4: Explain how the values of the free constants in the standard model of particle physics are chosen in nature

Problem 5: Explain dark matter and dark energy. Or, if they don't exist, determine how and why gravity is modified on large scales. More generally, explain why the constants of the standard model of cosmology, including the dark energy, have the values they do, (SmL1).

L. Susskind, A. Friedman

„Lorentz did know about the Michelson-Morley experiment. He came up with the same transformation equations but interpreted them differently. He envisioned them as effects on moving objects caused by their motion through the ether. Because of various kinds of ether pressures, objects would be squeezed and therefore shortened", (SuL) p. 61.

H. Weyl

Space, Time, Matter: „The theory of Maxwell and Lorentz cannot hold for the interior of the electron; therefore, from the point of view of ordinary theory of electrons we must treat the electron as something given a priori, as a foreign body in the field. A more general theory of electrodynamics has been proposed by Mie, by which it seems possible to derive the matter from the field", (WeH1) p. 206 ff., (vide) Ann. d. Physik, Bd. 37, 39, 40 (1912-1913).
Philosophy of Mathematics and Natural Science: "On the basis of rather convincing general considerations G. Mie in 1912 pointed out a way of modifying the Maxwell equations in such a manner that they might possibly solve the problem of matter, by explaining why the field possesses a granular structure and why the knots of energy remain intact in spite of the back-and-forth flux of energy and momentum. The Maxwell equations will not do because they imply that negative charges compressed in an electron explode; to guarantee their coherence in spite of Coulomb's repulsive forces was the only service still required of the substance by H. A. Lorentz's theory of electrons. The preservation of the energy knots must result from the fact that the modified field laws admit only of one state of field equilibrium", (WeH) p. 171.

E. Wigner

The Unreasonable Effectiveness of Mathematics in the Natural Sciences: „We now have, in physics, two theories of great power and interest: the theory of quantum phenomena and the theory of relativity. These two theories have their roots in mutually exclusive groups of phenomena. Relativity theory applies to macroscopic bodies, such as stars. The event of coincidence, that is, in ultimate analysis of collision, is the primitive event in the theory of relativity and defines a point in space-time, or at least would define a point if the colliding panicles were infinitely small. Quantum theory has its roots in the microscopic world and, from its point of view, the event of coincidence, or of collision, even if it takes place between particles of no spatial extent, is not primitive and not at all sharply isolated in space-time. The two theories operate with different mathematical concepts -

the four dimensional Riemann space and the infinite dimensional Hilbert space, respectively. So far, the two theories could not be united, that is, no mathematical formulation exists to which both of these theories are approximations. All physicists believe that a union of the two theories is inherently possible and that we shall find it. Nevertheless, it is possible also to imagine that no union of the two theories can be found. This example illustrates the two possibilities, of union and of conflict, mentioned before, both of which are conceivable.

In order to obtain an indication as to which alternative to expect ultimately, we can pretend to be a little more ignorant than we are and place ourselves at a lower level of knowledge than we actually possess. If we can find a fusion of our theories on this lower level of intelligence, we can confidently expect that we will find a fusion of our theories also at our real level of intelligence. On the other hand, if we would arrive at mutually contradictory theories at a somewhat lower level of knowledge, the possibility of the permanence of conflicting theories cannot be excluded for ourselves either. The level of knowledge and ingenuity is a continuous variable and it is unlikely that a relatively small variation of this continuous variable changes the attainable picture of the world from inconsistent to consistent. …

Let us consider a few examples of "false" theories which give, in view of their falseness, alarmingly accurate descriptions of groups of phenomena. With some goodwill, one can dismiss some of the evidence which these examples provide. The success of Bohr's early and pioneering ideas on the atom was always a rather narrow one and the same applies to Ptolemy's epicycles. Our present vantage point gives an accurate description of all phenomena which these more primitive theories can describe. The same is not true any longer of the so-called free-electron theory, which gives a marvelously accurate picture of many, if not most, properties of metals, semiconductors, and insulators. In particular, it explains the fact, never properly understood on the basis of the "real theory," that insulators show a specific resistance to electricity which may be 10 26 times greater than that of metals. In fact, there is no experimental evidence to show that the resistance is not infinite under the conditions under which the free-electron theory would lead us to expect an infinite resistance. Nevertheless, we are convinced that the free-electron theory is a crude approximation which should be replaced, in the description of all phenomena concerning solids, by a more accurate picture.

If viewed from our real vantage point, the situation presented by the free-electron theory is irritating but is not likely to forebode any inconsistencies which are unsurmountable for us. The free-electron theory raises doubts as to how much we should trust numerical agreement between theory and experiment as evidence for the correctness of the theory. We are used to such doubts", (WiE).

6. Perspectives and articles on mathematics, physics, and philosophy

Authors

Aristotle; Avenarius R.; Barbour J.; Bergson H.; Bethe H. A.; Böhme G.; Bohm D.; Cassirer E.; Capra F.; Chen F. F.; Courant R.; Davidson J.; Dee K.; Dehnen H. et al.; Deligne P.; Deleuze G; Derbyshire J.; Descartes R.; Dirac P. A. M.; Dürr H.-P.; Eco U.; Ehrenhaft F. (and W. Schauberger); Einstein A.; Euler L.; Fermi E.; Feynman R.; Goethe J. W. v.; Gödel K.; Hawking S. W.; Hegel G. W. F.; Heidegger M.; Heisenberg W.; Helmholtz H.; Hildebrandt S; Hübscher A.; Husserl E.; Kant I.; Kirchhoff G.; Klainerman S.; Kneser A.; Kramers H. A.; Leedskalnin E.; Leibniz G.-W.; Lorentz H. A.; Mach E.; Marx W.; Maupertuis P.; Mijajlovic Z.; Miyamoto K.; Müller O. L.; Nagel Th.; Neuenschwander D. E.; Nietzsche F.; Nussbaumer I.; Penrose R.; Peskin M.; Plaass P.; Planck M.; Plemelj J.; Poluyan P.; Robitaille P.-M.; Rollnik H.; Rovelli C.; Russel R.; Schauberger V.; Schiller F.; Schmicking D. A.; Schopenhauer A.; Schpolski E. W.; Schrödinger E.; Shaw B.; Shu F. H. ; Smolin L.; Spatschek K. H.; Treder H.-J.; Unzicker A.; Vagt C.; Weinberg S.; Weizsäcker C. F. v.; Welzer H.; Weyl H.; Wheeler J. A.; Whitehead A. N.; Wigner E.

Aristotle
Motion: the pair of the concepts *potential* and *actual*

(DrM) p. 189: „Aristotle*, on the other hand, derives time from motion in general; motion does not have to be cyclic. Motion, in turn, he derives from the pair of concepts* potential *and* actual*, fundamental for his philosophy. He defines motion thus: „The actuality of that which potentially is, as such, is motion." This formulation has often been missunderstood, still today some English translations (and most German ones!) give, instead of „actuality", e.g.: „the progress of its realization" or „realization of their potentiality". This translations look more plausible at the first sight, but it is of no use as a definition since the concept of „realization" presupposes the very* process *that is to be defined. – The definition by Aristotle, read correctly, is especially interesting because it associates* time *with* potentiality*, as we will do below as well."*

(BöG) S. 63: „Er (Aristoteles) selber hat wohl die Bezeichnung Physik für die zentrale (naturwissenschaftliche Disziplin) gehalten. Aus dem einfachen Grunde, weil dieses Wort noch an das griechische Wort physis = *Natur erinnert. ….*

… „Physik, so Aristoteles, sei „die Betrachtung der Wahrnehmbaren" und folglich der Versuch „in bezug auf die Wahrnehmbaren die Wesensbestimmungen zu definieren"".

(BöG) S. 64: „Physik heißt, daß man für den jeweiligen Gegenstand, der immer eine „Erscheinung" sein muß, vier „Ursachen", vier „Faktoren" angibt: den Stoff oder das Material, die Form oder das Wesen, die Wirkungursache oder den Produzenten, den Zweck. Die Angabe aller vier Faktoren ergibt dann die volle Wesensbestimmung im Sinne der Physik. Die Wesensbestimmung im engeren Sinne ist also nur ein Faktor in der Gesamtbestimmung."

Avenarius R.
Philosophie als Denken der Welt gemäß dem Prinzip des kleinsten Kraftmaßes
Prolegomena zu einer Kritik der reinen Erfahrung

(AvR) S. 3: *Diese Schrift versucht, die Entwicklung der Philosophie unter das Prinzip des kleinsten Kraftmaßes zu befassen. Freilich ist dies Prinzip zunächst ein Prinzip der Beharrung, welches hinsichtlich der Seele etwa so lauten würde: Die Änderung, welche die Seele ihren Vorstellungen bei dem Hinzutritt neuer Eindrücke erteilt, ist eine möglichst geringe; oder mit anderen Worten: Der Inhalt unserer Vorstellungen nach einer neuen Apperzeption ist dem Inhalt vor derselben möglichst ähnlich. – Insofern aber die Seele den Bedingungen organischer Existenz und deren Zweckmäßigkeitsanforderungen unterworfen ist, wird das angezogene Prinzip zu einem Prinzip der Entwicklung: Die Seele verwendet zu einer Apperzeption nicht mehr Kraft als nötig, und gibt bei einer Mehrheit möglicher Apperzeptionen derjenigen den Vorzug, welche die gleiche Leistung mit einem geringeren Kraftaufwand, mit welchem aber eine geringere Wirkungsdauer verbunden ist, eine zeitweilige Mehranstrengung vor, welche um so viel größere bez. andauerndere Wirkungsvorteile verspricht.*

(AvR) S. 6: *„Eine Auffassung, welche, gleich der hier niedergelegten, jede individuelle Gedankenbildung, also auch die eigene, mehr als ein Fremdes denn ein Eigenes betrachtet, da sie dieselbe als zum weitaus größeren*

Teil durch die allgemeine Gedankenentwicklung bestimmt anerkennt, - eine Auffassung, welche sich zugleich nicht verhehlt, wie in dem übrig bleibenden Teile scheinbar freier individueller Entfaltung noch so viele Einflüsse menschlich-subjektiver Befangenheit hemmend und trübend eingreifen: eine solche Auffassung hat wenig Grund, eine gerechte, rein von theoretischen Interessen geleitete Beurteilung zu scheuen. Vielmehr ist sie bereit, von der Kritik – und erst recht von der des Gegners – zu lernen, indem sie sich der Erkenntnis fügt, daß in den Regionen des Denkens, wo Exempel und Experiment versagen, es meist der Einwirkung gegensätzlicher Meinungen bedarf, um uns zu der vornehmensten Bedingung aller Selbstkritik und Selbstweiterbildung zu entwickeln: zu dem vollen Bewußtsein dessen, was unser Wissen war und was unser Wollen."

Barbour J.
The End of Time, The Two Big Mysteries

(BaJ1) p. 15: „*Physicists currently describe the world by means of two very different theories. Large things are described by classical physics, small things by quantum physics. There are two problems with this picture.*

First, general relativity, Einstein's theory of gravity, seems to be incompatible with the principles of quantum mechanics in a way Newtonian dynamics and the theory of electromagnetism, developed by Michael Faraday and James Clerk Maxwell in the nineteenth century, are not. For these theories, it proved possible to transform them, by a process known quantization, from classical into quantum theories. Attempts to apply the same process to general relativity and create quantum gravity *failed. It was this technical work, by Dirac and others, which brought to fore all problems about time with which this book is concerned.*

The second mystery is the relationship between quantum and classical physics. It seems that quantum physics is more fundamental and ought to apply to large objects, even the universe. There ought to be a quantum theory of the universe: quantum cosmology. *But quantum physics does not yet exist in such a form. And its present form is very mysterious. Part of it seems to describe the actual behavior of atoms, molecules and radiation, but another part consists of rather strange rules that act at the interface between microscopic and macroscopic worlds. Indeed, the very existence of a seemingly unique universe is a great puzzle within the framework of quantum mechanics. This is very unsatisfactory, since physicists have a deep faith in the unity of nature. Because general relativity is simultaneously a theory of gravity and the large-scale structure of the universe, the creation of quantum cosmology will certainly require the solution of the only slightly narrower problem of quantum gravity."*

Bergson H.
Creative Evolution

(BeH1) Intoduction: „*The history of the evolution of life, incomplete as it yet is, already reveals to us how the intellect has been formed, by an uninterrupted progress, along a line which ascends through the vertebrate series up to man. It shows us in the faculty of understanding an appendage of the faculty of acting, a more and more precise, more and more complex and supple adaption of the consciousness of living beings to the conditions of existence that we made for them. Hence should result this consequence that our intellect, in the narrow sense of a word, is intended to secure the perfect fitting of our body to its environment, to represent the relations of external things among themselves – in short, to think matter. Such will indeed be one of the conclusions of the present essay. ….*

But from this it must also follows that our ought, in its purely logical form, is incapable of presenting the true nature of life, the full meaning of the evolutionary movement. Created by life, in definite circumstances, to act on definite things, how can it embrace life, of which it is only an emanation or an aspect? Deposited by the evolutionary movement in the course of its way, how can it be applied to the evolutionary movement itself? As well contend that the part is equal to the whole, that the effect can reabsorb its cause, or that the pebble left on the beach displays the form of the wave that bought it there. In fact, we do indeed feel that not one of the categories of our thought – unity, multiplicity, mechanical causality, intelligent finality, etc. – applies exactly to the things of life: who can say where individuality begins and ends, whether the living being is one or many, whether it is the cells which associate themselves into the organism or the organism which dissociates itself into cells? In vain we force the living into this or that one of our molds. All the molds crack. They are too narrow, above all too rigid, for what we try to put into them. Our reasoning, so sure of itself among things inert, feels ill

at ease on this new ground. It would be difficult to cite a biological discovery due to pure reasoning. And most often, when experience has finally shown us how life goes to work to obtain a certain result, we find its way of working is just that of which we should never have thought."

Bethe H. A.
Elementary Nuclear Theory

The size of nuclei

(BeH) p. 7-12: *„The methods of determining the size of nuclei fall into two classes: those that indicate the presence of nuclear matter even if it is electrically neutral, and those that are purely electromagnetic and are influenced only by the electric charge distribution within the nucleus.*

1. *Nuclear methods*
    a. *Cross section for fast neutrons*
    b. *Lifetimes for radioactivity*
    c. *Cross sections for nuclear reactions involving charged particles*
2. *Electromagnetic methods*
    a. *Electrostatic interaction of protons in the nucleus*
    b. *Electron scattering*
    c. *μ-Mesonic atomic x-ray energies*
    d. *Electron energy levels*
    e. *General trend of nuclear Coulomb energies"*

γ-rays disintegration

(BeH) p. 14: *„Nuclei are found in nature (and more can be produced artifically) that emit electrons spontaneously according the the reaction schema (Z^A denotes a nucleus with mass number A)*

$$Z^A \rightarrow (Z+1)^A + \beta^-, \qquad (\beta^+, \beta^- \text{ denote a positron resp.an electron})"$$

(BeH) p. 17: *„Nuclei emit not only particles (heavy particles and electrons) but also γ-radiation (light quanta). Such emission is possible only when a nucleus goes from an excited energy state to a lower energy state. The half-lives for dipole radiation (nuclear spin change $\Delta I = 0$, or ± 1) are generally of the order of 10^{-17} second to about 10^{-13} second.*

Summary of decay processes

(BeH) p. 17: *„Consider a nucleus Z^A with mass number A in some quantum state;*

1. *it may be unstable to the emission of heavy particles*
    a. *Neutrons*
    b. *Protons*
    c. *α-Particles*
2. *Emission of γ-rays or K-electron capture: … Thus the unstable nuclei can be put into three groups*

    Group I: Lives unobservablely short

    Group II: Lives observable (10^{-8} second to 10^{12} years)
    Nearly all β-radioactive nuclei, many α-radioactiv ones, and many „nuclear isomer" emitting γ-rays

    Group III: Lives unobservable long (greater that 10^{14} years)"

Spin and its measurement

(BeH) p. 19: *„Each nucleus has an intrinsic angular momentum which interacts with angular momenta of electrons or other nuclei. It is measured in units of the Planck constant and, according to quantum mechanics, can take only integral or half-integral values. Three methods of determining nuclear spin are:*

- *Hyperfine structure of spectra*
- *Zeeman spectra*
- *Band spectra*

These observed spin values are another reason for rejecting a nuclear model composed of electrons and protons. Such a model for a nucleus Z^A has A protons and $A - Z$ elecrons or $2A - Z$ particles. On this basis, nuclei with odd Z should have half-integer spin; and nuclei with even Z (and therefore an odd total number of particles) should have half-integer spin."

Statistics

(BeH) p. 20: „Identical particles obey either Fermi statistics or Bose statistics; … Electrons obey Fermi statstics. To determine the statistics of nuclei, we shall investigate how an exchange of identical nuclei will affect the sign of the wave function for a molecule.

(BeH) p. 22: „now it was found experimentally that nuclei with even A obey Bose statistics, those with odd A Fermi statistics. This proves that the neutron must obey Fermi statistics, just as the proton for which this fact is known experimentally."

(BeH) p. 24: With no known exceptions, all nuclei of even Z and even A have total nuclear spin zero".

The structure of nuclei

(BeH) p. 157: „From a detailed knowledge of the forces between nucleons it would be possible to calculate the properties of all nuclei"

Böhme G.
Idee und Kosmos
Platons Zeitlehre – Eine Einführung in seine theoretische Philosophie

(BöG): *Dieses Buch ist eine Einführung in Platons theoretische Philosophie, nämlich seine Ideenlehre, seine Prinzipienlehre, seine ungeschriebene Lehre* und *seine Naturphilosophie als einer Einheit. Damit soll der unselige Zustand der Platonforschung überwunden werden, in dem diese Teile seiner Philosophie gegeneinander ausgespielt werden. Als Paradigma, an dem diese Einheit demonstriert werden soll, wurde die platonische Zeitlehre gewählt. Ihr kommt deswegen eine hervorragende Bedeutung zu, weil sich an ihr entscheidet, wie das Verhältnis von Idee und Kosmos zu denken ist. Es ist falsch, dies ist die These dieses Buches, den Kosmos – wie bei Kant – als den Bereich zeitlichen Seins zu verstehen.* Jede *Bestimmtheit im Kosmos ist Darstellung idealen Seins. Auch die Zeit ist, wie der Timaios lehrt, eine solche Darstellung.*

Das Verständnis der theoretischen Philosophie Platons als einer Einheit demonstriert zugleich, wie notwendig es ist, den Stand der – für Platon – zeitgenössischen Wissenschaft zu berücksichtigen. Philosophie heißt eben nicht nur Liebe zur Weisheit, sondern auch Liebe zu den Wissenschaften.

(BöG) S. 49: „*Wir können jetzt formulieren, in welchem eingeschränkten Sinne das Verhältnis von Idee und sinnlicher Wirklichkeit als ein Abbildungsverhältnis angesehen werden kann, und damit in welchem Sinne der ganze Kosmos als Bild zu verstehen ist. Das Darstellungsverhältnis von Idee und Ding ist von dem gewöhnlichen Urbild-Abbild-Verhältnis durch zweierlei unterschieden: Zum ersten ist das Original nicht ein Seiendes mit gewissen Bestimmungen, die dann auch am Bild erscheinen könnten. Zum zweiten enthält das Bild nicht qua Bild besondere Charaktere. Wie soll man ein so merkwürdiges Darstellungsverhältnis begreiflich machen?*

Die Darstellung der Ideen besteht offensichtlich darin, daß sie, die an sich nicht Bestimmungen von etwas sind, als Bestimmungen von etwas auftreten. Die Gerechtigkeit selbst, ist nicht jemandes Gerechtigkeit, sie erhält aber ihre Darstellung als eine Gerechtigkeit des Sokrates oder die des Staates. Der moderne Leser mag sich hierbei an den Darstellungsbegriff erinnern, der in der neueren Mathematik eine Rolle gespielt hat. Als man zu Anfang dieses Jahrhunderts dazu überging, in freierer Weise mathematische Strukturen zu entwickeln, forderte man zu ihrer Anerkennung immer eine „Darstellung" der betreffenden Struktur. Man verstand darunter den Aufweis eines Gegenstandsbereiches, der diese Struktur auch wirklich hat – in der Regel dienten dazu die natürlichen Zahlen oder eine bestimmte Teilmenge derselben. Man verlangte den Nachweis, daß die Struktur auch irgendwo „realisiert" sei. Diese Analogie ist allerdings mit Vorsicht zu benutzen, dennn die Forderung nach einer Darstellung entsprang sicherlich einer eher aristotelisch geprägten Ontologie, denn als das eigentlich

Reale sah man offenbar dasjenige an, was eine Struktur trägt, während man der Struktur als solcher kein Sein zubilligte.

Die Darstellung der Ideen ist eine Darstellung ohne spezifische Drstellungsprinzipien, sie ist – um dies noch einmal zu betonen – nicht die Erscheinung der Ideen in der Zeit. Die Zeit selber ist eine Darstellung, nämlich die des αίών."

(BöG) S. 68, ÄON (αίών): „*Als der erzeugende Vater das (Weltall) bewegt und lebendig erschaute, hervorgetreten als Heiligtum der ewigen Götter, war er entzückt und dachte daran, es dem Vorbild noch ähnlicher zu machen. So wie nun dieses selbst ein ewiges Lebewesen ist, versuchte er jenes All nach Möglichkeit als ein derartiges zu vollenden. Nun ist das Wesen des Lebendigen aber äonisch, und dies dem Hervorgetretenen ganz zu gewähren war allerdings nicht möglich: Er gedachte aber ein bewegliches Bild des Äon zu machen, und indem er zugleich den Himmel ordnete, machte er ein nach Zahlen gehendes, äonisches Bild des in einem bleibenden Äon, jenes (nämlich), das wir Zeit genannt haben." (37 c,d).*

(BöG) S. 69: ÄON (αίών): „*Ist also αίών das Wesentliche der Zeit, so gilt es, um das Wesen der Zeit zu erfassen, gerade diesen zu verstehen. Nun ist dieses Verstehen durch die Übersetzung von αίών durch Ewigkeit nur allzuschnell geleistet. Es bleibt nämlich dabei unausgemacht, welchen Sinn man mit dem Wort Ewigkeit verbindet, und es wird insbesondere verdeckt, daß es gerade Platon, daß es gerade diese Stelle war, die dem Wort αίών erst den Sinn von Ewigkeit verlieh. Die erste Aufgabe der Interpretation der vorgelegten Textstelle muß also darin bestehen, die gängige Übersetzung von αίών durch Ewigkeit zu destruieren. Das muß nicht zu einer Widerlegung führen, im Gegenteil wird dieser Weg erst wieder die Bedeutungsfülle von αίών erschließen, von der her sich die Übersetzung durch „Ewigkeit" mit Inhalt füllen läßt.*"

(BöG) S. 145, Die Zeit als Thema der Astronomie: „*Kosmologisch gesehen sind die Gestirne um der Zeit willen da, und nicht umgekehrt. Sie sind, wie es im Timaios heißt, Werkzeuge der Zeit (42 d 5) oder Werkzeuge der Zeiten (41 e 5). Die Zeit ist der kosmologische Sinn der Gestirne, sie ist deshalb das eigentliche Thema der Astronomie.*

Bohm D.
The Special Theory of Relativity

(BoD) Preface: „*Einstein's basically new step was in adoption of a relational approach to physics. Instead of supposing that the task of physics is the study of an absolute underlying substance of the universe (such as an ether) he suggested that it is only in the study of relationships between various aspects of this universe, relationships that are in principle observables. … Einstein's analysis of the concept of simultaneity, in which he regards time as a kind of „coordinate" expressing the relationship of an event to a concrete physical process in which this coordinate is measured. On the basis of the observed fact of the constancy of actually measured velocity of light for all observers, one sees that observers moving at different speeds cannot agree on the same time coordinate to be asccibed to distant events. From this conclusion, it also follows that they cannot agree on the lengths of objects or the rates of clocks. Thus, the essential implications of the theory of relativity are seen qualitatively, without the need for any formulas. The transformations of Lorentz are then shown to be the only ones that can express in precise quantitative from to be the only ones that can express in precise quantitative form the same conclusions that were initially obtained without mathematics*"

(BoD) p. 97: *„We have seen already that Newton's laws of motion are not invariant to a Lorentz transformation, and that the principle of relativity therefore implies (except in the limit as v/c approaches zero), these cannot be correct laws of mechanics. … our first problem with regard to these laws is therefore to generalize them so as to obtain a new set of equations that is invariant to a Lorentz transformation. … in an isolated system of bodies the total momentum P is related to the total mass M and the velocity V of the center of mass by the formula $P = V \cdot M$. It is a well known theorem in Newtonian mechanics that in such a system the total momentum P is a constant vector and the total mass is also a constant. … to generalize Newton's laws the basic idea behind our procedure is that it is essential in physical theories to be able to analyze a whole system into parts or components. Thus in a theory of a continuous medium, such as hydrodynamics, we regard the fluid as being constituted out of small elements of volume, and, in a theory which explains matter as having a discrete atomic structure, a whole system is likewise regarded as constituted out of small elements, now taken to be atoms. In both kinds of theories we can treat the total momentum of a system as the sum of momenta of its parts,*

likewise with total mass and the total energy. Moreover, at least in the domain where Newtonian theory applies, such systems are known by experiment (as well as from the theory) to satisfy the laws of conservation o momentum, conservation of mass, and conservation of energy"

Bohm D.
Wholeness and the implicate (and explicate) order in physical law

(BoD1) p. 111: *„What we usually call „particles" are relatively stable and conserved excitations on top of this vacuum. Such particles will be registered at the large-scale level, where apparatus is sensitive only to those features of the field that will last a long time, but not to those features that fluctuate rapidly. Thus, the „vacuum" will produce no visible effects at the large-scale level, since its fields will cancel themselves out on the average, and space will be effectively „empty" for an electron in the lowest band, even though the space is full of atoms*"

(BoD1) p. 186/188: *„What is being suggested here is that the considerations of the difference between lens and hologram can play a significant part in the perception of a new order that is relevant for physical law. … the word „implicit" (based on the verb „to implicate") means „to fold inward*"

(BoD1) p. 199: *„It is important to emphasize, however, that mathematics and physics are not being regarded here as separate but mutually related structures (so that, for example, one could be said to apply mathematics to physics as paint is applied to wood). Rather, it is being suggested that mathematics and physics are to be considered as aspects of a single undivided whole"*

(BoD1) p. 200: *„explicate order arises primarily as a certain aspect of snese of perception and of experience with the content of such sense perception*"

(BoD1) *p. 200: „What is common to the functioning of instruments generally used in physical research is that the sensibly perceptible content is ultimately describable in terms of a Euclidean system of order and measure, i.e., one that can adequately be understood in terms of ordinary Euclidean geometry. …"*

(BoD1) p. 200: *„In this discussions, we shall adopt the well-known view of the mathematician Klein, who considered the general transformations are considered to be the essential determining features of a geometry. Thus, in an Euclidean space of three dimensions, there are three displacement operator D_i. Each of these operators defines a set of parallel lines which transform into themselves under the operation in question. Then, there are three rotation operators R_i. Each of these define a set of concentric cylinders around the origin which transform into themselves under the operation in question. Together, they define concentric spheres which transform into themselves under the whole set of R_i. Finally, there is the dilation operator R_0, which transforms a sphere of a given into one of a different radius. Under this operation, the radial lines through the origin transform into themselves"*

(BoD1) p. 201: *„So we may describe displacements on a numerical scale. This gives not only an order, but also a measure (in so far we treat successive displacements as equivalent in size)"*

(BoD1) p. 202: *„Implicate order is generally to be described not in terms of simple geometric transformations, such as translations, rotations, and dilations, but rather in terms of a different kind of operations. …"*

(BoD1) p. 202: *„What happens in the broader context of implicate order we shall call a metamorphosis. …"*

(BoD1) p. 202: *„A hologram is an example of a similarity transformation (or a similarity metamorphosis). It is determed by the Green's function relating amplitudes at the illuminated structure to those at the photographic plate"*

(BoD1) p. 206: *„Of course, in the quantum theory, the algebraic terms are interpreted as standing for 'physical observables' to which they correspond. However, in the approach that is being suggested here, such terms are not to be regarded as standing for anything in particular. … This means, of course, that we do not regard terms like 'particle', 'charge', 'mass', 'position', 'momentum', etc., as having primary relevance in the algebraic language. Rather, at best, they will have to come out as high-level abstractions."*

Bohm D.
Quantum Theory
Spin and Angular Momentum

(BoD2) p. 387: „*In chapter 14 we studied the quantum properties of the angular momentum of single-particle systems. We wish now to extend this treatment to take into account the angular momentum of a system of particles. We shall also discuss the treatment of the additional angular momentum arising from the fact that the electron has an intrinsic spin.*

Electron spin

Although the Schrödinger wave equation gives excellent general agreement with experiment in predicting the frequencies of spectral lines, small discrepancies are found, which can be explained in terms of the postulate that the electron has, besides its usual orbital angular momentum, an additional intrinsic angular momentum that acts as if it came from a spinning solid body (*). It was found that agreement with experiment could be obtained by means of the assumption that the magnitude of the additional angular momentum was $\frac{1}{2}\frac{h}{2\pi}$. *The magnetic moment needed to obtain agreement with the Zeeman effect was, however,* $\mu = e\frac{h}{2\pi}(2mc)$, *which is exactly the same as that arising from an orbital angular momentum of* $\frac{h}{2\pi}$ *(It should be noted that because it is the order of* $\frac{h}{2\pi}$, *spin is an essentially quantum-mechanical property). The gyromagnetic ratio, i.e., the ratio of magnetic moment to angular momentum is therefore twice as great for electron spins as it is for orbital spins*."

(*) H. A. Kramers, Die Grundlagen der Quantentheorie

Cassirer E.
Kants Leben und Lehre
Die Kritik der Urteilskraft

(CaE) S. 305: „*Das Reich der Kunst und das der organischen Naturformen stellt nur darum eine andere Welt als die der mechanischen Kausalität und der sittlichen Normen dar, weil die Verknüpfung, die wir in beiden zwischen den Einzelgebilden annehmen, unter einer eigentümlichen Gesetzesform steht, die weder durch die theoretischen „Analogien der Erfahrung", durch die Verhältnisse von Substanz, Ursächlichkeit und Wechselwirkung, noch durch die ethischen Imperative ausdrückbar ist. Welches ist diese Gesetzesform und worauf gründet sich die Notwendigkeit, die wir auch ihr zusprechen? Ist sie eine „subjektive"oder „objektive" Notwendigkeit: beruht sie auf einem Zusammenhang, der lediglich in unserer menschlichen Vorstellung besteht und von hier aus fälschlich den Gegenständen angeheftet wird, oder ist sie im Wesen dieser Gegenstände selbst gegründet? Ist der Zweckgedanke, wie Spinoza will, lediglich ein „asylum ignorantiae" oder bildet er, wie Aristoteles und Leibniz behaupten, das objektive Fundament jeder tieferen Naturerklärung? Oder, wenn wir alle diese Fragen vom Gebiet der Natur auf das der Kunst übertragen: steht die Kunst im Zeichen der „Naturwahrheit" oder im Zeichen des „Scheins"; ist sie die Nachahmung eines Bestehenden oder eine freie Schöpfung der Phantasie, die mit dem Gegebenen nach Belieben und Willkür schaltet? Durch die gesamte Entwicklung der organischen Naturlehre, wie durch die der Ästhetik ziehen sich diese Probleme hindurch: — jetzt aber gilt es, ihnen einen festen systematischen Platz anzuweisen und sie dadurch zur Hälfte bereits zur Lösung zu bringen.*"

Capra F.
The Tao of Physics

(CaF1) PREFACE: „*Five years ago, I had a beautiful experience which set me on a road that has led to the writing of this book. I was sitting by the ocean one late summer afternoon, watching the waves rolling in and feeling the rhythm of my breathing, when I suddenly became aware of my whole environment as being engaged in a gigantic cosmic dance. Being a physicist, I knew that the sand, rocks, water and air around me were made of vibrating molecules and atoms, and that these consisted of particles which interacted with one another by creating and destroying other particles. I knew also that the Earth's atmosphere was continually bombarded by showers of 'cosmic rays', particles of high energy undergoing multiple collisions as they penetrated the air. All this was familiar to me from my research in high-energy physics, but until that moment I had only experienced it through graphs, diagrams and mathematical theories. As I sat on that beach my former experiences came to life; I 'saw' cascades of energy coming down from outer space, in which particles were created and destroyed in*

rhythmic pulses; I 'saw' the atoms of the elements and those of my body participating in this cosmic dance of energy; I felt its rhythm and I 'heard' its sound, and at that moment I knew that this was the Dance of Shiva, the Lord of Dancers worshipped by the Hindus.

(CaF1) THE WAY OF PHYSICS: „*The Greek atomists drew a clear line between spirit and matter, picturing matter as being made of several 'basic building blocks'. These were purely passive and intrinsically dead particles moving in the void. The cause of their motion was not explained, but was often associated with external forces which were assumed to be of spiritual origin and fundamentally different from matter. In subsequent centuries, this image became an essential element of Western thought, of the dualism between mind and matter, between body and soul.*

…

The birth of modern science was preceded and accompanied by a development of philosophical thought which led to an extreme formulation of the spirit/matter dualism. This formulation appeared in the seventeenth century in the philosophy of Rene Descartes who based his view of nature on a fundamental division into two separate and independent realms; that of mind (res cogitans), and that of matter (res extensa). The 'Cartesian' division allowed scientists to treat matter as dead and completely separate from themselves, and to see the material world as a multitude of different objects assembled into a huge machine. Such a mechanistic world view was held by Isaac Newton who constructed his mechanics on ist basis and made it the foundation of classical physics. From the second half of the seventeenth to the end of the nineteenth century, the mechanistic Newtonian model of the universe dominated all scientific thought. It was paralleled by the image of a monarchical Cod who ruled the world from above by imposing his divine law on it. The fundamental laws of nature searched for by the scientists were thus seen as the laws of God, invariable and eternal, to which the world was subjected.

…

The philosophy of Descartes was not only important for the development of classical physics, but also had a tremendous influence on the general Western way of thinking up to the present day. Descartes' famous sentence 'Cogito ergo sum'- 'I think, therefore I exist'-has led Western man to equate his identity with his mind, instead of with his whole organism. As a consequence of the Cartesian division, most individuals are aware of themselves as isolated egos existing 'inside' their bodies. The mind has been separated from the body and given the futile task of controlling it, thus causing an apparent conflict between the conscious will and the involuntary instincts. Each individual has been split up further into a large number of separate compartments, according to his or her activities, talents, feelings, beliefs, etc., which are engaged in endless conflicts generating continuous metaphysical confusion and frustration.

…

The Cartesian division and the mechanistic world view have thus been beneficial and detrimental at the same time. They were extremely successful in the development of classical physics and technology, but had many adverse consequences for our civilization. It is fascinating to see that twentieth-century science, which originated in the Cartesian split and in the mechanistic world view, and which indeed only became possible because of such a view, now overcomes this fragmentation and leads back to the idea of unity expressed in the early Greek and Eastern philosophies.

…

In contrast to the mechanistic Western view, the Eastern view of the world is 'organic'. For the Eastern mystic, all things and events perceived by the senses are interrelated, connected, and are but different aspects or manifestations of the same Physics ultimate reality. Our tendency to divide the perceived world into individual and separate things and to experience ourselves as isolated egos in this world is seen as an illusion which comes from our measuring and categorizing mentality. It is called avidya, or ignorance, in Buddhist philosophy and is seen as the state of a disturbed mind which has to be overcome:

> *When the mind is disturbed, the multiplicity of things is produced, but when the mind is quieted, the multiplicity of things disappears.*

…

Before studying the parallels between modern physics and Eastern mysticism, we have to deal with the question of how we can make any comparison at all between an exact science, expressed in the highly sophisticated language of modern mathematics, and spiritual disciplines which are mainly based on meditation and insist on the fact that their insights cannot be communicated verbally.

What we want to compare are the statements made by scientists and Eastern mystics about their knowledge of the world. To establish the proper framework for this comparison, we must firstly ask ourselves what kind of 'knowledge' we are talking about; does the Buddhist monk from Angkor Wat or Kyoto mean the same thing by 'knowledge' as the physicist from Oxford or Berkeley? Secondly, what kind of statements are we going to compare? What are we going to select from the experimental data, equations and theories on the one side, and from the religious scriptures, ancient myths, or philosophical treatises on the other? This chapter is intended to clarify these two points: the nature of the knowledge involved and the language in which this knowledge is expressed. ….

…

Rational knowledge is derived from the experience we have with objects and events in our everyday environment. It belongs to the realm of the intellect whose function it is to discriminate, divide, compare, measure and categorize. In this way, a world of intellectual distinctions is created; of opposites which can only exist in relation to each other, which is why Buddhists call this type of knowledge 'relative'.

Abstraction is a crucial feature of this knowledge, because in order to compare and to classify the immense variety of shapes, structures and phenomena around us we cannot take all their features into account, but have to select a few significant ones.

…

The realm of rational knowledge is, of course, the realm of science which measures and quantifies, classifies and analyses. The limitations of any knowledge obtained by these methods have become increasingly apparent in modern science, and in particular in modern physics which has taught us, in the words of Werner Heisenberg, 'that every word or concept, clear as it may seem to be, has only a limited range of applicability."

…

What the Eastern mystics are concerned with is a direct experience of reality which transcends not only intellectual thinking but also sensory perception. In the words of the Upanishads:

> *What is soundless, touchless, formless, imperishable, Likewise tasteless, constant, odourless,*
> *Without beginning, without end, higher than the great, stable- By discerning That, one is liberated from the mouth of death.*

Knowledge which comes from such an experience is called 'absolute knowledge' by Buddhists because it does not rely on the discriminations, abstractions and classifications of the intellect which, as we have seen, are always relative and approximate. It is, so we are told by Buddhists, the direct experience of undifferentiated, undivided, indeterminate 'suchness'. Complete apprehension of this suchness is not only the core of Eastern mysticism, but is the central characteristic of all mystical experience.

…

Absolute knowledge is thus an entirely non-intellectual experience of reality, an experience arising in a non-ordinary The state of consciousness which may be called a 'meditative' or mystical state. That such a state exists has not only been testified by numerous mystics in the East and West but is also indicated by psychological research.

…

Although physicists are mainly concerned with rational knowledge and mystics with intuitive knowledge, both types of knowledge occur in both fields. This becomes apparent when we examine how knowledge is obtained and how it is expressed, both in physics and Eastern mysticism.

…

In physics, knowledge is acquired through the process of scientific research which can be seen to proceed in three stages. The first stage consists in gathering experimental evidence about the phenomena to be explained. In the second stage, the experimental facts are correlated with mathematical symbols and a mathematical scheme is worked out which interconnects these symbols in a precise and consistent way. Such a scheme is usually called a mathematical model or, if it is more comprehensive, a theory. This theory is then used to predict the results of further experiments which are undertaken to check all its implications. At this stage, physicists may be satisfied when they have found a mathematical scheme and know how to use it to predict experiments. But eventually, they will want to talk about their results to non-physicists and will therefore have to express them in plain language. This means they will have to formulate a model in ordinary language which interprets their mathematical scheme. Even for the physicists themselves, the formulation of such a verbal model, which constitutes the third stage of research, will be a criterion of the understanding they have reached.

…

Intuitive insights, however, are of no use to physics unless they can be formulated in a consistent mathematical framework, supplemented by an interpretation in plain language. Abstraction is a crucial feature of this framework. It consists, as mentioned before, of a system of concepts and symbols which constitute a map of reality. This map represents only some features of reality; we do not know exactly which these are, since we started compiling our map gradually and without The critical analysis in our childhood. The words of our language are thus not clearly defined. They have several meanings, many of which pass only vaguely through our mind and remain largely in our subconscious when we hear a word.

…..

Eastern mysticism is based on direct insights into the nature of reality, and physics is based on the observation of natural phenomena in scientific experiments. In both fields, the observations are then interpreted and the interpretation is very often communicated by words. Since words are always an abstract, approximate map of reality, the verbal interpretations of a scientific experiment or of a mystical insight are necessarily inaccurate and incomplete. Modern physicists and Eastern mystics alike are well aware of this fact.

In physics, the interpretations of experiments are called models or theories and the realization that all models and theories are approximate is basic to modern scientific research. Thus the aphorism of Einstein, 'As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.' Physicists know that their methods of analysis and logical reasoning can never explain the whole realm of natural phenomena at once and so they single out a certain group of phenomena and try to build a model to describe this group. In doing so, they neglect other phenomena and the model will therefore not give a complete description of the real situation. The phenomena which are not taken into account may either have such a small effect that their inclusion would not alter the theory significantly, or they may be left out simply because they are not known at the time when the theory is built."

(CaF1) THE DYNAMIC UNIVERSE: „*The central aim of Eastern mysticism is to experience all phenomena in the world as manifestations of the same ultimate reality. This reality is seen as the essence of the universe, underlying and unifying the multitude of things and events we observe. The Hindus call it Brahman, the Buddhists Dharmakaya (the Body of Being), or Tathata (Suchness), and the Taoists Tao; each affirming that it transcends our intellectual concepts and defies further description. This ultimate essence, however, cannot be separated from its multiple manifestations. It is central to its very nature to manifest itself in myriad forms which come into being and disintegrate, transforming themselves into one another without end. In its phenomenal aspect, the cosmic One is thus intrinsically dynamic, and the apprehension of its dynamic nature is basic to all schools of Eastern mysticism.*

….

The more one studies the religious and philosophical texts of the Hindus, Buddhists and Taoists, the more it becomes apparent that in all of them the world is conceived in terms of movement, flow and change. This dynamic quality of Eastern philosophy seems to be one of its most important features. The Eastern mystics see the universe as an inseparable web, whose interconnections are dynamic and not static. The cosmic web is alive; it moves, grows and changes continually. Modern physics, too, has come to conceive of the universe as such a web of relations and, like Eastern mysticism; has recognized that this web is intrinsically dynamic. The dynamic aspect of matter arises in quantum theory as a consequence of the wave-nature of subatomic particles, and is even more essential in relativity theory, as we shall see, where the unification of space and time implies that the being of matter cannot be separated from its activity. The properties of subatomic particles can therefore only be understood in a dynamic context; in terms of movement, interaction and transformation.

…

In physics, we recognize the dynamic nature of the universe not only when we go to small dimensions - to the world of atoms and nuclei - but also when we turn to large dimensionsto the world of stars and galaxies. Through our powerful telescopes we observe a universe in ceaseless motion. Rotating clouds of hydrogen gas contract to form stars, heating up in the process until they become burning fires in the sky. When they have reached that stage, they still continue to rotate, some of them ejecting material into space which spirals outwards and condenses into planets circling around the star. Eventually, after millions of years, when most of ist hydrogen fuel is used up, a star expands, and then contracts again in the final gravitational collapse. This collapse may involve gigantic explosions, and may even turn the star into a black hole. All these activities the formation of stars out of interstellar gas clouds, their contraction and subsequent expansion, and their final collapse-can all actually be observed somewhere in the skies.

The spinning, contracting, expanding or exploding stars cluster into galaxies of various shapes-flat discs, spheres, spirals, etc. - which, again, are not motionless but rotate. Our galaxy, the Milky Way, is an immense disc of stars and gas turning in space like a huge wheel, so that all its stars - including the Sun and its planets - move around the galaxy's centre. The universe is, in fact, full of galaxies strewn through all the space we can see; all spinning like our own.

When we study the universe as a whole, with its millions of galaxies, we have reached the largest scale of space and time; and again, at that cosmic level, we discover that the universe is not static-it is expanding! This has been one of the most important discoveries in modern astronomy. A detailed analysis of the light received from distant galaxies has shown that the whole swarm of galaxies expands and that it does so in a well orchestrated way; the recession velocity of any galaxy we observe is proportional to the galaxy's distance. The more distant the galaxy, the faster it moves away from us; at double the distance, the recession velocity will also double. This is true not only for distances measured from our galaxy, but applies to any point of reference. Whichever galaxy you happen to be in, you will observe the other galaxies rushing away from you; nearby galaxies at several thousand miles per second, farther ones at higher speeds, and the farthest at velocities approaching the speed of light. The light from galaxies beyond that distance will never reach us, because they move away from us faster than the speed of light."

(CaF1) EMPTINESS AND FORM: „*With the concept of the quantum field, modern physics has found an unexpected answer to the old question of whether matter consists of indivisible atoms or of an underlying continuum. The field is a continuum which is present everywhere in space and yet in its particle aspect has a discontinuous, 'granular' structure. The two apparently contradictory concepts are thus unified and seen to be merely different aspects of the same reality. As always in a relativistic theory, the unification of the two opposite concepts takes place in a dynamic way: the two aspects of matter transform themselves endlessly into one another. Eastern mysticism emphasizes a similar dynamic unity between the emptiness and the forms which it creates.*

The field theories of modern physics have led not only to a new view of subatomic particles but have also decisively modified our notions about the forces between these particles. The field concept was originally linked to the concept of force, and even in quantum field theory it is still associated with the forces between particles. The electromagnetic field, for example, can manifest itself as a 'free field' in the form of travelling waves/photons, or it can play the role of a field of force between charged particles. In the latter case, the force manifests itself as the exchange of photons between the interacting particles. The electric repulsion between two electrons, for example, is mediated through these photon exchanges.
...
The conception of physical things and phenomena as transient manifestations of an underlying fundamentalentity is not only a basic element of quantum field theory, but also a basic element of the Eastern world view. Like Einstein, the Eastern mystics consider this underlying entity as the only reality: all its phenomenal manifestations are seen as transitory and illusory.

This reality of the Eastern mystic cannot be identified with the quantum field of the physicist because it is seen as the essence of a// phenomena in this world and, consequently, is beyond all concepts and ideas. The quantum field, on the other hand, is a well defined concept which only accounts for some of the physical phenomena. Nevertheless, the intuition behind the physicist's interpretation of the subatomic world, in terms of the quantum field, is closely paralleled by that of the Eastern mystic who interprets his or her experience of the world in terms of an ultimate underlying reality. Subsequent to the emergence of the field concept, physicists have attempted to unify the various fields into a single fundamental field which would incorporate all physical phenomena. Einstein, in particular, spent the last years of his life searching for such a unified field. The Brahman of the Hindus, like the Dharmakaya of the Buddhists and the Tao of the Taoists, can be seen, perhaps, as the ultimate unified field from which spring not only the phenomena studied in physics, but all other phenomena as well.

In the Eastern view, the reality underlying all phenomena is beyond all forms and defines all description and specification. It is therefore often said to be formless, empty or void. But this emptiness is not to be taken for mere nothingness. It is, on the contrary, the essence of all forms and the source of all life.
....

Buddhists express the same idea when they call the ultimate reality Sunyata 'Emptiness', or 'the Void-and affirm that it is a living Void which gives birth to all forms in the phenomenal world. The Taoists ascribe a similar infinite and endless creativity to the Tao and, again, call it empty. The Tao of Heaven is empty and formless' says the Kuan-tzu and Lao-tzu uses several metaphors to illustrate this emptiness. He often compares the Tao to a hollow valley, or to a vessel which is for ever empty and thus has the potential of containing an infinity of things.

In spite of using terms like „empty" and „void", the Eastern sages make it clear that they do not mean ordinary emptiness when they talk about Brahman, Sunyata or Tao, but, on the contrary, a Void which has an infinite creative potential. Thus, the Void of the Eastern mystics can easily be compared to the quantum field of subatomic physics. Like the quantum field, it gives birth to an infinite variety of forms which it sustains and, eventually, reabsorbs.
…..
It is interesting to note that physicists have used the same analogy in the context of field theory to point out the illusion of a material substance created by a moving particle. Thus Hermann Weyl writes:

> *According to the Field theory of matter a material particle such as an electron is merely a small domain of the electrical field within which the field strength assumes enormously high values, indicating that a comparatively huge field energy is concentrated in a very small space. Such an energy knot, which by nomeans is clearly delineated against the remaining field, propagates through empty space like a water wave across the surface of a lake; there is no such thing as one and the same substance of which the electron consists at all times."*

Chen F. F.
Plasma physics

(ChF) p.1: „*It has often been said that 99% of the matter in the universe is in the plasma state; that is, in the form of an electrified gas with the atoms dissoviated into positive ions and negative electrons*."

Definition of plasma

(ChF) p. 3: „*Any ionized gas cannot be called a plasma, of cource; there is always some small degree of ionization in any gas. A useful definition is as follows:*

A plasma is a quasineutral gas of charged and
neutral particles which exhibits collective behavior.

We must now define „quasineutral" and „collective behavior". …."

Phase vs. group velocity of waves in a plasma exceeding vs. not exceeding the velocity of light c

(ChF) p. 81: „*The phase velocity of a wave in a plasma often exceeds the velocity of light c. This does not violate the theory of relativity, because an infinitely long wave train of constant amplitude cannot carry information. The carrier of a radio wave, for instance, carries no information until it is modulated. The modulation information does not travel at the phase velocity but at the group velocity, which is always less than c. To illustrate this, we may consider a modulated wave formed by adding („beating") two waves of nearly equal frequencies. Let these waves be*

$$E_1 = E_0 cos[(k + \Delta k)x - (\omega + \Delta\omega)t]$$
$$E_2 = E_0 cos[(k - \Delta k)x - (\omega - \Delta\omega)t]$$

E_1 and E_2 differ in frequency by $2\Delta\omega$. Since each wave must have the phase velocity ω/k appropriate to the medium in which they propagate, one must allow for the difference $2\Delta\omega$ in propagation constant. Using the abbreviation $a = kx - \omega t$, $b = (\Delta k)x - (\Delta\omega)t$ we have

$$E_1 + E_2 = E_0\, cos(a+b) + E_0\, cos(a-b)$$
$$= E_0[cos\, a\, cos\, b - sin\, a\, sin\, b + cos\, a\, cos\, b + sin\, a\, sin\, b]$$
$$= 2E_0\, cos\, a\, cos\, b$$

i.e., $E_1 + E_2 = 2E_0(coskx - \omega t)cos(\Delta k)x - (\Delta\omega)t$. *This is a sinusoidally modulated wave. The envelope of the wave, given by* $cos(\Delta k)x - (\Delta\omega)t$, *is what carries information; it travels at velocity* $\Delta\omega/\Delta t$. *Taking the limit* $\Delta\omega \to 0$, *we define the group velocity to be*

$$v_g = d\omega/dk.$$

It is this quantity that cannot exceed c.

Linear vs. nonlinear Landau damping phenomena
arising from different physical effects

(ChF) p. 245: Landau damping is a characteristic of collisionsless plasmas, but it may also have application in other fields. For instance, in the kinetic treatment of galaxy formation, stars can be considered as atoms of a plasma interacting via graviational rather than electromagnetic forces. Instabilities of the gas of stars can cause spiral arms to form, but this process is limited by Landau damping."

(ChF) p. 248-249: „*There are actually two kinds of Landau damping: linear Landau damping, and nonlinear Landau damping. Both kinds are independent of dissipative collisional mechanisms. If a particle is caught in the potential well of a wave, the phenomenon is called „trapping". Particles can indeed gain or lose energy in trapping. However, trapping does not lie within the purview of the linear theory. …. Trapping is not in the linear theory. When a wave grows to a larger amplitude, collisonless damping with trapping occurs. One then finds that the wave does not decay monotonically; rather the amplitutes fluctuates during the decay as the trapped particles bounce back and forth in the potential wells. This is nonlinear Landau damping. .. Since the linear Landau damping is derived from a linear theory, … the nonlinear Landau damping must arise from a different physical effect. The question is: Can untrapped electrons moving close to the phase velocity of the wave exchange energy with the wave?"*

(ChF) P. 254: „*Neither the untrapped particles nor particle trapping are responsible for linear Landau damping. … Indeed, there are particles in the original distribution which have velocities so close to* v_φ *that at the time* t *they have not yet gone half-wavelength relative to the wave. For these particles, one cannot take the average* $\langle \Delta W_k \rangle$. *These particles can absorb energy from the wave and are properly called the „resonant" particles. As time goes on, the number of resonant electrons increases, since an increasing number will have shifted more than* $\frac{1}{2}\lambda$ *from their original positions. The damping rate, however, can stay constant, since the amplitude is now smaller, and it takes fewer electrons to maintain a constant damping rate*."

(ChF) p. 260: The resonant particles
„*We are now in a position to see precisely which are the resonant particles that contribute to linear damping. … These particles rapidly become spread out in phase, so that they contribute little to the average; the initial distribution is forgotten. … Those particles may include both trapped and untrapped particles. This phenomenon is unrelated to particle trapping."*

(ChF) p. 260: Two paradoxes resolved
„*The function which describes the relative contribution of various velocity groups to Landau damping is an even function of* $\omega - ku$ *so that the particles going both faster than the wave and slower than the wave add to Landau damping. On the other hand, the slope of the curve of this function, … is an odd function of* $\omega - ku$; *and one would infer from this particles traveling faster than the wave give energy up to it, while those traveling slower than the wave takes energy from it. The two descriptions differ by an integration by parts. Both descriptions are correct; which one is the be chosen depends on whether one wishes to have* $\hat{f}_0(v)$ *or* $\hat{f}_0'(v)$ *in the integrand (of the formula of the rate of change of the wave energy density function).*

A second paradox concerns the question of Galilean invariance. If we take the view that damping requires there be fewer particles traveling faster the wave than slower, there is no problem as long as one is in the frame in which the plasma is at rest. However, if one goes into another frame moving with velocity than V, there would appear to be more particles faster than the wave than slower, and one would expect the wave to grow instead of decay. This paradox is removed by reinserting the second term $\frac{2ku}{\omega - ku}$, which we neglected. This term can make $\langle \Delta W_k \rangle$ negative … and the wave appears to have negative energy (that is, there is more energy in the quiescent, drifting Maxwelllian distribution than in the presence of an oscillation). The wave „growth", but adding energy to negative energy wave makes ist amplitude decrease."

(ChF) p. 261: „*We have seen that Landau damping is directly connected to the requirement that $f_0(v)$ be initially uniform in space. On the other hand, one can generate undampted electron waves if $f_0(v, t = 0)$ is made to be constant along the particle trajectories initially. Those particled will neither gain nor lose energy, on the average, if the plasma is initially prepared to that the density is constant along each trajectory. Such a wave is called a BGK mode (I. B. Bernstein, J. M. Green, M. D. Krustal)."*

Microwave radiation pressure to plasma
Ponderomotive force

(ChF) p. 305, 307: „*Light waves exert radiation pressure which is usually very weak and hard to detect. … When high-powered microwaves or laser beams are used to heat or confine plasmas the radiation pressure can reach several hundred thousand atmospheres! When applied to plasma, this force is coupled to the particles in a somewhat subtle way and is called the ponderomotive force. Many nonlinear phenomena have a simple explanation in terms of the ponderomotive force."*

Nonlinear Landau damping or growth
Potential due to ponderomotive force

(ChF) p. 328: „*When the amplitude of an electron or ion wave excited, say by a grid is followed in space, it is often found that the decay is not exponential, as predicted by linear theory, if the amplitude is large. Instead, one typically finds that the amplitute decays, grows again, and then oscillates before settling down to a steady value. … although other effects may also be operative, these oscillations in amplitutes are exactly what would be expected from the nonlinear effect of particle trapping discussed in section 7.5. Trapping of velocity occurs when its energy in the wave frame is smaller than the wave potential. Small waves will trap only these particles moving at high speeds near v_φ. … When the wave is large, its linear behavior can be exspected to be greatly modified. .. There is a bounce frequency ω_B of a sinusoidal potential well with corresponding potential and equation of motion, where the frequency is not constant unless x is small, and the potential is approximattely parabolic. … When the resonant particles are reflected by the potential, they give kinetic energy back to the wave, and the amplitude increases. When the particles bounce again from the other side, the energy goes back into the particles, and the wave is damped. Thus, one would expect oscillations in amplitutde at the frequence ω_B in the wave frame. … The condition $\omega_B \geq \omega$ turns out to define the breakdown of linear theory even when other processes besides particle trapping are responsible. Another typ of nonlinear Landau damping involves the beating of two waves. Suppose there are two high-frequency electron waves (ω_1, k_1) and (ω_2, k_2). These would beat to form an amplitude envelope traveling at a velocity $\frac{\omega_2 - \omega_1}{k_2 - k_1} \approx \frac{d\omega}{dk} = v_g$. This velocity may be low enough to lie within the ion distribution function. There can then be an energy exchange with the resonant inos. The potential the ions see is the effective potential due to the ponderomotive force, and Landau damping or growth can occur. Damping provides an effective way to heat ions with high-frequency waves, which do not ordinary interact with ions. If the ion distribution is double-humped, it can excite the electron waves, Such an instability is called a modulational instability."*

The Korteweg-de Vries and the Schrödinger equations
of nonlinear plasma physics
Ponderomotive force forming isolated structures
called envelope solitary waves

(ChF) p. 330: „*There are two nonlinear equations that have been treated extensively in connection with nonlinear plasma waves: The Korteweg-de Vries equation and the nonlinear Schrödinger equation. Each concerns a different type of nonlinearity. When an ion acoustic wave gains large amplitude, then main nonlinear effect is wave steeping, whose physical explanation was given in section 8.3.3. This effect arises from the* $\boldsymbol{v} \cdot \nabla \boldsymbol{v}$ *term in the ion equation of motion and is handled mathematically by the Korteweg-de Vries equation. The wave-train and the soliton solutions are also predicted by this equation.*

When an electron plasma wave goes nonlinear, the dominant new effect is that the ponderomotive force of the plasma waves causes the background plasma to move away, causing a local depression in density called a caviton*. Plasma waves trapped in this cavity then form an isolated structure called an* envelope soliton *or* envelope solitary wave*. Such solutions are described by the nonlinear Schrödinger equation. Considering the difference in both the physical model and the mathematical form of the governing equations, it is surprising that solitons and envelope solitons have almost the same shape.*“

The Korteweg-de Vries equation

(ChF) p. 331: „*This equation occurs in many physical situations including that of a weakly nonlinear ion wave:*

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial \xi} + \frac{1}{2}\frac{\partial^3 U}{\partial \xi^2} = 0$$

where U *is the amplitude, and* τ *and* ξ *are timelike and spacelike variables, respectively. Although several transformations of variables will be necessary before this form is obtained, two physical features can already be seen. The second term is easily recognized as the convective term* $\boldsymbol{v} \cdot \nabla \boldsymbol{v}$ *leading to wave steepening. The third term arises from wave dispersion; that is, the* k *dependence of the phase velocity.*“

The nonlinear Schrödinger equation

(ChF) p. 336: „*This equation has the standard dimensionless form*

$$i\frac{\partial \psi}{\partial t} + p\frac{\partial^2 \psi}{\partial x^2} - q|\psi|^2\psi = 0$$

where ψ *is the wave amplitude,* $i = (-1)^{1/2}$*, and* p *and* q *are coefficients whose physical significance will be explained shortly. This equation differs from the usual Schrödinger equation*

$$ih\frac{\partial \psi}{\partial t} + \frac{h^2}{2m}\frac{\partial^2 \psi}{\partial x^2} - V(x,t)\psi = 0$$

in that the potential $V(x,t)$ *depends on* ψ *itself, making plane waves of the non-linear Schrödinger equation in the form. Note however, that* V *depends only on the magnitude and not on the phase of* ψ*. This is to be expected, as far as electron plasma waves are concerned, because the nonlinearity comes from the ponderomotive force, which depends on the gradient of the wave intensity*.

Plane wave solutions of the (standard dimensionless) equation are modulationally unstable if $pq > 0$*; that is, a ripple on the envelope of the wave will tend to grow. For plasma waves, it is easily to see how the ponderomotive force can cause a modulational instability. The ponderomotive force moves both electrons and ions toward the intensity minima, forming a ripple in the plasma density. Plasma waves are trapped in regions of low density.*“

Courant R.
Empirical evidence and mathematical existence

(HiS) p. 148: „*Empirical evidence can never establish mathematical existence – nor can the mathematician's demand for existence be dismissed by the physicist as useless rigor. Only a mathematical existence proof can ensure that the mathematical description of a physical phenomenon is meaningful*."

Davidson J.
Das Geheimnis des Vakuums

(DaJ1) p. 17: „*Obwohl die Energiedichte dieser Vakuum-Oszillationen immens hoch ist, entstehen grundlegende Schwierigkeiten, wenn man versucht, diese Form der Energie in brauchbare Arbeit umzuwandeln. Es ist jedoch vorstellbar, daß die in den Quantenfluktuationen des Vakuums enthaltenen Energie eine gewisse Form von interaktiven „Raumpartikeln" hervorbringt, die eine geordnete, dynamische Vakuumstruktur erzeugen, welche sowohl elektromagnetische als auch gravitative Aspekte enthalten und außerdem wesentlich mit der Entstehung, dem Erhalt und der Gestaltung von biologischen Lebensformen verknüpft sind".*

Dee's Story

key words: steps toward thermodynamic & gravitational initiation", „two movements of mass toward aggregation (gravity) and toward dispersion (thermodynamics)" (*).
(*) Those two kinds of movements put the spot on Viktor Schauberger's two kinds of mechanical energies governing centrifugal the (linear movements) and (the much stronger acting) centripetal forces

(DeK) p. 3*: „Consider the Big Bang Theory. A mass the size of our Universe appeared out of nowhere. Even though it was the biggest black hole ever, it then exploded. Our universe is a real thing but the Big Bang Theory is a fairy tale."*

(DeK) p. 98: *Steps toward thermodynamic/gravitational initiation*
„As an universe matures, the movement of mass goes in two directions: toward aggregation and toward dispersion. These translate into gravity and thermodynamics, the two great opposites. … To make thermodynamics work, space beyond gravity must be present. With gravity's constant force, space creation must also be a constant process.

In view of the described shrinking at high speed with the appearance of increased space between, gravity, the great attractor, is also the great space creator. If gravity is, as the general relativity proposes, just a field in space, expansion and contraction are just the plus and the minus of the same thing. For the concentration of mass suitable to make the environment we know, both directions have no future unless there is a way to initialize or restart them at their primodial, more mixed arrangement. … The implosion theory of universe creation process is that initializer. Of course, there are provisional arrangements such as slower development (e.g. small stars), explosions and orbiting to stop failing. But current theory has no place for any long term thermodynamic/gravitational reset.

Considering the character of the flow between gravity and thermodynamic tendency, nature seems to favor stepwise energy changes. Each of these steps is separated from the next by some conditional barrier and, often, a catalyst-like agent needed to make the change to the next step. In animals, for example, metabolic energy release from otherwise stable molecules is made in a stepwise fashion and is made possible by enzymes, catalyst-like agents. In stars, there are also stable steps separated by change periods that only occur under special circumstances."

(PeR) p. 444: „*In order to produce an universe resembling the one in which we live, the Creator would have to aim for an absurdly tiny volume of phase space of possible universes – about* $1/10^{10^{123}}$ *of the entire volume, for the situation under consideration*".

Dehnen H , Hönl H., and Westphal K.
A heuristic approach to the General Relativity
extracts from (DeH) translated by DeepL

Table of Contents

A heuristic approach to the general theory of relativity is presented, which, while avoiding covariant theory, addresses the three Einstein effects that can still be tested today: redshift of the Sun's spectral lines, light deflection in the Sun's gravitational field, and perihelion motion of planetary orbits, in connection with the Einstein–Mach principle of the "relativity of inertia." Finally, a historical-critical overview of the significance of the Mach principle in general relativity.

§ 1. Introduction

The primary aim of this study is not to provide a didactically convenient "elementary" introduction to the basic concepts and most important facts of general relativity; rather, it attempts, through a new arrangement of the arguments, to make the internal structure of the complex of general relativity theory more manageable and, by this heuristic approach, to facilitate and, if possible, deepen the physical understanding of the theory. In doing so, we limit ourselves to the presentation of first- and second-order effects (shift of spectral lines and light deflection in a gravitational field, perihelion motion of planetary orbits). The exact form of the general covariant field equations of the metric field is not the subject of our considerations; on the contrary, it is our intention to show how far one can advance in the understanding of general relativity—or, more precisely, metric gravitational theory—without using covariant theory.

To achieve this goal, it is helpful to distinguish from the outset between "physically measurable" quantities and those that are merely "fictitious." The former refer to measurements that have actually been carried out or are at least possible in principle, regardless of whether these can be performed "directly" or whether the experimental result - the measurement of a specific physical quantity - is obtained "indirectly" through a series of inferences using physical laws. Direct natural measurements include applying a ruler to a real object to determine its dimensions, measuring a time interval between two physical events using a standard clock at rest, determining a mass using a set of weights, or measuring a velocity by simultaneously using a ruler and a standard clock. Indirect natural measurements are all procedures that serve, in a known manner, to determine physical quantities - such as the constants of nature. The validity of physics as an empirical science rests on the consistency of results obtained by various independent methods. The fact that this consistency is in principle achievable—apart from the assumed logical consistency of the theory - stems from the fact that all measurements are based on calibrations and all calibrations can ultimately be traced back to the calibration of a standard of length, time, and mass (C.G.S. system). There is therefore no fundamental difference between direct and indirect natural measurements.

On the other hand, physics cannot do without certain fictional concepts. Such fictions played an important heuristic role in the extension of special relativity to general relativity. For example, the influence of gravitational fields on rigid rulers and standard clocks can be described such that the "length" of the ruler and the "period" of the clock undergo certain "fictitious" changes in the gravitational field, which apply in the same way to all bodies characterized as "rigid" and all "standard" clocks i n t h e s a m e w a y. These "changes" cannot, of course, be determined directly- i.e., by natural measurement at the location where the rulers, clocks, etc., are situated (i.e., through ordinary laboratory experiments) - since all measuring instruments are subject to the same changes. Nevertheless, such fictions yield physical consequences that can be directly verified by experiment. Thus, the (fictitious) assumption that the period of oscillation of a standard clock (atom) on the surface of the Sun is greater than the period of oscillation of the same clock on Earth leads to the experimentally verifiable Red-shift in the Sun's spectral lines. Similarly, the (fictitious) assumption of a location-dependent "length" of measuring rods (which varies according to Newton's gravitational potential) implies that the theorems of Euclidean geometry no longer apply; we have here a simple "model" of a non-Euclidean three-dimensional manifold). The idea of a spatially variable speed of light in a gravitational field leads directly (according to Huygens' principle) to the deflection of light in the gravitational field; nevertheless, the "naturally measured" speed of light at every point in space would yield the normal value c.

These examples demonstrate the usefulness of such assumptions. Their fictional nature is reflected in the fact that all statements refer to a purely fictitious Euclidean reference spacex_1, x_2, x_3 and a location-independent

fictitious coordinate time t . Since we assume in the following that the special theory of relativity is well-known and no longer problematic in its structure, the influence of gravitational fields on geometry and the course of natural processes can be described without restriction in the manner of such fictitious assumptions. The influence of the gravitational field must then, however, not be limited to measuring rods and clocks, but must consequently be extended to all physical quantities such as mass, charge, force, etc.

Since, as the above examples show, the fictitiously introduced quantities lead directly to an interpretation of physical effects, one is tempted to call the quantities altered in the gravitational field "effective" as well. For example, a clock that has been placed for some time at a location of lower Newtonian potential (the surface of the Sun) and then returned to its original location would run "effectively" slow compared to a clock that did not make the journey; the rate of the moved clock must therefore have been slower on average. In the following, to avoid ambiguity, we will generally refrain from using the term "effective" and will only resort to it when discussing specific results (e.g., for mass and charge).

Naturally, the behavior of rulers, clocks, and masses plays a key role in these considerations, since the structure of mechanics since Newton has been based on measured lengths and times and on mass values (inertia) established through experiments. It is therefore surprising, at first glance, that the behavior of masses in the gravitational field has not been given the same attention as the behavior of measuring rods and clocks. The approach we wish to take to the problem consists in conceiving the "rigid" measuring rods and standard clocks from the outset as composed of elementary constituents such as electrons, protons, and neutrons (§2), whereby all the constants that characterize these elementary particles and their physical behavior - such as mass, charge, interaction forces, and the action quantum - are subject to fictitious changes in the gravitational field. A necessary condition for the feasibility of our method is that the interaction of the various fictitious changes is internally consistent. Furthermore, fictitious changes in the measured lengths and time intervals should result, which can at least in a first approximation be interpreted in terms of the metric line element of general relativity. Since the form of the line element is an exact solution to the general covariant field equations in simple cases is known (e.g., for the centrally symmetric field), we have here a criterion for the reliability of the method, against which we can subsequently test our heuristic approach. It should be explicitly stated, however, that - which is actually self-evident – the derivation of general-covariant field equations by Einstein and the search for solutions to them is, of course, far superior to the method described here, as is already evident from the fact that our method can reproduce only physical effects of the first and, at most, second order. But since these effects in themselves are of great physical interest (and are so far the only ones by which the general theory of relativity has been tested in the realm of the planetary system), one may regard this alone as sufficient justification for our presentation.

In particular, the problem of the influence of the gravitational field on mass is now closely connected with that complex of questions which - following the example of A. Einstein - is usually summarized under the name "Mach's principle." According to the application of Mach's principle to the "relativity of inertia" of ponderable matter, the mass of a test body (planet) should increase as it approaches large mass concentrations (the Sun). The discussion in §§3 and 4 of the dependence of mass on the (static) gravitational field confirms this assumption; however, the increase in mass is three times the amount assumed by Einstein. This result will be further corroborated, in particular, by a detailed analysis of the perihelion motion of the planets (second-order effect) in §5. We we will not go into detail on the physical significance of Mach's principle until §6 (Concluding Remarks).

It should be emphasized once again, as a matter of principle, that the method of introducing fictitious quantities actually yields only a specific "picture" or "model" of the actual conditions. Through such a model of the realization of non-Euclidean metric relations of the spacetime continuum, for example, the influence of the gravitational field on the metric is immediately and clearly interpreted. On the other hand, the description of physical effects within the framework of this concept makes it clear that this is merely a stopgap measure, by means of which it is possible to translate a state of affairs - which in itself can be adequately represented only in the conceptual language of a more perfect theory - back into the more primitive language of the older theory. For example, the theorem regarding the equality of inertial and gravitational mass has an immediately comprehensible meaning only within the framework of Newtonian theory; in general relativity, the phenomenon corresponding to this theorem has been so completely subsumed into the foundations of the theory that the distinction between inertial and gravitational mass has become irrelevant in the motion of a test body in a gravitational field has become irrelevant. In principle, the introduction of fictitious quantities in the gravitational field should be regarded as a straightforward method for expressing a context that is inherently

more complex in the more familiar terminology of the simpler Newtonian theory – or to represent them in their generalized form in the Lagrangian equations of motion. One must simply bear in mind that this simplified formulation cannot fully do justice to the process being described in every respect. In this sense, the realization of non-Euclidean metric relations as a consequence of fictitious changes in physical constants corresponds entirely to those other "images" through which one has attempted (here too, successfully) to interpret the Lorentz contraction and time dilation in special relativity. Such images, of course, are destined to become superfluous in the final, but largely formal, formulation of the theory. They can, however, very well promote physical understanding because they immediately reveal the reasons for the extension of the earlier theory.

§2 Atomic Standards and Clocks

§3 The heuristic method

§4 Behavior of mass and charge in a static gravitational field according to the general theory of relativity

§ 5 Analysis of the perihelion motion of the planets

§ 6. Historical Background on Mach's Principle

In the preceding sections, we have shown that the investigation of the influence of the gravitational field on physical quantities necessarily also results in a change in mass within the gravitational field. Such an influence of the gravitational field on the "effective" mass of a body has been anticipated by E i n s t e i n since the formulation of the general theory of relativity and interpreted in terms of the "relativity of inertia." He clearly stated that "inertia must have its origin in a kind of interaction between bodies" and that it therefore exists in a fully - "empty" space cannot impart any inertia. It is therefore to be expected that the inertia of a body increases as it approaches larger concentrations of mass. However, as already mentioned (p. 394), Einstein's own investigations into this question are, in quantitative terms, not very (the actual increase in effective mass in a gravitational field is three times the amount deduced by Einstein from the equations of motion, § 3-5). Nor does effective mass have the property of tending toward zero). Nevertheless, the effect hypothesized by E i n s t e i n exists with the modifications specified in this work.

Einstein was inclined from the outset to interpret the expected change in the inertial mass of a body in the gravitational field of other gravity-generating masses in the spirit of Mach's ideas on the critique of Newtonian mechanics. However, since the significance of what is known as the "Mach principle" according to Einstein is greatly complicated for the theory of natural phenomena by the fact that quite different things are conventionally grouped under this title, it seems appropriate to us to recall the historical development of the so-called Mach principle, in order thereby to distinguish the original and proper meaning of this principle from later additions and interpretations.

The emergence of the Mach principle is based on a simple empirical fact: namely, that inertial systems, for which Newtonian mechanics holds approximately, are, with a high degree of approximation, rotation-free with respect to the "fixed-star system." E. Mach, in particular, drew attention to this fact; he rightly emphasized that such a striking fact is not anchored in the foundations of Newtonian mechanics and made this circumstance the starting point of his critique of Newton's concept of "absolute space" (and likewise "absolute time"). Mach expressed the conjecture that this coincidence of a "dynamically" defined reference frame and, on the other hand, a purely "kinematically" defined reference frame could not be accidental. In this conjecture (and also in the subsequent critique), one must undoubtedly see the origin of Mach's principle. When Einstein, after completing the special theory of relativity - which had demonstrated the untenability of Newton's conception of absolute space and absolute time - proceeded to formulate the general theory of relativity, it was natural for him to use the "Mach principle" as a heuristic principle for the extension of his theory. But the further development of the general theory of relativity has shown that the Mach principle was dispensable for its foundation. Neither the formulation of the general covariant field equations of gravitation has anything to do with the Mach principle, nor are the known solutions to these equations consistently in agreement with this principle. Consequently, the significance of the Mach principle for physics has become doubtful, and it has even fallen into disrepute among many physicists.

Despite these valid objections, the fact remains that the aforementioned astronomical phenomenon cannot be explained by either Newtonian mechanics or the general theory of relativity. Modern astronomical research has exacerbated rather than alleviated the situation. Current knowledge of the arrangement of galactic systems in

space has increasingly demonstrated how precisely the "inertial compass" and the "fixed-star compass" (oriented toward extragalactic systems) coincide. We would like to see this circumstance as decisive evidence that Mach's principle, in its original cosmological significance, cannot be set aside even today. This, however, simultaneously raises the question of what the physical basis of this fact is. In our view, Einstein has already provided the correct answer to this question. For the determination of the metric (inertial systems) is generally not determined solely by the distribution of matter in space, but also by boundary conditions , for example, of the metric field in the ly infinite space, a determination of the metric in accordance with Mach's ideas can only take place if the boundary conditions in infinity are eliminated altogether. It then goes without saying that the metric and thus the phenomena of inertia (as well as the propagation of light) are, for given field equations (with or, better yet, without a cosmological Λ term), determined exclusively by the "matter tensor" $T_{\mu\nu}$, whereby Mach's principle also acquires a directly comprehensible concrete meaning. According to this, the universe must be finite and closed in its spatial extent. It suffices here to emphasize that the Mach principle, in its true sense - in contrast to later interpretations - is a cosmological principle that refers to the global (topological) structure of the universe as a whole. Since there are countless cosmological models but only one real universe, the Mach principle will assume the role of a selection principle that, from the large number of possible models, identifies a few as physically viable is set apart. This also implies that, in this sense, the Mach principle cannot be a strictly "constitutive" principle of experience - one that (like, for example, the requirement of general covariance of the field equations) can be used to derive laws of nature - but rather that it possesses the character of a "regulatory" principle (in Kant's sense). It is above all in this that the special status of Mach's principle vis-à-vis other physical principles seems to us to be grounded. On the other hand, however, there is initially no logically compelling reason to view the effects understood as an expression of the "relativity of inertia" (change in mass in a gravitational field) as being in direct connection with the cosmological question. We must rather regard this as a free interpretation of Mach's ideas by Einstein, whereas no hints of this kind are to be found in Mach. To dispel all doubt as to how Einstein interpreted Mach's ideas, here are a few characteristic sentences from his summary presentation "Fundamentals of the Theory of Relativity":

"Secondly, however, the theory of relativity makes it probable that Mach was on the right track with his idea that inertia is based on a certain interaction of matter. For we shall demonstrate in what follows that, according to our equations, inertial masses interact with one another (albeit very weakly) in the sense of the relativity of inertia. What must be expected in the light of Mach's idea?

1. *The inertia of a body must increase when ponderable masses are accumulated in its vicinity.*

2. *A body must experience an accelerating force when masses in its vicinity are accelerated, and this force must be in the same direction as that acceleration.*

3. *A rotating hollow body must generate a "Coriolis field" within itself, which deflects moving bodies in the direction of rotation, as well as a radial centrifugal field.*

We will now show that, according to our theory, these three effects expected based on Mach's ideas must indeed exist, albeit to such a small extent that confirmation through laboratory experiments is out of the question."

Point 1 addresses the problem discussed in detail here of the influence of the gravitational field on mass. The final remark regarding the smallness of the effects is not entirely correct, insofar as "laboratory experiments" are understood to include astronomical observations. Since the changes in all physical quantities in a gravitational field are interrelated, one can certainly relate the first-order general relativistic effects (redshift and light deflection in a gravitational field) to the small change in mass.

Point 3, however, is of particular interest (point 2 will not be discussed further here). This concerns the so-called Thirring effect, according to which, inside a rotating hollow body, a mass point moving perpendicular to the axis of rotation is deflected in the direction of the hollow body's rotation. The centrifugal effect mentioned above within rotating hollow bodies also follows from the theory, as Mr. Thirring has shown. This effect seems to point directly to Mach's principle, and some have sought to see in it a confirmation of Mach's ideas (analogy to Newton's bucket experiment). However, the unsatisfactory aspect of this interpretation lies in the fact that H. Thirring frames the problem in terms of his approximation method such that the metric field merges into the limit values of the pseudo-Euclidean metric in infinite space, so that the hollow body actually rotates relative to empty "absolute" space.

To make the connection with Mach appear more compelling, Mrs. Ch. Soergel-Fabricus has modified Thirring's calculation as follows: Instead of starting from the Euclidean metric of empty space, one starts from the metric in the closed Einstein universe as the "basic metric." Then, isolate a small fraction of the matter at every point and let it rotate "rigidly" at a constant angular velocity along a previously chosen spatial geodesic. After determining the gravitational field in a first approximation to the Einstein metric, one discusses the motion of a test body to see whether the influence of the rotation can be described by Coriolis and centrifugal forces and how these additional forces arise. One can now very simply summarize the results of Ms. Sörgel -Fabricus be summarized very simply as follows: the form and magnitude of these additional forces are exactly what one would expect quantitatively based on Mach's principle. Boundary conditions do not apply in the Einstein universe, so that the total inertial effects are attributable to the internal interaction of matter.

Ms. S o e r g e l - F a b r i c i u s 's calculations seem significant to us not only because they give Mach's ideas a more concrete form, but above all because this example clearly shows how the two lines of thought traditionally associated with the Mach principle, are intrinsically connected: the effects understood as the "relativity of inertia" with the cosmological problem. It thus also becomes apparent that the entirety of Einstein's views does indeed stem from a single source.

In this context, it should be particularly noted that even when assuming the hypothesis of the relativity of inertia, it makes good physical - and at the very least practical - sense to speak of "absolute rotation." For there are cosmological models with non-zero density (e.g., static Einstein -cosmos), in which, essentially (i.e., apart from trivial coordinate transformations), an axially symmetric coordinate system can be introduced in only one way, such that, with respect to this system, all matter is at rest and no Coriolis or centrifugal forces occur. Such a system approximately assumes the role of an inertial system in the standard theory; with respect to any other reference frame "rotating" relative to this distinguished system, the familiar "apparent forces" immediately arise. However, in such a cosmological model, it poses no fundamental difficulty to attribute the occurrence of these forces to interactions of matter. - In many models, it even makes concrete sense to retain the concept of "absolute velocity." As examples, let us consider the non-stationary world models with isotropic expansion or contraction. In these models, the following simple law applies to peculiar motion (translational inertial motion of isolated masses): $pR = const$ *, where* $p = p(t)$ *denotes the momentum measured "relative to the substrate," and* $R = R(t)$ *denotes the time-varying (average) radius of curvature of the cosmos. The substrate itself has a specific state of motion at every location and at every time, which is equated with that of rest can be defined. This "rest frame" is apparently physically well-defined; for mass points that have zero velocity relative to this frame do not change their state of motion (*$p(t) = 0$ *). The fact that the momentum (and thus also the velocity) relative to the substrate generally (for* $p > 0$*) and increases during contractions can, in the sense of Mach's principle, very well be understood as the effect of an induction of the surrounding matter (the material substrate) on the mass point (this is an effect as described above under point 2 (p. 400). In the expansion of a cosmos without matter (such as the de Sitter cosmos, in which the state of motion of the substrate is nevertheless determined everywhere), such a physical interpretation would of course be invalid. The paradox of the concept of "absolute velocity" lies in the fact that even in this case such a concept can still be defined kinematically (i.e., with respect to the "substrate" that has become immaterial). With this conceptualization, which is possible "in itself," one has, however, already strayed so far from the original concept of velocity - which is, of course, a relational concept, i.e., based on the perception of a materially detectable reference body - that such concepts and ideas cannot be perceived as anything other than "unnatural" or "unphysical."*

From these considerations, we may well conclude that, among the great variety of exact solutions to the field equations of general relativity, there are only a few that come into consideration for describing the actual conditions, both in terms of empirical facts and in terms of the a priori requirements to be placed on a theory. In making the necessary selection here, however, Mach's principle will be able to provide an important guide. -

We could conclude our discussion here, were it not for the fact that views have recently been put forward that are entirely contrary to the spirit of the general theory of relativity and that regard Mach's principle in a purely negative light. Since this is a body of thought that should not be abandoned without good reason, a few general remarks may follow here.

All effects predicted by the general theory of relativity with respect to the planetary and Milky Way systems are extraordinarily "small" (and therefore have not yet been determined experimentally with sufficient precision), i.e., the deviations from the laws known since Newton, Maxwell, and the special theory of relativity are very slight. From an empirical standpoint, it is therefore natural to proceed from the fact of inertial systems and the

Lorentz invariance of the fundamental equations applicable to them, and to incorporate the gravitational effects into the framework of a complete conceptual language provided thereby. This procedure was also followed in the preceding discussion. The same standpoint has recently been taken in an interesting paper, albeit in a decidedly polemical stance against the general theory of relativity and especially against Mach principle. W. Thirring shows in particular that in this way a far-reaching adaptation of Einstein's theory of gravitation - at least as far as first-order effects are concerned - to the formalism of field theory for elementary particles can be accomplished, so that gravitation does not appear to occupy any special position vis-à-vis other types of elementary interactions. W. Thirring's general view, however, goes further in that the Mach principle is not only entirely superfluous but even contradicts the fundamental concept of quantum field theory. In contrast, it must be emphasized that the concept of "vacuum" in quantum field theory also presupposes that of inertial systems - and that it makes good physical sense to ask about the origin of inertial frames and the Lorentz group. This question need not, of course, be asked within quantum field theory (it is, so to speak, "bracketed out" there); but to fail to ask it in principle means attributing no significance to the cosmological problem.

The above-mentioned viewpoint, suggested by experience, becomes insufficient, however, as soon as one considers larger portions of the physical world. For example, the cosmic redshift of extragalactic systems is a powerful effect that can only be satisfactorily interpreted within the framework of general relativity (i.e., on the basis of the non-linear field equations). From this perspective, the true domain of general relativity is cosmology. Just as special relativity only begins to yield results significantly only when one moves to "large" speeds comparable to the speed of light, so general relativity becomes of decisive importance only in regions of "cosmic" scale, whose dimensions are no longer small compared to the mean radius of curvature of the universe. With the question of the nature of the world on a large scale, however, Mach's principle is also restored to its former prominence.

Perhaps this latter view also sheds some light on the much-discussed question of the relationship between general relativity and quantum theory. We have shown in §3 that the action quantum h is entirely independent of the gravitational field, i.e., also of arbitrarily strong is not influenced by gravitational fields. This seems to indicate that the scope of the laws of the theory of relativity and those of quantum theory are far apart; or, to be more precise: that the aspect under which the physical world appears in general relativity is fundamentally different from the one under which it appears in quantum theory, in such a way that there is no hope of reducing the laws of quantum theory to those of the theory of relativity (while the opposite does not appear entirely hopeless). This contrast is famously in the profound difference between "macrophysics" and "microphysics," or between classical-deterministic continuum physics and statistical-discontinuous quantum phenomena. It would therefore be consistent to limit the scope of general relativity to the macrophysical behavior of bodies and to refrain from extending the space-time structure of general relativity into the dimensions of elementary particles and atoms. This view is suggested precisely by Mach's principle: for according to it, space and time can only have meaning as conceivable interrelationships between bodies and between events, but cannot be conceived as absolute, physically effective realities. Furthermore, rulers and clocks are always finitely extended measuring bodies whose properties cannot be readily transferred to the differentials of the metric field. Therefore, the space-time continuum of the theory of relativity is likely to possess the physical significance of a continuum approximation of interactions between bodies (elementary particles), which are governed by the laws of quantum theory. This approximation will be all the more accurate the more material bodies are involved in its construction. The space-time continuum, accordingly, it is merely the "stage" (res extensa) on which the actual events of the world - quantum events - unfold. In this sense, the gravitational field does indeed occupy a special position relative to all other types of fields (electromagnetic field, meson field, etc.).

A (well-known) example may clarify this further. Let a large number of small bodies that generate gravity be arranged in a plane in a lattice-like pattern and firmly connected to one another; other small bodies of various masses m strike the lattice from a specific direction (roughly perpendicular to the lattice) and with equal initial velocities v ; furthermore, let the field of the lattice be superimposed by an external gravitational field. As long as the mutual distance between the lattice points is very large compared to the de Broglie wavelength $\frac{h}{m}v$, the motion of the free mass points according to the law of gravitation (principle of equivalence), and all bodies move along identical trajectories regardless of their mass. However, if the de Broglie wavelength is of the same order of magnitude as the lattice spacing, diffraction occurs and there is a spectral decomposition of the incident masses according to their momenta, even though all interactions are gravitational effects. In the latter case, we find ourselves outside the scope of classical mechanics and gravitational theory, and the laws of

quantum theory apply. It thus becomes apparent that only the laws of gravitational theory are occasionally violated, but never the laws of quantum theory. Quantum theory therefore takes precedence over classical theory, the most complete formulation of which is general relativity. Consequently, the laws of the metric field, which are in principle independent of the elementary laws of quantum theory, cannot possess absolute validity. The regularity of the metric field would thus be linked - albeit in a statistical sense - to elementary interactions, as is, incidentally, in accordance with Mach's principle.

Deleuze G.
Differenz und Wiederholung

(PfM) S. 100: *„Deleuze will in einem Werk die konstitutive Differenz untersuchen: diese Differenz ist die Entstehungsbedingung quasi im Hintergrund. … Man kann dies auch transzendentale Differenz (d.h. die Bedingung der Ermöglichung von Differenz) nennen. … für jede Individuation (nach Simondon und Deleuze) braucht es immer eine transzendentale Differenz bzw. Disparation. .. „Eine jedwede Individuation erfordert die Existenz eines metastabilen Zustandes, eines Zustandes der Disparation. … Der Unterschied zwischen einer physikalischen und einer vitalen Individuation ist nach Simondon folgender: Der wachsende Kristall bekommt ein einziges Mal die eine Information und fängt dann an zu wachsen. Ein Lebewesen reagiert wiederholt auf verschiedenste Informationen auf verschiedenen Ebenen. Deleuze in seiner Simondon-Rezension: „Insbesondere sind die Differenzen zwischen der physikalischen und der vitalen Individuation Gegenstand tiefgreifender Ausführungen. Der Bereich der inneren Resonanz erscheint in beiden Fällen als verschieden; das physische Individum begnügt sich damit, ein einziges Mal eine Information zu erhalten, und wiederholt eine anfängliche Singularität, während das Lebendige nacheinander mehrere Informationen erhält und mehrere Singularitäten verbucht; und vor allem entsteht und hält sich die physische Individuation an der Grenze des Körpers, z.B. des Kristalls, während der Lebendige von innen und von außen aus wächst, da der gesamte Inhalt seines inneren Raums „topologisch" in Kontakt zum Inhalt des äußeren Raums steht."*

(PfM) S. 174: *„Transzendentaler Empirismus: „In Wirklichkeit wird der Empirismus transzendental und die Ästhetik eine apodiktische Disziplin, wenn wir im Sinnlichen direkt das erfassen, was nur empfunden werden kann, das Sein selbst des Sinnlichen: die Differenz, die Differenz im Potential, die Intensitätsdifferenz als Ratio des qualitativ Verschiedenen."*

Deligne P. et. al.
Quantum fields and strings
A course for mathematicians

(DeP) p. 551: *"… the behavior of a physical system depends on a scale (of energies, distances, momenta, etc.) at which the behavior is studied."*

(DeP) pp. 553/544: *"Very generally speaking, the method of renormalization group is a method designed how to describe how the dynamics of some system changes when we change the scale (distance, energies) at which we probe it,. …*

0. Scale dependence*. Physics is scale dependent. For example, consider a fluid. At each scale of distances, we need a different theory to describe its behavior:*

At ~ 1cm – classical continuum mechanics (Navier-Stokes equations)
At ~ 10^{-5} cm – theory of granular structure
At ~ 10^{-8} cm – theory of atom (nucleus + electronic cloud)
At ~ 10^{-13} cm – nuclear physics (nucleons)
At ~ 10^{-13} - 10^{-18} cm – quantum chromodynamics (quarks)
At ~ 10^{-33} cm string theory

At each scale, we have different degrees of freedom and different dynamics.

1. Decoupling*. Physics at large scale (largely) decouples from the physics at a smaller scale. For example, to describe the behavior of fluid at the scale ~ 1cm, we don't need to know about the granular structure,*

nor about the atoms or nucleons. The only things we need to know are the viscosity and the density of the fluid. Of course, these values can be computed from the physics at a smaller scale, but if we fould them out in some way (for example, measurement), we can do without smaller scale theories at all. Similiary, if we want to describe atoms, we don't need to know anything about the nucleus except its mass and electric charge.

Thus, a theory at a larger scale remembers only finitely many parameters from the theories at smaller scale to a larger scale, we average over irrelevant degrees of freedom. Mathematically, this means that they become integration variables and thus disappear in the answer.

This decoupling is the reason why we are able to do physics. If there was no decoupling, it would be necessary for Newton to know string theory to describe the motion of a viscous fluid.

***Remark 1**: In quantum theory, decoupling of scales is not at all obvious. Indeed, because of the uncertainty principle, we have to work at all scales at once. The renormalization group explains why decoupling survives in quantum theory, (DeP) p. 554*

***Remark 2**: In classical mechanics, there are 3 basis units of measurement (distance D, time T, mass M), and all others can be expressed through them. Thus, in classical mechanics we deal with three scales. In nonrelativistic quantum theory and in classical relativity there remains only two of them, as in the first case we can express M through T and D using the Planck constant, and in the second T can be expressed via D using the speed of light. Thus, in relativistic quantum theory we only have one scale – the scale of distances. Equivalently, we can use the inverse scale – the scale of momenta. Thus we have:*

SMALL distances, times = LARGE momenta, energies, masses".

Derbyshire J.
The Montgomery-Odlyzko law

(DeJ): p. 280 ff.*: „The eigenvalues (of Gaussian-random Hermitian matrices)… are struggling to keep their distance from each other. … The statistical properties of spacings between long non-uniform string of numbers are encapsulated in a creature called „pair correlations function" and a certain ratio associated with this function is called its „form factor". … The form factor for the pair correlation of random Hermitian matrices is the conjectured distribution function for the differences between the non-trivial zeros of Riemann's zeta function. …"*

(DeJ): p. 285 ff.: „The following points look pretty plausible on the basis of related comparing figures of „the eigenvalues of a 269-by-269-random matrix."

(DeJ): p. 289*: „The first 269 values of t, where $\frac{1}{2} + it$, is a non trivial zero of the zeta function."*

(DeJ): p. 292: "The distribution of the spacings between successive non-trivial zeros of the Riemann zeta function (suitable normalized) is statistically identical with the distribution of eigenvalue spacing in a Gaussian Unitary Ensemble (i.e. a collection of Gaussian unitary operators that share some common statistical properties)."

(DeJ) p. 295: *„What on earth does the distribution of prime numbers have to do with the behavior of subatomic particles?"*

Descartes R.
Abhandlungen über die Methode, richtig zu denken und Wahrheit in den Wissenschaften zu suchen

(DeR1) S. 5: „*Meine Absicht ist also hier nicht, die Methode zu lehren, die Jeder zur richtigen Leitung seines Verstandes zu befolgen habe, sondern ich will nur zeigen, wie ich den meinigen zu leiten gestrebt habe. Wer Lehren geben will, muss sich für klüger halten als die, an welche er sich richtet, und bei dem geringsten Versehen trifft ihn der Tadel. Ich biete daher diese Schrift nur als eine Erzählung oder, wenn man lieber will, als eine Fabel dar, wo neben nachahmenswerten Beispielen sich vielleicht auch manche finden, denen man mit Recht nicht folgen mag. So hoffe ich, dass sie Manchem nützen und Niemanden schaden werden, und dass Alle mir für meine Offenheit Dank wissen werden.“*

(DeR2) iii: „*My present design, then, is not to teach the method which each ought to follow for the right conduct of his reason, but solely to describe the way in which I have endeavored to conduct my own. They who set themselves to give precepts must of course regard themselves as possessed of greater skill than those to whom they prescribe; and if they are in the slightest particular, they subject themselves to censure. But as this tract is put forth merely as a history, or, if you will, as a tale, in which, amid some examples worthy of imitation, there will be found, perhaps, as many more which it were advisable not to follow, I hope it will prove useful to some without being hurtful to any, and that my openess will find some favor with all.“*

Dirac P. A. M.
Classical Theory of Radiation

(DiP1) „*One of the most attractive ideas in the Lorentz model of the electron, the idea that all mass is of electromagnetic origin, appears at the present time to be wrong, for two separate reasons. First, the discovery of the neutron has provided us with a form of mass which it is very hard to believe could be of electromagnetic nature. Secondly, we have the theory of the positron a theory in agreement with experiment so far it is known – in which positive and negative values for the mass of an electron play symmetrical roles. This cannot be fitted in which the electromagnetic idea of mass, which insists on all mass being positive, even in abstract theory. … We are faced with the difficulty that, if we accept Maxwell's theory, the field in the immediate neighborhood of the electron has an infinite mass.*“

A new basis for cosmology

(DiP2): „*The modern study of cosmology is dominated by Hubble's observations of a shift to the red in the spectra of the spiral nebulae—the farthest parts of the universe—indicating that they are receding from us with velocities proportional to their distances from us. These observations show us, in the first place, that all the matter in a particular part of space has the same velocity (to a certain degree of accuracy) and suggest a model of the universe in which there is a natural velocity for the matter at any point, varying continuously from one point to a neighbouring point. Referred to a four-dimensional space-time picture, this natural velocity provides us with a preferred time-axis at each point, namely, the time-axis with respect to which the matter in the neighbourhood of the point is at rest. By measuring along this preferred time-axis we get an absolute measure of time, called the epoch. …. Such ideas of a preferred time-axis and absolute time depart very much from the principles of both special and general relativity and lead one to expect that relativity will play only a subsidiary role in the subject of cosmology. This first point of view, which differs markedly from that of the early workers in this field, has been much emphasized recently by Milne*.

To get this (Dirac's principle of Nature) *principle in its most general form we should not make the assumption, which we made at the beginning of this section, that the velocity of recession of each spiral nebula is roughly constant. Without this assumption we can still talk about the epoch of an event, but we have no natural zero from which to measure it, so that only the difference of two epochs can enter into laws of nature. We must now use Hubble's constant, namely, the coefficient of proportionality between the red-shift and the distance, as one of the quantities from w hich very large dimensionless numbers are to be constructed (to replace our previous use of the present epoch as one of these numbers) and express our principle in the form:*

Any two of the very large dimensionless numbers occurring in Nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity.

If we can deduce from elementary considerations that some of these very large numbers vary with the epoch (as we shall find in the next section is the case), then they must all do so to preserve the mathematical relations between them."

Dürr H.-P.
Geist, Kosmos und Physik

(DüH1), S. 122: „*Die Biologen und Hirnforscher sind immer noch an die Vorstellungen der alten Physik gebunden, die versuchen, auch das Lebendige auf die reduzierbare materiell-energetische Realität zurückzuführen. Sie halten das was von der Quantenphysik aufgedeckte urlebendige Dazwischen-Beziehungsgefüge für irrelevant wegen des vermutlich unvermeidbaren Ausmittelungseffekts für die Billionen mal größeren Lebensformen und interessieren sich, gleichnishaft gemeint, wieterhin nur für die messbare Hardware.*"

(DüH) S. 442: „*In der Quantentheorie ist dabei zu beachten, daß Translation und Rotation nicht miteinander vertauschbar sind. Man kann daher den Drehimpuls eines Systems um eine Achse nur dann durch eine Quantenzahl charakterisieren, wenn der Translationsimpuls des Systems senkrecht zu dieser Ache entweder verschwindet oder unbekannt ist*."

(DüH) S. 446*: „Wenn es sich als unmöglich erweist, einen voll symmetrischen Zustand „Vakuum" zu konstruieren, so kann dies anschaulich wohl nur so gedeutet werden, daß es sich bei dem unsymmetrischen Grundzustand nicht eigentlich um ein Vakuum, sondern um einen Zustand „Welt" handelt, der die Grundlage für die Existenz der Elementarteilchen bildet. Dieser Zustand muß dann entartet sein; er kann z. B. einen sehr hohen Isospin besitzen. Wenn man — gewissermaßen als Idealisierung des realen Zustandes der Welt — die Translationseigenschaften des Vakuums beibehalten will, so muß er sogar unendlich hoch entartet sein.“*

(DüH) S. 446: „*Der Symmetrieverlust wäre in der vorliegenden Theorie also ähnlich zu deuten wie das Auftreten einer Zentrifugalkraft in der allgemeinen Relativitätstheorie, das ja auch einen Symmetrieverlust anzeigt. Die Zentrifugalkraft kann dort nur als Folge der unendlich fernen Massen im Weltall angesehen werden, obwohl diese Massen in der mathematischen Formulierung schließlich nur als eine Art Randbedingung im Unendlichen erscheinen, die eben die Zentrifugalkraft indirekt hervorruft.“*

(DüH) S. 446: „*Der Grundzustand hätte in der vorliegenden Theorie also einen praktisch unendlich großen Isospin (die Welt enthält neben Protonen und Elektronen beliebig viele Neutronen!) und es würde verständlich, daß die Zustände Neutron und Proton eine etwas verschiedene Masse erhalten. Sie wären gewissermaßen die beiden Dublettkomponenten eines Zustandes „Nukleon + Welt", bei dem sich der hinzukommende Isospin parallel oder antiparallel zu dem der „Welt" stellen kann, und der als Ganzes wieder einen beliebig hohen Isospin trüge.*"

Eco U.
Die Geschichte der Schönheit

(EcU) p. 62, Die Zahl und die Musik*: „Wir beurteilen etwas als schön, wenn es wohl proportioniert ist. Das ist insofern verständlich, als Schönheit seit der Antike mit der Proportion identifiziert wird – auch wenn daran zu erinnern ist, daß die in der griechischen und lateinischen Welt übliche Definition von Schönheit neben den Proportionen immer auch die angenehme Wirkung der Farbe (und des Lichts) enthielt.*

Als in Griechenland die sogenannten Vorsokratiker – wie Thales, Anaximander und Aniximenes – um die Wende des 7. zum 6. Jahrhundert über das Wesen des Ursprungs aller Dinge zu diskutieren begannen, wollten sie die Welt als ein Ganzes definieren, das durch ein einziges Gesetz geordnet und regiert wird. Das bedeutet auch, die Welt als eine Form zu denken, und die Griechen erkennen deutlich die Identität von Form und Schönheit. Explizit wird dies allerdings erst ab dem 6. Jahrhundert v. Chr. bei Pythagoras und seiner Schule durch die Verknüpfung von Kosmologie, Mathematik, Naturwissenschaften und Ästethik. Pythagoras (der wahrscheinlich die mathematischen Überlegungen der Ägypter kennengelernt hatte) stellte als erster die Behauptung auf, daß die Zahl das Grundprinzip aller Dinge sein. Die Pythagoräer empfinden eine Art heiligen Schauer vor dem Unendlichen und allem, was nicht in Grenzen gehalten werden kann, und deshalb versuchen sie in der Zahl das

Gesetz zu finden, um die Wirklichkeit zu begrenzen, ihr Ordnung und Verständlichkeit zu geben. Mit Pythagoras entsteht eine ästhetische mathematische Sicht des Universums: Alle Dinge existieren, weil sich in ihnen mathematische Regeln realisieren, die zugleich Bedingungen für die Existenz von Schönheit sind."

Kant und das Schnabeltier

(EcU1) S. 86: *„Rorty,* (RoR), (RoR1), *hat überzeugend dargelegt, daß Kant nicht am* knowlede of *interessiert war, sondern am* knowledge that*; anders ausgedrückt, nicht an den Bedingungen des Erkennes (und damit des Benennens) der Objekte, sondern eher an der Möglichkeit, die Wahrheit unserer Sätze über Objekte zu begründen. So daß man, will man als „Erkenntnis" das* knowledge of *bezeichnen und als Wissen das* knowledge that*, durchaus sagen könnte, das Problem der Erkenntnis habe ihn nicht interessiert. Ihm ging es vor allem darum, inwiefern eine reine Mathematik und eine reine Physik möglich seien, bzw. wie man aus der Mathematik und der Physik zwei theoretische Erkenntnisbereiche machen könne, die ihre Gegenstände a priori bestimmen müssen. Der Kern der ersten Kritik ist die Suche nach der Garantie für eine Gesetzgebung des Verstandes in bezug auf jene Sätze, die ihre Vorbilder in Newtons Gesetzen haben – und die man zuweilen in Sätzen exemplifiziert, die verständlicher und ehrwürdiger sind, wie etwa:* Alle Körper sind schwer*. Kant möchte die Erkenntnis jener Gesetze garantieren, die der Natur als dem* Inbegriff der Gegenstände der Erfahrung *zugrunde liegen; daß diese Gegenstände der Erfahrung auch die sind, mit deren Erkennen die Empiristen sich herumschlugen, also Hunde, Pferde, Steine, Bäume, hat Kant nie bezweifelt; indes scheint er (zumindest seit der* Kritik der Urteilskraft*) außerordentlich desinteressiert zu sein an der Klärung der Frage, wie jene Gegenstände, die man heute als* natural kinds*, als natürliche Typen, bezeichnet, wie Kamel, Buche und Käfer. Husserl, ein am* knowledge of *interessierter Philosoph, stellte dies mit deutlicher Enttäuschung fest (HuE1) VI, §66; doch wird diese Enttäuschung zur Genugtuung für Denker, die der Ansicht sind, das Problem der Erkenntnis (oder des Wissens) lasse sich nur innerhalb der Sprache, also in Hinsicht auf Kohärenz zwischen Sätzen lösen."*

Ehrenhaft F.
Photophoresis

(EhF): *„Particles of matter irradiated by light between electrodes behave as if they carry positive or negative electric charges. Therefore we can say that through the action of the light uncharged particles obtain unipolar charges, either negative or positive."*

It is unlikely, that all those movement phenomena in light with or without the action of a field can be explained with the helf of today's hypothesis; we may be forced to reach for something new, (*).
(*) Acta Physicia Austriaca, Band 4, 1950 and Band 5, 1951

(AlO) p. 222: *„Completely new and amazing is the fact, that the movements of the particles in the field do not run in straight lines, but run in paths in extremly regular forms, sizes and orbital frequencies."*
Note: this is in line with V. Schauberger's implosion (cycloidal) movement in the context of the movements of planets/stars, and (sub-) atomic particles

It was also interesting too, that a centripetal force occured, which acted on the particles 130 times stronger than the gravity force. Among other things, Ehrenhaft's comment on this experiment was
„Es ist unwahrscheinlich, daß alle diese Bewegungsphenomene im Licht mit oder ohne Einwirkung eines Feldes erklärt werden können mit Hilfe heutiger Hypothesen; wir werden vielleicht gezwungen, nach Neuem zu greifen."

(AlO) p. 223: *W. Schauberger hat diesen Versuch so gedeutet:*

„Jedes Energieteilchen in Bewegung erzeugt ein Feld – einen Energieraum -, der von der Bewegung abhängig ist, und je dichter dieses Feld ist, desto mehr wirkt es auf die Umgebung ein, sodaß auch Teilchen mit größerer Masse als die felderzeugenden Teilchen in dieses Feld hineingezogen werden. Diese Teilchen aus Silver, Nickel oder Kohle müssen im Verhältnis zu den Lichtphotonen wie riesige Felsblöcke gewesen sein. Dennoch wurden sie in den Wirbeltanz der Photone hineingezogen. Wir müssen also lernen, Medien, die zur Verfügung stehen, in so eine Bewegung zu bringen, wie sie Elektronen und Photonen anzuwenden pflegen. … dann können wir auch mit einem relativ kleinen Energieeinsatz „Berge versetzen."

Kurz gesagt, Ehrenhafts Versuche deuten darauf hin, daß das Grundelement der Natur sich mittels Schraubenbewegung bewegt und, wie V. Schauberger meinte, daß die Technik diese kopieren sollte."

(EhF) p. 243: *„In order to explain the phenomena of photophoresis one conclusion is drawn from the movement of illuminated particles in the homogeneous electric and magnetic fields. The light induces electric and magnetic charges (poles) upon the particles if they are illuminated by concentrated light preponderantly shorter wave lengths. …. For the magnetic charges this conclusion is new, but is justified because of the complete analogy of this phenomenon with the electric phenomenon.“*

(EhF1)*: „…. light beams must have electric stationary components in the direction of the wave front normal, and that consequently there must be stationary electric potential differences between different points along the beam; and that there must be also a stationary magnetic field in the beam of light with potential differences. Hence, the light beam must have a magnetizing effect, and the charge of a magnet should be changed by light.“*

Einstein A.
Mein Weltbild, (EiA)
The World as I See it, (EiA1)
Lichtgeschwindigkeit und die Statik des Gravitationsfeldes, (EiA2)

(EiA1) p. 19: Religion and Science
„Everything that the human race has done and thought is concerned with the satisfaction of felt needs and the assuagement of pain. One has to keep this constantly in mind if one wishes to understand spiritual movements and their development. Feeling and desire are the motive forces behind all human endeavour and human creation, in however exalted a guise the latter may present itself to us. Now what are the feelings and needs that have led men to religious thought and belief in the widest sense of the words? A little consideration will suffice to show us that the most varying emotions preside over the birth of religious thought and experience. With primitive man it is above all fear that evokes religious notions--fear of hunger, wild beasts, sickness, death. Since at this stage of existence understanding of causal connexions is usually poorly developed, the human mind creates for itself more or less analogous beings on whose wills and actions these fearful happenings depend. One's object now is to secure the favour of these beings by carrying out actions and offering sacrifices which, according to the tradition handed down from generation to generation, propitiate them or make them well disposed towards a mortal. I am speaking now of the religion of fear. This, though not created, is in an important degree stabilized by the formation of a special priestly caste which sets up as a mediator between the people and the beings they fear, and erects a hegemony on this basis. In many cases the leader or ruler whose position depends on other factors, or a privileged class, combines priestly functions with its secular authority in order to make the latter more secure; or the political rulers and the priestly caste make common cause in their own interests.

The social feelings are another source of the crystallization of religion. Fathers and mothers and the leaders of larger human communities are mortal and fallible. The desire for guidance, love, and support prompts men to form the social or moral conception of God. This is the God of Providence who protects, disposes, rewards, and punishes, the God who, according to the width of the believer's outlook, loves and cherishes the life of the tribe or of the human race, or even life as such, the comforter in sorrow and unsatisfied longing, who preserves the souls of the dead. This is the social or moral conception of God.

The Jewish scriptures admirably illustrate the development from the religion of fear to moral religion, which is continued in the New Testament. The religions of all civilized peoples, especially the peoples of the Orient, are primarily moral religions. The development from a religion of fear to moral religion is a great step in a nation's life. That primitive religions are based entirely on fear and the religions of civilized peoples purely on morality is a prejudice against which we must be on our guard. The truth is that they are all intermediate types, with this reservation, that on the higher levels of social life the religion of morality predominates.

Common to all these types is the anthropomorphic character of their conception of God. Only individuals of exceptional endowments and exceptionally high-minded communities, as a general rule, get in any real sense beyond this level. But there is a third state of religious experience which belongs to all of them, even though it is rarely found in a pure form, and which I will call cosmic religious feeling. It is very difficult to explain this feeling to anyone who is entirely without it, especially as there is no anthropomorphic conception of God corresponding to it.

The individual feels the nothingness of human desires and aims and the sublimity and marvellous order which reveal themselves both in nature and in the world of thought. He looks upon individual existence as a sort of prison and wants to experience the universe as a single significant whole. The beginnings of cosmic religious feeling already appear in earlier stages of development--e.g., in many of the Psalms of David and in some of the Prophets. Buddhism, as we have learnt from the wonderful writings of Schopenhauer especially, contains a much stronger element of it.

The religious geniuses of all ages have been distinguished by this kind of religious feeling, which knows no dogma and no God conceived in man's image; so that there can be no Church whose central teachings are based on it. Hence it is precisely among the heretics of every age that we find men who were filled with the highest kind of religious feeling and were in many cases regarded by their contemporaries as Atheists, sometimes also as saints. Looked at in this light, men like Democritus, Francis of Assisi, and Spinoza are closely akin to one another.

How can cosmic religious feeling be communicated from one person to another, if it can give rise to no definite notion of a God and no theology? In my view, it is the most important function of art and science to awaken this feeling and keep it alive in those who are capable of it.

We thus arrive at a conception of the relation of science to religion very different from the usual one. When one views the matter historically one is inclined to look upon science and religion as irreconcilable antagonists, and for a very obvious reason. The man who is thoroughly convinced of the universal operation of the law of causation cannot for a moment entertain the idea of a being who interferes in the course of events--that is, if he takes the hypothesis of causality really seriously. He has no use for the religion of fear and equally little for social or moral religion. A God who rewards and punishes is inconceivable to him for the simple reason that a man's actions are determined by necessity, external and internal, so that in God's eyes he cannot be responsible, any more than an inanimate object is responsible for the motions it goes through. Hence science has been charged with undermining morality, but the charge is unjust. A man's ethical behavior should be based effectually on sympathy, education, and social ties; no religious basis is necessary. Man would indeed be in a poor way if he had to be restrained by fear and punishment and hope of reward after death.

It is therefore easy to see why the Churches have always fought science and persecuted its devotees. On the other hand, I maintain that cosmic religious feeling is the strongest and noblest incitement to scientific research. Only those who realize the immense efforts and, above all, the devotion which pioneer work in theoretical science demands, can grasp the strength of the emotion out of which alone such work, remote as it is from the immediate realities of life, can issue. What a deep conviction of the rationality of the universe and what a yearning to understand, were it but a feeble reflection of the mind revealed in this world, Kepler and Newton must have had to enable them to spend years of solitary labour in disentangling the principles of celestial mechanics! Those whose acquaintance with scientific research is derived chiefly from its practical results easily develop a completely false notion of the mentality of the men who, surrounded by a sceptical world, have shown the way to those like-minded with themselves, scattered through the earth and the centuries. Only one who has devoted his life to similar ends can have a vivid realization of what has inspired these men and given them the strength to remain true to their purpose in spite of countless failures. It is cosmic religious feeling that gives a man strength of this sort. A contemporary has said, not unjustly, that in this materialistic age of ours the serious scientific workers are the only profoundly religious people."

(UnA) p. 217*: „In a reasonable theory there are no numbers which can be only determined empirically."*

(EiA) S.130: *"Nach unserer bisherigen Erfahrung sind wir nämlich zum Vertrauen berechtigt, daß die Natur die Realisierung des mathematisch denkbar Einfachsten ist. Durch rein mathematische Konstruktion vermögen wir nach meiner Überzeugung diejenigen Begriffe und diejenige gesetzliche Verknüpfung zwischen ihnen zu finden, die den Schlüssel für das Verstehen der Naturerscheinungen liefern. Die brauchbaren mathematischen Begriffe können durch Erfahrung wohl nahegelegt, aber keinesfalls aus ihr abgeleitet werden. Erfahrung bleibt natürlich das einzige Kriterium der Brauchbarkeit einer mathematischen Konstruktion für die Physik. Das eigentlich schöpferische Prinzip liegt aber in der Mathematik. In einem gewissen Sinn halte ich es also für wahr, daß dem reinen Denken das Erfassen des Wirklichen möglich sei, wie es die Alten geträumt haben."*

(EiA) S.131 *„... daß all diese Bildungen und deren gesetzliche Verknüpfungen sich nach dem Prinzip des Aufsuchens der mathematisch einfachsten Begriffe und deren Verknüpfungen gewinnen lassen"*

(EiA2) S. 368 *„Damit ist also erwiesen, daß man auch für unendlich kleine Raum-Zeitgebiete nicht an der Lorentztransformation festhalten kann, sobald man die universelle Konstanz von c aufgibt“*

„The principle of the constancy of the speed of light can be maintained only by restricting to space-time regions with a constant gravitational potential“

Einstein A.
The meaning of relativity

(EiA4) p. 28: „Maxwell's equations determine the electromagnetic field when the distribution of electric charges and currents is known. But we do not know the laws which govern the currents and charges. We do know, indeed, that electricity consists of elementary particles (electrons, positive nuclei), but from a theoretical point of view we cannot comprehend this. We do not know the energy factors which determine the distribution of electricity in particles of definite size and charge, and all attempts to complete the theory in this direction have failed. If then we can build upon Maxwell's equations at all, the energy tensor of the electromagnetic field is known only outside the charged particles [(*)]

[(*)] It has been attempted to remedy this lack of knowledge by considering the charged particles as proper singularities. But in my opinion this means giving up a real understanding of the structure of matter. It seems to me much better to admit our present inability rather than to be satisfied by a solution that is only apparent.

Einstein A.
Ether and the theory of relativity

(EiA5): *„Lorentz succeeded in reducing all electromagnetic happenings to Maxwell’s equations for free space.*

As to the mechanical nature of the Lorentzian ether, it may be said of it, in a somewhat playful spirit, that immobility is the only mechanical property of which it has not been deprived by H. A. Lorentz. It may be added that the whole change in the conception of the ether which the special theory of relativity brought about, consisted in taking away from the ether its last mechanical quality, namely, its immobility. …

Generalizing we must say this: -- There may be supposed to be extended physical objects to which the idea of motion cannot be applied. They may not be thought of as consisting of particles which allow themselves to be separately tracked through time. In Minkowski’s idiom this is expressed as follows: -- Not every extended conformation in the four-dimensional world can be regarded as composed of world-threads. The special theory of relativity forbids us to assume the ether to consist of particles obserbale through time, but the hypothesis of ether in itself is not in conflict with the special theory of relativity. Only we must be our guard against ascribing a state of motion to the ether.“

Euler L.
The division of the truths within the boundaries of human cognition into
the truths of experience, reason, and believe

(HiS1) S. 15, 23: *„Die Vermischung (mixing) von Vernunft- (reason) und Erfahrungswahrheiten (experience) ist schon älteren Datums und liefert immer wieder Anlaß zu Streitigkeiten (disput) zwischen Mathematikern und Physikern. Ein prominentes Beispiel liefert das sogenannte Dirichletsche Prinzip, das später an Hand des isoperimetrischen Problems näher erläutert werden soll. Dieses Beweisprinzip ist deshalb in die Geschichte der Mathematik eingegangen, weil hier im vorrigen Jahrhundert die Begriffe Minimum und kleinste untere Schranke verwechselt wurden, die heute jeder Mathematik- und Physikstudent schon in den ersten Vorlesungsstunden auseinanderzuhalten gelernt hat. Nachdem Weierstraß diesen Fehler, der selbst Gauß, Dirichlet und Riemann unterlaufen war, aufgedeckt und kritisiert hatte, wurde von Mathematikern anerkannt, daß es in jedem speziellen Fall nötig sei, die Existenz eines Minimums zu beweisen Physiker hingegen meinten, die Existenzfrage erledige sich durch „Evidenz.“*

Fermi E.
Quantum Theory for Radiation

(FeE): *„Dirac's theory of radiation is based on a very simple idea; instead of considering an atom and the radiation field with which it interacts as two distinct systems, he treats them as a single system whose energy is the sum of three terms: one representing the energy of the atom, a second representating the electromagnetic energy of the radiation field, and a small term representing the coupling energy of the atom and the radiation field.*

If we neglect this last term, the atom and the field could not affect each other in any way; that is, no radiation energy could be either emitted or absorbed by the atom. A very simple example will explain these relations. Let us consider a pendulum which corresponds to the atom, and an oscillating string in the neighborhood of the pendulum which represents the radiation field. If there is no connection between the pendulum and the string, the two systems vibrate quite independently of each other; the energy is in this case simply the sum of the energy of the pendulum and the energy of the string with no interaction term. To obtain a mechanical representation of this term, let us tie the mass M of the pendulum to a point A of the string by means of a very thin and elastic thread a. The effect of this thread is to perturb slightly the motion of the string and of the pendulum. Let us suppose for instance that at the time $t = 0$*, the string is in vibration and the pendulum is at rest. Through the elastic thread a the oscillating string transmits to the pendulum very slight forces having the same periods as the vibrations of the string. If these periods are different from the period of the pendulum, the amplitude of its vibrations remains always exceedingly small; but if a period of the string is equal to the period of the pendulum, there is resonance and the amplitude of vibration of the pendulum becomes considerable after a certain time. This process corresponds to the absorption of radiation by the atom. If we suppose, on the contrary, that at the time* $t = 0$ *the pendulum is oscillating and the string is at rest, the inverse phenomenon occurs. The forces transmitted through the elastic thread from the pendulum to the string put the string in vibration; but only the harmonics of the string, whose frequencies are very near the frequency of the pendulum reach a considerable amplitude. This process corresponds to the emission of radiation by the atom.“*

Feynman R.

(GlJ) p. 433:

> *„When a historian of particle physics pressed him (R. Feynman) on the question of unification, he resited.*
> *„Your career spans the period of the construction of the standard model,“ the interviewer said.*
> *„'The standard model'“ Feynman repeated dubiously.*
> *“*$SU(3) \times SU(2) \times U(1)$*. From renormalization (*), to quantum electrodynamics to now?“*
> *„The standard model, standard model,“ Feynman said. „The standard model ---- is that the one that says that we have electrodynamics, we have weak interaction, and we have strong interaction? Okay, Yes.“*
> *The interviewer said, „That was quite an achievement, putting them together.“*
> *„They're not put together.“*
> *„Linked together in a single theoretical package?“*
> *„No.“*
> *The interviewer was having trouble getting his question onto the table.*
> *„What do you call* $SU(3) \times SU(2) \times U(1)$*?“*
> *„Three theories,“ Feynman said. „Strong interactions, weak interactions, and electromagnetic … The theories are linked because they seem to have similar characteristics … Where does it go together? Only if you add some stuff that we don't know. There isn't any theory today that has* $SU(3) \times SU(2) \times U(1)$ *--- whatever the hell it is --- that we know is right, that has experimental check … Now, these guys are all trying to put all this together. They're trying to. But they haven't. Okay?“*

„Somebody makes up a theory: The proton is unstable. They make a calculation and find that there would be no protons in the universe any more! So they fiddle around with their numbers, putting higher mass into the new particle, and after much effort they predict that the proton will decay at a rate slightly less than the last measured rate the proton has shown not to decay at. When a new experiment comes along and measures the proton more carefully, the theories adjust themselves to squeeze out from the pressure“, (UnA) p. 162.

Feynman R.
The character of physical laws

(FeR1) p. 13: "*There is a rhythm and a pattern between the phenomena of nature which is not apparent to the eye, but only to the eye of analysis; and it is these rhythms and patterns which we call Physical Laws.*"

(FeR1) p. 36: "*The strange thing about physics is that for the fundamental laws we still need mathematics.*"

(FeR1) p. 39: "*There is no model of the theory of gravitation today, other than the mathematical form. … „Every one of our laws is a purely mathematical statement in rather complex and abstruse mathematics*."

(FeR1) p. 54: "*The best law, as at present understood, is really a combination of the two in which we use minimum principles plus local laws.*"

(FeR1) P. 57: "*This shows again that mathematics is a deep way of expressing nature, and any attempt to express nature in philosophical principles, or in seat-of-the-pants mechanical feelings, is not an efficient way.*"

(FeR1) p. 58: "*To summarize, I would use the words of Jeans, who said that „the Great Architect seems to be a mathematician". To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature. ….If you want to learn about nature, to appreciate nature, it is neccessary to understand the language that she speaks in. She offers her information only in one form; we are not so unhumble as to demand that she change before we pay any attention*."

(FeR1) p. 173: "*What is it about nature that lets this happen, that it is possible to guess from one part what the rest is going to do? That si an unscientific question: I do not know how to answer it, and therefore I am going to give an unscientific answer. I think it is because nature has a simpicity and therefore a great beauty.*"

(FeR1) p. 66: "*Charge is the source of a field; in other words, electricity is related to charge. Thus the particular quantity which is conserved here has two other aspects which are not connected with the conservation directly, but are interesting anyway. One is that it comes in units, and the other that it is the source of the field.*"

(FeR1) p. 149: "*First of all there is matter – and, remarkably enough, all matter is the same.*"

(FeR1) p. 155: "*If we put all this principles together, we discover that there are too many. They are inconsistent with each other. It seems that if we take quantum mechanics, plus relativity, plus the proposition that everything has to be local, plus a number of tacit assumptions, we get inconsistency, because we get infinity for various things when calculate them, and if we get infinity how can we ever say that this agrees with nature?*"

(FeR1) p. 163: "*Any schemes – such as „think of symmetric laws", or „put the information in mathematical form", or „guess equations" – are known to everybody now, and they are all tried all the time. When you are stuck, the answer cannot be one of these, because you will have tried these right away. There must be another way next time. Each time we get into this log-jam of too much trouble, too much problems, it is because the methods that we are using are just like the ones we have used before. The next scheme, the new discovery, is going to be made in a complete different way. So history does not help us much*."

Goethe J. W. v.
Zur Farbenlehre
Sechste Abteilung – Achtzehntes Jahrhundert
Erste Epoche: Von Newton bis auf Dolland
Isaak Newton

(GoJ) S. 324 ff.: *„Brief an den Sekretär der Londoner Sozietät:*

Im Jahre 1671 wird er Mitglied der Londoner Sozietät und legt ihr sein neuestes katoptrisches Teleskop vor und zugleich seine Farbentheorie, aus welcher gefolgert wird, daß die dioptrischen Fernröhre nicht zu verbessern seien. Dieser Brief eigentlich beschäftigt uns hier, weil Newton den Gang, den er genommen, sich von seiner Theorie zu überzeugen, darin ausführlich erzählt, und weil er überhaupt hinreichnend wäre, uns einen vollkommenen Begriff von der Newotonischen Lehre zu geben.
An diesen Brief schließen sich auch die ersten Entwürfe gegen die Newtonische Lehre, welche nebst den Antworten des Verfassers bis 1676 reichen.

Die Optik
Seit gedachtem Jahre läßt sich Newton in weiter keine Kontrovers ein, schreibt aber die Optik, welche 1705 herauskommt, da seine Autorität am höchsten gestiegen und er zum Präsidenten der Sozietät ernannt war. In diesem Werke sind die Erfahrungen und Versuche so gestellt, daß sie allen Einwendungen die Stirn bieten sollen. Um nunmehr dasjenige, worauf es bei der Sache ankommt, historisch deutlich zu machen, müssen wir einiges aus der vergangenen Zeit nachholen.

Die Wirkung der Refraktion war von den ältesten Zeiten her bekannt, ihre Verhältnisse aber bis in das sechzehnte Jahrhundert nur empirisch bestimmt.Snellius entdeckte das Gesetzlich daran und bediente sich zur Demonstration des subjektiven Versuchs, den wir mit dem Namen Erhebung bezeichnet haben. Andere wählten zur Demonstration den objektiven Versuch, und das Kunstwort Brechung wird davon ausschließend gebraucht. Das Verhältnis der beiden Sinus des Einfalls- und Brechungswinkels wird rein ausgesprochen, als wenn kein Nebenumstand dabei zu beachten wäre.

Die Refraktion kam ausschließlich bei Gelegenheit der Fernröhre zur Sprache. Diejenigen, die sich mit Teleskopen und deren Verbesserung beschäftigten, mußten bemerken, daß durch Objektivgläser, die aus Kugelschnitten bestehen, das Bild nicht rein in einen Punkt zu bringen ist, sondern daß eine gewise Abweichung stattfindet, wodurch das Bild undeutlich wird. Man schrieb sie der Form der Gläser zu und schlug deswegen hyperbolische und elliptische Oberflächen vor.

So oft von Refraktion, besonders seit Antonius De Dominis, die Rede ist, wird auch immer der Farberscheinung gedacht. Man ruft bei dieser Gelegenheit die Prismen zu Hülfe, welche das Phänomen so eminent darstellen. Als Newton sich mit Verbesserung der Teleskope beschäftigte und, um jene Aberration von seiten der Form wegzuschaffen, hyperbolische und elliptische Gläser arbeitete, untersuchte er auch die Farberscheinungen und überzeugte sich, daß diese gleichfalls eine Art von Abweichung sei wie jene, doch von weit größerer Bedeutung, dergestalt, daß jene dagegen gar nicht zu achten sei, diese aber wegen ihrer Größe, Beständigkeit und Untrennbarkeit von der Refraktion alle Verbesserung der dioptrischen Teleskope unmöglich machte.
Bei Betrachtung dieser die Refraktion immer begleitenden Farberscheinungen fiel hauptsächlich auf, daß ein rundes Bild wohl seine Breite behielt, aber in der Länge zunahm. Es wurde nunmehr eine Erklärung gefordert, welche im siebzehnten Jahrhundert oft versucht worden, niemandem aber gelungen war.

Newton scheint, indem er eine solche Erklärung aufsuchte, sich gleich die Frage getan zu haben: ob die Ursache in einer inneren Eigenschaft des Lichts oder in einer äußern Bedingtheit desselben zu suchen sei? Auch läßt sich aus seiner Behandlung der Sache, wie sie uns bekannt worden, schließen, daß er sich sehr schnell für die erstere Meinung entschieden hat.

Das erste, was er also zu tun hatte, war, die Bedeutsamkeit aller äußeren Bedingungen, die bei dem primatischen Versuche vorkamen, zu schwächen oder ganz zu beseitigen. Ihm waren die Überzeugungen seiner Vorgänger wohl bekannt, welche eben diesen äußeren Bedingungen einen großen Wert beilegte. Er führt ihrer sechs auf, um eine nach der andern zu veneinen. Wir tragen sie in der Ordnung vor, wie er sie selbst aufführt, und als Fragen, wie er sie gleichfalls gestellt hat.

Erste Bedingung. Trägt die verschiedene Dicke des Glases zur Farberscheinung bei?
….
Zweite Bedingung. Inwiefern tragen größere oder kleinere Öffnungen im Fensterladen zur Gestalt der Erscheinung, besonders zum Verhältnis ihrer Lage zur Breite bei?
….
Dritte Bedingung. Tragen die Grenzen des Hellen und Dunkeln etwas zur Erscheinung bei?

> *Das ganze Kapitel unseres Entwurfs, welches die Farben abhandelt, die bei Gelegenheit der Refraktion entstehen, ist durchaus bemüht zu zeigen, daß eben die Grenzen ganz allein die Farberscheinungen hervorbringen. Wir wiederholen hier nur das Hauptmoment.*
> *Es entspringt keine prismatische Farbenerscheinung, als wenn ein Bild verrückt wird, und es kann kein Bild ohne Grenze sein. Bei dem gewöhnlichen prismatischen Versuch geht durch die kleinste Öffnung das ganze Sonnenlicht durch, das ganze Sonnenlicht wird verrückt; bei geringer Brechung nur an den Rändern, bei stärkerer aber völlig gefärbt.*
>
> *Durch welche Art von Untersuchung jedoch Newton sich überzeugt habe, daß der Grenze kein Einfluß auf die Farberscheinung zuzuschreiben sei, muß jenen, der nicht verwahrlost ist, zum Erstaunen, ja zum Entsetzen bewegen, und wir fordern alle günstige und ungünstige Leser auf, diesem Punkte die größte Aufmerksamkeit zu widmen.*

Bei jenem bekannten Versuche, bei welcher sich das Prisma innerhalb der dunklen Kammer befindet, geht das Licht, oder vielmehr das Sonnenbild, zuerst durch die Öfnung und dann durch das Prisma, da denn auf der Tafel das farbige Spektrum erscheint. Nun stellt der Experimentator, um gleichsam eine Probe auf seinen ersten Versuch zu machen, das Prisma hinaus vor die Öffnung und findet in der dunklen Kammer, vor wie nach, sein gefärbtes verlängertes Bild. Daraus schließt er, die Öffnung habe keinen Einfluß auf die Färbung desselben. Wir fordern alle unsere gegenwärtigen und künftigen Gegner auf diese Stelle. Hier wird von nun an um die Haltbarkeit oder Unhaltbarkeit des Newtonischen Systems gekämpft, hier, gleich am Eingange des Labytinths und nicht drinnen in den verworrenen Irrgängen, hier, wo uns Newton selbst aufbewahrt hat, wie er zu seinen Überzeugung gelangt ist.

Wir wiederholen daher was schon oft von uns didaktisch und polemisch eingeschärft worden: das gebrochene Licht zeigt keine Farbe, als bis es begrenzt ist; das Licht nicht als Licht, sondern insofern es als ein Bild entstehe, das nachher gebrochen wird, oder ob eine Brechung vorgehe, innerhalb welcher man ein Bild begrenzt.
Man gewöhne sich, mit dem großen Wasserprisma zu operieren, welches uns ganz allein über die Sache einen vollkommnen Aufschluß geben kann, und man wird nicht aufhören, sich zu wundern, durch welch einen unglaublichen Fehlschluß sich ein so vorzüglicher Mann nicht alleine zu Anfang getäuscht, sondern den Irrtum so bei sich festverwurzeln lassen, daß er wider allen Augenschein, ja wider besser Wissen und Gewissen, in der Folge dabei verharrt und einen ungehörigen Versuch nach dem anderen ersonnen, um seine erste Unaufmerksamkeit vor unaufmerksamen Schülern zu verbergen. Man sehe, was von uns im polemischen Teile, besonders zum zweiten Teil des ersten Buchs der Optik, umständlicher ausgeführt worden, und erlaube uns hier den Triumpf der guten Sache zu feiern, den ihr die Schule, mit aller ihrer Halsstarrigkeit, nicht lange mehr verkümmern wird.

Jene drei nunmehr abgehandelten Fragepunkte beziehen sich auf Äußerungen älterer Naturforscher. Der erste kam vorzüglich durch Antonius De Dominis, der zweite und dritte durch Kirchner und Descartes zur Sprache. Außerdem waren noch andre Punkte zu beseitigen, andere äußere Bedingungen zu leugnen, die wir nun der Ordnung nach vorführen, wie sie Newton beibringt.

Vierte Bedingung. Sind vielleicht Ungleichheiten und Fehler des Glases schuld an der Erscheinung?
….
Fünfte Bedingung. Hat das verschiedene Einfallen der Strahlen, welche von verschiedenen Teilen der Sonne herabkommen, Schuld an der farbigen Abweichung?
….
Sechste Bedingung. Ob vielleicht die Strahlen nach der Refraktion sich in krummen Linien fortpflanzen und also das so seltsam verlängerte Bild hervorbringen?
….
Da nunmehr Newton diese sechs äußeren Bedingungen völlig removiert zu haben glaubt, so schreitet er unmittelbar zu dem Schlusse: es sei die Farbe dem Licht nicht nur eingeboren, sondern die Farben in ihren

spezifischen Zuständen seien in dem Licht als ursprüngliche Lichter enthalten, welche nur durch die Refraktion und andre äußere Bedingungen manifestiert, aus dem Lichte hervorgebracht und in ihrer Uranfänglichkeit und Unveränderlichkeit nunmehr dargestellt würden.

Daß an diesen dergestalt entwickelten und entdeckten Lichtern keine weitere Veränderung vorgehe, davon sucht er sich und andere durch Exprimentum Crucis zu überzeugen; worauf er denn in dreizehn Propositionen seine Lehre mit allen Klauseln und Kautelen, wie sie hernach völlig stehen geblieben, vorträgt, und da er die Farben zuerst aus dem weißen Licht entwickelt, zuletzt sich genötigt sieht, das weiße Licht wieder aus ihnen zusammenzusetzen.

Dieses glaubt er vermittelst der Linse zu leisten, die er ohne weitre Vorbereitung einführt und sich für vollkommen befriedigt hält, wenn er das im Brennpunkt aufgehobene farbige Bild für das wieder zusammengebrachte, vereinigte, gemischte ausgeben kann.

Die Folgerung, die er aus allem diesem zieht, ist sodann, daß es unnütz sei, sich mit Verbesserungen der diotrischen Fernröhre abzugeben, daß man vielmehr bloß an die katoptrischen halten müsse, wozu er eine neue Vorrichtung ausgesonnen.

Diese ersten Konfessionen und Behauptungen Newtons wurden in jenem von uns angezeigten Briefe an die königliche Sozietät der Wissenschaften gebracht und durch die Transaktionen öffentlich bekannt. Sie sind das erste, was von Newtons Lehre im Publikum erscheint und uns in manchem Sinne merkwürdig, besonders deshalb, weil die ersten Einwendungen seiner Gegner vorzüglich gegen diesen Brief gerichtet sind.
Nun haben wir gesehen, daß sein Hauptfehler darin bestanden, daß er jene Fragen, die sich hauptsächlich darauf beziehen: ob äußere Bedingungen bei der Farberscheinung mitwirken? zu schnell und übereilt beseitigt und verneint, ohne auf die näheren Umstände genauer hinzusehen.Deswegen haben wir ihm bei einigen Punkten völlig, bei andren zum Teil, und abermals bei andern nicht widersprechen müssen und können, und wir haben deutlich zu machen gesucht, welche Punkte, und inwiefern sie haltbar sind oder nicht. Widerstrebt nun einer seiner Gegner irrigerweise den haltbaren Punkten, so muß er bei der Kontrovers verlieren, und es entsteht ein guten Vorurteil für das Ganze; widerstrebt ein Gegner den unhaltbaren Punkten, aber nicht kräftig genug und auf die unrechte Weise, so muß er wieder verlieren, und das Falsche erhält die Sanktion des Wahren.
Schon in diesem Briefe, wie in allen Beantwortungen, die er gegen seine ersten Gegner richtet, finde sich jene von uns in der Polemik angezeigte Behandlungsart seines Gegenstandes, die er auf seine Schüler fortgepflanzt hat. Es ist ein fortdauerndes Setzen und Aufheben, ein unbedingtes Aussprechen und augenblickliches Limitieren, so daß zugleich alles und nichts wahr ist.

Diese Art, welche eigentlich bloß dialektisch ist und einem Sophisten ziemte, der die Leute zum besten haben wollte, findet sich, so viel mir bekannt geworden, seit der scholastischen Zeit wieder zuerst bei Newton. Seine Vorgänger von den wiederauflebenden Wissenschaften an, waren, wenn auch oft beschränkt, doch immer treulich dogmatisch, wenn auch unzulänglich, doch redlich didaktisch; Newtons Vortrag hingegen besteht aus einem ewigen Hinterstzuvörderst, aus den tollsten Transpositionen, Wiederholungen und Verschränkungen, aus dogmatisierten und didaktisierten Widersprüchen, die man vergeblich zu fassen strebt, aber doch zuletzt auswendig lernt und also etwas wirklich zu besitzen glaubt.

Und bemerken wir nicht im Leben, in manchen andren Fällen: wenn wir ein falsches Apercu, ein eigenes oder fremdes, mit Lebhaftigkeit ergreifen, so kann es nach und nach zur fixen Idee werden und zuletzt in einen völligen partiellen Wahnsinn ausarten, der sich hauptsächlich dadurch manifestiert, daß man nicht allein alles einer solchen Vorstellungsart Günstige mit Leidenschaft festhält, alles zart Widersprechende ohne weiteres beseitigt, sondern auch das auffallend Entgegengesetzte zu seinen Gunsten auslegt.“

Gödel K.
A new type of cosmological solutions of the gravity field equations

(GöK): *„All cosmological solutions with non-vanishing density of matter known at present have the common property that, in a certain sense, they contain an „absolute" time coordinate, owing to the fact that there exists a one-parametric system of three-spaces everywhere orthogonal on the world lines of matter. It is easily seen that the non-existence of such a system of three-spaces is equivalent with a rotation of matter relatively to the compass of inertia. In this paper I am proposing a solution (with a cosmological term* $\neq 0$*) which exhibits such a rotation. This solution, or rather the four-dimensional space* S *which it defines, has the further properties*

(10) S *is homogeneous*

(11) …. so that any two world lines of matter are equidistant

(12) S *has rotational symmetry*

(13) … That is, a positive direction of time can consistently be introduced in the whole solution

(14) It is not possible to assign a time coordinate to each space-time point in such a way that the coordinate always increases, if one moves in a positive time-like direction; …

(15) … it is theoretically possible in these worlds to travel into the past, or otherwise influence the past

(16) There exist no three-spaces which are everywhere space-like and intersect each world line of matter in one point

(17) … an absolute time does not exist, even if it is not required to agree in direction with the times of all possible observers (where absolute means: definable without reference to individual objects, such as e.g. a particular galactic system).

(18) *Matter everywhere rotates relatively to the compass of inertia with the angular velocity* $2\sqrt{\pi\mu\rho}$*, where* ρ *is the mean density of matter and* μ *Newton's gravitational constant*."

Hawking S. W.
A Brief History of Time
Elementary Particles and the Forces of Nature

„All known particles in the universe can be divided into two groups: particles of spin ½, which make up the matter in the universe, and particles of spin 0, 1, and 2, which give rise to forces between matter particles."

"A particle of spin 0 is like a dot: it looks the same from every direction. A particle of spin 1 is like an arrow: it looks different from different directions. Only if one turns it round a complete revolution (360 degrees) does the particle look the same. A particle of spin 2 is like a double-headed arrow: it looks the same if one turns it round half a revolution (180 degrees). Similarly, higher spin particles look the same if one turns them through smaller fractions of a complete revolution. ... there are particles that do not look the same if one turns them through just one revolution: one has to turn them through two revolutions! Such particles are said to have spin 1/2."
„The matter particles obey what is called Pauli's exclusion principle. … It says that two similar particles cannot exist in the same state; that is, they cannot have both the same position and the same velocity, within the limits given by the uncertainty principle. The exclusion principle is crucial because it explains why matter particles do not collapse to a state of very high density under the influence of the forces produced by the particles of spin 0, 1, and 2; if the matter particles have very nearly the same positions, they must hve different velocities, which means that they will not stay in the same position any longer. If the world had been created without the exclusion principle, quarks would not form separate, well-defined protons and neutrons. Nor would these, to gether with electrons, form separate, well-defined atoms. They would all collapse to form a roughly uniform, dense „soup"."

Hawking S. W.
The theory of everything
Open questions

(HaS) p. 77: „*This picture of a universe that started off very hot and cooled as it expanded is in agreement with all the observational evidence that we have today. Nevertheless, it leaves a number of important questions unanswered.*

First, why was the universe so hot?
Second, why is the universe so uniform on a large scale – why does it look the same at all points of space and in all directions?

Third, why did the universe start out so nearly the critical rate of expansion to just avoid recollapse? If the rate of expansion one second after the big bang had been smaller by even one part in a hundred thousand million million, the universe would have recollaped before it ever reached ist present size. On the other hand, if the expansion rate at one second had been larger by the same amount, the universe would have expanded so much that it would be effectively empty now.

Fourth, despite the fact the universe is so uniform and homogenous on a large scale, it contains local lumps such as stars and galaxies. These are thought to have developed from small differences in the density of the early universe from one region to another. What was the origin of these density fluctuations?

The general theory of relativity, on its own, cannot explain these features or answer these questions. This is because it predicts that the universe started off with infinite density at the big bang singularity. At the singularity, general relativity and all other physical laws would break down. One cannot predict what would come out of the singularity.“

Hegel G. W. F.
Rother W., Vorlesungen über Hegels Phänomenologie des Geistes

(RoW) S. 9: „Einführung
Was ist Phänomenologie? Für Hegel ist die Phänomenologie sowohl eine Methode philosophischer Forschung als auch eine Methode philosophischer Darstellung. Phänomenologie als Forschungsmethode ist durch das gekennzeichnet, was Husserl später als ἐποχή, als Zurückhaltung des subjektiven Urteils bezeichnete (vgl. Ideen zu einer reinen Phänomenologie, 56-57). In der Phänomenologie des Geistes schaut Hegel der Entwicklung der Phänomene gewissermassen zu. Als Methode der Darstellung ist Phänomenologie aber nicht bloss Deskription der Phänomene, sondern auch Konstruktion und Rekonstruktion, das heisst philosophischer Nachvollzug ihrer systematischen und dialektischen Ordnung.

Gegenstand der Hegelschen Phänomenologie ist der Geist, dessen erste Gestalt das Bewusstsein ist. Im späteren System der Enzyklopädie der philosophischen Wissenschaften reduziert Hegel den Geist, der in der Phänomenologie des Geistes thematisiert wird, auf das Bewusstsein, auf eine Gestalt des subjektiven Geistes - aber der Geistbegriff der Phänomenologie des Geistes umfasst in gleicher Weise den objektiven und den absoluten Geist, also nicht nur die Gestalten des Bewusstseins, sondern auch die Gestalten der konkreten Welt und ihrer Geschichte. Hegel fasst diese Gestalten des Geistes nicht als statische Entitäten, die zu analysieren wären, sondern als Entitäten, die sich entwickeln und denen wir bei ihrer Entwicklung zuschauen - ich benutze diesen Ausdruck durchaus mit Blick auf Husserls Wesenserschauung und seine eidetische Reduktion (vgl. Ideen zu einer reinen Phänomenologie, 10-13, 108-119), die in gewisser Hinsicht in Hegels phänomenologischer Methode präformiert sind.

(RoW) S. 10-11: Zum Aufbau der Phänomenologie des Geistes

Die Entwicklung des Bewusstseins und seiner Gestalten verläuft, wie ein Blick auf die Gliederung der Phänomenologie des Geistes zeigt, vom unmittelbaren, sinnlichen Bewusstsein über das Selbstbewusstsein zur Vernunft.

A. BEWUSSTSEIN
I. Die sinnliche Gewissheit oder das Diese und das Meinen
II. Die Wahrnehmung oder das Ding und die Täuschung
III. Kraft und Verstand, Erscheinung und übersinnliche Welt

B. SELBSTBEWUSSTSEIN
IV. Die Wahrheit der Gewissheit seiner selbst

C. VERNUNFT

(AA) VERNUNFT
V. Gewissheit und Wahrheit der Vernunft
(BB) DER GEIST
VI. Der Geist
(CC) DIE RELIGION
VII. Die Religion
(DD) DAS ABSOLUTE WISSEN
VIII. Das absolute Wissen

Was sehen wir an dieser Gliederung? Bewusstsein und Selbstbewusstsein sind phänomenologische Entwicklungsphasen auf dem Weg hin zur Vernunft - das ist der Dreischritt A.-B.-C. Hegels Phänomenologie des Geistes ist also keine blosse Bewusstseinsphilosophie. Das Bewusstsein (A.), das sich selbst zum Gegenstand hat, das sich seiner selbst vergewissert hat (B.), ist Voraussetzung für den Schritt in die Vernunft (C).

Die Vernunft ist das Resultat, der End- und Kulminationspunkt (C.) der Entwicklung, die das Bewusstsein durchläuft, und zugleich der Anfang (AA) der neuer, spezifischer Gestalten, nämlich Geist (BB) - Religion (CC) - absolutes Wissen (DD). Unter Geist versteht Hegel hier nicht mehr Gestalten nur des Bewusstseins, sondern die konkreten und wirklichen Gestalten der Welt: von der Sittlichkeit, Bildung, Aufklärung und Moralität über die Religion und Kunst zum absoluten Wissen.

(BB) DER GEIST
VI. Der Geist
A. Der wahre Geist. Die Sittlichkeit
a. Die sittliche Welt. Das menschliche und göttliche Gesetz, der Mann und das Weib
b. Die sittliche Handlung. Das menschliche und göttliche Wissen, die Schuld und das Schicksal
c. Der Rechtszustand
B. Der sich entfremdete Geist. Die Bildung
I. Die Welt des sich entfremdeten Geistes
II. Die Aufklärung
III. Die absolute Freiheit und der Schrecken
C. Der sich seiner selbst gewisse Geist. Die Moralität
a. Die moralische Weltanschauung
b. Die Verstellung
c. Das Gewissen. Die schöne Seele, das Böse und seine Verzeihung

Um hier nicht zu sehr vorzugreifen und nur die grossen Entwicklungslinien zu verfolgen: Dem Dreischritt Bewusstsein (A.) - Selbstbewusstsein (B.) - Vernunft (C.) korrespondiert auf «höherer» Ebene der Dreischritt wahrer Geist (IV. A.) - sich entfremdeter Geist (IV. B.) - sich seiner selbst gewisser Geist (IV. C.). Die Dynamik geht in der Hegelschen Dialektik immer vom Mittelglied aus, das das Unmittelbare, das erst einmal Gegebene negiert. Die Negativität ist der Motor der Dynamik. In der Dynamik des Geistes ist das zentrale Moment die Entfremdung und in der Entfremdung ist es die Aufklärung (IV. B. I.), die zur absoluten Freiheit und zum Schrecken führt.“

Heidegger M.
The Age of the World Picture

(HeM) p.70: *„In metaphysics reflection is accomplished concerning the essence of what is and a decision takes place regarding the essence of truth. Metaphysics grounds an age, in that through a specific interpretation of what is and through a specific comprehension of truth it gives to that age the basis upon which it is essentially formed. This basis holds complete dominion over all the phenomena that distinguish the age. Conversely, in order that there may be an adequate reflection upon these phenomena themselves, the metaphysical basis for them must let itself be apprehended in them. Reflection is the courage to make the truth of our own presuppositions and the realm of our own goals into the things that most deserve to be called in question."*

(HeM) p.72: *„Modern physics is called mathematical because, in a remarkable way, it makes use of a quite specific mathematics. But it can proceed mathematically in this way only because, in a deeper sense, it is already itself mathematical.“*

(HeM) p.73: „*The rigor of mathematical physical science is exactitude. Here all events, if they are to enter at all into representation as events of nature, must be defined beforehand as spatiotemporal magnitudes of motion. Such defining is accomplished through measuring, with the help of number and calculation. But mathematical research into nature is not exact because it calculates with precision; rather it must calculate in this way because its adherence to its object-sphere has the character of exactitude. The humanistic sciences, in contrast, indeed all the sciences concerned with life, must necessarily be inexact just in order to remain rigorous. A living thing can indeed also be grasped as spatiotemporal magnitude of motion, but then it is no longer apprehended as living. The inexactitude of the historical humanistic sciences is not a deficiency, but is only the fulfillment of a demand essential to this type of research. It is true, also, that the projecting and securing of the object-sphere of the historical sciences is not only of another kind, but is much more difficult of execution than is the achieving of rigor in the exact sciences.*“

(HeM) Heidegger M., The Age of the World Picture, Cambridge University Press

Heidegger M.
Sein und Zeit

„Was gibt den Anlaß, Zeit und Sein zusammen zu nennen? Sein besagt seit der Frühe des abendländisch-europäischen Denkens bis heute dasselbe wie Anwesen. Aus Anwesen, Anwesenheit spricht Gegenwart. Diese bildet nach der geläufigen Vorstellung mit Vergangenheit und Zukunft die Charakteristik der Zeit. Sein wird als Anwesenheit durch die Zeit bestimmt. Daß es sich so verhält, könnte schon genügen, um eine unablässige Unruhe in das Denken zu bringen. Diese Unruhe steigert sich, sobald wir uns aufmachen, dem nachzudenken, inwiefern es diese Bestimmung des Seins durch die Zeit gibt. Inwiefern? Dies fragt: Weshalb, auf welche Weise und woher spricht im Sein dergleichen wie Zeit? Jeder Versuch, das Verhältnis von Sein und Zeit mit Hilfe der landläufigen und ungefähren Vorstellungen von Zeit und Sein hinreichend zu denken, verstrickt sich alsbald in ein unentwirrbares Geflecht kaum durchdachter Beziehungen. Wir nennen die Zeit, wenn wir sagen: Jedes Ding hat seine Zeit. Dies meint: Jegliches, was jeweilen ist, jedes Seiende kommt und geht zur rechten Zeit und bleibt eine Zeit lang während der ihm zugemessenen Zeit. Jedes Ding hat seine Zeit.

Aber ist das Sein ein Ding? Ist das Sein so wie ein jeweilig Seiendes in der Zeit? Ist das Sein überhaupt? Würde es sein, dann müßten wir es unweigerlich als etwas Seiendes anerkennen und demzufolge unter dem übrigen Seienden als ein solches vorfinden. Dieser Hörsaal ist Der Hörsaal ist beleuchtet. Den beleuchteten Hörsaal werden wir ohne weiteres und ohne Bedenken als etwas Seiendes anerkennen. Aber wo im ganzen Hörsaal finden wir das »ist«? Nirgends unter den Dingen finden wir das Sein. Jedes Ding hat seine Zeit. Sein aber ist kein Ding, ist nicht in der Zeit. Gleichwohl bleibt Sein als Anwesen, als Gegenwart durch Zeit, durch Zeithaftes bestimmt. …

… Sein und Zeit« ist der Versuch einer Interpretation des Seins auf den transzendentalen Horizont der Zeit hin. Was meint hier transzendental«? Nicht die Gegenständlichkeit eines Gegenstandes der Erfahrung als konstituiert im Bewußtsein, sondern der aus der Lichtung des Da-seins erblickte Entwurfbereich für die Bestimmung des Seins, d. h. des Anwesens als eines solchen. In dem Vortrag »Zeit und Sein« wird der bislang ungedachte, im Sein als Anwesen liegende Sinn von Zeit in ein ursprünglicheres Verhältnis zurückgeborgen. Die

Rede von einem Ursprünglicheren ist hier leicht mißverständlich. Wenn wir aber auch zunächst unausgemacht lassen, wie das Ursprünglichere zu verstehen, und «las heißt, nicht zu verstehen ist, bleibt es dennoch bestehen, daß das Denken — und zwar sowohl in dem Vortrag selbst als auch im Ganzen des Weges von Heidegger — den Charakter eines Rückgangs hat. Das ist der Schritt zurück. Zu beachten bleibt die Mehrdeutigkeit des Titels. Nötig wird die Erörterung des Wohin und des Wie in der Rede vom »zurück«."

Martin Heidegger: „Sein und Zeit"
Luckner A.

§5. Die ontologische Analytik des Daseins als Freilegung des Horizontes für eine Interpretation des Sinnes von Sein überhaupt

(LuA) S. 20-22: „*In diesem Paragraphen stellt Heidegger das Programm für den ersten Teil von* Sein und Zeit *vor, von dessen drei Abschnitten wiederum nur die ersten beiden veröffentlicht sind. Nachdem der Gegenstandsbereich der Analyse eingegrenzt ist, stellt sich nun die Frage, wie das Dasein einer existenzialen Analyse zugänglich ist.*

Ontisch ist das Dasein uns das nächste, denn wir sind als Fragende selbst ein Seiendes von der Art des Daseins. Ontologisch aber sind wir uns selbst denkbar fern, so wie für das Auge eine aufgesetzte Sonnenbrille am nächsten ist, aber wir sie normalerweise selbst nicht sehen, wenn wir durch sie schauen.

Wie soll nun vorgegangen werden bei dieser Analyse, wenn wir hierfür nicht auf die Wissenschaften vom Menschen zurückgreifen können? Zunächst sind wir erst einmal auf eine phänomenologische *Beschreibung des Daseins angewiesen. Nur eine phänomenologische Beschreibung – im Unterschied zu einer immer schon eine bestimmten Seinsweise fraglos voraussetzenden wissenschaftlichen Erklärung – kann gewährleisten, daß das Dasein „sich an ihm selbst von ihm selbst her zeigen kann" (16). Hierfür ist es notwendig, das Dasein so zu beschreiben, wie es sich gerade in seiner Alltäglichkeit zeigt, ohne vorherige Unterscheidung in wesentliche und unwesentliche Züge.*

Heidegger nimmt in diesem Paragraphen thetisch das Ergebnis der vorbereitenden Analytik des Daseins (= 1. Abschnitt von Sein und Zeit*) vorweg, ohne daß wir es hier schon überprüfen könnten: Als Sinn des Seins des Daseins, der Existenz, wird sich die* Zeitlichkeit *erweisen. Das heißt: Dasein* existiert zeitlich*, auf diese Weise „ist" das Dasein. Wenn dies so ist, lassen sich alle Strukturmomente des Daseins, die in der Analytik zutage gefördert wurden, auf diesem Programmhintergrund als Modi der Zeitlichkeit interpretieren: das ist das Programm des zweiten Abschnitts von* Sein und Zeit*.*

Dasein ist (wesentlich) Zeitlichkeit. Dieses Ergebnis kann uns den Boden bereiten, die Seinsfrage sinnvoll zu stellen, denn wenn 1. Das Dasein ontologisch an ihm selbst ist, d.h. ein Seinsverständnis immer schon besitzt, 2. Dasein wesenhaft zeitlich existiert, dann ist jedes Verständnis von Sein nur auf dem Hintergrund der Zeit verständlich. Wenn wir vom Sein reden, sprechen wir von der Zeit. Die Frage nach dem Sinn von „Sein" ist also immer auch und von vornherein die Frage nach der Zeit. Die Zeit ist der Horizont *alles Seinsverständnisses und daher auch, als zentrale Problematik aller Ontologie, Fluchtpunkt der Fundamentalontologie. Die Zeit als Thema der Ontologie ist freilich nicht neu, im Gegenteil, sie ist ja von Anaximander bis heute so etwas wie ein Dauerbrenner der Metaphysik. Aber der Zeitbegriff wurde immer in Opposition zum Ewigen, Unzeitlichen abgehandelt und daher auf einer Ebene, die Heidegger aus gutem Grund unterlaufen möchte. Eine Darstellung und Analyse des, wie er es nennt, „vulgären" Zeitverständnisses – solches, welches die Zeitlichkeit auf ein „in der Zeit sein" verkürzt – kann zeigen, daß gerade das Unzeitliche und Ewige sich überhaupt nur als ein Modus der Zeit denken läßt. Bis zu diesem Punkt reicht das Textfragment* Sein und Zeit*.*

Von dem Befund ausgehend, daß jedes Seinsverständnis im Horizont der Zeit steht, wäre nun die Aufgabe des dritten Abschnitts gewesen, die Temporalität *des Seins herauszuarbeiten. Im Unterschied zum Ausdruck „Zeitlichkeit", der für die Seinsweise des Daseins reserviert ist, betrifft der Ausdruck „Temporalität" alle möglichen Seinsweisen. Die Zeitlichkeit ist also wiederum, als spezifische Seinsweise des Daseins, fundiert in der Temporalität des Seins selbst. Deren Bestimmung wäre die Beantwortung der Frage nach dem Sinn von „Sein". Es ist klar, daß hiermit die Sphäre der Daseinsanalyse verlassen werden muß: Der Weg verläuft vom Sein des Daseins zum Aufweis seiner fundamentalen Zeitlichkeit, danach sollte der Weg im Abschnitt „Zeit und Sein" eine* Kehre *machen."*

Heisenberg W.
The degeneracy of the ground state seeming to be closely
connected with the existence of long-range forces

In (HeW) the deviation from iso-spin-symmetry in electrodynamics is taken as indication for an asymmetry of the ground state, (DüH):

(HeW) vi*: „The mathematical formalism contains some unconventional features which formerly have rendered its understanding somewhat difficult: the indefinite metric in Hilbert space and the degeneracy of the ground state. But in recent years the indefinite metric has been studied in connexion with the Bleuler-Gupta version of quantum electrodynamics and with the Lee-model, the degeneracy of the ground state plays an important part in modern solid state physics."*

(HeW) p. 90*: „In fact the number of protons in the world seems to be very different from the number of neutrons, the number of electrons is very different from the number of neutrinos. Even the matter and antimatter should be distributed in the universe with equal average density – many galaxies might be consist of matter, equally many of antimatter – and if total isospin should be small in this way, the big asymmetry would remain, since in matter the total isospin would point in one direction, in antimatter in the opposite direction. Hence there would be a macroscopic deviation from symmetry in isospace.*

An asymmetry of the ground state and therefore a degeneracy of this state is a well-known phenomenon in many systems discussed in conventional quantum mechanics. Ferromagnetism, superfluidity, superconductivity, crystal structure are obvious examples. In such cases two important new phenomena appear ….: The degeneracy of the ground state enforces the existence of bosons of rest mass zero, as has been pointed out in a mathematical form by Goldstone (the Goldstone theorem). Some property of the ground state can be attached to the particles thereby changing normal particles into strange particles."

(HeW) p. 108: „*The asymmetry of the ground state with respect to the isospin group has been used in chapter 7 as explanation for the strange particle poles in the Green's functions and as basis for the spurion formalism. … It has been emphasized already in earlier papers on this subject, that empirically the asymmetry of the ground state seems to be closely connected with the existence of long-range forces, i.e. of particles with rest mass zero, (DuH). The asymmetry with respect to the isospin group comes in through the long-range forces of electrodynamics, the asymmetry of with respect to the the space reflection parity appears in the weak interactions, and this is the first interaction which affects neutrinos. It can be well understood that short-range forces allow a clear separation of the particles from the rest of the world, while long-range forces may lead to a dependence of the properties of the particles on the state of the world in large dimensions. This connexion has been found a mathematical expression in the theorem of Goldstone. .. In the present theory the Goldstone theorem is the basis for an understanding of quantum electrodynamics.*"

Helmholtz H.
Atome der Elektrizität

(ScW) S. 2: „*Wenn wir Atome der chemischen Elemente annehmen, so können wir nicht umhin, weiter zu schließen, daß auch die Elektrizität positiv sowohl wie negativ in bestimmte elementare Quanta geteilt ist, die sich wie Atome der Elektrizität verhalten*".

Hildebrandt S.
Calculus of variations
The link between mathematics and physics

(HiS) X: „*The Greek word mathema – which means knowledge, cognition, understandaing, perception – suggests that the study of mathematics started about 3000 years ago with asking questions about the world.The historical sections of our account show that a large part of the development of mathematics was the result of a desire to comprehend nature. Mathematics, however, is more than the handmaiden of other sciences. It is, as C. F. Gauss stated, irrelevant whether one applies mathematical knowledge to number theory or to the movement of a lump of matter such as a planet.*

(HiS) p. 17: Our goal will be to find an easy-to-understand link between mathematics and physics. The mathematical theory that provides this link is called the calculus of variations."

The Euler-Lagrange equations

(HiS) 29 ff.: *The mathematical (indispensible for physics) „principle of the horizontal tangent plane"*

> Question*: how, in principle, you can locate the summits in a mountainous area in the dark equipped with only a small flashlight and a level?*
> Answer*: „by employing the idea of the horizontal tangent.*

This is how the mathematician locates maxima and minima, by first reducing the questions of best and worst to a geometric question of finding summits or pits in some mathematically constructed mountain range. However these are usually not mountains in a three dimensional world, but a „higher-dimensional" space. … The mathematician uses a strategy to focus attention on a small number of points suspected to be maxima (peaks) or minima (pits). This procedure is similar to that of a detective, who uses all available circumstantial evidence to reduce the number of suspects who might have committed a crime.

From the „principle of the horizontal tangent plane" the mathematician establishes a system of differential equations (called the Euler-Lagrange equations)"

The geometric description of minimal surfaces

(HiS) P. 166: „*Now we can discuss the theorem of Lagrange in which he stated the minimal-surface equation. This theorem will provide the geometric characterization of least-area surfaces that we are seeking:*

At each regular point, as surface of minimal area must have a mean curvature of zero.

That is, the surfaces of minimal area satisfy the equation $H \coloneqq \frac{\kappa_1+\kappa_2}{2} = 0$, κ_1, κ_2 *denote the largest and smallest (principle) curvatures that a normal section at point* P *can have defining the mean curvature* H*, and the Gaussian curvature* $K = \kappa_1 \cdot \kappa_2$."

(HiS) p. 241: „*Planets, rotating drops, and the nuclei of atoms*
Three (other) phenomena can be explained by a single variational principle founded by Bernoulli's principle of virtual work. These phenomena belong to the fields of astronomy, hydrodynamics, and nuclear physics, which, at first sight, do not seem to have very much in common. Specifically, we will consider rotating and self-gravitating liquid masses of homogeneous density, then rotating liquid drops endowed with surface tension, and finally the nuclei of atoms with or without an angular momentum."

(HiS) p. 262: „*The geometry of crystals*
Among the most-admired forms in nature are those of crystalline structures. … A natural question to ask is whether the shapes of crystals can be explained by variational principle. …

We then ask for the structure or shape that, for a fixed volume, has a minimum toal surface energy. .. G. Wulff's discovery is that, given some further reasonable assumptions about the mathematical character of the surface energy, the following holds:

> *For every given volume, there is a unique convex body whose boundary consists of planar faces, such that this boundary surface has less energy than does the boundary surface of any other piecewise smooth body of the same volume.*

This theorem is remarkable in two ways. First, there is an infinite number of possible surface energies; nevertheless, for each such admissible energy, the unique minimum is a convex region bounded by planes. Second, unlike most problems in mathematics in which explicit solutions are impossible to find, the solution to our minimum problem, the optimal crystalline region, can be determined by a simple procedure known as the Wulff construction.

Maupertius' principle, calculus of variations,
and Newton's dynamics

(HiS) p. 279 ff.: „*In the Principia, the entire program of modern mechanics is formulated, not only in content but also in style. Newton began like a mathematician by first giving definitions of the basic notions, such as mass and momentum, and then formulated three basic laws or axioms from which everything else was to follow:*

> *First law: Every body remains in its state of rest or uniform motion in the same direction unless it is compelled by impressed forces to change this state*
> *Second law: The change in motion is proportional to the impressed moving force and, secondly, it will occur along the straight line in which that force is impressed.*
> *Third law: To an action there is always an equal and opposite reaction, or, the mutual actions of two bodies upon each other are equal and point in opposite directions.*

… The three laws are only the formal framework of dynamics and do not say anything about the nature of the acting forces. In fact, the second law has occasionally been considered tautogical. If we want to apply the dynamical laws to concrete cases, we must specify the acting forces. Attraction is one of the basic forces, and Newton stated how this force acts:

Every particle of matter attracts every other particle with a force proportional to the mass of each, and inversely proportional to the square of the distance between them.

This is known as Newton's universal law of gravitation. (Actually, Newton never formulated the law in this general form; instead he gave different versions in different places, which have been combined into the preceding statement). He named the attractive force of mass gravitas, meaning heaviless or weight. Today we speak of gravity or gravitation.

Hence the gravitation law expresses the difference between mass and weight: a body's mass is independent of its location in space; but if it is placed in a field of gravitation, it experiences weight caused by gravitation. Theoretically, gravitation exists everywhere, but, in practice, the gravitational effect of one body on another is zero if they are far enough apart."

(HIS) p. 286 ff.: „*A faily precise version of this principle, as currently accepted, for the simplest case, that of the motion of a single point-mass is the following:*

Consider a point-mass m that moves from time t_1 and t_2 in a field of conservative forces*, such as gravitation. At each point in a force field, a force of a given magnitide and direction acts on each point-mass m moving in a field according to Newton's second law:*

$$F = m \cdot a.$$

A field of forces F is conservative*, if it posseses a potential energy U. This is a rule that attaches a numerical value $U(P)$ to each point P in space, in such a way that the „negative gradient" of these values U at P equals the force F at P. This means the following:*

Suppose the function U describes a landscape in a four-dimensional world above the three-dimensional space. Let us now consider a point P in space and the point P^* on the energy landscpae above P. If P^* is not a stationary point on the mountain range, we can find a direction in the three-dimensional space at P that indicates the direction of steepest ascent of the landscape at the point P^* above P; opposite to it, we have the direction of steepest descent of the function U. Let us attach an arrow to P, pointing in either one of these two directions, whose size equal the rate of change of U on corresponding direction. In this way, we define two vectors whose feet lie at P. The direction of steepest ascent defines the „gradient (vector) of U," dnotes by $gradU$, and the vector pinting in the opposite direction defines the „negative gradient"," which is the arrow opposite to $gradU$ and therefore is denoted by $-gradU$.

If P^* is a stationary point, say, the top of a mountain, then the function U has a maximum at P, and there is neither a direction of strongest ascent nor one of strongest descent (this expresses the feeling that, on top of a mountain, the ground is practically horizontal, without ascent or descent). Therefore we set $gradU$ and $-gradU$ equal to zero at a stationary point.

Then our assumption that F is a conservative field of forces with the potential energy U is expressed by the equation

$$F = -gradU$$

which is to hold at each point of space.

Suppose now that the point-mass m moves with some velocity of absolute value v through space. Since v can change in time, it has to be considered as a function of the time t. Then we can define the kinetic energy T of the point-mass at each instant of time by

$$T = \frac{m}{2}\vec{v}^2.$$

The expression

$$E = T + U$$

is called the total energy of the point-mass at each instant.

With some infinitesimal calculus, we can prove from the equations $F = m \cdot a$ and $F = -gradU$ that the total energy E is a constant; in other words, a point-mass in a conservative field of forces moves in such a way that its total energy has a numerical value h that is the same at each instant. This is the law of the conservation of energy, and it is expressed by the equation $E = h$.

This law explains the term conservative to a field of forces. We look at two important cases of conservative fields of forces.

The first one is the gravitational field on the surface of the Earth, where one considers the problem of the trajectory of a stone thrown in the air. Here it is assumed that the force of attraction is the same size at each point and that it always points perpendiculary towards the surface of the Earth, which for simplicity is supposed to be a plan. The potential energy U of this field of forces is a linear function of the height h above the ground, and it increases with increasing height. This is described by the rule

$$U(P) = mgh + c.$$

Here m is the mass of the point moving in the gravitational field, c is an arbitrarily chosen gauge constant, and g is a given positive constant that, according to measurements, has a value of approximately 9.81 m/sec^2.

Another model is the gravitational field of a large mass M, which rests at some fixed point Q and attracts some point-mass m that moves around Q. The potential energy $U(P)$ of this field at some point P is given by the expression

$$U(P) = -\frac{GmM}{r} + c,$$

where $r = \overline{PQ}$ is the distance of P from the center Q, G is Newton's gravitation constant, which has, with great precision, been measured by experiments, and c is a number that can be chosen arbitrarily.

The action A performed by a point-mass during its motion between times t_1 and t_2 is defined by the integral

$$A = \int_{t_1}^{t_2} 2T dt.$$

Suppose now that the point-mass m moves under the influence of a conservative field of forces. What distinguishes the actual motion from all the other motions that, in principle, were possible but actually do not occur?

According to Newton, the actual motion can be determined from the equation $F = ma$ provided that its initial data are known. There is also another way to determine the true motion which is conceptually totally different from the first.

According to Maupertuis, the actual motion of the point-mass m from P_1 to P_2 under the influence of the force field $F = -gradU$ is distinguished among all the other motions by the property that it provides a stationary value for the action A. Actually, here we have replaced Maupertuis' minimum principle.

Seek a motion that minimizes A,

with the following somewhat weaker requirement:

Seek a motion that is stationary for A.

But, as it stands, this principle is pure nonsense, because the acting forces do not appear anywhere in the expression for A. The correct statement would be that we do not need to seek a stationary value for A among all motions from P_1 to P_2, but only amoung those which at each instant have the same constant total energy E, say $E = E_0$ as the actual motion. (We need to know the actual motion to compute the energy constant E_0; it can, for instance, be obtained from the initial data or from any other complete set of data.)

There is one more complication: in general it is not possible to get from a given point P_1 to another point P_2 by means of a motion of fixed total energy if we prescribe the initial time t_1 and the final time t_2. Therefore, the demand to minimize the value of action amoung all possible motions of the point-mass from P_1 to P_2 which have the same total energy point P_1 to another point E, the same initital time t_1, and the same final time t_2, does not make sense, since there might be not any suchmotion. The way out of this dilemma is to minimize action amoug all possible (or virtual) motions from P_1 to P_2 of fixed total energy, for which neither the time of departure nor the time of arrival is fixed.

In this form, the action principle turns out to be correct, but often it is not easy to handle. Therefore, we shall state another version, which was devised by Lagrange.

First, we define P_1 to $L = T - U$, the so-called Lagrangian *or* action density*; it is the difference between kinetic and potential energy. We can then infer from $E = T + U$ that $2T = L + E$, and thus we find for the motions with a constant total energy of the value E_0 that*

$$2T = L + E_0.$$

Now we consider the areas under the graps of two functions $2T$ and $L + E_0$ between two t −values t_1 and t_2. As the two graphs coincide, we infer that

$$\int_{t_1}^{t_2} 2T dt = \int_{t_1}^{t_2} (L + E_0) dt = \int_{t_1}^{t_2} L dt + \int_{t_1}^{t_2} E_0 dt.$$

The *integral on the left-hand side is the action integral A, while the integral $\int_{t_1}^{t_2} E_0 dt$ is the are area of a rectangle of the height E_0 above the t-axis which has the interval between t_1 and t_2 on the t-axis as one of its edges.Therefore, this integral has the value $E_0(t_2 - t_1)$, and we obtain*

$$\int_{t_1}^{t_2} L dt = A + E_0(t_1 - t_2).$$

From this equation we guess correctly that the integral also can be used to formulate a variational principle for the actual motion from P_1 to P_2. This variational principle says:

If the actual motion begins at P_1 at time t_1, and ends at P_2 at time t_2, then it gives the integral $\int_{t_1}^{t_2} L dt$
A stationary (and sometimes even a minimal) value, among all motion beginning at P_1 and ending at P_2 at the same time as the actual motion.
The new features of this variational principle are that, contrary to the first one, no subsidary condition $E = E_0$ and no variation of the limits are needed! In other wrds, the virtual motions are not restricted to those with constant energy E, and we may fix t_1 and t_2. This makes the integral $\int L dt$ much easier to handle than $\int 2T dt$. In fact, the variational principle

Seek a stationary motion for $\int_{t_1}^{t_2} \mathrm{Ldt}$

is the form of the action principle that can easily be generalized to more difficult situations and to other physical problems.

Hübscher A.
Um Schopenhauers Farbenlehre
Ein Brief *(von O. Volger)* und Bericht

(HüA) S. 84: *Für heute möchte ich jenen Punkt in Anregung bringen, in Betreff dessen Sie mit Goethe in Widerspruch traten: das Weiß. Hier war Goethe im Irrthum – aber mir scheint, Sie sind es gleichfalls.*
…
Die Unklarheit liegt nun ganz einfach in einer, allerdings sehr gebräuchlichen, höchst bemerkenswerthen und für die Unvollkommenheit der menschlichen Sinnesübung bezeichnenden Verwechslung: der Verwechslung nämlich des Lichtes, welches als Klarheit den quantitativen Gegensatz zur Finsterniß, also zum Schwarz, bildet, und des Weiß, welches eben selber den neutralen Punkt zwischen den polarisch kontrastirenden Quantitäten einnimmt.
…
Daher ist auch Ihre Aufstellung (p. 24) der intensiven Theilung der Thätigkeit der Retina zu verändern.

Licht	*-*	*Halbschatten*	*-*	*Finsternis*
Klarheit	*-*	*Grau*	*-*	*Schwarz*

Bei eigenen Versuchen werden Sie Sich überzeugen, daß Weiß und Schwarz nie das Grau des Halbschattens, sondern vielmehr das, vom Grau im gewöhnlichen Leben zwar nicht genügend unterschiedene, aber wesentlich verschiedene Greis geben.
….
Die Sonne ist nicht weiß, sondern klar. Die Quantität des Lichtes blendet hier unser Auge. Weiß als solches ist nicht blenden. …
Weiß gehört also mit in die Reihe der Farben, aber freiich ohne einen Gegensatz zu haben, als Neutralität.

Husserl E.
Phenomenology
Logische Untersuchungen
(providing a new foundation for pure logic and epistemology)

(ZaD) S. 9: „*The fundamental mistake of psychologism is that it does not distinguish correctly between the object of knowledge and the act of knowing. Whereas the act is a psychical process that elapses in time and that has a beginning and an end, this does not hold true for the logical principles or mathematical truths that are known (Hua 24/141). When one speaks of a law of logic or refers to mathematical truths, to theories, principles, sentences, and proofs, one does not refer to a subjective experience with a temporal duration, but to something atemporal, objective, and eternally valid. Although the principles of logic are grasped and known by consciousness, we remain conscious of something ideal that is irreducible to and utterly different from the real psychical acts of knowing. This distinction between the ideal and real is so fundamental and urgent to Husserl, that in his criticism of psychologism he occasionally approaches a kind of (logical) Platonism: The validity of the ideal principles are independent of anything actually existing.2 No truth is a fact, i.e. something determined as to time. A truth can indeed have as its meaning that something is, that a state exists, that a change is going on etc. The truth itself is, however, raised above time: i.e. it makes no sense to attribute temporal being to it, nor to say that it arises or perishes (Hua 18/87 [109-110]). The truth that 2 + 3 = 5 stands all by itself as a pure truth whether there is a world, and this world with these actual things, or not (Hua 9/23).“*

The Lifeworld and the Crisis of Science

(ZaD) S. 126: „*According to Husserl's diagnosis, this crisis is a direct consequence of the objectivism that has dominated since the Scientific Revolution in the Renaissance, a revolution characterized by its quantitative ideal of method, its sharp distinction between facts and values, and its insistence that science and science only can describe reality as it is in itself. To quote Galileo, who, according to Husserl, personifies this entire enterprise:*

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.

According to Husserl, the only way to overcome the present scientific crisis and to heal the disastrous rupture between the world of science and the world of everyday life is by criticizing this reigning objectivism. This is why Husserl commences his analysis of the lifeworld, a lifeworld which, although it constitutes the historical and systematical foundation of science, has been forgotten and repressed by it."

(ZaD) S. 128: „*Husserl does acknowledge the validity of scientific theories and descriptions, and would even concede that they attain a higher degree of objectivity than our daily observations. But, as he repeatedly points out, we are faced with a faulty inference if against that background, we conclude that 1) only scientific accounts can capture true reality, or that 2) these accounts manage to grasp something which, in a very radical sense, is independent of our experiential and conceptual perspective. To think that science can give an absolute description of reality, that is, a description from a view from nowhere, is simply a misunderstanding. We must reject the assumption that physics is the sole arbiter of what there is, and that all notions to be taken seriously should be reducible to the vocabulary and the conceptual apparatus of the exact sciences.*

As Husserl points out, natural science by itself undermines the categorical distinction between the sensuously given and the physically described. After all, it does insist that it investigates the water I am drinking, or the diamond I am admiring, rather than a completely different object. It maintains that it is the true nature of the experienced object that it seeks to capture.

The physical thing which he [the physicist] observes, with which he experiments, which he continually sees, takes in his hand, puts on the scale or in the melting furnace: that physical thing, and no other, becomes the subject of the predicates ascribed in physics, such as weight, temperature, electrical resistance, and so forth (Hua3/ii3).

According to Husserl, physics does not present us with an entirely new physical object, but rather with a different, higher, and more exact objective determination of the very same object that we encounter in our daily life (Ms. A III 9 8b). In contrast to my own estimation of whether the water is warm or hot or whether it tastes strange, a definition of water as H_2O is not only valid for me personally, but for all subjects. Even the most exact and abstract scientific results, however, are rooted in the intuitively given subject-relative evidence of the lifeworld—a form of evidence that does not merely function as an unavoidable, but otherwise irrelevant, way point toward scientific knowledge, but as a permanent and quite indispensable source of meaning and justification (Hua 6/142).

In its urge toward idealization, in its search for exact and objective knowledge, science has made a virtue out of its decisive showdown with subject-relative evidence, but it has thereby overlooked that its own more refined measurements inevitably continue to draw on the contribution of intuition, as when one sets up the experiment, reads the measuring instruments, or interprets, compares, and discusses the results with other scientists. We should not forget that empirical theories are based on experimental and experiential evidence (Hua 6/128). Although scientific theory in its idealization transcends the concrete, intuitively given lifeworld, the latter remains as a reference point and meaning-foundation (Hua 6/129)."

(Hua) Husserliana

Kant I.
The Critique of Teleological Judgement
Objective Purposiveness

(KaI) § 62: *On Merely Formal, as Distinguished from Materlal, Objective Purposiveness*

(KaI) § 63: *On Relative as Distinguished from the Inner, Purposiveness of Nature*
„Only in one case does experience lead our power of judgment to the concept of a purposiveness that is both objective and material purposiveness, i.e., to the concept of a purpose of nature-namely, when we have to judge a relation of cause to effect which is such that we can see it as law-governed only if we regard the cause's action as based on the idea of the effect, with this idea as the underlying condition under which the cause itself can produce that effect. We can do this in two ways: we may regard the effect either as directly the product of art, or as only the material that other possible natural beings employ in their art; in other words, we may regard the effect either as a purpose, or as a means that other causes employ purposively. The second purposiveness is called either usefulness (for human beings) or benefit (for any other creature), and this second purposiveness is merely relative, whereas the first is an intrinsic purposiveness of the natural being."

(KaI) § 64: *On the Character Peculiar to Things (Considered) as* („intrinsic“ as distinguished from „relative“) *Natural Purposes*

„To say that a thing is possible only as a purpose is to say that the causality that gave rise to it must be sought, not in the mechanism of nature, but in a cause whose ability to act is determined by concepts. And seeing that a thing is possible only as a purpose requires that the thing's form could not have arisen according to mere natural laws, laws we can cognize by understanding alone as applied to objects of sense, but requires that even empirical cognition of this form in terms of its cause and effect presupposes concepts of reason. [Therefore the form of such a thing is, as far as reason is concerned, contingent in terms of all empirical laws. But reason, even if it tries to gain insight only into the conditions attached to the production of a natural product, must always cognize not only the product's form but the form's necessity as well. And yet in that given form it cannot assume that necessity. Hence that very contingency of the thing's form is a basis for regarding the product as if it had come about through a causality that only reason can have. Such a causality would be the ability to act according to purposes (i.e., a will), and in presenting an object as possible only through such an ability we would be presenting it as possible only as a purpose.

Suppose that someone coming to a seemingly uninhabited country perceived a geometric figure, say a regular hexagon, traced in the sand. As he reflected on this figure, working out a concept for it, reason would make him aware, even if obscurely, of the unity of the principle (required) for producing this concept. And so, following reason, he would not judge that such a figure is made possible by the sand, the adjoining sea, the wind, or even animals that leave footprints familiar to him. or by any other nonrational cause; for it would seem to him that coming across such a concept (a regular hexagon), one that is possible only in reason, is so infinitely contingent that there might as well be no natural law for it at all, and hence that such an effect could also not have been caused by anything in nature, which operates merely mechanically, but could have been caused only by the concept of such an object, a concept that only reason can provide and compare the object with. It would seem to him therefore that, although this effect can be considered a purpose, it cannot be considered a natural purpose, but can be considered only a product of art (vestigium hominis video). …

If, on the other hand, we cognize something as a natural product and yet are to judge it to be a purpose, and hence a natural purpose -unless perhaps the very (thought) is contradictory-then we need more (than the above example provided). I would say, provisionally, that a thing exists as a natural purpose if it is both cause and effect of itself (although (of itself) in two different senses). For this involves a causality which is such that we cannot connect it with the mere concept of a nature without regarding nature as acting from a purpose; and even then, though we can think this causality, we cannot grasp it. Before we analyze this idea of a natural purpose in full, let me elucidate its meaning by the example of a tree. ….“

(KaI) *§ 65: Things (Considered) as Natural Purposes Are Organized Beings*
„We said in the preceding section that if a thing is a natural product but yet we are to cognize it as possible only as a natural purpose, then it must have this character: it must relate to itself in such a way that it is both cause and effect of itself. But this description is not quite appropriate and determinate and still needs to be derived from a determinate concept.“
(McP) *p. 28: „Wir haben gerade gesehen, wie Kant im Anhang zur Dialektik in der KdrV die Betrachtung des Naturganzen als deistisches System zum regulativen Prinzip erhebt. In der Einleitung zur KdUk greift er die Frage der Systematisierung von empirisch konstatierten Gesetzmäßigkeiten auf: Wir sollen solche empirischen Gesetze als Teile eines Systems von Gesetzen betrachten, “als ob gleichfalls ein Verstand (wenn gleich nicht der unsrige) sie zum Behuf unserer Erkenntnisvermögen, um ein System der Erfahrung nach besonderen Naturgesetzen möglich zu machen, gegeben hätte” (Bxxvii; W,253). Wir machen es zum regulativen Prinzip, dass die Natur als so strukturiert zu beurteilen ist, wie unser Bedürfnis nach Ordnung es verlangt; indem wir einzelne empirische Gegenstände klassifizieren und gesetzmäßig ordnen, setzen wir voraus, dass die Natur eine Ordnung hat. Die subjektive Zweckmäßigkeit der Natur, d.h. die Übereinstimmung der Natur mit unserem Ordnungsbedürfnis ist ein Prinzip der reflektierenden Urteilskraft.*

> *Diese Zusammenstimmung der Natur zu unserem Erkenntnisvermögen wird von der Urteilskraft, zum Behuf ihrer Reflexion über dieselbe, nach ihren empirischen Gesetzen, a priori vorausgesetzt; ... weil wir, ohne diese vorauszusetzen, keine Ordnung der Natur nach empirischen Gesetzen, mithin keinen Leitfaden für eine mit diesen nach aller ihrer Mannigfaltigkeit anzustellende Erfahrung und Nachforschung derselben haben würden”. (Bxxxvi; W,258–9)*

(McP) *p. 28: Die Analytik der teleologischen Urteilskraft*
„In der Kritik der teleologischen Urteilskraft unternimmt es Kant, die Grenzen der mechanistischen Erklärungsweise und die Berechtigung teleologischer Prinzipien in der Naturwissenschaft systematisch zu untersuchen. Es geht ihm darum zu bestimmen, inwiefern und unter welchen Bedingungen die Zweckmäßigkeit von Dingen, Beziehungen oder Vorgängen selbst irgendeinen Erklärungswert hat bzw. legitim in einer wissenschaftlichen Erklärung benutzt werden darf. Es geht auch darum, ob und wann man teleologische Annahmen als heuristische Mittel, um dem verborgenen Mechanismus auf die Spur zu kommen, einführen darf und soll. Es ist von vornherein klar, dass die teleologischen Annahmen bloß regulative Prinzipien sind. Es ist ausgeschlossen, dass die Zweckmäßigkeit auf einem zwecktätigen Subjekt beruht, bzw. dass man wirkliche Absichten dabei unterstellt. Es handelt sich aber auch nicht um die subjektive Zweckmäßigkeit der Natur oder einzelner Naturdinge für unser Erkenntnisvermögen oder unsere ästhetischen Gefühle, sondern um eine "objektive" Zweckmäßigkeit, d.h. eine Mittel-Zweck-Beziehung, die im Objekt der Erkenntnis selbst liegen soll und nicht in der Beziehung des Objekts zum Subjekt. Gefragt wird nicht, ob etwas für unser Erkenntnisvermögen zweckmäßig eingerichtet ist, sondern ob ein Ding oder ein Teil-System für ein anderes Ding bzw. Teil-System (oder beide gegenseitig) zweckmäßig sein kann, und was es für ein Ding bedeutet, dass etwas für es zweckmäßig sein soll."

Kirchhoff G.
On the Distribution of Electricity Between Two Conductive Spheres

(KiG): „Poisson *addressed the problem of determining the distribution of electricity across two conducting spheres in two famous papers [(*)], and Plana further developed the formulas given by* Poisson *in those works [(**)]. One can view the problem as consisting of two parts; in the first, a function must be determined, which Poisson denotes by* $f(x)$*, and which specifies the potential of the electricity distributed over one sphere for all points on the central line; in the second, a function must be formed from this* $f(x)$ *that represents the potential of the same electricity for points outside the central line, and from which the density of the electricity at all points on the sphere can be easily determined. However, to calculate the quantities that are of primary interest to the physicist, knowledge of the function* $f(x)$ *is sufficient. For, if the radius of the sphere is set to* 1*,* $f(0)$ *is the total amount of electricity present on the sphere, furthermore, the expression* $\frac{1}{2\pi}(f(x) + 2x\frac{df(x)}{dx}$ *- when* $x = 1$ *or* $x = -1$ *is set in it - gives the density of electricity at one or the other of the two points on the spheres lying on the central line; and the force with which the two spheres attract or repel each other can also be expressed by* $f(x)$*. The considerations to be discussed below refer only to this function* $f(x)$*. Poisson has found a series for it that always converges, and indeed the greater the distance between the two spheres, the faster it converges. I have derived this series in a way that seems to me to be superior to the method used by* Poisson*. The series bears a resemblance to certain series that appear in the theory of elliptic functions; I have observed that some of the quantities dependent on* $f(x)$ *can be represented by elliptic functions in closed-form expressions. Among these is the electric density at the point on the sphere that lies on the line connecting the two centers, in the case where the total potential in both spheres has the same value. … I have attempted to derive a series that yields the value of when the distance between the spheres is small.*

Let a *and* b *be the radii of the two spheres, and* c *the distance between their centers, which is assumed to be greater than* $a + b$*; let the potential of all free electricity in the first sphere be* h*, that in the second sphere be* g*, and* $f(x)$ *the potential of the electricity located on the first sphere relative to a point lying within that sphere, on the central line, between the two centers, at a distance* x *from the center of the first sphere. … For convenience, let us set* $a = 1$*, which yields*

$$f(x) = h\frac{b}{1+b}\frac{1}{1-x}\left[\psi\left(\frac{b}{1+b}\frac{x}{1-x}\right) - \psi\left(\frac{b}{1+b}\frac{1}{1-x} - 1\right)\right]$$

The equation agrees with equation (115.) in Planas's treatise."

(*) Mém. De la classe des sc. math. et. ph. De l'Institut imperial de France, année 1811, première et seconde partie
(**) Memorie della reale accademia delle scienze die Torino, tomo VII, 1845

Klainerman S.
The global nonlinear stability of the Minkowski space

(ChD) pp. 1, 10-13: „Einstein's field equations is about an unified theory of space-time and gravitations; the space-time (M, g) is the unknown, where M denotes a 4-dimensional manifold; one has to find an Einstein metric g, fulfilling the Einstein field equations. This is basically the equality $G = T$, whereby G denotes the Einstein tensor and T denotes the energy momentum tensor (e.g. the Maxwell equations). The Einstein-Vacuum equations (in the absense of matter, i.e. $T = 0$) are given by $R = 0$, whereby R denotes the Ricci tensor. Its simplest solution is the Minkowski space-time with its canonical coordinate system. Apart from Minkowski space-time it is not known, if there are any smooth, geodesically complete solution, which becomes flat at the infinity on any given spacelike direction. The main difficulties one encounters in the proof for the Cauchy Einstein-Vacuum equations with given initial data are:

(1) the problem of coordinates
(2) the strongly nonlinear hyperbolic features of the Einstein equations.

The problem of coordinates comes along with the concept of manifolds. To write the equations in a meaningful way, one seems forced to introduce coordinates. Such coordinates seem to be necessary even to allow the formulation of well-posed Cauchy problems and a proof of a local in time existence result. Nevertheless, as the particular case of wave coordinates illustrates, the coordinates may lead, in the large, to problems of their own."

Kneser A.
Das Prinzip der kleinsten Wirkung von Leibniz bis zur Gegenwart
The principle of least action from Leibniz until present (1928)

(KnA) p. 1: „*Die Leibnizsche Teleologie, die Vorstellung, daß der Weltverlauf ein Maximum des Guten gewähre, hat bei Leibniz selbst, abgesehen von anderen Anwendungen, den bestimmten Sinn, daß die Naturvorgänge aus Integralprinzipien nach der Methode des Größten und des Kleinsten abgeleitet werden können. Das bedeutet folgendes. Bei einem beliebig definierten, beliebigen Kräften unterworfenen Massensystem wird jeder in einer kleinen Zeit dt vor sich gehenden Bewegung durch besondere Definition ein Wirkungselement wdt zugeordnet. Betrachtet man nun die Bewegung in einem endlichen Zeitintervall, das durch Summierung der Elemente dt entsteht, so summieren sich die Elemente wdt zu einer Größe*

$$A = \int w dt \,,$$

der Wirkung oder dem Aufwande von Wirkung für das betrachtete Intervall. Und nun besteht das Prinzip darin, daß, wenn man die wirkliche Bewegung mit gewissen fingierten, näher zu definierenden Nachbarbahnen, Nachbarbewegungen vergleicht, die Größe A bei bei ersterer, verglichen mit ihren Werten A' bei fingierten Bewegungen, ein Maximum oder Minimum wird; allgemeiner braucht auch nur die Differenz $A' - A$ im Verhältnis zu den Dimensionen der Abweichung der fingierten von der wirklichen Bahn klein zu sein; A braucht nur, wie schon Leibniz sagt, ein ausgezeichneter Wert zu sein. Natürlich sind alle hier ziemlich unbestimmt bezeichneten Größen und Operationen exakt mittels der Begriffe der Infinitesimalrechnung zu definieren."

(KnA) p. 2: „*..(bei der allgemeinen Teleologie) handelt es sich um ein Prinzip der ausgezeichneten Fälle, um das Prinzip, daß der Fall der Natur gegenüber den möglichen fingierten Vorgängen ein ausgezeichneter ist, der aber den Vorgang vollständig charakterisiert, wenn man nur die nötigen mathematischen Hilfsmittel heranzieht, und das Wesen der Auszeichnung genügend definiert.*

Die Weisheit Gottes besteht nun für ein gewisses Gebiet von Erscheinungen oder, wie wir auch sagen können, für gewisse Wissenschaften darin, daß für jedes Erscheinungsgebiet ein im angegebenen Sinne beherrschendes Integralprinzip da ist; alle diese Prinzipien haben nur die angegebene allgemeine Form gemein; die konkrete Form der Größen w und A ist in den verschiedenen Gebieten ganz verschieden, auch nicht ohne weiteres aus beherrschenden allgemeinen Formen durch Spezifikation ableitbar.

Man denke an die einfache Aufgabe der Bewegung eines materiellen Punktes ohne wirkende Kräfte in der Ebene oder auf einer beliebigen gekrümmten Fläche. Die Bahnlinie wird erhalten, wenn man das Prinzip der kleinsten Wirkung in der von Leibniz geforderten Form ansetzt, daß der Aufwand, dessen Extrem man sucht, das

Zeitintegral der lebendigen Kraft ist. Letztere ist konstant nach dem Satze der lebendigen Kraft, der vorausgesetzt werden muß, aber nicht ausreicht, um die Bahnlinie zu bestimmen. Das Wirkungsprinzip fordert, wenn v die Geschwindigkeit ist, das Extrem der Größe $A = \int v^2 dt = v^2 \int dt$, also das Extrem der Zeit, mithin auch, wegen der konstanten Geschwindigkeit, der Extrem der Länge; die Bahnlinie ist die kürzeste Linie zwischen ihren Endpunkten, in der Ebene also die Gerade. Hierbei wird die gekennzeichnete Eigenschaft der Geraden, daß ihre Krümmung in jedem Punkt $= 0$ ist, abgeleitet aus der Betrachtung eines endlichen Bogens und eines endlichen entsprechenden Zeitintervalls, in welchem der bestimmte Punkt mitten inne liegt, das also auch zeitlich nachfolgende Lagen des bewegten Punktes mit enthält. Bei der Ableitung der gekennzeichneten Eigenschaft der Bahnkurve für einen bestimmten Raum-Zeitpunkt, bei der Ableitung der Richtung und der Geschwindigkeit in diesem besonderen Punkte wird also die Zukunft, ein nachfolgendes Zeitintervall, im Ansatz und Beweis benutzt, hat also den Charakter des logischen Prius. Dagegen wird das einzelne Naturgesetz etwas bei Newton so formuliert, daß aus gegebenen Zuständen des Massensystems nur der Zustand in einem späteren Zeitpunkte gefolgert wird. Der Planet findet sich in einer Anfangslage mit einer gewissen Anfangsgeschwindigkeit; seine Lage wird durch das Gravitationsgesetz gegeben für jede spätere Zeit; die Zukunft wird durch die Vergangenheit und die Gegenwart bestimmt. Bei Verwendung des Integralprinzips wird die Gegenwart durch die Vergangenheit und die Zukunft betimmt; hierin liegt das teleologische, eine entfernte Erinnerung an das Handeln mit vorbestimmten Zweck. Die Krümmung der Bahn zur Zeit t wird, wenn $t_1 < t < t_2$ ist, abgeleitet aus der Betrachtung der Lagen auf der ganzen Zeitstrecke von t_1 bis t_2; diese Lagen sind natürlich unbekannt, aber sie werden bei dem Ausgang vom Integralprinzip hypothetisch benutzt. Aus der Minimums- oder Extemaleigenschaft der Aufwandsgröße A auf der Strecke von t_1 bis t_2 folgert man, was man braucht, für den Zeitpunkt t.

Hier sieht man einen logischen Unterschied zwischen Newtons klassischer Methode, der Methode der Effizienten nch Leibniz, bei der man aus der Wirkung der bekannten Kräfte alles ableitet, und der Leibnizischen Methode der Finalen, der Endursachen, wie wir sie definiert habe, der Integralprinzipien.

(KnA) p. 55: *„... so dürfen wir endgültig als Beziehung unseres Prinzips der zur Kantischen Urteilskraft feststellen: Das Prinzip der kleinsten Wirkung in seiner modernsten Allgemeinheit ist eine Maxime der reflektierenden Urteilskraft“* (*)

(*) The Einstein field equations can be derived from the Einstein-Hilbert action by using the principle of least action

Kramers H. A.
Bohrs Komplementaritätsbegriff

(KrH) S. 4: *„So haben die Grenzen der Anwendungsmöglichkeit des klassischen Partikelbegriffs eine einfache Formulierung erhalten durch die Heisenbergsche Ungestimmtheitsrelationen, und die Eigenart der heutigen Quantentheorie wird in durchsichtiger Weise durch den von Bohr eingeführten Komplementaritätsbegriff aufgedeckt, nach dem Gesetzmässigkeiten, welche sich auf raumzeitliche Zusammenhänge, und diejenigen, die sich auf Energie- und Impulszusammenhänge beziehen (oder allgemeiner auf den kausalen Zusammenhang der Erscheinungen), sich komplementär zueinander verhalten, d.h. einander ausschließen, soweit es sich um ihre genaue quantitative Feststellbarkeit durch messende Beobachtung handelt. Der Kern dieser neuen Auffassungen liegt vor allem in einer Kritik des Beobachtungsbegriffes, und die früheren Schwierigkeiten hatten besonders ihren Grund darin, daß man sich durch Extrapolation der klassischen Begriffe ein Weltbild zu schaffen suchte, nach dem man widerspruchslos von einem „objektiven“, wirklichen Geschehen in Raum und Zeit reden konnte. Die Diskussion der empirischen physikalischen Gesetze hat uns tatsächlich gelehrt, daß eine solche Extrapolation unerlaubt ist, daß vielmehr eine jedliche Messung mit einer objektiv nicht beschreibbaren, also – wenn man so will – irrationellen Wechselwirkung zwischen dem Beobachtungsapparat und dem Beobachteten behaftet ist*.“

Leedskalnin E.
Magnetic current is the same as electric current

(LeE) p. 31: *A magnetic current is the same as an electric current, those are two currents, which are made up of individual North Pole particles and South Pole particles. One current runs against one another with high velocity in a whirling helical form.*

In order to let a current flow, it must must be necessarily run against the other current.

Leibniz G.-W.
Preestablished harmony

(HiS) p. 22: „*Leibniz developed the idea that our world is organized to be the best of all possible worlds. …. By very much simplifying it, we may describe Leibniz's theory as follows. God does not interfere, like a clumsy clockmaker, from time to time with the affairs of this world, to regulate the hands of His clock. On the contrary, God created his world in preestablished harmony. Like a skilled clockmaker looking after his clockwork, God brought the nature of each single part of His world for all eternity into agreement with the nature of all the others; thus all parts are forever in complete harmony with each other. This alone is worthy of God, the most intelligent and almighty being. …*

Leibniz understood perfectly well that this world, being merely the best selection out of what is possible, may be much worse than what we might hope for. However, popular misunderstandings of Leibniz's views reduced his thoughts to this oversimplification: all that exists is good."

Lorentz H. A.

(SuL) 1.6.2*: „Light speed is caused by the movements of bodies through the ether. Because of various kinds of ether pressures, objects are squeezed and therefore shortened"*

Mach E.
Beziehungen der Mechanik zur Physik

(MaE) S. 519: „1. Rein mechanische Vorgänge gibt es nicht. Wenn Massen gegenseitige Beschleunigungen bestimmen, so scheint dies allerdings ein reiner Bewegungsvorgang zu sein. Allein immer sind mit diesen Bewegungen in Wirklichkeit auch thermische, magnetische und elektrische Änderungen verbunden, und in dem Maße, als diese hervortreten, werden die Bewegungsvorgänge modifiziert. Umgekehrt können auch thermische, magnetische, elektrische und chemische Umstände Bewegungen bestimmen. Rein mechanische Vorgänge sind also Abstraktionen, die absichtlich oder notgedrungen zum Zwecke der leichtern Übersicht vorgenommen werden. Dies gilt auch von den übrigen Klassen der physikalischen Erscheinungen. Jeder Vorgang gehört genau genommen allen Gebieten der Physik an, welche nur durch eine teils konventionelle, teils physiologische, teils historisch begründete Einteilung getrennt sind.

2. Die Anschauung, dass die Mechanik als Grundlage aller übrigen Zweige der Physik betrachtet werden müsse und dass alle physikalischen Vorgänge mechanisch *zu erklären seien, halten wir für ein Vorurteil. Das historisch Ältere muss nicht immer die Grundlage für das Verständnis des später Gefundenen bleiben. In dem Maße, als mehr Tatsachen bekannt und geordnet werden, können auch ganz neue leitende Anschauungen Platz greifen. Wir können jetzt noch gar nicht wissen, welche von den physikalischen Erscheinungen am* tiefsten *gehen, ob nicht die mechanischen gerade die oberflächlichsten sind, ob nicht alle* gleich tief *gehen. Auch in der Mechanik betrachten wir ja nicht mehr das älteste Gesetz, das Hebelgesetz, als die Grundlage aller übrigen".*

(MaE) S. 482: „*Die Vorstellung von der Art, wie die Summe der Bewegung zu rechnen sei, hat sich von Descartes auf Leibniz und später bei den Nachfolgern sehr bedeutend modifiziert, und es ist nach und nach das entstanden, was man heute „Gesetz der Erhaltung der Energie" nennt*."

(MaE) S. 483: „*Er (Lagrange) führt einen Neubau der Mechanik (die ganze Mechanik gründet auf dem Eulerschen Prinzip der kleinsten Wirkung) auf anderen Grundlagen aus, und kein Sachverständiger kann dessen Vorzüge verkennen. Alle späteren bedeutenden Naturforscher haben sich der Auffassung von Lagrange angeschlossen, und damit was im wesentlichen die heutige Stellung der Physik zur Theologie gegeben*."

(UnA1) pp. 62,65, 66*: Mach's hypothesis*

- *the laws of dynamics could depend only on the motion of masses relatively to each other*
- *the laws of nature are independent to accelerated motion.*

The Mach hypothesis is that distant celestial objects must be responsible for masses having gravitational properties. It anticipates Einstein's later comparison of inertial and gravitational mass known as the equivalence principle.

The Mach principle has two different aspects. First, and qualitatively, just as the (Einstein) equivalence of principle, it says that inertia and gravitational mass are mystereriously connected. Secondly, Mach also claimed that inertia (i.e. the resistance to acceleration) must have its origin in the relative acceleration with respect to all other masses in the universe.This meant that the strength of gravity was also determined by every other celestial body – and suddenly we have a quantitiative statement".

Marx W.
Hegels „Phänomenologie des Geistes"

(MaW2) S. 17-18: „Um der Bedeutung des Begriffs bei Hegel gerecht zu werden, darf man aber auch nicht übersehen, was ihn von Kants Verständnis der Subjekt-Objekt-Identität in der transzendentalen Apperzeption trennt. Für Kant war das reine Selbstbewußtsein in seiner einigenden Funktion für die Möglichkeit synthetischer Urteile a priori von Bedeutung, d.h. letztlich für die Grrundlegung der Erfahrungserkenntnis der Naturwissenschaft und ihrer „Objektivität". Das Problem des deutschen Idealismus, vor allem Hegels, war prnizipieller. Die Subjektivität war ihm die Bewegung, die das Ganze des Seins „logifiziert". Sie erhielt damit die Bedeutung, die der Logos für die griechische Philosophie besessen hatte.

Für die Griechen war die Bestimmung des Logos bereits eine Identität von Denken und Sein gedacht, insofern der Logos zugleich die Ordnung selbst und das Wissen der Ordnung besagte. Diese Identität bedeutete für sie somit ein Partizipieren des Denkens an der Ordnung des Kosmos. Demgegenüber ist das neuzeitliche Philosophieren seit Kant das wissende Subjekt der Ursprung der Ordnung der als Begriff formierenden (kategorialen) Gegenständlichkeit der Gegenstände. Diese „Wende" behält Hegel bei. Der Begriff ist für ihn der sich als Subjekt vollziehende Logos, der die Ordnung und Intelligibilität von allem, was ist, konstituiert. Dabei bedeutet „Subjekt" freilich nicht das menschliche oder gar individuelle Erkennen. „Subjekt" ist auch die sich in Formen und Gesetzen der Natur widerspiegelnde Ordnung, es ist auch der ordnende sittlich-objektive Geist sowie der sich in den geordneten Gebilden von Kunst, Religion und Wissenschaften darstellende „absolute Geist". Das Erkennen des Menschen stellt alle diese Ordnungen nicht her, sondern vollzieht sie begreifend nach. Weil diese Logifizierung schon immer geschehen ist, ist das Erkennen kein Formieren eines zuvor Formlosen, wie bei Kant, sondern ein Sich-durchsichtig-Werden der als Logos in alle waltenden Bewegung des Begriffes. Der Begriff gelangt zu einer eigenen vollen Durchsichtigkeit, nachdem er alles ihm „andere" durchdrungen hat. Der Weg dieser zunehmenden Selbstdurchdringung ist die sich im Unterschied des wissenden und wissend handelnden Selbst und seines „Gegenstandes" vollziehenden Bewegung. Die Darstellung eben dieses Weges ist die Phänomenologie des Geistes.

Dies genügt, um deutlich zu machen, dass sich Hegels Metaphysik in die Tradition der Logosphilosophie einfügt, wenngleich sie den Logos in seiner spezifisch neuzeitlichen, maßgeblich von Kant bestimmten Fassung aufnimmt. In diesem Hegel leitenden traditionellem Sinn des Logos liegen aber noch weitere Bestimmungen, die auch in der Phänomenolgie wirksam geworden sind. Der Logos bedeutet traditionell nicht nur die Identität von Denken und Sein – neuzeitlich: von Subjektivität und Objektivität -; er hatte bereits für die Griechen den Sinn einer Ordnung, die – jedenfalls potentiell – total offenbar, für jeden nachvollziehbar sein muß. Der Logos als Gedanke, als Denken, gewährt eben diese Durchsichtigkeit. Der nous *– der „Geist" oder die „Vernunft" – ist das lichtgebende Prinzip, das Denken als* noesis *die dem Menschen gegebene Möglichkeit eines intuitiven, niemals dem Irrtum unterlegenen, ans Licht bringenden Erfassens; der Vollzug des Logos als* dianosia *– der „Verstand" – vollzieht sich als Wissen, das begreift, urteilt, schließt, induziert und deduziert, Definitionen und Wesensbestimmung zu geben vermag. Diese Macht des* nous *und des Logos vollendete sich für die Griechen in der Philosophie, die sich als „Ontologie" verstand, als ein Suchen nach den letzten kategorialen Bestimmungen des Seienden, wie nach derjenigen des höchsten Seienden, des* theos*, insofern Ontologie immer zugleich Theologie war. Als die wichtigste Kategorie in dieser onto-theologischen Ordnung dachte Aristoteles diejenige der* ousia*, der Substanz, die sich in vielerlei Arten artikulierte, insbesondere als* telos*. Es ist von grosser Wichtigkeit an diese Bestimmung des* telos *eigens zu erinnern, weil der Gedanke des „erreichten Ziels" oder des „erfüllten Zweckes", der alles, was ist, vom Anfang her zu sich hinordnet, damit sowohl dem Seienden als*

solchem, wie dem Zusammenhang alles Seienden, dem Kosmos, eine bestimmte Art von „Notwendigkeit" zu verleihen vermochte.

Es wird sich zeigen, daß die Phänomenologie Hegels in all den genannten Hinsichten der Logos-Tradition verpflichtet geblieben ist, nicht nur in dem Gedanken der Identität von Denken und Sein, sondern vor allem darin, daß die Macht des Logos und des nous *totale Durchsichtigkeit gewähren muß. Wenn auch in einer durch die Geschichte der Philosophie weitgeändert veränderten Form bleibt Hegels Auffassung des Wesens der Philosophie onto-theologisch orientiert, und die immer noch kategorial gedachte Ordnung ist auch bei ihm entscheidend bestimmt durch die Kategorie der „Substanz", die freilich selber – neuzeitlich gedacht – sich dem bereits angedeutenen Sinne als „Subjekt" erfaßt. In der Kategorie der Substanz aber sieht auch Hegel die Bestimmung des* telos *und die in der teleoloschen Kreisbewegung liegende „Notwendigkeit", die für ihn das Ganze der Ordnung beherrscht; diese ist freilich keine am Leitbild des* uranos *gedachte „kosmologische", sondern die zum System entfaltete Ordnung des Begriffs.*

Maupertuis P.
The general (least-action) principle of nature

(HiS) p. 20: „*The „metaphysical principle" of Maupertuis is the assumption that nature always operates with the greatest possible economy. For example, in a homogeneous medium, light would take the shortest possible path. From this idea he drew the following conclusion, which he stated as his general principle:*

> If there is some change in nature, the amount of action necessary for this change must be as small as possible.

What is this „action" that nature is supposed to consume so thriftily?

We shall define action as the product of distance, velocity, and mass:

$$Action = Mass \times Distance \times Velocity.$$

Moreover, according to Leibniz, the kinetic energy E is given by the formula

$$E = \frac{1}{2} \times Mass \times (Velocity)^2;$$

So action has the same physical dimension as $Energy \times Time$, because velocity is distance divided by time. … Actually, our preceding reasoning to motivate this definition of action is taken from one of the Leibniz's letters (To Bernoulli, March 1696)."

Mijajlovic Z., et.al.
Regularity varying solutions of Friedman acceleration equation

(MiZ): „*The Friedman acceleration equation together with the fluid equation and the Friedman equation (which are all just Ordinary Differential Equations determines the expansion scale factor $\alpha(t)$ of the Universe. The nature of the solution strongly depends on the sign of the energy density term. In order to explain the expansion of the universe the cosmological constant is added (Einstein's „grösste Eselei"). It is well known that there are significant discrepancies in the prediction of what order should be the value of the cosmological constant. The reason may lay in the course tuned asymptotic description of the scale of the acceleration factor $\alpha(t)$ such as $\alpha(t) = t^{\alpha}$. The theory of regularly varying function provides the means for such an analysis, particularly for solutions of the the Friedmann (accelaration) equation.*

(MiZ): „*The 'standard' model of cosmology is founded on the basis that the expansion rate of the universe is accelerating at present — as was inferred originally from the Hubble diagram of type Ia supernovae. There exists now a much bigger database of supernovae so we can perform rigorous statistical tests to check whether these 'standardisable candles' indeed indicate cosmic acceleration. Taking account of the empirical procedure by which corrections are made to their absolute magnitudes to allow for the varying shape of the light curve and extinction by dust, we find, rather surprisingly, that the data are still quite consistent with a constant rate of expansion*".

Miyamoto K.
Fundamentals of Plasma Physics and Controlled Fusion
Charge neutrality and Landau damping

(MiK) p. 1*: „One of the fundamental property of plasma is the shielding of the electric potential applied to the plasma. When a probe is inserted into a plasma and positive (negative) potential is applied, the probe attracts (repulses) electrons and the plasma tends to shield the electric disturbance.*

(MiK) p. 3: *The other fundamental process of plasma is collective phenomena of charged particles. Waves are associated with coherent motions of charged particles. When the phase velocity v_{ph} of wave or perturbation is much larger than the thermal velocity v_T of charged particles, the wave propagates through the plasma media without damping or amplification. However when the refractive index N of plasma media becomes large and plasma becomes hot, the phase velocity $v_{ph} = c/N$ (c is light velocity) of the wave and the thermal velocity v_T become comparable ($v_{ph} = \frac{c}{N} \sim v_T$), then the exchange of energy between the wave and the thermal energy of plasma is possible. The existence of a damping mechanism of wave was found by L. D. Landau. The process of Landau damping involves a direct wave-particle interaction in collisionless plasma without necessity of randamizing collision. This process is fundamental mechanism in wave heatings of plasma (wave damping) and instabilities (inverse damping of perturbations).*

Müller O. L.
Mehr Licht, Goethe mit Newton im Streit um die Farben

(MüO) S. 9: „Was wäre geschehen, wenn sich Newton und Goethe ans Prisma gestellt hätten, um zusammen zu experimentieren? Diese Frage hält mich seit fünfzehn Jahren auf Trab. Sie hat mein Leben verändert und zu diesem Buch geführt. Selbstverständlich habe ich keine definitive Antwort auf die Frage gefunden, doch die tentative Vermutung, zu der ich gelangt bin, ist beunruhigend genug: Möglicherweise sähe heute unsere Physik komplett anders aus*."*

(MüO) S. 10: „Goethe und Newton waren einander in optischen Angelegenheiten ebenbürtig. Sie hätten sich gegenseitig ernst nehmen müssen, jeder hätte vom anderen lernen können, und das Ergebnis ihres rationalen Gedankenaustauschs zur Optik wäre nicht auszudenken.

Da die naturwissenschaftlich informierte Welt Newton als den rechtmäßigen Gewinner im Streit über das Licht und die Farben ansieht, steckt in meiner These eine Provokation: Nicht nur hätte Goethe von Newton viel lernen können (geschenkt, geschenkt), sondern Newton auch von Goethe – und zwar, wie gesagt, in seinem ureigensten Metier, in der Optik. Goethe hat dort eine faszinierende Symmetrie entdeckt, die Newtons Argusaugen entgangen war und die das gesamten Reich der newtonischen Experimente verdoppelt. Hier in modernen Worten eine erste grobe Fassung dessen, worauf Goethes Entdeckung hinausläuft: Man nehme die Farbfotografie eines beliebigen Experiments von Newton; dann kann man auch das Negativ dieses Fotos als Bild eines Experimentes deuten – und zwar eines Experiments, das wirklich so ausgeht, wie das Negativ zeigt. Jedes Experiment Newtons hat also ein komplementäres Gegenstück (das bei Newton und an unseren Schulen unter den Tisch fällt). Das Gegenstück entsteht aus dem ursprünglichen Experiment durch Umkehrung der Beleuchtung – durch Vertauschung der Rollen von Licht und Dunkelheit. Daher rede ich oft von einer Symmetrie zwischen Helligkeit und Finsternis. Diese Symmetrie ist bis heute nicht recht gewürdigt worden; vermutlich hat man sie noch nicht einmal richtig verstanden. Beides möchte ich mit meinem Buch ändern. Und da gutes Verständnis vor jeder Würdigung kommt, werde ich alles tun, um Ihnen Goethes Entdeckung Schritt für Schritt zu erklären. Irgendwelche besonderen Vorkenntnisse werden Sie für meinen Gedankengang nicht brauchen".

Nagel Th.
Mind & Cosmos

(NaT) p. 14: Antireductionism and the Natural Order, *„We and other creatures with mental lives are organisms, and our mental capacities apparently depend on our physical constitution. So what explains the existence of organisms like us must also explain the existence of mind. … If evolutionary biology is a physical theory – as it is generally taken to be – then it cannot account for the appearance of consciousness and of other phenomena*

that are not physically reducible. *So if mind is a product of biological evolution – if organisms with mental life are not miraculous anomalies but integral part of nature – then biology cannot be a purely physical science*. *The possibility opens up of pervasive conception of the natural order very different from materialism – one that makes mind central, rather than a side effect of physical law".*

(NaT) p. 55: Consciousness, „*The existence of consciousness is both one of the most familar and one of the most astounding things about the world. No conception about natural order than does not reveal it as something to be expected can expire even to the outline of completeness. And if physical science, whatever it may have to say about the origin of life, leaves us necessarily in the dark about consciousness, that shows that it cannot provide the basic form of intelligibility for this world. There must be a very different way in which things as they are make sense, and that includes the physical world is, since the problem cannot be quarantined in the mind."*

(NaT) p. 92: Cognition, „*The teleology I want to consider would be an explanation not only of the appearence of physical organisms but of the development of consciousness and ultimately of reason in those organisms. But its form can be described even if we stay at the physical level. Natural teleology would require two things. First, that the nonteleological and timeless laws of physics - those governing the ultimate elements of the physical universe, whatever they are – are not fully deterministic. Given the physical state of the universe at any moment, the laws of physics would have to leave open a range of alternative successor states, presumably with a probability distribution over them.*

Second, among those possible futures there will be some that are more elegible than others are possible steps on the way to the formation of more complex systems, and ultimately of the kinds of replicating systems characteristic of life. The existence of teleology requires that successor states in this subset have a significantly higher probability that is entailed by the laws of physics alone – simply because they are on the path toward a certain outcome. Teleological laws would assign higher probability to steps on the paths in the state space that have higher „velocity" toward certain outcomes. They would be laws of the self-organization of matter, essentially – or whatever is more basic than matter."

Neuenschander D. E.
Emmy Noether's wonderful theorem
Symmetry, invariance, and conservation laws

(NeD) pp. 1, 4: „*The conservation principles of energy, linear momentum, angular momentum, and electric charge are among the most fundamental principles of physics. … „Conservation" as in „conservation of energy" is not the same as „invariant". They are related, …, but they are not synonymous. The momentum or energy of a system of particles may be conserved but not necessarily invariant. For example, imagine one billiard table, prior to the collision one ball moves and the other sits at rest, and the momentum of the system is nonzero. But in the center-of-mass reference frame the system's total momentum sums to zero because the balls approach one another with opposite momentum. In both frames, the collision is analyzed using conservation of momentum within that frame. The table frame sees nonzero momentum, but the center-of-mass frame sees zero momentum. Momentum is conserved within each frame but is not invariant between these two frames. „Invariant" means that a quantity's numerical value is not altered by a coordinate transformation. „Conserved", in contrast, means that within a given coordinate system the quantity does not change throughout a process. „Invariant" compares a quantity between reference frames. „Conservation" compares the quantity before and after collision or reaction or process within a reference frame. Noether's theorem relates conservation to invariance, and thus to symmetry.*

We will see that conservation of energy, conservation of linear momentum, and conservation of angular momentum are related to invariance under time translations, space translations, and rotations, respectively. These invariances, signify underlying symmetries: the homogeneity of time, the homogeneity of space, and the isotropy of space. The conservation of electric charge emerges from a more abstract symmetry called „gauge invariance". … The invariant quantities in the conservation laws of mechanics and electrodynamics are called „functionals".

(NeD) p. 194*: „there is no continuous infinitesimal transformation for charge conjugation. No states exist that carry charge values in a continuum from the -e electric charge of an electron to the +e of the positron, or between the* $I_z = \pm 1/2$ *isospin eigenvalues. How do we define invariance for discrete symmetries?"*

Nietzsche F.
Nietzsche, Biographie seines Denken
Das Schopenhauer-Erlebnis

(SaR) S. 38: „*In der ein halbes Jahrzehnt später verfaßten Abhandlung über Schopenhauer spricht Nietsche deutlich aus, daß ihm Schopenhauer nicht nur ein Lehrer, sondern vor allem ein* Erzieher *gewesen ist. Den wahren Erzieher definiert er dort als* Befreier *der einer* jungen Seele *dabei hilft, das* Grundgesetz des eigentlichen Selbst *zu entdecken. Der Befreier ist auch ein Erwecker, und wie erweckungsbedürftig und erweckungsbereit der junge Nietsche zum Zeitpunkt seiner ersten Begegnung mit dem Werk Schopenhauers gewesen ist, schildert er 1872 in dem fünften seiner Vorträge „Über die Zukunft unserer Bildungsanstalten". Der Student, so resümiert er in eigener Erfahrung, lebt scheinbar frei und unabhängig und kommt sich vor wie in einem Traum, wenn man glaubt fliegen zu können, aber doch durch unerklärliche Hindernisse sich zurückgezogen fühlt. Er merkt,* daß er sich selbst nicht führen, sich selbst nicht helfen kann*. Zwar wachsen in ihm stolze und edle Entschlüsse, aber es fehlt ihnen die Duchsetzungskraft. So taucht er sich* hoffnungsarm in die Welt der Tagesarbeit*, wovor es ihm nach einer kleinen Weile graut: er will nicht so früh in* enger kleinlicher Fachmäßigkeit *versinken. Aber dies müßte sein Schicksal sein, wenn es an dem* Mangel eines Führers zur Bildung *bliebe. Für Nietsche war Schopenhauer ein solcher* Führer*, von dem jene Wirkung ausging, die er von einem* wahren Philosophen *erwartete, nämlich, daß man ihm* gehorchen könnte, weil man ihm mehr vertrauen würde als sich selbst*. Solches Vertrauen muß nicht die Zustimmung zu den Lehren im einzelnen bedeuten. Die persönliche Glaubwürdigkeit ist ihm wichtiger als der Sachgehalt der Lehre. Deshalb bliebt das Vertrauen zu Schopenhauer auch erhalten, nachdem sich bei einer zweiten, kritischen Lektüre einige Zweifel und Einwände ergeben haben.*

Diese zweite Lektüre war beeinflußt durch ein anderes großes Leseerlebnis dieser Jahre: Friedrich Albert Langes „Geschichte des Materialismus", ein damals wirkungsmächtiger Versuch, materialistisches und idealistisches Denken miteinander zu verbinden. Durch Lange hatte Nietsche die Erkenntiskritik Kants, den antiken und den modernen Materialismus, den Darwinismus und die Grundzüge der neueren Naturwissenschaften kennengelernt, und mit einer geschärften Aufmerksamkeit entdeckte er nun einige theoretische Bruchstellen in Schopenhauers System. Man dürfte, so notierte er, vom unerkennbaren „Ding an sich" keine Aussagen machen, auch nicht die, daß alle Prädikate der erscheinenden Welt – wie Raum, Zeit, Kausalität – diesem „Ding an sich" entzogen werden müßten. Das Unerkennbare darf nicht zum Negativbild des Erkennbaren umgedeutet werden, denn auch mit der Logik des Gegensatzes werden Bestimmungen der erkennbaren Welt fälschlich ins Unbestimmbare hineingetragen. Schon gar nicht dürfe man das „Ding an sich" als Wille interpretieren, was eine viel zu bestimmte Aussage über das unbestimmbare Wesen der Welt sei. Daß der „Wille" eine elementare, vielleicht sogar die primäre Lebensmacht ist, das leuchtet ihm zwar ein, aber er kritisiert, daß man den „Willen" jenen kategorialen Ort einnehmen läßt, den Kant für das „Ding an sich" freigehalten hat."

(RuB1) p. 728: *„Nietzsche (1844-1900) regarded himself, rightly, as the successor of Schopenhauer, to whom, however, he is superior in many ways, particularly in the consistency and coherence of his doctrine. Schopenhauer's oriental ethics of reunciation seems out of harmony with his metaphysics of the omnipotence of will; in Nietzsche, the will has ethical as well as metaphysical primacy."*

(RuB1) p. 734: *„Nevertheless there is a great deal in him that must dismissed as merely megalomaniac. Speaking of Spinoza he says: „How much of personal timidity and vulnerability does this masquerade of a sickly recluse betray!" Exactly the same may be said of him, with the less reluctance since he has not hesitated to say it of Spinoza. It is obvious that in his day-dreams he is a warrior, not a professor; all the men he admires were military. His opinion of woman, like every man's, is an objectification of his own emotion of women towards to them, which is obviously one of fear. „Forget not the whip" – but nine women out of ten would get the whip away from him, and he knew it, so he kept away from women, and soothed his wounded vanity with unkind remarks."*

Nussbaumer I.
Zur Farbenlehre, Entdeckung der unordentlichen Spektren

(NuI) S. 69/70: „Für Goethe ist nicht der Grad der Ablenkung entscheidend für die Erscheinungen der Farben, den man primär der optischen Dichte zuschreibt, sondern insbesondere ein Faktor, den er die Trübe in einem durchsichtigen Medium nennt. Inwieweit dieser Faktor mit dem Begriff der optischen Dichte zusammenfällt oder sich diesem subordiniert, bleibe hier offen. Sofern die optische Dichte auch das Maß für die Abschwächung des Lichts ist, das dieses im Durchschnitt durch die Materie erfährt, könnte hier durchaus eine Verbindung angenommen werden.

Goethe spricht hier von einer möglichen dritten und der Refraktion und Reflexion bloß verwandten Kraft, mit anderen Worten: von einem noch nicht entdeckten oder noch nicht näher eingegrenzten Wirkungsfaktor. Inwieweit sich hier Goethe den Faktor der Trübe verantwortlich denkt, bleibt jedoch ungeklärt.

Der Faktor der Trübe wohnt seiner Ansicht nach aber allen durchsichtigen Mitteln bei, wie es brechende Substanzen sind. Der Wirkungsfaktor der Trübe tritt in Kraft, wenn Licht durch ein Medium fällt und an diesem gebrochen wird. Er stellt sich dem einfallenden und abgelenkten Licht entgegen und entfaltet daraus eine bestimmte Wirkung. Eben dadurch erst werden nach Goethe die Farben am Licht erregt. Den Nachweis, daß der Faktor der Trübe in durchsichtigen Mitteln gegeben ist, erbringt er auf folgende Art:

> „Physische Farben nennen wir diejenigen, zu deren Hervorbringung gewisse materielle Mittel nötig sind, welche aber selbst keine Farben haben und teils durchsichtig, teil trüb und durchscheinend, teils völlig undurchsichtig sein können."

Die nähere Eingrenzung der physischen Farben führt Goethe zu den dioptrischen Farben, von denen er sagt:

> Man nennt dioptrische Farben diejenigen, zu deren Entstehung ein farbloses Mittel gefordert wird, dergestalt daß Licht und Finsternis hindurchwirken, entweder aufs Auge oder auf entgegenstehende Flächen. Es wird aber gefordert, daß das Mittel durchsichtig oder wenigstens bis auf einen gewissen Grad durchscheinend sei".

Nach diesen Bedingungen teilt Goethe die dioptrischen Farben in zwei Klassen, in solche, bei durchscheinenden trüben Mitteln und in solche, die bei durchsichtigen Mitteln entstehen.

…. In Hinblick auf durchsichtige Mittel, wie es eben ein ungetrübtes Glas ist, schreibt nun Goethe:

> *Da aber jedes empirisch Durchsichtige an sich schon als trüb angesehen werden kann, wie uns jede vermehrte Masse eines durchsichtig genannten Mittels zeigt, so ist die nahe Verwandtschaft beider Arten genugsam einleuchtend" Oder „Das Durchsichtige selbst, empirisch betrachtet, ist schon der erste Grad des Trüben. Die ferneren Grade des Trüben bis zum undurchsichtigen Weißen sind unendlich."*

In diesem Sinne wohnt nach Goethe jedem durchsichtigen Mittel eine gewisse, obgleich kaum merkliche Trübe inne. Dies zum goetheschen Nachweis des Faktors der Trübe in durchsichtigen Mitteln, wie es auch ein Prisma ist.

(NuI) S. 85/86: „Bei Goethe tritt an die Stelle einer Stahlentheorie des Lichtes so etwas wie eine Feldtheorie des Lichtes. Stets ist es ein Lichtfeld oder – weiter ausgeholt – ein Lichtkörper und kein Lichtstrahl, den Goethe im Auge hat. Newton fasst das Licht im wesentlichen aus Lichtstrahlen, das heißt: aus kleinsten Teilstücken bestehend auf. Wie in der geometrischen Optik, denkt sich Newton eben das Licht aus Lichtstrahlen zusammengesetzt. In der phänomenologischen Optik Goethes spielt ein solcher Gedanke überhaupt keine Rolle. Unter einem Strahl läßt sich höchstens die (mehr oder weniger scharfe) Begrenzung eines Feldes denken, mit anderen Worten: eine Erscheinung, die sich aus der natürlichen Begrenzung eines Phänomens ergibt. Der Strahl im goetheschen Verständnis ist eine sichtbare Grenze, die auf bestimmte Art verläuft und ihren empirisch typischen Verlauf nimmt. Desgleichen ist auch die Farbe stets ein begrenztes Feld."

Penrose R.
The emperor's new mind

(PeR) p. 444:

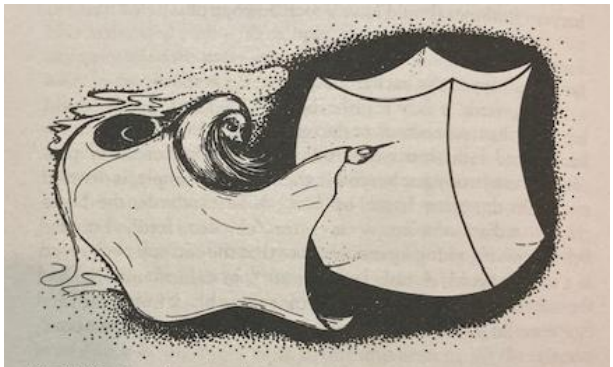


Fig. 7.19. In order to produce a universe resembling the one in which we live, the Creator would have to aim for an absurdly tiny volume of the phase space of possible universes – about $1/10^{10^{123}}$ of the entire volume, for the situation under consideration. (The pin, and the spot aimed for, are not drawn to scale!)

444

How do nerve signals works?

(PeR) p. 506 *„When a signal reaches a synaptic knob, it emits a chemical substance known as a neurotransmitter. This substance travels across the synaptic cleft to another neuro – either at a point on one of its dendrites or on the soma itself. Now some neurons have synaptic knobs which emit a neurotransmitter chemical with a tendency to encourage th:e soma of the next neuron to „fire", i.e. to initiate a new signal out along its axon. These synapses are called excitatory. Others tend to discourage the next neuron from firing and are called inhibitory. The total effect of the excitatory synapses which are active at any moment is added up, and the total of the active inhibitory ones substracted from this, and if the net result reaches a certain critical threshold, the next neuron is indeed induced to fire. (The excitatory ones cause positive electrical potential difference between the inside and the outside of the next neuron and the inhibitory ones cause a negative potential difference. These potential differences add up appropriately. The neuron will fire when this potential differences reaches a critical level on the attached axon, so that the potassium can't got out fast enough to restore equilibrium) ."*

Penrose R.
The road to reality
Dirac's route to the positron

(PeR4) p. 622: *„Not only is the electron's charged-particle behaviour correctly described; in addition Dirac's electron responds in accordance with its possessing a* magnetic moment *of very little specific amout, namely* $\left(\frac{h}{2\pi}\right) e/(4\mu c)$*, where* $-e$ *is the electron's charge and* μ *is its mass*."
(PeR4) p. 623: *„There is a strong physical need for the electron's two spin states. Indeed, the very subject of chemistry, as we know it, depends upon this. In an atom, the electrons sourrounding the nucleus are constrained to orbit the nucleus in particular states known as „orbitals". By Pauli's exclusion principle, it would seem that each electron orbital can be occupied by no more than* one *electron, yet we find that a second electron is always allowed in each of the orbitals. The pair of them can coexist and still satisfy the exclusion principle because their states are* not *identical but have opposite spins. There can be no more than two electrons in any one orbital, however, because there are only two independent spin states for the electron. The chemical notion of „covalent bond" depends upon the same phenomenon, two shared electrons seeming to coexist in the same state, because their spins are opposite*."

Penrose R.
The road to reality
The electroweak symmetry group

(PeR4) p. 641: In the standard model the weak and the electromagnetic interactions are unified in what is called electroweak theory, where there is a special symmetry related to W^+, W^-, Z^0, and the photon γ, according to the groups $SU(2) \times U(1)$ or, more correctly, $U(2)$, (*).

(*) (PeR4) p. 654: The group might be expressed as $SU(2) \times U(1)/Z_2$, where the $'/Z_2'$ means „factor out by a Z_2 subgroup". However, there is more than one such subgroup, so this notation is not fully explicit. The notation $'U(2)'$ automatically picks out the correct one. (I am grateful to Florence Tsou for this observation.) It seems that the reason that the electroweak symmetry group is not conventionally referred to as $'U(2)'$ is that this does not easily extend to the symmetry of the full standard model, which also incorporates the strong symmetry group $SU(3)$, the full group being a version $SU(3) \times SU(2) \times U(1)/Z_6$.

Peskin M.
The Parton Model of Hadron Structure

(PeM) p. 473: „*Which particular quantum field theories describe the interactions of elementary paricles?*

Since the mid-1970s, most high-energy physicists have agreed that the elementary particles that make up matter are a set of fermions, interacting primarily through the exchange of vector bosons. The elementary fermions include the leptons (the electron, its heavy counterparts μ and τ, and n neutral, almost massless neutrino corresponding to each of these species), and the quarks, whose bound states form the particles with nuclear interactions, mesons and baryons (collectively called hadrons). These fermions interact through three forces: the strong, the weak, and the electromagnetic interactions. Of these, the strong interaction is responsible for nuclear binding and the interactions of constituents of nuclei, while the weak interaction is responsible for the radioactive beta decay processes. The electromagnetic interaction is the familar Quantum Electrodynamics, coupled minimally to all charged quarks and leptons. It is not clear that these three forces suffice to explain the most subtle properties of the elementary fermions, but these three forces are certainly the most prominent. All three are now understood to be mediated by the exchange of vector bosons."

(PeM) p. 474 ff: *„How can a model of noninteracting quarks represent the behavior of a force that, under other circumstances, is extremely strong?*

In fact, there are many circumstances in the study of the strong interaction at high energy in which this force has unexpectedly weak effect. Historically, the first of these appeared in proton-proton collisions. At high energy, above 10GeV or so in the center of mass, collisions of protons (or any other hadrons) product large number of pions. One might have imagined that these pions would fill all of the allowed phase space, but, in fact, they are mainly produed with momenta almost collinear with the collision axis. The probability of producing a pion with a large component of momentum transverse to the collision axis falls off exponentially in the value of this transverse momentum, suppressing the production substancially for transverse momenta greater than a few hundered MeV.

This phenomenon of limited transverse momentum led to a picture of a hadron as a loosly bound assemblage of many components. In this picture, a proton struck by another proton would be torn into a cloud of pieces. These pieces would have momenta roughly collinear with the original momentum of the proton and would eventually reform into hadrons moving along the collision axis. By hypothesis, these pieces could not absorb a large momentum transfer. We can characterize this hypothesis mathematically as follows: In a high-energy collison, the momenta of the two initial hadrons are almost lightlike. The scattered pieces of the hadrons, arrayed along the collision axis, also have lightlike momenta parallel to the original momentum vectors. This final state can be produced by exchanging momenta q among other pieces in such a way that, though the components of q might be large, the invariant q^2 is always small. The ejection of a hadron at large transverse momentum would require large (spacelike) q^2, but such a process was very rare. Thus it was hypothesized that hadrons were loose clouds of constituents., like jelly, which could not absorb a large q^2."

(PlP) p. 242: „The most general concept under which the objects of external nature fall is „matter". … In order to establish the metaphysical foundations of natural science proper, one must be primarily concerned with the a priori aspects of matter, i.e., its „form", its necessary and universal determinations and laws that govern (predictively describe) its behavior. This corresponds to „nature" in its formal meaning," which Kant defines as „the first inner principle of everything that belongs to the existence of a „thing". Science is dependent on this senses of „nature", he argued, because „nature" necessarily involves the concept of laws and hence the necessarity of all the determinations that belong to its existence, and therefore natural science must take must take its legitimacy from its pure part, which consists of the a priori principles that constitute (underlie) those determinations. Thus it is a formal sense of „nature" that must be the primary subject matter .. and must serve as the basis of analysis for establishing the a priori foundations of natural science. … the „empirical concept of matter" under which all objects of the external senses fall (and from which metaphysical construction starts) must be determined a priori (i.e., formally) in its content and is „empirical" only in regard to the proof of its objective reality"", (PlP) pp. 29/30, appendix.

„ Therefore the principles of natural science cannot derive from mathematics alone since it does not in itself deal with existence but only with the possibility of things. While mathematics is also crucial for the a priori foundation of physics, it is not sufficient; only metaphysics deals with the question of existence. Therefore, a metaphysical foundation of natural science that simultaneously includes the basis for the application of mathematics to nature is necessary to assure that mathematics can be applied to what belongs to the existence of natural things – and not just their possibility. The mathematizability of nature (which Galileo and Newton simply presupposed in order to ground physics mathematically) is itself dependent on a metaphysics of nature. If the mathematizability of nature is simply hypothesized and left unexamined, the result is to fall back onto uncritical metaphysical assumptions that fail to deal with the underlying problems and hence leave one's position open to the kind of skeptical undermining disclosed by Hume", (PlP) p. 92, appendix.

3.0.4. Heuristic Preliminary Consideration: „Suppose we want to ground the physics of the electron. To do this, one must undoubtedly use mathematics somehow. One is declined to say: to construct the concept of the electron. Now, what can that mean? What lies in the concept of the electron that one could present in pure intuition? Which schemata (directions for construction; A 718, B 746) could derive from the synthesis contained in the concept (A 220, B 267)? Of what kind of synthesis of intuition does one have to think when one wants to think „electron"? It's size? It's form? About these we know almost nothing, and perhaps we cannot even give a clear meaning to questions about them. There is, however, a mathematical physics of the electron, isn't there? Then, on what is such a physics based? Now, one sees that nothing can be undertaken if one stops with the mere concept of the electron as such. One must take as a point of departure how the electron appears, how it manifests itself as such. That is to say, one must investigate what the criteria are for judging whether a given appearance is the appearance of an electron. That is to say, one must know exactly what necessarily belongs to the self-manifestation of the existence of the electron as such. And there one immediately remembers that it is indeed necessary to start from its nature. The nature of an electron is the first inner principle of everything that belongs to the existence of an electron. Existence means the temporal determination of the apprearance according to the Analogies: causality, substance: action, force (A 204, B 249f). The electron is determinable in regard to its existence as something in an appearance that on its part, e.g., causality determines changes by means of its specific, distinguishing, definite (electrostatic) force and its mass (reaction). And these concepts that I have to combine a priori with its existence (not with the mere concept of the electron as noumenon), these one can indeed construct and can do so by means of the concept of motion that underlies them all and that offers the key for this construction. One is even obliged to construct them because the „dynamic principles" (the Analogies and Postulates), from which all this flows in the final analysis, are merely regulative in respect to perceptions, i.e., they offer only an a priori rule to look for perceptions, which must themselves be given, however. That such perceptions are possible at all must necessarily be proved by providing the intuition a priori that corresponds to each concept in question. By means of such intuitions one demenstrates that at least the condition for the perception's being given is fulfilled a priori.

But how does one find what necessarily belongs to the existence of an electron? What is given is only the empirical concept of an electron. Obviously one must designate a path from here, by way of which one can develop a priori and synthetically the entire complex of concepts of what belongs to the nature of the electron. For this one needs to know what belongs to existence in general, and from that one must deduce what, as a

consequence, necessarily belongs to the existence of the special object that is characterized by a given concept. Through „application" of the concept of the nature in general to the empirical concept of the electron, the concept of nature of the electron is generated, and from it one must then analytically develop the required determinations. Thus, from the mere concept of the existence of this object – insofar as its empirical concept is given – without taking anything more from experience, one gains a rich system of entirely pure physics about the electron. It is this system „which makes the concept" of that object „fit a priori for application to … experience" (S xiii, p. 10). It is not easy to see how this pure part can then be the basis of the entire physics of the electron and – in spite of the necessary empirical principles in it – can give apodeictic certainty. We return to this question later", (PlP) p. 242.

(PlP) p. 247*: 3.1.1.3. Reality and Knowledge*
„A representation (an intuiton, a ceoncpet, a judgement) can only be knowledge of things in the proper sense if it has objective reality*, and that means* that it can be related to an object in the narrower sense*."*

(PlP) p. 253: *„The property of a concept of a thing indicating that the real possibility that belongs to the concept exists is called the objective reality of the concept."*

(PlP) p. 253: *„The proof of objective reality is to be conducted in different ways according to the concept in question. From this perspective one must distinguish between the following kinds of concepts:*

1. *Categories as original pure concepts of understanding (transcendentalia), which are given a priori.*
2. *Concepts (intentionally) produced (gemacht) in regards to their content. Among these:*
   (a) produced concepts that contain nothing but a synthesis of pure intuition, thus determinations of space and time.
   (b) produced concepts that are produced out of the material of empirically given features, thus by the voluntary connecting of features that (as these partial concepts) are formed from corresponding intuitions by abstraction, etc.
3. *Empirical concepts that manifest a combination of features that are gained by abstraction, etc., from intuitions that contain the features in just this combination, thus are given a posteriori.*

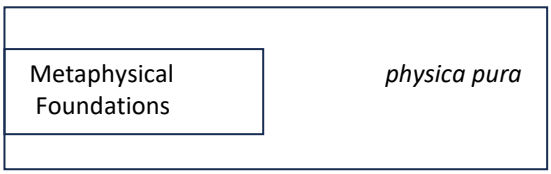
For group (1) it is clear that the objective reality of these concepts presents no problem. Their origin in experience simultaneously assures the possibility of their objects since even their actuality is not in question (comp. B xxvi footnote). This is dealt with here therefore only for delimitation. To be sure, the concept „empirical concept" must later still be subjected to a careful critique (comp. No. 4. below).

The groups (a) and (b) are the actually problematic ones because here the concept is available but not the object, and one therefore must start the concept in order to demonstrate the possibility of the object. If in addition the possibility is to be proved a priori (and not derived from actuality as for group (c)), then it is clear that this may not be accomplished by producing the object (e.g., in an experiment) on the basis of the concept in order to prove the possibility in this way."

The „Pure Part" Within The Doctrine of Nature
(PlP) p. 314-315

„The pure part has a structure that can be indicated as follows:

Pure Part (*physica generalis*)



Here only the actual core, i.e., the metaphysics of material nature itself, is of finite scope, while the pure part (the physica pura) is in principle infinite.

Thus the pure part contains nothing but pure apodeictic knowledge, whose objective reality is also guaranteed in the mathematical part by the metaphysics that serves as its basis – sometimes that does not hold for all

concepts of pure mathematics. For „all mathematical concepts are not in themselves knowledge; except insofar as one presupposes that there are things that allow being presented to us only in accord with the form of that pure sensible intuition". This presupposition, however, is given to mathematics in this application in the physica generalis precisely by the core of the pure part, i.e., the Metaphysical Foundation, and this, as already noted, is the proper content of the assertion, „that the art of measurement (Meßkunst, i.e., applied mathematics) and philosophy are two completely different things, although in natural science they do indeed shake hands with each other".

Kant make the application of mathematics into the criterion for how much „science proper" is contained in a special doctrine of nature. From this, one sees that the „application of mathematics" is not thought of as restricted to the pure part but rather by means of the pure part obviously extends to the entire empirical, but apodeictic (proper) science."

Planck M.
The dynamical and the statistical type of law

(PlM) S. 90: *„… Auch die Physik hat, wie schon lange vorher die sozialen Wissenschaften, die hohe Bedeutung einer von der rein kausalen gänzlich verschiedenen Betrachtungsweise kennengelernt und hat dieselbe seit etwa der Mitte des vorigen Jahrhunderts mit immer steigendem Erfolge angewendet; es ist dies die statistische Methode, mit deren Ausbildung die ganze neuere Entwicklung der theoretischen Physik aufs engste zusammenhängt. Statt den zur Zeit noch völlig im Dunkeln liegenden dynamischen Gesetzen eines Einzelvorganges ohne eine Aussicht auf greifbaren Erfolg nachzuforschen, werden zunächst einmal nur die an einer großen Zahl von Einzelvorgangen einer bestimmten Art gemachten Beobachtungen zusammengestellt und aus ihnen Durchschnitts- oder Mittelwerte gebildet. Für diese Mittelwerte ergeben sich dann je nach den besonderen Umstanden des Falles gewisse erfahrungsmäßige Regeln, und die so gewonnenen Regeln gestatten, allerdings niemals mit absoluter Sicherheit, aber doch mit einer Wahrscheinlichkeit, die sehr häufig der Gewißheit praktisch gleichkommt, den Ablauf auch zukünftiger Vorgänge im voraus anzugeben, zwar nicht in allen Einzelheiten, wohl aber - und darauf kommt es bei den Anwendungen oft gerade am meisten an - in ihrem durchschnittlichen Verlauf …*
… Immerhin erhellt aus der geschilderten Sachlage wohl hinreichend deutlich die überaus hohe Bedeutung, welche die Durchführung einer sorgfältigen und grundsatzlichen Trennung der beiden besprochenen Arten von Gesetzmaßigkeit: der dynamischen, streng kausalen, und der lediglich statistischen, für das Verständnis des eigentlichen Wesens jeglicher naturwissenschaftlichen Erkenntnis besitzt".

Plemelj J.
Potentialtheoretischen Untersuchungen

§ 4 Über eine Erweiterung der Gültigkeit der Greenschen Sätze

Die Greenschen Sätze sind an einige einschränkenden Voraussetzungen gebunden, die bei mancher Untersuchung sich als sehr störend erweisen. Es war zunächst die Voraussetzung der Stetigkeit und eindeutigen Bestimmbarkeit der Normalenrichtung, wodurch erst $\frac{\partial V}{\partial n}$ *eine klare Bedeutung gewinnt.* Der Herleitung liegt die Annahme zugrunde, daß das Potential mit seinen ersten Ableitungen endlich und stetig ist bis zum Rande, wo ein stetiger Anschluß an stetige Werte am Rande vorhanden ist. *Nun zeigt aber eine genauere, im übrigen ganz einfache Überlegung bei der Herleitung der Greenschen Formeln, daß sie ungestört auch dann noch weiter bestehen, wenn die Normale in einzelnen Punkten einen Sprung ihrer Richtung erleidet, wofern es nur sicher ist, daß die ersten Ableitungen der Potentiale in der Umgebung solcher Punkte nirgends über endliche Werte hinausgehen. In den Formeln kommen aber von den ersten Ableitungen nur jene vor, die der normalen Richtung am Rande entspricht. Es liegt nun nahe, zu vermuten, daß die Existenz und Stetigkeit dieser einzigen Ableitung nebst der Stetigkeit des Potentials selbst am Rande zur Gültigkeit der Greenschen Formeln ausreicht. In der Tat läßt sich die Gültigkeit derselben leicht einsehen, wenn man über die Form der Berandung die sehr einschränkende Annahme macht, daß nebst der durchgängigen Stetigkeit der Normalen, auch überall ein Krümmungskreis vorhanden ist, der zwar nicht an die Vorausetzung der Stetigkeit gebunden ist, doch aber überall der Krümmungsradius über einer endlichen von Null verschiedenen Größe* a *liegt. Da zwei Nachbarnormalen sich erst im Krümmungsmittelpunkt treffen, so wird die Gesamtheit der Punkte, deren*

Distanz vom Rande (in der Normalen) überall dieselbe Größe $\lambda < a$ ist, eine Parallelkurve ergeben, die sich nirgends durchsetzt, und deren Punkte den Punkten der gegebenen Randkurve eindeutig zugeordnet sind so, daß entsprechende Punkte eine gemeinsame Normale besitzen, (PlP) p. 9.

§ 5 Der Begriff der Masse und der Strömung

Nach Gleichung (6) $\int \frac{\partial U}{\partial n}(s) = 0$ ergibt sich bei der Erstreckung des Integrals über die Berandung eines geschlossenen Feldes, in dessen Innern und auf dem Rande das Potential stetig ist, und auch die normalen Ableitungen $\frac{\partial U}{\partial n}$ einen stetigen Verlauf haben. Versucht man aber die Gleichung (6) auf unendliche Felder und auf solche Funktionen auszudehen, die bei hinreichend großer Entfernung R eine Gestalt

$$U = mlog\left(\frac{1}{r}\right) + u,$$

mit im Unendlichen regulärem u haben, so ergibt sich das Integral $\int \frac{\partial U}{\partial n}(s)$, nur über den im Endlichen liegenden Rand genommen, nicht mehr Null. Man kann nun, wie oben, nur jenen Teil des Regularitätsgebietes betrachten, der innerhalb eines sehr grossen Kreises vom Radius R liegt und hat dann den Beitrag zum Integral $\int \frac{\partial U}{\partial n}(s)$ zu untersuchen, der aus der Integration über den grossen Kreis sich ergibt. Im Grenzfall $lim\, R = \infty$ bekommt man aber dafür $2\pi m$ (beim Newton potential würde 4π statt 2π stehen). Man hat also für unendliche Felder

(11) $$m = -\frac{1}{2\pi}\int \frac{\partial U}{\partial n}(s)ds,$$

wo jetzt die Integration nur über die im Endlichen liegende Berandung zu erstrecken ist.

Besteht die Berandung aus mehreren getrennten Kurvenzügen, so wird man zweckmäßig jenen Teil des Integrales (11), der einem für sich geschlossenem Stück der Berandung entspricht, als die Mase auf diesem Kurvenzuge bezeichnen. *So ergibt sich die Gesamtmasse m als Summe der Massen auf den einzelnen geschlossenen Kurvenzügen. Bei einem im Unendlichen regulären Potential ist die Summe der Massen gleich Null, sie brauchen aber einzeln nicht zu verschwinden. Das Verschwinden der Masse auf jedem einzelnen geschlossenen Kurvenzug ist eine sehr ausgezeichnete Eigenschaft, weshalb solche Potentiale in der Potentitaltheorie eine hervorragende Rolle einnehmen.*

Vom Integral $\int \frac{\partial U}{\partial n}(s)ds$ auf einer nichtgeschlossenen Kurve ergibt sich aus der Gleichung (6) eine Eigenschaft von großer Wichtigkeit. Das Integral hängt nämlich nur von den Endpunkten ab und nicht von der näheren Form der sie verbindenden Integrationskurve in der Weise, daß die Integrale alle einander gleich sind, welche Integrationswegen entsprechen, die durch stetige Deformation im Regularitätsgebiete auseinander hervorgehen. Sind also und zwei Punkte im Regularitätsgebiete und verbindet man sie durch irgend eine Kurve (die Tangenten hat), so ist $\int_p^q \frac{\partial U}{\partial n}(s)ds$ (p und q gehören nicht zur Berandung) wohl definiert und hat einen von der näheren Form der Kurve nicht abhängigen Wert. Dies erkennt man, wenn man durch p und q eine geschlossene Kurve legt und auf diese die Gleichung (6) anwendet (beim Newtonschen Potential ist die volle Analogie vorhanden).

Das Integral zwischen den Punkten p und q

$$\overline{U}(q) = -\int_p^q \frac{\partial U}{\partial n}(s)ds$$

ist, weil von der Kurve unabhängig, eine wohldefinierte Funktion der Grenzen p und q und soll in seiner Abhängigkeit von q mit $\overline{U}(q)$ bezeichnet werden. Diese Funktion ist stetig bei einer stetigen Änderung der oberen Grenze q. Läßt man nun den Punkt q aus dem Innern des Regularitätsgebietes gegen einen Randpunkt σ rücken, so wird sich der Integralwert stetig ändern. Nun kann es vorkommen, daß trotz der Nichtexistenz eines bestimmten Wertes für die Ableitungen von q doch das Integral $\int_p^q \frac{\partial U}{\partial n}(s)ds$ gegen einen ganz bestimmten Grenzwert strebt, der dann mit $\overline{U}(\sigma) = -\int_p^\sigma \frac{\partial U}{\partial n}d\sigma$ zu bezeichnen wäre. Ganz analoges gilt vom Einrücken des zweiten Punkten gegen einen Randpunkt σ_0. So erklärt sich die Bedeutung des Integrals

$$\bar{U}(\sigma) = -\int_{\sigma_0}^{\sigma} \frac{\partial U}{\partial n} ds$$

zwischen zwei Randpunkten selbst dann, wenn es längs der Berandung nicht zu bilden ist wegen Nichtexistenz von $\frac{dU}{dn}$*. Wenn das Integral* $\bar{U}(\sigma)$ *bei stetiger Verlegung des* σ *eine stetige Änderung erfährt, so wird infinitesimalen Stücken* $d\sigma$ *der Berandung auch eine infinitesimale Änderung von* $\bar{U}(\sigma)$ *entsprechen, die wir mit* $d\bar{U}(\sigma)$ *bezeichnen können. Der Zusammenhang mit den normalen Ableitungen ist jetzt leicht herzustellen. Ist nämlich die Größe* $\bar{U}(\sigma)$ *längst des Randes differenzierbar, d.h. besteht* $\frac{d\bar{U}(\sigma)}{d\sigma}$*, so ist dieser Differentialquotient gleich* $-\frac{dU}{dn}$*. Wir haben also im neuen Begriff tatsächlich einen Ersatz für die normalen Ableitungen, aber von einer weit größeren Leistungsfähigkeit. Es kann nämlich* $d\bar{U}(\sigma)$ *einen guten Sinn haben, ohne dass ein solcher für* $\frac{d\bar{U}(\sigma)}{d\sigma}$ *besteht.* Es handelt sich hier um eine Verallgemeinerung, wie es die Erweiterung differentiierbarer Funktionen auf die stetigen ist*. Die größere Tragweite des neuen Begriffes ist nicht schwer zu erkennen, denn jetzt ist man unabhängig sowohl von der Existenz der Normalen als auch der Ableitungen am Rande (selbst die Annahme der Rektifizierbarkeit der Randkurve ist unwesentlich). Man wird also z.B. eine Verallgemeinerung der Aufgabe, normale Ableitungen eines Potentials am Rande vorzugeben, darin haben, jedem Element* $d\sigma$ *des Randes den ihm entsprechenden Betrag* $d\bar{U}(\sigma)$ *vorzuschreiben.*

Dieser neue Begriff hat in der physikalischen Anwendung bei der Strömung der Elektrizität bereits den Namen „Stärke des Stromes*“ der durch dieses Stück der Berandung fließt. Ich will also in der Folge, mangels einer von der Anwendung freien Benennung die Bezeichung* „Strom*“ oder* „Strömung*“ auf dem Randstücke gebrauchen, (PlJ), pp. 10-12.*

§ 7 Die Lage der Extremwerte

Die Gleichung

$$U(p) = \frac{1}{2\pi}\int U(s) \cdot \frac{\partial log\frac{1}{r_{ps}}}{\partial n_s} ds - \frac{1}{2\pi}\int log\frac{1}{r_{ps}} \cdot \frac{\partial U}{\partial n_s} ds$$

gestattet in einfacher Weise den Wert eines Potentials im Mittelpunkt eines Kreises durch die Peripheriewerte des Potentials darzustellen. Es ergibt sich der Mittelpunktwert

$$U_o = \frac{1}{2\pi r}\int U(s) \cdot ds$$

Darin ist der Satz von Gauß enthalten:

Der Wert eines Potentials im Mittelpunkt eines Kreises ist ein Mittelwert der Peripheriewerte, (PlP) p. 15.

§ 8 Die Potentiale der einfachen und der doppelten Schicht

Bisher war es üblich für das Potential der einfachen Schicht *oder* Belegung *die Form*

$$V(p) = \int log\frac{1}{r_{ps}}\mu'(s)ds$$

vorauszusetzen, wobei dann $\mu'(s)$ *die* Massendichte *der Belegung genannt wurde.* Eine solche Annahme erweist sich aber als eine derart folgenschwere Einschränkung, daß dadurch dem Potential der gößte Teil seiner Leistungsfähigkeit hinweg genommen wird. *Für tieferliegende Untersuchungen erweist sich das Potential* V *nur in der Form*

$$V(p) = \int log\frac{1}{r_{ps}} \cdot d\mu_s$$

verwendbar. Die infinitesimale Größe $d\mu_s$ *kann man dann das* Massenelement *nennen, von Massendichtigkeiten wird man aber nur dann sprechen können, wenn der Differentialquotient* $\frac{d\mu_s}{ds} = \mu'(s)$ *besteht, (PlJ) p. 17.*

§ 9 Die Masse der Potentiale von V und W und das Verhalten im Unendlichen

Es wurde bereits im § 6 das Verhalten der Potentiale von V und W im Unendlichen erörtert. Setzt man

$$m = \int d\mu_s ,$$

indem man die Integration über die ganze Berandung erstreckt, so hat bei allen großen Distanzen R das Potential V eine Entwicklung der Form

$$V = mlog\frac{1}{R} + A\frac{x}{R^2} + B\frac{y}{R^2} + \cdots ,$$

analog hat man für das Potential W der Doppelschicht eine Reihenentwicklung

$$W = A\frac{x}{R^2} + B\frac{y}{R^2} + \cdots.$$

Die weggelassenen Glieder sind von höherer Ordnung in $\frac{x}{R^2}$ und $\frac{y}{R^2}$. Aus dieser Betrachtung folgt, daß das Potential V stets im Unendlichen im allgemeinen nicht regulär ist, während das Potential W im Unendlichen regulär ist und dort den Wert Null hat. Wenn in den Potentialen V und W die Integration sich über mehrere für sich geschlossene Kurven erstreckt, so kann man die Potentiale als Summe von Potentialen darstellen, die aus der Integration über je eine Kurve entstehen. Aus der Tatsache , daß das Integral $\int \frac{\partial U}{\partial n_s} ds$ über jede Kurve, die ein für das Potential U reguläres Flächenstück einschließt, sich Null ergibt, folgert man unschwer, daß die Masse von V auf einem geschlossenen Kurvenzuge, über den das Integral $V(p) = \int log\frac{1}{r_{ps}} \cdot d\mu_s$ zu verstehen ist, gleich ist jenem Anteil am Integral $\int d\mu_s$, welcher aus der Integration über dieses geschlossene Kurvenstück sich ergibt. Unsere Definition der Masse stimmt also überein mit der klassischen, wonach geradezu $\int d\mu_s$ als Masse bezeichnet wird. Eine analoge Erwägung über das Potential zeigt, daß die Masse des Potentials der Doppelschicht auf jedem einzelnen der geschlossenen Kurvenstücke der Berandung Null ist. *Diese ausgezeich-nete Eigenschaft der Potentiale tritt bei den Randwertaufgaben der Potentialtheorie, sobald es sich um mehfach zusammenhängende Gebiete handelt, sehr stark hervor, ist aber bisher zu wenig betont worden, (PlP) pp. 20.*

§ 10 Die Randwerte der Potentiale V und W

Ist das Potential der einfachen Schicht $V(p) = \int log\frac{1}{r_{ps}} \cdot d\mu_s$ in jeder beliebigen Nähe eines Randpunktes unter einer angegebenen Grenze gelegen, so sit es auch stetig und geht durch die Berandung stetig hindurch.

Das Potential der Doppelschicht erleidet in einem Punkte der Randkurve, wo die Belegung stetig ist, Stetigkeitssprünge, (PlP) pp. 20-22.

Poluyan P.
Non-standard analysis of non-classical motion
do the hyperreal numbers exist in the quantum-relative universe?

(PoP): *„In Einstein´s theory the rule of speed addition is used, when adding units does not lead to endless increase of the sum, it is limited by the maximum velocity-of-light limit. But in this case the matter is not in the breaking up of the Eudocks-Archimedean axiom, but in the special features of Lorentz transformations, actual for pseudo-Euclidean continuum of space-time. Obviously, it can be admitted, that the analogical rule of addition will work when dealing with simple quantities, such as the length or the time space. But still, it is not clear why we must limit the endless space with some set of radius, to which the sum of the added quantities would aspire. The prospect law exists, but we do understand that lessening of length within the distance is the optic illusion, but not the characteristic of the spacial metrics.*

Now let us stake the quantum mechanics. It is known, that the so-called „ultra-violet-catastrophe" was the direct consequence from the formulae of the classical mathematical analysis – for the balance of radiation in the field of high frequencies the result was endless quantity of energy. But the way out was found not in the

modification of mathematical principles, but in realizing experimental data: Max Planck´s hypothesis put the limit to the endless energy subdivision $E = h\nu$ appered to be non-divided. And at the moment the clinical formulae of analysis being used, and what concerns all „disturbing" modern physic-theoretic learnt as Richard Feynman said, to „sweep them under the rug".

There is no absolute motion, two points can be move only with regard to each other. If we take one of them for standard point, we believe it is stable, and the second one moves with regard to the first one. And vice versa: we can take the second moving point for the stable starting point and consider the first one to be moving. The notion of motion quite naturally and necessarily requires the principle of relativity as the distance change between these two points BETWEEN THEM with some time. Sketchily the principle of relativity is explained with the example of two points A and C. We take one of them for the starting point, the other moves with regards to the starting point, and vice versa. Let us imagine, in space there are two points (mathematically size less), separated by some distance. Now let us try to imagine that the distance changes… But how can we check this „change"? Henri Poincare, illustrating these cases, made the imaginary experience- he asked: what would happen if the distance between the two points becomes twice bigger? And he answered: the world would not notice it. I think it is clear. To be able to speak of the change of the distance between two points, there must be one more point which would be stable with regard to one of the two given points".

Robitaille P.-M.
Fourty lines of evidence for condensed matter
The Sun on trial, Liquid metallic hydrogen as a solar building block

(RoP): „*Our Sun has confronted humanity with overwhelming evidence that it is comprised of condensed matter. Dismissing this reality, the standard solar models continue to be anchored on the gaseous plasma. In large measure, the endurance of these theories can be attributed to 1) the mathematical elegance of the equations for the gaseous state, 2) the apparent success of the mass-luminosity relationship, and 3) the long-lasting influence of leading proponents of these models. Unfortunately, no direct physical finding supports the notion that the solar body is gaseous. Without exception, all observations are most easily explained by recognizing that the Sun is primarily comprised of condensed matter. However, when a physical characteristic points to condensed matter, a postori arguments are invoked to account for the behavior using the gaseous state. In isolation, many of these treatments appear plausible. As a result, the gaseous models continue to be accepted. There seems to be an overarching belief in solar science that the problems with the gaseous models are few and inconsequential. In reality, they are numerous and, while often subtle, they are sometimes daunting. The gaseous equations of state have introduced far more dilemmas than they have solved. Many of the conclusions derived from these approaches are likely to have led solar physics down unproductive avenues, as deductions have been accepted which bear little or no relationship to the actual nature of the Sun. It could be argued that, for more than 100 years, the gaseous models have prevented mankind from making real progress relative to understanding the Sun and the universe. Hence, the Sun is now placed on trial. Forty lines of evidence will be presented that the solar body is comprised of, and surrounded by, condensed matter. These 'proofs' can be divided into seven broad categories: 1) Planckian, 2) spectroscopic, 3) structural, 4) dynamic, 5) helioseismic, 6) elemental, and 7) earthly. Collectively, these lines of evidence provide a systematic challenge to the gaseous models of the Sun and expose the many hurdles faced by modern approaches. Observational astronomy and laboratory physics have remained unable to properly justify claims that the solar body must be gaseous. At the same time, clear signs of condensed matter interspersed with gaseous plasma in the chromosphere and corona have been regrettably dismissed. As such, it is hoped that this exposition will serve as an invitation to consider condensed matter, especially metallic hydrogen, when pondering the phase of the Sun*".

Blackbody radiation and the loss of universality,
Implications for Planck's formulation and Boltzmann's constant

(RoP1): „*Through the reevaluation of Kirchhoff's law (Robitaille P. M. L. IEEE Trans. Plasma Sci., 2003, v. 31(6), 1263–1267), Planck's blackbody equation (Planck M. Ann. der Physik, 1901, v. 4, 553–356) loses its universal significance and becomes restricted to perfect absorbers. Consequently, the proper application of Planck's radiation law involves the study of solid opaque objects, typically made from graphite, soot, and carbon black. The extension of this equation to other materials may yield apparent temperatures, which do not have any physical meaning relative to the usual temperature scales. Real temperatures are exclusively obtained from objects which are known solids, or which are enclosed within, or in equilibrium with, a perfect absorber. For this*

reason, the currently accepted temperature of the microwave background must be viewed as an apparent temperature. Rectifying this situation, while respecting real temperatures, involves a reexamination of Boltzman's constant. In so doing, the latter is deprived of its universal nature and, in fact, acts as a temperature dependent variable. In its revised form, Planck's equation becomes temperature insensitive near 300 K, when applied to the microwave background".

Water, Hydrogen Bonding, and the Microwave Background

(RoP2): „*In this work, the properties of the water are briefly revisited. Though liquid water has a fleeting structure, it displays an astonishingly stable network of hydrogen bonds. Thus, even as a liquid, water possesses a local lattice with short range order. The presence of hydroxyl* ($O-H$) *and hydrogen* ($H\cdots OH_2$) *bonds within water, indicate that it can simultaneously maintain two separate energy systems. These can be viewed as two very different temperatures. The analysis presented uses results from vibrational spectroscopy, extracting the force constant for the hydrogen bonded dimer. By idealizing this species as a simple diatomic structure, it is shown that hydrogen bonds within water should be able to produce thermal spectra in the far infrared and microwave regions of the electromagnetic spectrum. This simple analysis reveals that the oceans have a physical mechanism at their disposal, which is capable of generating the microwave background*".

Rollnik H.
Der Spin des Elektrons und die Gruppe $SU(2)$

(RoH) S. 214: „*In der klassischen Physik gibt es keinen Magnetismus! Denn dazu müßte ein Stück Materie, das aus vielen Atomen besteht, im thermodynamischen Gleichgewicht ein resultierendes magnetisches Moment besitzen*"

(RoH) S. 217 ff.: „*Bei der genaueren experimentellen Untersuchung der Atomspektren und ihrer Aufspaltung in magnetischen und elektrischen Feldern stieß man in der Mitte der zwanziger Jahre des vorherigen Jahrhunderts auf eine Reihe gravierender, da qualitativer Widersprüche zu den theoretischen Erwartungen. Sie lassen sich in der Feststellung zusammenfassen:*

Man beobachtete die Aufspaltung von Spektrallinien oder von Elektronenstrahlen in eine gerade Anzahl von Komponenten, während die Drehimpulsmultipletts nur ungerade Multipletts (die zu einer festen Energie gehörenden Eigenzustände des Hamiltonoperators von physikalischen Zuständen), nämlich mit der Anzahl $2l+1$ *erwarten lassen. Im einzelnen fand man:*

i) Es gibt Spektren mit einer geradzahligen Multiplettstruktur

ii) Die Zahl der Zeeman-Terme und deren Aufspaltungsregeln widersprechen in vielen Fällen dem Experiment, insbesondere beim Wasserstoff und den Alkali-Atomen. Es gilt wieder die Multiplizitätsregel: eine ungerade Elektronenzahl ist mit einer geraden Anzahl von Zeeman-Termen verbunden und umgekehrt

iii) Der Stern-Gerlach Versuch bestätigt die in den Spektren gefundenen Multiplizitätsregeln.

Diese Phänomene legen aufgrund der Drehimpulsregel „Multiplizität $= 2l+1$ *das Auftreten von* $j = 1/2$ *nahe. Konkret wurde nach vielen tastenden Vorüberlegungen im Herbst 1925 von Uhlenbeck und Goudsmit die Hypothese des Elektronenspins eingeführt. In moderner Sprache lautet sie:*

Hypothese des Elektronenspins

Neben den Observablen $\boldsymbol{Q}$ *und* $\boldsymbol{P}$ *besitzt ein Elektron eine neue Observable, einen inneren Drehimpuls, genannt Spin* $\hbar\boldsymbol{S}$ *mit den folgenden Eigenschaften*

a) S *ist ein Drehimpuls und es gilt* $\boldsymbol{S}\times\boldsymbol{S} = i\boldsymbol{S}$
b) Für jede Komponente von $\boldsymbol{S}$ *gibt es zwei mögliche Eigenwerte, daher gehört* $\boldsymbol{S}$ *zur Drehimpulsquantenzahl* $j = 1/2$*, und sein Quadrat hat den Wert* $\boldsymbol{S}^2 = \frac{1}{2}\left(\frac{1}{2}+1\right) = \frac{3}{4}$
c) Die Komponenten des Spins kommutieren mit den Bahnvariablen $\boldsymbol{Q}$ *und* $\boldsymbol{P}$*,* $[S_j, Q_k] = 0$*,* $[S_j, P_k] = 0$*, so daß z.B. der Ort* $\boldsymbol{Q}$ *und die dritte Komponente des Spins* S_3*, gleichzeitig gemessen werden können*

d) *Der Gesamtdrehimpuls eines Elektrons* $\boldsymbol{J}$ *wird durch die Summe von Bahndrehimpuls* $\boldsymbol{L}$ *und des Spins* $\boldsymbol{S}$ *gegeben,* $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$

e) *Der Spin* $\boldsymbol{S}$ *ist mit einem magnetischen Moment der Größe* $\boldsymbol{\mu}_s = g_s \frac{eh}{4\pi m_e c}\boldsymbol{S} = -g_s \frac{|e|h}{4\pi m_e c}\boldsymbol{S}$ *verbunden. Dabei wird der* g_s*-Faktor – das gyromagnetische Verhältnis – durch* $g_s \sim 2$ *gegeben (für die Bahnbewegung gilt lediglich der Wert* $g_s \sim 1$*). Dieser Wert* $(g_s \sim 2)$ *ist notwendig, um die Aufspaltung der Atomniveaus quantentheoretisch richtig zu beschreiben.*

Rovelli C.
Quantum gravity

(RoC) p. 9: *„The physical meaning of general relativity (GR): GR is the discovery that spacetime and the gravitational field are the same entity. What we call „spacetime" is itself a physical object, in many respects similar to the electromagnetic field. We can say that GR is the discovery hat there is no spacetime at all. What Newton called „space", and Minkowski called „spacetime", is unmasked: it is nothing but a dynamic object – the gravitational field – in a regime in which we neglect its dynamics. …., the universe is not made up of fields on spacetime; it is made up of fields on fields."*

(RoC) p. 34: „*I call „gravitational field" the tetrad field rather than Einstein's metric field.*

In General Relativity (GR) a frame field (also called a tetrad field) is a set of four (one time-like and three space-like) orthogonal vector fields, defined on a Lorentz manifold. All tensorial quantities defined on the manifold can be expressed by the frame field and its dual coframe field. The related gravitational field e *is a one-form* $e^I(x) = e^I_\mu(x)dx^\mu$ *with values in Minkowski space. A tetrad field* e *determines uniquely a torsion-free spin connection* $\omega = \omega[e]$*. Its compatibility condition with* e*, (RoC) (2.6), and the Einstein equations, (RoC) (2.11), are the field equations of GR in the absence of other fields. They are the Euler-Lagrange equations of the action* $S[e, \omega]$*, (RoC) (2.12). Replacing* ω *with* $\omega[e]$ *leads to the second order action formalism* $S[e]$*, (RoC) (2.16). The two Lagrange formalisms are not equivalent in the presence of fermions.*

There are three reasons for this

(1) the standard model cannot be written in terms of g *because fermions require the tetrad formalism*
(2) the tetrad field e *is nowadays more utilized than* g *in quantum gravity, and*
(3) I think that e *represents the gravitational fields in a more conceptually clean way than* g *(see section 2.2.3*)"

(RoC) p. 36*: „the formalism in (2.12) where* e *and* ω *(the spin connection, which is also a one-form with values in the Lie algebra of the Lorentz group* $SO(3,1)$*) are independent is called the first-order formalism. The two formalism are not equivalent in the presence of fermions; we do not know which one is physically correct, because the effect of gravity on single fermions is hard to measure."*

(RoC) p. 143: *The thermal time hypothesis: In Nature, there is no preferred physical time variable t. There are no equilibrium states* ρ_0 *preferred a priori. Rather, all variables are equivalent: we can find the system in an arbitrary state* ρ*; if the system is in a state* ρ*, then a preferred variable is singled out by the state of the system. This variable is what we call time. …. In other words, it is the statistical state that determines which variable is physical time, and not any a priori hypothetical „flow" that drives the system to a preferred statistical state."*

Russel B.
The philosophy of Leibniz & history of modern philosophy

(RuB) p. 108: „*Leibniz rejected atoms, the vacuum, and action at a distance*."

(RuB1) p. 211: „*"Substance", when taken seriously, is a concept impossible to free from difficulties. A substance is supposed to be the subject of properties, and to be something distinct from all its properties. But when we take away the properties, and try to imagine the substance itself, we find that there is nothing left. To put the matter in another way: What distinguishes one substance from another? Not difference of properties, for,*

according to the logic of substance, difference of properties presupposed numerical diversity between the substances concerned. Two substances, therefore, must be just *two, without being, in themselves, in any way distinguish-able. How, then, are we are ever to find out that they* are *two?"*

(RuB1) p. 212: „*"Substance", in a word, is a metaphysical mistake, due to transference to the world-structure of the structure of sentences composed of a subject and a predicate."*

(RuB1) p. 680: „*Hume had proved that the law of causality is not analytic, and had inferred that we could not be certain of its truth. Kant accepted the view that it is synthetic, but nevertheless maintained that it is known* a priori*. He maintained that arithmetic and geometry are synthetic, but are likewise* a priori*. He was thus led to formulate his problem in these terms:*
How are synthetic judgements a priori *possible?*
The answer to this question, with its consequences, constitutes the main theme of The Critique of Pure Reason*.*

(RuB1) p. 680/681: „*According to Kant, the outer world causes only the matter of sensation, but our own mental apparatus orders this matter in space and time, and supplies the concepts by means of which we understand experience. Things themselves, which are the causes of our sensations, are unknowable; they are not in space and time, they are general concepts which Kant calls „categories". Space and time are subjective, they are part of our apparatus of perception. But just because of this, we can be sure that whatever we experience will exhibit the characteristics dealt with by geometry and the science of time. If you always ore blue spectables, you could be sure of seeing everything blue (this is not Kant's illustration). Similarly, since ayou always wear spatial spectacles in your mind, you are sure of always seeing everyting in space. Thus geometry is* a priori *in the sense that it must be true of everything experienced, but we have no reason to suppose that anything analogous is true of things in themselves, which we do not experience.*

Space and time, Kant says, are not concepts; they are forms of „intuition". (The German word is „Anschauung", which means literally „looking at" or „view". The word „intuition", though the accepted translation, is not altogether a satisfactory one.) There are also, however, a priori *concepts; these are twelve „categories", which Kant derives from the forms of the syllogism. The twelve categories are divides into four sets of three: (1) of quantity: unity, plurality, totality; (2) of quality: reality, negation, limitation; (3) of relation: substance-and accident, cause-and-effect, reciprocity; (4) of modality: possibility, existence, necessity. These are subjective in the same sense in which space and time are – that is to say ,our mental constitution is such that they are applicable to whatever we experience, but there is no reason to suppose them applicable to things themselves. As regards cause, however, there is an inconsistency, for things in themselves are regarded by Kant as causes of sensations, and free volitions are held by him to be causes of occurrences in space and time. This inconsistency is not an accidential oversight; it is an essential part of his system."*

Schauberger V.
Implosion als Abbild planetarer oder atomarer Bewegung

(LaS) S. 226: „Der eine Pfeiler, auf dem Schaubergers Implosionsprinzip ruht, ist das Prinzip der „planetaren Bewegung". In einer schraubenartigen Bewegung sollen sich nach Kepler die Planeten unseres Sonnensystems um ihre eigenen Achse drehen (Kreiseln), und sich in ellipsoiden Bahnen um die in einem Brennpunkt der Ellipse befindlichen Sonne kreisen.

Nach dem Motto: Wie im Großen so im Kleinen, können wir dieses Bewegungsmodell auch in kleinsten Teilchen, den Atomen, feststellen. Im Bohrschen Atommodell stellt der Atomkern die Sonne dar, um den sich die Elektronen als Planeten drehen. Sie bewegen sich nach Arnold Sommerfeld auch auf ellipsenförmigen Bahnen um den Atomkern. Man nennt die kreiselnde Bewegung der Elektronen auch „Spin".

Wenn diese Bewegungsform im Makrokosmos (Universum) und im Mikrokosmos (Atom) feststellbar ist, dann muß sie auch in den materiellen Zwischenformen unserer physischen Realität feststellbar sein, überlegte Schauberger. Und sie muß eine besondere Bedeutung haben, da sich in der Natur scheinbar alles Aufbauende in dieser Weie bewegt oder bewegt wird, schloß er weiter. Er sollte später noch erkennen, daß er dem evolutiven Prinzip der Natur auf die Spur gekommen war. Aufgrund dieser Erkenntnisse und seiner Naturbeobachtungen kam Schauberger zu dem Schluß, daß Mensch (die Wissenschaft) und Natur verschiedene Wege gehen. Wir bleiben im Sinne der Schauberger'schen Dialektik bei der Wissenschaft als Antithese zur Natur. Die

Wissenschaft, so Schauberger, arbeitet in jeder Richtung gegen die Natur und ihre Intensionen. Er sah diesen Unterschied vornehmlich in der Diskrepanz zweier Bewegungsformen: Die Natur tendiert dazu, ihre Massen planetar zu bewegen (spiral-konzentrisch), die Wissenschaft hingegen tendiert dazu, Massen gleichförmig-geradlinig zu bewegen. Wobei bekannt ist, daß es keine geradlinige Bewegung gibt, sondern aufgrund der Raumkrümmung und Endgravitation jede geradlinig intendierte Bewegung gekrümmt verläuft.
Schauberger bezeichnet die der Intelligenz der Natur entspringende Bewegung „Implosion" und die der Intelligenz der Wissenschaft entspringende Bewegung „Explosion". Die Natur, der Kosmos, Planeten, Sterne, Atome, Moleküle, Wasser, Wellen, Wind (vor allem Wirbelstürme), Wolken, Blut und Pflanzensäfte, folgen der implosiven Bewegung. Die konzentrisch-spiralförmige „Implosionsbewegung" hat saugenden, ziehende Charakter. Man bedenke, welche Saugkräfte der Rüssel eines Tornados entwickelt. Nur der Mensch (die Wissenschaft) fociert die „widernatürliche" geradlinige (drückenden) Bewegungsform, die im Widerstand Wärme erzeugt und abbauende Eigenschaften hat, so Schauberger. Dazu Ludwig Boltzmann: Nur die geradlinige Bewegung steigert den Druck und die Temperatur. Bei der Implosionsbewegung soll durch die Saugwirkung eine minimale Reibung entstehen und eine Abkühlung erfolgen, da die Wärmeenergie in Bewegungsenergie umgewandelt wird, durch die zum Beispiel der Wirbelsturm auf Touren gebracht wird.

(LaS) S. 230: Viktor Schauberger sah also einen engen Zusammenhang zwischen der Bewegung der Planeten, der Atome und der materiellen Zwischenstufen (Moleküle, Wasser, Wellen, Wind (vor allem Wirbelstürme), Wolken, Blut und Pflanzensäfte usw.) Er benütze für die Planetenbewegung auch den schwierigen Begriff „zykloide Raumkurvenbewegung". … Nach Schauberger spiralen sie um die Sonne. Er begründete dies damit, daß alles, was sich im Kreise dreht, nicht von Fleck kommt. Statt „zykloide Raumkurvenbewegung" sagte er auch „planetare Bewegung". Also dreht sich auch die Erde in dieser Weise. Die „planetare Bewegung" der Erde hat auch Einfluß auf die Massenbewegung, sagte Schauberger. Das Wasser, Blut, Pflanzensäfte, Winde und Meereswellen, Gase, Rauchschwaden, etc. werden in dieser Weise bewegt.

Univ. Prof. Felix Ehrenhaft, Vorstand des ersten Physikalischen Institutes der Universität Wien, machte mit seinen Mitarbeitern die Entdeckung, daß kleinste freischwebende Materieteilchen in einem konzentrierten Lichtstrahl sich auf Schraubenbahnen bewegen. Teils in der Fortpflanzung des Lichts, teils in der entgegengesetzten Richtung. Dieser Versuch wird Photophorese genannt.

Grundsätzlich neuartig und aufregend, so Prof. Ehrenhaft, ist das Phänomen, daß die Bewegung von Materieteilchen in Feldern nicht auf geraden Bahnen, sondern auf Schraubenbahnen der regelmäßgsten Form, Größe und Umlauffrequenz erfolgt. Zu der Bewegung um die Schraube, kommt oft noch eine Bewegung um die eigene Achse.

Nach Dipl.-Ing. Walter Schauberger spielt sich die Erscheinung in allen Gasen, insbesondere auch in Edelgasen (Argon) und bei allen Drücken ab. Der Inder Satyendra Nat Ray bewies, daß auch in Flüssigkeiten derartige Bewegungen auftreten. G. Fachini in Italien hat ebenfalls Photophorese in Flüssigkeiten festgestellt. W. W. Barkas im Porterschen Laboratorium zu London hat auch in Röntgenstrahlen Photophorese gefunden … Die Photophorese – die schraubenförmige-spiralige Bewegung kleinster Materieteilchen – würde Viktor Schaubergers Implosionstheorie (planetarer Bewegung) im Prinzip bestätigen."

(LaS) S. 232: „Schauberger verstand unter „Implosion" also zentripetale Massenbewegungsform, die auf einer konzentrisch-spiralförmigen Bahn von außen nach innen verläuft, deren Zentrum saugend ist. … Diesen Naturvorgang (Wirbel) versuchte Schauberger technisch zu kopieren. Massen, wie Luft oder Wasser, die in diesen Bewegungsvorgang geraten, werden aufgrund des immer enger werdenden Raumes der konzentrischen Bahn verdichtet. Dabei wird auf atomarer Ebene Kernenergie frei, jene Bindungsenergie, welche die Atome im Innersten zusammenhält. Zugleich erfolgt eine qualitative Veränderung des ursprünglichen Stoffes. In der Physik wird dieser Vorgang Massendefekt bezeichnet: Paarbildung – Massendefekt – Freie Energie. Zum Beispiel bei der Umwandlung von Wasserstoff in Helium. Das Ursprüngliche wird quantitativ leichter, verliert an Masse, wird aber auf ein höheres Ordnungsniveau gehoben und gewinnt dadurch an Qualität. Jeder Stoff hat eine spezifische Eigenfrequenz und Struktur. Wenn nun die Eigenfrequenz beziehungsweise Struktur eines Stoffes durch Zufuhr von Energie verändert (erhöht) wird, verändert sich auch seine Qualität. Endprodukt eines solchen Bewegungs (=Veredelungs)-vorganges ist beispielsweise Edelwasser."

(LaS) S. 233: „Der Wirbel sorgt für Ordnung. Schauberger nannte diesen Vorgang auch „Atomumwandlung statt Atomzertrümmerung". Bei der Atomzertrümmerung werden Zerfallsprozesse eingeleitet. Uranatome werden gespalten, wodurch Energie frei wird. Uran ist bereits ein hochwertiges Element, das durch den

Kernspaltungsvorgang in ein minderwertiges, hochgiftiges Abfallprodukt verwandelt wird. Aus Ordnung wird Chaos.

Beim Verfahren der natürlichen Atomumwandlung nach Schauberger werden Atome nicht gespalten, sondern durch die saugende, verdichtende Implosionskraft auf ein höheres Ordnungsnivau gebracht. Die ursprüngliche Atomstruktur wird aufgelöst und neu gruppiert, eingespeicherte Energie wird frei. Ein Trennen und Wiedervereinen auf höherer (qualitativer) Ebene. Aus Chaos wird Ordnung.

Schauberger meinte, daß man das Chaos „überchaotisieren" müsse, um Ordnung zu schaffen. Jedenfalls verhalten sich Atome nach einer „Wirbelbehandlung", oder – neuesten Erkenntnissen zufolge – auch nach einer elektromagnetischen Beeinfußung nicht mehr chaotisch, sondern kohärent. Man kann auch sagen, wenn man ihnen von außen Energie zuführt, „erinnern" sie sich wieder an ihre Ordnung."

(LaS) S. 244: „Nach Schauberger ist Wachstum das Ergebnis eines Druckausgleiches zweier bipolarer (gegengeschlechtlicher) feinstofflicher Energien, die sich gegensinnig kreuzen (vermählen, vereinen), woraus ein Drittes entsteht. Diese feinstofflichen Energien sind für unser menschliches Auge unsichtbar. … Erst das Produkt aus der Kreuzung dieser feinstofflichen Energien, die Auswirkung, das sogenannte „Dritte", das „Grobstoffliche" (summa summarum unsere gesamte materielle Welt) ist für uns sichtbar und greifbar. ….

Durchlebte Materie zerfällt und eingespeicherte Energie wird frei, die sich laut Schauberger wiederum mit einfallender, gegenpoliger („kosmischer") Energie trifft, um erneut Ausgleiche einzugehen."

(LaS) S. 250: „Ein bißchen Lichteinfluß, ein bißchen Lichtabschluß, ein bißchen Wärme, ein bißchen Kälte rhythmisch geordnet genügt, um das edelste Wasser, das durch Mutatoren fließt, hochwertig aufzuladen (zu ionisieren").

Ich löse in neuester Zeit aus edelsten Bergkristallen die eingebauten Kraftstoffe, und die freiwerdenden Strahlungsenergien akkumuliere ich dann in das Wasser. Auf diese Weise bekomme ich die Ur-Eiweißstoffe oder die lebensanfachenden Vitamine."

Schiller F.
On the aesthetic education of man
Eleventh Letter, (ScF) p. 48 ff.
This is about ~*„the sensuous-rational nature of Man"*

„When abstraction mounts as high as it is possible can, it arrives at two final concepts, at which it must halt and recognize its limits. It distinguishes in Man something that edures and something that perpetually alters. The enduring is called person*, the changing is his* condition*.*

Person and condition – the self and its determinations – which we think of in the absolute Being as one and the same, are eternally two in the finite. Throughout the persistence of the person the condition changes, through every change of condition the person persists. We pass from rest to activity, from passion to nidifference, from assent to contradiction; but we always exist, and what springs immediately from our self remains. In the absolute Person alone all the determinations persist alongside the personality, since they flow out of personality. All that Divinity is, it is just because it is; consequently it is everything to eternity, because *it is eternity.*
Since in Man, as finite being, person and condition are distinct, neither can the condition be derived from the person nor the person from the condition. In the latter case, the person would have to alter; in the former case, the condition would habe to persist, and thus in each case either the personality or the finiteness would cease. Not because we think and will and feel do we exist; not because we exist and think and will do we feel. We exist because we exist; we feel, think and will because there is something other besides ourselves.

The person must therefore be its own ground, for the enduring cannot issue from alteration; and so we have in the first place the idea of absolute being grounded in itself, that is to say of freedom*. Condition must have a ground; since it does not exist through the person, and is thus not absolute, it must* result*; and so we have in the second place the qualification of all depending being and becoming,* time*. ‚Time is the condition of all* Becoming' *is an identical proposition, for it merely asserts that the result is the condition of something resulting.*

The person that is revealed in the eternally persisting ego, and only there, cannot become, cannot have a beginning in time; the reverse is rather the case – time must begin in it, because something constant must form the basis of change. There must be something that alters, if alternation is to occur; this something cannot therefore itself be alternation. In saying that the flower blooms and fades, we make the flower the thing that persits through the transformation and lend it, so to say, a personality in which both those conditions are manifested. It is no objection that Man has first to become; For Man is not simply person in general but person situated in a particular condition. But every condition, every definite instance arises in time, and so Man as a phenomenon must have his beginning, although the pure intelligence in him is eternal. Without time, that is to say without becoming it, he would never be a definite existence; his personality would certainly exist in potentiality, but not in fact. Only through the succession of its perceptions does the persisting ego itself come to appear.

The subject matter of activity, therefore, or the reality which the supreme Intelligence creates out of itself, must first be received *by Man, and he does in fact receive it as something eternal to himself in space and as something changing within himself in time, through the medium of perception. This changing substance in him is accompanied by his never-changing ego – and to remain remain perpetually himself throughout all change, to turn every perception into experience, that is, into unity of knowledge, and to make each of his manifestations in time a law for all time, is the rule which is prescribed for him by his rational nature. Only as he alters does* he *exist; only as he remains unalterable does* he *exist. Man conceived in his perfection would accordingly be the constant unity which admidst the tides of change remains eternally the same.*

Now although an infinite being, a divinity, cannot become*, we must surely call the divine a tendency which has for its infinite task the proper characteristic of divinity, absolute realization of capacity (actually of all that is possible) and absolute unity of manifestation (necessity of all that is actual). Beyond question Man carries the potentiality for divinity within himself; the path to divinity, if we may call a path what never reaches its goals, is open to him in his* sense*.*

His personality, regarded in itself alone and independently of all sense material, is merely the potentiality of a possible infinite expression; and so long as he neither contemplates nor fells he is still nothing but form and empty capacity. His sense faculty, regarded in itself and dissociated from all spontaneous activity of the mind, can do nothing beyond making himself material – for without it he is mere form – but by no means uniting him to matter. So long as he only perceives, only desires and acts from mere appetite, he is still nothing but world*, if we understood by this simply the formless content of time. It is indeed his sense faculty alone which turns his capacity into operative power; but it is only his personality which makes his operation really his own. Thus in order not to be merely world, he must lend form to his material; in order to be not merely form, he must make actual the potentiality which he bears within hinself. He realizes form when he create time, and opposes constancy with alteration, the eternal unity of his ego with diversity of the world; he gives form to matter when he proceeds to annul time, affirms persistence within change, and subjects the diversity of the world to the unity of his ego.*

Hence flow two contrary demands upon Man, the two fundamental laws of his sensuous-rational nature. The first insists upon absolute reality*; he is to turn everything that is mere form into world, and realize all his potentialities; the second insists upon absolute* formality*: he is to eradicate in himself everything that is merely world, and produce harmony in all its mutations; in other words, he is to turn outwards into internal, and give form to everything external. Both tasks, considered in their supreme fulfilment, lead back to the conception of divinity from which I started."*

Outlook to the Twelfth Letter, which is *„on the fulfilment of this twofold tasks"* regarding the sensuous impulse and the formal impulse*: If the first impulse only furnishes* cases*, the other gives* laws*.*

Schmicking D. A.
Die Subjekt-Objekt-Beziehung bei Schopenhauer
(ReT) S. 32-35

Schopenhauers System entfaltet sich auf erkenntnistheoretischer Ebene ausgehend von einer Struktur, die er als „Zerfallen in Objekt und Subjekt" charakterisiert. Das Subjekt ist da zwar „Träger der Welt, die durchgängige stets voraussetzende Bedingung alles Erscheinenden, alles Objekts". Aber das Subjekt ist in Beziehung auf die

Welt als Vorstellung eben auch nur eine von zwei Hälften, die andere Hälfte bildet das Objekt: „Die Hälften sind (…) unzertrennlich, selbst für den Gedanken: denn jede von beiden hat nur durch und für die andere Bedeutung und Daseyn, ist mit ihr da und verschwindet mit ihr". In den formal-ontologischen Kategorien der LU Husserls kann man die „Hälften" geeigneter als „abstrakte Momente" charakterisieren, da letzterer Terminus keine Ablösbarkeit suggeriert, die Schopenhauer ja bestreitet.

Mit dem Zerfallen in Objekt und Subjekt unterscheide sich seine Methode „ganz und gar von allen je versuchten Philosophien, als welche alle entweder vom Objekt oder vom Subjekt ausgingen, und demnach das eine aus dem anderen zu erklären suchten". Beide letzteren Ansätze führen auf ein Verkennen des Verhältnisses von Subjekt und Objekt, indem sie deren Verhältnis auf der Basis des Satzes vom Grunde, also kausal zu erklären versuchen. Nimmt man an, das Objekt sei unabhängig von und Ursache für das Subjekt, resultiert der Realismus bzw. die Naturphilosophie bzw. der Materialismus. Nimmt man an, das Subjekt sei unabhängig und erzeuge das Objekt, führt dies auf den transzendentalen Idealismus Fichtes. Damit begehe sowohl ein materialistisches als auch ein idealistisches System den Fehler, „zum voraus anzunehmen, was es erst abzuleiten vorgiebt, nämlich das nothwendige Korrelat seines Ausgangspunktes".

Die Subjekt-Objekt-Korrelation ist bereits in der Dissertation formuliert:

> *Wie mit dem Subjekt sofort das Objekt gesetzt ist (..) und auf gleiche Weise mit dem Objekt das Subjekt, und also Subjektseyn gerade soviel bedeutet, als ein Objekt haben, und Objektseyn so viel, als vom Subjekt erkannt werden: genau eben so nun ist auch mit einem auf irgend eine Weise bestimmten Objekt sofort auch das Subjekt als auf eben solche Weise erkennend gesetzt".*

Interessant ist, wie Schopenhauer die Strukturen von Subjekt und Objekt bestimmt, besonders im Hinblick auf das Erstere, denn obwohl das Subjekt als erkennendes sich nicht selbst erkennen könne, da es notwendiges Korrelat, und damit immer Bedingung aller Vorstellungen bleibt, ist es eben doch in bestimmten Formen oder Ausprägungen erschließbar, nämlich in Form der Korrelate der vier Vorstellungsklassen. Es ergeben sich aus diesen: Verstand, Vernunft, reine Anschauungsformen und Selbstbewusstsein. Die korrelativen Formen, so sagt Schopenhauer, werden

> *erschlossen, oder richtiger: sie sind allgemeine Ausdrücke für die aufgestellten Klassen der Vorstellungen (..) sie sind mit Rücksicht auf das als Bedingung nothwendige Korrelat jener Vorstellungen, das Subjekt, von ihnen abstrahirt, verhalten sich zu folglich zu den Klassen der Vorstellungen gerade so, wie das Subjekt überhaupt zum Objekt überhaupt".*

Das Erschließen dieser Ausdrücke bildet ein Verfahren, das, verglichen mit Kants Versuch einer Deduktion, eher als ein induktives zu charakterisieren ist.

Hierzu ein erster vergleichender Blick: Husserl reflektiert kontinuierlich und mit Blick auf zeitgenössische Kritiker die Möglichkeiten und Grenzen phänomenologischer Reflexion des eigenen Erlebens bzw. des Selbst. Er kommt zu einem positiven Ergebnis. In den Ideen I *etwa lesen wir: „Jedes Erlebnis, das nicht im Blicke ist, kann nach idealer Möglichkeit zum „erblicken" werden, eine Reflektion des Ich richtet sich darauf, es wird nun Objekt* für *das Ich." Mittels eidetischer Variation gelangt die Phänomenologie dann zu einer reichen Formenlehre der Akte, ihrer Teile und abstrakten Momente. Während also Schopenhauer die Operationen des Objekts aus dessen korrelativen Vorstellungen erschließt, erweist sich Husserls Zugang zur Subjektivität als vergleichsweise direkt. So steht Husserls Morphologie intentionaler (und prä-intentionaler) Leistungen und Gegenstandstypen der eher kargen Taxonomie der vierfachen Wurzeln des Satzes vom Grunde gegenüber. Schopenhauer mangelt nicht etwa der analytische psychologische Blick. Aber trotz seiner reichen Beobachtungen und Einsichten kennt Schopenhauer nicht eine vergleichbare, systematisch durchforschte Morphologie der intentionalen Akte und Objekte mit all ihren Stufungen, Komplexionen und Fundierungen, wie sie sich in der Husserlschen Lehre über Jahre ausdifferenziert.*

Welche allgemeinen Strukturen nimmt das Objekt bei Schopenhauer ein? Hier sind zunächst die vier Vorstellungsklassen: 1. Die anschaulichen, vollständigen, empirischen Vorstellungen bzw. realen Objekte, 2. die abstrakten Vorstellungen bzw. Begriffe (Vorstellungen von Vorstellungen), 3. die reinen bzw. a priori gegebenen Anschauungsformen des Raumes und der Zeit und 4. das Subjekt des Wollens bzw. alle Gefühls- und Willenszustände. Weitere Unterklassen werden von Schopenhauer teils detailliert behandelt, so neben den Wahrnehmungen Erinnerungen und Phantasmen, die verschiedenen Gattungen von Begriffen, Gegenstände der

Arithmetik und Geometrie, und schließlich die verschiedenen Arten der Gefühls- und Willensregungen. All diese bilden offensichtlich Unterklassen bzw. Spezies der vier Gattungen von Vorstellungsklassen.

Im Zusammenhang der ersten Vorstellungsklasse (der anschaulichen Vorstellungen bzw. realen Objekte) stößt man auf einen erkenntnistheoretisch entscheidenden Punkt. Wenn Schopenhauer manchmal vom „Bild" spricht, das der Intellekt (respektive das Gehirn) vom Gegenstand bzw. der Welt erzeugt, klingt das nach einer repräsentionalistischen Konzeption. Er erklärt jedoch, dass zwischen Gegenstand und Vorstellung kein Unterschied bestehe, dass wir nicht Vorstellungen haben, die von vermeintlich außerhalb des Bewusstseins liegenden Dingen verschieden sind, dass wir nicht ein „bloßes Abbild" der Dinge anschauen. Phänomenologisch gewendet: die äußere Wahrnehmung ist ein unmittelbares Wahrnehmen des originär, leibhaftig erscheinenden Dings selber. Damit liegt kein Bildbewusstsein vor, bei dem ein leibhaftig erscheinendes Ding als Bild eines anderen, nicht gegenwärtigen Gegenstands aufgefasst wird.

Bekanntlich liegt dem gesamten Schopenhauerschen System die Unterscheidung von Erscheinung und Ding an sich zugrunde. Aber die anschaulichen Vorstellungen lassen sich nach Schopenhauers Verständnis gerade nicht als Abbilder des Willens bzw. des Dings begreifen, sondern als Objektivität bzw. die sich über viele Stufen der Natur entwickelnden Objektiviationen des Willens. Dieser Repräsentation liegt keine repräsentationale Beziehung zugrunde, da der Wille an sich zur Erscheinung die Relation einer „Äußerung" hat, ähnlich wie sich das Verstandesvermögen, Kausalität zu erkennen, in unterscheidbare „Formen" äußert. Von der Anschauung unterscheidet Schopenhauer die signitiven Akte der Vernunft. Letztere bilden begriffliche Vorstellungen von anschaulichen Vorstellungen, den unmittelbaren Objekten unserer Anschauungen und Handlungen. Husserls Kritik an philosophischen Positionen, die annehmen, „die Transzenden des Dinges sei die eines Bildes *oder* Zeichens*", trifft damit wohl den Wortlaut, aber nicht den Gehalt der Schopenhauerschen Lehre.*

Man könnte nun einwenden, dass Schopenhauer den Leib als unmittelbares Objekt *unterscheidet, das die Anschauung aller übrigen Objkte „vermittelt", wobei der Verstand die „dumpfe, nichtssagende Empfindung" in einen Anschauung formt. Aber dies geschieht ohne Schließen in Begriffen, ohne Reflexion und Willkür. Auch diese Erklärung Schopenhauers weist auf eine Operation hin, die nicht ein Bewusstsein eines Etwas umfasst, das für ein Anderes steht. Schopenhauers „Anschauung" ist das originär Erscheinende, das unmittelbar und gegenwärtig apperzipiert wird. Die „Vermittlung" spielt sich sozusagen hinter den Kulissen ab, gehört nicht in den Bereich der Vorstellungen und führt in der Wahrnehmung nicht zu einer Distinktion in einerseits originär wahrgenommene und andererseits durch Bildbewusstsein apperzipierte Objekte. Hier kann also die phänomenologische Explikation Schopenhauer zur Seite springen und zeigen, dass Husserls berechtigte Kritik an repräsentationalistischen Konzeptionen nicht Schopenhauers Theorie trifft.*

Schopenhauer A.
Die Welt als Wille und Vorstellung
Die Vorstellung unterworfen dem Satze vom Grunde:
Das Objekt der Erfahrung und Wissenschaft

(ScA) §2: *„Dasjenige, was Alles erkennt und von Keinem erkannt wird, ist das Subjekt. Es ist sonach der Träger der Welt, die durchgängige, stets vorausgesetzte Bedingung alles Erscheinenden, alles Objekts: denn nur für das Subjekt ist, was nur immer da ist. Als dieses Subjekt findet Jeder sich selbst, jedoch nur sofern er erkennt, nicht sofern er Objekt der Erkenntniß ist. Objekt ist aber schon sein Leib, welchen selbst wir daher, von diesem Standpunkt aus, Vorstellung nennen. Denn der Leib ist Objekt unter Objekten und den Gesetzen der Objekte unterworfen, obwohl er unmittelbares Objekt ist. Er liegt, wie alle Objekte der Anschauung, in den Formen alles Erkennens, in Zeit und Raum, durch welche die Vielheit ist. Das Subjekt aber, das Erkennende, nie Erkannte, liegt auch nicht in diesen Formen, von denen selbst es vielmehr immer schon vorausgesetzt wird: ihm kommt also weder Vielheit, noch deren Gegensatz, Einheit, zu. Wir erkennen es nimmer, sondern es eben ist es, das erkennt, wo nur erkannt wird.*

Die Welt als Vorstellung also, in welcher Hinsicht allein wir sie hier betrachten, hat zwei wesentliche, nothwendige und untrennbare Hälften. Die eine ist das Objekt: dessen Form ist Raum und Zeit, durch diese die Vielheit. Die andere Hälfte aber, das Subjekt, liegt nicht in Raum und Zeit: denn sie ist ganz und ungetheilt in jedem vorstellenden Wesen; daher ein einziges von diesen, eben so vollständig, als die vorhandenen Millionen, mit dem Objekt die Welt als Vorstellung ergänzt: verschwände aber auch jenes einzige; so wäre die Welt als Vorstellung nicht mehr. Diese Hälften sind daher unzertrennlich, selbst für den Gedanken: denn jede von beiden

hat nur durch und für die andere Bedeutung und Daseyn, ist mit ihr da und verschwindet mit ihr. Sie begränzen sich unmittelbar: wo das Objekt anfängt, hört das Subjekt auf. Die Gemeinschaftlichkeit dieser Gränze zeigt sich eben darin, daß die wesentlichen und daher allgemeinen Formen alles Objekts, welche Zeit, Raum und Kausalität sind, auch ohne die Erkenntniß des Objekts selbst, vom Subjekt ausgehend gefunden und vollständig erkannt werden können, d.h. in Kants Sprache, a priori in unserm Bewußtseyn liegen. Dieses entdeckt zu haben, ist ein Hauptverdienst Kants und ein sehr großes. Ich behaupte nun überdies, daß der Satz vom Grunde der gemeinschaftliche Ausdruck für alle diese uns a priori bewußten Formen des Objekts ist, und daß daher Alles, was wir rein a priori wissen, nichts ist, als eben der Inhalt jenes Satzes und was aus diesem folgt, in ihm also eigentlich unsere ganze a priori gewisse Erkenntniß ausgesprochen ist."

(ScA) §4*: „Wer die Gestaltung des Satzes vom Grunde, welche in der reinen Zeit als solcher erscheint und auf der alles Zählen und Rechnen beruht, erkannt hat, der hat eben damit auch das ganze Wesen der Zeit erkannt. Sie ist weiter nichts, als eben jene Gestaltung des Satzes vom Grunde, und hat keine andere Eigenschaft. Succession ist die Gestalt des Satzes vom Grunde in der Zeit; Succession ist das ganze Wesen der Zeit. - Wer ferner den Satz vom Grunde, wie er im bloßen rein angeschauten Raum herrscht, erkannt hat, der hat eben damit das ganze Wesen des Raumes erschöpft; da dieser durch und durch nichts Anderes ist, als die Möglichkeit der wechselseitigen Bestimmungen seiner Theile durch einander, welche Lage heißt. Die ausführliche Betrachtung dieser und Niederlegung der sich daraus ergebenden Resultate in abstrakte Begriffe, zu bequemerer Anwendung, ist der Inhalt der ganzen Geometrie, - Eben so nun, wer diejenige Gestaltung des Satzes vom Grunde, welche den Inhalt jener Formen (der Zeit und des Raumes), ihre Wahrnehmbarkeit, d.i. die Materie, beherrscht, also das Gesetz der Kausalität erkannt hat; der hat eben damit das ganze Wesen der Materie als solcher erkannt: denn diese ist durch und durch nichts als Kausalität, welches Jeder unmittelbar einsieht, sobald er sich besinnt. Ihr Seyn nämlich ist ihr Wirken: kein anderes Seyn derselben ist auch nur zu denken möglich. Nur als wirkend füllt sie den Raum, füllt sie die Zeit: ihre Einwirkung auf das unmittelbare Objekt (das selbst Materie ist) bedingt die Anschauung, in der sie allein existirt: die Folge der Einwirkung jedes andern materiellen Objekts auf ein anderes wird nur erkannt, sofern das letztere jetzt anders als zuvor auf das unmittelbare Objekt einwirkt, besteht nur darin. Ursache und Wirkung ist also das ganze Wesen der Materie: ihr Seyn ist ihr Wirken. …*

… Nun aber erhält das Gesetz der Kausalität seine Bedeutung und Nothwendigkeit allein dadurch, daß das Wesen der Veränderung nicht im bloßen Wechsel der Zustände an sich, sondern vielmehr darin besteht, daß an dem selben Ort im Raum jetzt ein Zustand ist und darauf ein anderer, und zu einer und der selben bestimmten Zeit hier dieser Zustand und dort jener: nur diese gegenseitige Beschränkung der Zeit und des Raums durch einander giebt einer Regel, nach der die Veränderung vorgehn muß, Bedeutung und zugleich Nothwendigkeit. Was durch das Gesetz der Kausalität bestimmt wird, ist also nicht die Succession der Zustände in der bloßen Zeit, sondern diese Succession in Hinsicht auf einen bestimmten Raum, und nicht das Daseyn der Zustände an einem bestimmten Ort, sondern an diesem Ort zu einer bestimmten Zeit. Die Veränderung, d. h, der nach dem Kausalgesetz eintretende Wechsel, betrifft also jedesmal einen bestimmten Theil des Raumes und einen bestimmten Theil der Zeit zugleich und im Verein. Demzufolge vereinigt die Kausalität den Raum mit der Zeit. Wir haben aber gefunden, daß im Wirken, also in der Kausalität, das ganze Wesen der Materie besteht: folglich müssen auch in dieser Raum und Zeit vereinigt seyn, d.h. sie muß die Eigenschaften der Zeit und die des Raumes, so sehr sich Beide widerstreiten, zugleich an sich tragen, und was in jedem von jenen Beiden für sich unmöglich ist, muß sie in sich vereinigen, also die bestandlose Flucht der Zeit mit dem starren unveränderlichen Beharren des Raumes, die unendliche Theilbarkeit hat sie von Beiden. Diesem gemäß finden wir durch sie zuvörderst das Zugleichseyn herbeigeführt, welches weder in der bloßen Zeit, die kein Nebeneinander, noch im bloßen Raum, der kein Vor, Nach oder Jetzt kennt, seyn konnte. Das Zugleichseyn vieler Zustände aber macht eigentlich das Wesen der Wirklichkeit aus: denn durch dasselbe wird allererst die Dauer möglich, indem nämlich diese nur erkennbar ist an dem Wechsel des mit dem Dauernden zugleich Vorhandenen; aber auch nur mittelst des Dauernden im Wechsel erhält dieser jetzt den Charakter der Veränderung, d.h. des Wandels der Qualität und Form, beim Beharren der Substanz, d.i. der Materie. Im bloßen Raum wäre die Welt starr und unbeweglich: kein Nacheinander, keine Veränderung, kein Wirken: eben mit dem Wirken ist aber auch die Vorstellung der Materie aufgehoben. In der bloßen Zeit wiederum wäre alles flüchtig: kein Beharren, kein Nebeneinander und daher kein Zugleich, folglich keine Dauer: also wieder auch keine Materie. Erst durch die Vereinigung von Zeit und Raum erwächst die Materie, d.i. die Möglichkeit des Zugleichseyns und dadurch der Dauer, durch diese wieder des Beharrens der Substanz, bei der Veränderung der Zustände. Im Verein von Zeit und Raum ihr Wesen habend, trägt die Materie durchweg das Gepräge von Beiden. Sie beurkundet ihren Ursprung aus dem Raum, theils durch die Form, die von ihr unzertrennlich ist, besonders aber (weil der Wechsel allein der Zeit angehört, in dieser allein und für sich aber nichts Bleibendes ist) durch ihr Beharren (Substanz), dessen Gewißheit a priori daher ganz und gar von der des Raumes abzuleiten ist: ihren Ursprung aus der Zeit aber offenbart sie an der

Qualität (Accidenz), ohne die sie nie erscheint, und welche schlechthin immer Kausalität, Wirken auf andere Materie, also Veränderung (ein Zeitbegriff) ist. Die Gesetzmäßigkeit dieses Wirkens aber bezieht sich immer auf Raum und Zeit zugleich und hat eben nur dadurch Bedeutung. Was für ein Zustand zu dieser Zeit an diesem Ort eintreten muß, ist die Bestimmung, auf welche ganz allein die Gesetzgebung der Kausalität sich erstreckt. Auf dieser Ableitung der Grundbestimmungen der Materie aus den uns a priori bewußten Formen unserer Erkenntniß beruht es, daß wir ihr gewisse Eigenschaften a priori zuerkennen, nämlich Raumerfüllung, d.i. Undurchdringlichkeit, d.i. Wirksamkeit, sodann Ausdehnung, unendliche Theilbarkeit, Beharrlichkeit, d.h. Unzerstörbarkeit, und endlich Beweglichkeit: hingegen ist die Schwere, ihrer Ausnahmslosigkeit ungeachtet, doch wohl der Erkenntniß a posteriori beizuzählen, obgleich Kant in den „Metaphys. Anfangsgr. d. Naturwiss.", S. 71 (Rosenkranz. Ausg., S. 372) sie als a priori erkennbar aufstellt.

Wie aber das Objekt überhaupt nur für das Subjekt da ist, als dessen Vorstellung; so ist jede besondere Klasse von Vorstellungen nur für eine eben so besondere Bestimmung im Subjekt da, die man ein Erkenntnißvermögen nennt. Das subjektive Korrelat von Zeit und Raum für sich, als leere Formen, hat Kant reine Sinnlichkeit genannt, welcher Ausdruck, weil Kant hier die Bahn brach, beibehalten werden mag; obgleich er nicht recht paßt, da Sinnlichkeit schon Materie voraussetzt. Das subjektive Korrelat der Materie oder der Kausalität, denn Beide sind Eines, ist der Verstand, und er ist nichts außerdem. Kausalität erkennen ist seine einzige Funktion, seine alleinige Kraft, und es ist eine große, Vieles umfassende, von mannigfaltiger Anwendung, doch unverkennbarer Identität aller ihrer Äußerungen. Umgekehrt ist alle Kausalität, also alle Materie, mithin die ganze Wirklichkeit, nur für den Verstand, durch den Verstand, im Verstande. Die erste, einfachste, stets vorhandene Aeußerung des Verstandes ist die Anschauung der wirklichen Welt: diese ist durchaus Erkenntniß der Ursache aus der Wirkung: daher ist alle Anschauung intellektual. Es könnte dennoch nie zu ihr kommen, wenn nicht irgend eine Wirkung unmittelbar erkannt würde und dadurch zum Ausgangspunkte diente. Dieses aber ist die Wirkung auf die thierischen Leiber. Insofern sind diese die unmittelbaren Objekte des Subjekts: die Anschauung aller andern Objekte ist durch sie vermittelt. Die Veränderungen, welche jeder thierische Leib erfährt, werden unmittelbar erkannt, d.h. empfunden, und indem sogleich diese Wirkung auf ihre Ursache bezogen wird, entsteht die Anschauung der letzteren als eines Objekts. Diese Beziehung ist kein Schluß in abstrakten Begriffen, geschieht nicht durch Reflexion, nicht mit Willkür, sondern unmittelbar, nothwendig und sicher. Sie ist die Erkenntnißweise des reinen Verstandes, ohne welchen es nie zur Anschauung käme; sondern nur ein dumpfes, pflanzenartiges Bewußtsein der Veränderungen des unmittelbaren Objekts übrig bliebe, die völlig bedeutungslos auf einander folgten, wenn sie nicht etwan als Schmerz oder Wollust eine Bedeutung für den Willen hätten. Aber wie mit dem Eintritt der Sonne die sichtbare Welt dasteht; so verwandelt der Verstand mit einem Schlage, durch seine einzige, einfache Funktion, die dumpfe, nichtssagende Empfindung in Anschauung. Was das Auge, das Ohr, die Hand empfindet, ist nicht die Anschauung: es sind bloße Data. Erst indem der Verstand von der Wirkung auf die Ursache übergeht, steht die Welt da, als Anschauung im Raume ausgebreitet, der Gestalt nach wechselnd, der Materie nach durch alle Zeit beharrend: denn er vereinigt Raum und Zeit in der Vorstellung Materie, d.i. Wirksamkeit. Diese Welt als Vorstellung ist, wie nur durch den Verstand, auch nur für den Verstand da. Im ersten Kapitel meiner Abhandlung „Ueber das Sehn und die Farben" habe ich bereits auseinandergesetzt, wie aus den Datis, welche die Sinne liefern, der Verstand die Anschauung schafft, wie durch Vergleichung der Eindrücke, welche vom nämlichen Objekt die verschiedenen Sinne erhalten, das Kind die Anschauung erlernt, wie eben nur dieses den Aufschluß über so viele Sinnenphänomene giebt, über das einfache Sehn mit zwei Augen, über das Doppeltsehn beim Schielen, oder bei ungleicher Entfernung hinter einander stehender Gegenstände, die man zugleich ins Auge faßt, und über allen Schein, welcher durch eine plötzliche Veränderung an den Sinneswerkzeugen hervorgebracht wird."

Schopenhauer's will & (Einstein's) cosmic energy

(ZiR) S. 110: „*Der Wille ist das verbindende Band zwischen allen Lebewesen; … Alles was ist, ist nur Erscheinung von Willen, verkörperter Wille. … Die Welt ist für uns Vorstellung, in Wahrheit aber ist sie Wille, die Erscheinung einer in allem Leben wirkende Kraft, eine irrational kosmische Energie, die sich im Prisma unserer Erkenntnis in unendlichen Gestalten bricht, deren einzigen Zweck es ist: zu leben, also Ausdruck des Willens zu sein. Die Welt des Willen ist zwar die „wahre" Welt, aber sie ist nicht, wie Platons Welt der Ideen, jenseitig und transzendent. Es ist die Welt, in der wir leben: Sie erscheint uns als Vorstellung, aber die Vorstellung ist nur die Form, in der der Mensch die Welt des Willens erkennt. Damit hat Schopenhauer die Welt auf jene beiden Begriffe gebracht, um die sich seine ganze Philosophie dreht: Wille und Vorstellung, die Tiefendimensionen der Welt und ihre Form der Erscheinung.*

Die Idee einer Tiefenrealität in Form einer in der Natur allseits wirkenden Kraft war keineswegs neu (Alexander v. Humboldt, Ansichten über die Natur", „ewige, all-verbreitete Kraft").

Der Schopenhauersche Wille hat keinen Urheber, er darf also nicht mit dem Willen einer Person verwechselt werden. Er ist auch keine Ursache von irgendetwas – den Zusammenhang zwischen Ursache und Wirkung gibt es nur in der Welt der Vorstellungen. Schopenhauers Wille ist schlicht die letzte Realität, eine kosmische Energie, die keine Frage nach dem Warum oder Wozu mehr zulässt."

(EiA1) p. 19: „But there is a third state of religious experience which belongs to all of them, even though it is rarely found in a pure form, and which I will call cosmic religious feeling. It is very difficult to explain this feeling to anyone who is entirely without it, especially as there is no anthropomorphic conception of God corresponding to it.
The individual feels the nothingness of human desires and aims and the sublimity and marvellous order which reveal themselves both in nature and in the world of thought. He looks upon individual existence as a sort of prison and wants to experience the universe as a single significant whole. The beginnings of cosmic religious feeling already appear in earlier stages of development--e.g., in many of the Psalms of David and in some of the Prophets. Buddhism, as we have learnt from the wonderful writings of Schopenhauer especially, contains a much stronger element of it."

Welt und Mensch
Eine Auswahl aus dem Gesamtwerk von Arthur Hübscher

(ScA1) S. 16: *„Die Philosophie ist wesentlich Weltweisheit; ihr Problem ist die Welt. Mit dieser allein hat sie es zu tun und läßt die Götter in Ruhe, erwartet aber dafür, auch von ihnen in Ruhe gelassen zu werden."*

(ScA1) S. 17: *„Die wahre Philosophie der Geschichte besteht in der Einsicht, daß man, bei allen diesen endlosen Veränderungen und ihrem Wirrwarr, doch stets nur dasselbe, gleiche und unwandelbare Wesen vor sich hat, welches heute dasselbe treibt wie gestern und immerdar."*

(ScA1) S. 19: *„Alle solche historische Philosophie, sie mag auch noch so vornehm tun, nimmt, als wäre Kant nie dagewesen, die Zeit für eine Bestimmung der Dinge an sich, und bleibt daher bei dem stehn, was Kant die Erscheinung, im Gegensatz des Dinges an sich, und Platon das Werdende, nie Seiende, im Gegensatz des Seienden, nie Werdenden nennt, oder endlich, was bei den Indern das Gewebe der Maja heißt: es ist eben die dem Satz vom Grunde anheimgegebene Erkenntnis, mit der man nie zum inneren Wesen der Dinge gelangt, sondern nur Erscheinungen ins Unendliche verfolgt, sich ohne Ende zum Ziel bewegt, dem Einhörnchen im Rade zu vergleichen, bis man etwa endlich ermüdet, oben oder unten, bei irgendeinem beliebigen Punkte stillesteht und nun für denselben auch von andern Respekt ertrotzen will."*

(ScA1) S. 38: *Ding an sich = Wille*

(ScA1) S. 43-44: *Geist und Natur*
„Ihr glaubt eine tote d.h. vollkommen passive und eigenschaftslose Materie zu erkennen, weil ihr alles das wirklich zu verstehn wähnt, was ihr auf mechanische *Wirkung zurückführen vermögt. Aber wie die physikalischen und chemischen Wirkungen euch eingeständlich begreiflich sind, solange ihr sie nicht auf mechanische zurückführen wißt; geradeso sind diese mechanischen Wirkungen selbst, also die Äußerungen, welche aus der Schwere, der Undurchdringlichkeit, der Kohäsion, der Härte, der Starrheit, der Elastizität, der Fluidität usw. hervorgehn, ebenso geheimnisvoll, wie jene, ja , wie das Denken im Menschenkopf. … Das wirklich rein und durch und durch, bis auf das Letzte, Verständliche in der Mechanik geht nicht weiter, als das rein Mathematische in jeder Erklärung, ist also beschränkt auf Bestimmungen des Raumes und der Zeit. Nun sind aber diese beiden, samt ihrer ganzen Gesetzlichkeit, uns a priori bewußt, sind daher bloße Formen unsers Erkennens und gehören ganz allein unsern Vorstellungen an. Ihre Bestimmungen sind also im Grunde subjektiv und betreffen nicht das rein Objektive, das von unserer Erkenntnis Unabhängige, das Ding an sich selbst. Sobald wir aber, selbst in der Mechanik, weiter gehen als das rein Mathematische, sobald wir zur Undurchdringlichkeit, zur Schwere, zur Starrheit oder Fluidität oder Gaseität kommen, stehn wir schon bei Äußerungen, die uns ebenso geheimnisvoll sind wie das Denken und Wollen der Menschen, also beim direkt Unergründlichen: denn ein solches ist jede Naturkraft. Wo bleibt nun also die* Materie*, die ihr so intim kennt und versteht, daß ihr alles aus ihr erklären, alles auf sie zurückführen wollt? – Rein begreiflich und ergründlich ist immer nur das*

Mathematische; weil es das im Subjekt, in unserem eigenen Vorstellungsapparat, Wurzelnde ist: sobald aber etwas eigentlich Objektives auftritt, etwas nicht a priori Bestimmbares; da ist es auch sofort in letzter Instanz unergründlich. Was überhaupt Sinne und Verstand wahrnehmen, ist eine ganz oberflächliche Erscheinung, die das wahre und innere Wesen der Dinge unberührt läßt. Das wollte Kant*."*

(ScA1) S. 49-50: *Der Intellekt ist unvollkommen*
„Unser Selbstbewußtsein hat nicht den Raum, sondern allein die Zeit zur Form: deshalb geht unser Denken nicht wie unser Anschauen nach drei Dimensionen vor sich, sondern bloß nach einer, also auf einer Linie, ohne Breite und Tiefe. Hieraus entspringt die größte der wesentlichen Unvollkommenheiten unseres Intellekts. Wir können nämlich allles nur sukzessive erkennen und nur Eines zur Zeit uns bewußt werden, ja auch dieses Einen nur unter der Bedingung, daß wir derweilen alles andere vergessen, also uns desselben gar nicht bewußt sind, mithin es solange aufhört, für uns dazusein. In dieser Eigenschaft ist unser Intellekt einem Teleskop mit einem sehr engen Gesichtfelde zu vergleichen; weil eben unser Bewußtsein kein stehendes, sondern ein fließendes ist. Der Intellekt apprehendiert nämlich nur sukzessive und muß, um das eine zu ergreifen, das andere fahren lassen, nichts als die Spuren von ihm zurücklassend, welche immer schwächer werden. Der Gedanke, der mich jetzt lebhaft beschäftigt, muß mir nach einer kurzen Weile ganz entfallen sein: tritt nun noch eine wohl durchschlafene Nacht dazwischen, so kann es vorkommen, daß ich ihn nie mehr wiederfinde: es sei denn, daß er an mein persönliches Interesse, d.h. an meinen Willen geknüpft wäre, als welcher stets das Feld behauptet. Auf dieser Unvollkommenheit des Intellekts beruht das Rhapsodische und oft Fragmentarische unseres Gedankenlaufs, und aus diesem entsteht die unvermeindliche Zerstreuung unsers Denkens …

(ScA1) S. 55: *Was ist die Zeit?*
Wir können die Zeit einem endlos drehenden Kreise vergleichen: die stets sinkende Hälfte wäre die Vergangenheit, die stets steigende Hälfte die Zukunft, oben aber der unteilbare Punkt, der die Tangente berührt, wäre die ausdehnungslose Gegenwart …, der Berührungspunkt des Objekts, dessen Form die Zeit ist, mit dem Subjekt, das keine Form hat, weil es nicht zum Erkennbaren gehört, sondern Bedingung alles Erkennbaren ist. Oder: die Zeit gleicht einem unaufhaltsamen Strom und die Gegenwart einem Felsen, an dem sich jener bricht, aber nicht ihn mit fortreißt.

(ScA1) S. 58 ff.: *V. Der Stufenbau der Wirklichkeit*

- *Die Welt als Spiegel des Willens*
- *Der Wille zum Leben*
- *Die Einheit des Willensaktes*
- *Organ und Umwelt*
- *Die Objektivationen des Willens*
- *Die niedrigste Stufe: die Naturkräfte*
  Als niedrige Stufe der Objektivation des Willens stellen sich die allgemeinsten Kräfte der Natur dar, welche teils in jeder Materie ohne Ausnahme erscheinen, wie Schwere, Undurchdringlichkeit, teil sich untereinander in die überhaupt vorhandene Materie geteilt haben, so daß einige über diese, andere über jene eben dadurch spezifisch verschiedene Materie herrschen, wie Starrheit, Flüssigkeit, Elastizität, Elektrizität, Magnetismus, chemische Eigenschaften und Qualitäten jeder Art. …
- *Pflanze, Tier und Mensch*
- *Das Erkennen: Charakter der Tierheit*
- *Vom Gattungscharakter zum Individualcharakter*
- *Der Mensch ein animal metaphysicum*
- *Keine Sicherheit für ihn*

Schposki E. W.
Atomphysik

(ScW) S. 187: *„In der Literatur findet man hin und wieder die Behauptung, daß sich bei Prozessen, die mit einer Freisetzung von Energie einhergeht (z.B. bei Kernreaktionen) „Masse in Energie umwandelt". Eine solche Formulierung ist nicht exakt und deshalb abzulehnen. Masse und Energie sind untrennbar miteinander verbunden, sie stellen sozusagen zwei Seiten derselben universellen Eigenschaft der Materie dar und können sich daher nicht ineinander „umwandeln". Es ist natürlich richtig, daß bei Prozessen, bei denen die kinetische*

Energie zunimmt, die Ruhmasse $\sum m_0$ eine entsprechende Verminderung erfährt. Aber dem Überschuß an kinetischer Energie, also ΔE_{kin} , der bei der Reaktion entsteht, entspricht die Masse $\Delta E_{kin}/c^2$, die die Verminderung von $\sum m_0$ exakt kompensiert, ebenso wie dieser letzteren Größe die Energie $\sum m_0\, c^2$ entspricht, die zusammen mit ΔE_{kin} exakt gleich $\sum m\, c^2$ vor der Reaktion ist."

Schrödinger E.
Statistical Thermodynamics (ScE)

(ScE) p. 1-2*: „There is, essentially, only one problem in statistical thermodynamics: the distribution of a given amount of energy E over N identical systems. Or perhaps better: to determine the distribution of an assembly of N identical systems over the possible states in which this assembly can find itself, given that the energy of the assembly is a constant E. The distinguished role of the energy is, therefore, simply that it is a constant of the motion – the one that always exists, and, in general, the only one. The generalization to the case, that there are others besides (momenta, moments of momenta), is obvious; it has occasionally been contemplated, but in terrestrial, as opposed to astrophysical, thermodynamics it has hitherto not acquired any importance. "To determine the distribution" .. means in principle to make oneself familiar with any possible distribution-of-the-energy (or state-of-the-assembly), to classify them in a suitable way, i.e. in the way suiting the purpose in question and to count the numbers in the classes, as as to be able to judge of the probability of certain features or characteristics turning up in the assembly. The question that can arise in this respect are of the most varied nature, especially in relation to the fineness of classification. At one end of the scale we have the general question of finding out those features which are common to almost all possible states of the assembly so that we may safely contend that they „almost always" obtain. In this case we have well-nigh only one class – actually two, but the second one has a negligibly small content. At the other end of the scale we have such a detailed question as: volume (=number of states of the assembly) of the „class" in which one individual member is in a particular one of its states. Maxwell's law of velocity distribution is the best-known example.*

This is the mathematical problem – always the same; we shall (soon) present its general solution, from which in the case of every particular kind of system enery particular classification that may be desirable can be found as a special case.
But there are two different attitudes as regards the physical application of the mathematical result. We shall later, for obvious reasons, decidedly favour one of them; for the moment, we must explain them both.

The older and more naive application is to N actually existing physical systems in actual physical interaction with each other, e.g. gas molecules or electrons or Planck oscillators or degrees of freedom ("ether oscillators") of a "hohlraum". The N of them together represent the actual physical system under consideration. This original point of view is associated with the names of Maxwell, Boltzmann and others.

But it suffices only dealing with a very restricted class of physical systems – virtually only with gases. It is not applicable to a system which does not consist of a great number of identical constituents with "private" energies. …"

(ScE) p. 3*: Hence a second point of view (or rather, a different application of the same mathematical result) which we owe to Willard Gibbs, has been developed. It has a particular beauty of its own, is applicable quite generally to every physical system, and has some advantages to be mentioned forthwith. Here the N identical systems are mental copies of the one system under consideration – of the one macroscopic device that is actually erected on our laboratory table. Now what on earth could it mean, physically, to distribute a given amount of energy E over these N mental copies? The idea is, in my view, that you can, of course, imagine that you really had N copies of your system, that they really were in "weak interaction" with each other, but isolated from the rest of the world. Fixing your attention on one of them, you find it in a peculiar kind of "heat-bath" which consists of the $N-1$ others.*

Now you have on the one hand, the experience that in thermodynamical equilibrium the behavior of a physical which you place in a heat-bath is always the same whatever be the nature of the heat-bath that keeps it at constant temperature, provided, of course, that the bath is chemically neutral towards your system, i.e., that there is nothing else but heat exchange between them. On the other hand, the statistical calculations do not refer to the mechanism of interaction: they only assume that it is "purely mechanical", that it does not affect the nature of the single systems (e.g., that it never blows them to pieces), but merely transfers energy from one to the other.

These considerations suggest that we may regard the behavior of any one of those N systems as describing the one actually existing system when placed in a heat-bath of given temperature. Moreover, since N systems are a likely and number similar conditions, we can then obviously, from their simultaneous statistics, judge of the probability of finding our system, when placed in a heat-bath of given temperature, in one or other of its private states. Hence all questions concerning the system in a heat-bath can be answered.

We adopt this point of view in principle – though all the following considerations may, with due care, also be applied to the other. The advantage consists not only in the general applicability, but also in the following two points:

- *(i)* *N can be made arbitrarily. In fact, in case of doubt, we always mean $limN = \infty$ (infinitely large heat-bath). Hence the applicability, for example, of Stirling's formula for $N!$, or for the factorials of „occupation numbers" proportional to N (and thus going with N to infinity), need never be questioned.*
- *(ii)* *No question about the individuality of the members of the assembly can ever arise – as it does, according to the „new statistics", with particles. Our systems are macroscopic systems, which we could, in principle, furnish with labels. Thus two states of the assembly differing by system No. 6 and system No 13 having exchanged their roles are, of course, to be counted as different states – while the same way not be true when two similar atoms within No. 6 have exchanged their roles; but the latter is merely a question of enumeration correctly the states of the single system, of describing correctly its quantum-mechanical nature."*

(ScE) p. 44*: „The different cases in the evaluation of the sum over states „Z" arise thus: The values admitted for every n_s may be*

$n_s = 0,1,2,3,4,$ …. *(Bose-Einstein gas);*
$n_s = 0,1$ *(Fermi-Dirac gas, Pauli´s exclusion principle).*

There may or may not be condition that the total number of particles is constant, $n = \sum_s n_s$.

(ScE) p. 50: *Not until the idea of photons had gained considerable ground did Bose (about 1924) point out that we could, alternatively to the „holhraum" oscillator statistics, speak of photon statistics, but then we ad to make it „bose statistics". Very soon after, Einstein applied the same to the particles of an ideal gas. And thereupon I pointed out that we could also in this case speak of ordinary statistics, applied to the wave-mechanical proper vibrations which correspond to the motion of the particles of the gas.*

The wave point of view in both cases, or at least in all Bose cases, raises another interesting question. Since in the Bose case we seem to be faced, mathematically, with simple oscillator of the Planck type, of which the n_s is the quantum number, we may ask whether we ought not to adopt for n_s half-odd integers

$$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots n + \frac{1}{2}, \ldots$$

rather then integers. One must, I think, call that an open dilemma. From the point of analogy one would very much prefer to do so. For, the "zero point energy" of a Planck oscillator is not only borne out by direct observation in the case of crystal lattices, it is also so intimitely linked up with the Heisenberg uncertainty relation that one hates to dispense with it. On the other hand, if we adopt it straightaway, we get into serious trouble, especially on contemplating changes of the volume (e.g. adiabatic compression of a given volume of black-body radiation), because in this process the (infinite) zero-point energy seems to change by infinite amounts! So we do not adopt it, and we continue to take for the n_s the integers, beginning with 0."

(ScE) pp. 76-82*: „According to physical laws the regular course of events is never the consequence of one well-ordered configuration of atoms. … On the contrary, in biology a single group of atoms existing only in one copy produces orderly event, marvellously tuned in with each other and with the environment according to most subtle laws. … It appears that there are two different „mechanisms" by which orderly events can be produced: the „statistical mechanism" which produces „order from disorder" and the biological „mechanism", producing „order from order". …. According to Schrödinger the latter principle is nothing else that the principle of quantum theory over again and the distinction between M. Planck's physical-statistical type of laws and „dynamical" laws, (PlM), is precisely the one being labbelled as „order from order" and „order from disorder."*

Schrödinger E.
My View of the World

(ScE2) pp. 12-13: SEEK FOR THE ROAD, I*V, The problem: Self - The World - Death - Plurality*
„I think that one of the principle problems, if not the *principle problem without whose solution there can be no final peace for the metaphysical urge, can be be quite briefly characterised as follows:*

Consider these four questions, which cannot, as a whole, be satisfactorily answered with any combination of „yes" and „no", but rather lead one on in an endless circle.

1. *(1) Does there exist a Self?*
2. *(2) Does there exist a world outside Self?*
3. *(3) Does this Self cease with bodily death?*
4. *(4) Does the world cease with my bodily death?*

If we start with Self, then all the facts of physiology assure us that there is so intimate and necessary a connection between all the sensations of this Self and the material modifications of my own body that it is impossible to doubt that destruction of the body implies dissolution of the Self. With equal certainty we must reject a world existing outside Self, because both consist of the same empirical „elements", and in fact that to which the term „world" is applied consists entirely *of elements which also belong to Self. In any case, that to which we give the name „world" is only a complex within the Self, but my own body is only a complex within the world-complex. Hence what is known as „world" would be completely eliminated by a descructive attack on one small part of itself – of which, furthermore, it contains millions of examples: a dreadful piece of nonsense!*

If on the other hand we start from the world alone*, this naturally does away with the grounds for supposing that the world ceases with the destruction of one's own body. But there then arises the following paradox, which has up till now only been recognised, I think, in Indian Samkhya philosophy:*

Assume two human bodies, A and B. Put A in some particular external situation so that some particular imagine is seen, let us say the view of a garden. At the same time B is placed in a dark room. If A is now put into the dark room and B in the situation in which A was before, there is no view of the garden: it is completely dark (because A is my *body, B someone else's!). This flagrant contradiction, for there is no more adequate ground (zureichender Grund) for this phenomenon, considered in general and as a whole, than there would be for one side of a symmetrically loaded balance to go for."*

p. 16: *„Mach, for instance, has said (Analyse der Empfindungen, 3rd ed., p. 274) that he draws „no essential distinction between my sensations and someone else's. The same* element *(his italics) cohere at a number of points of combination, which are selves." Avenarius and, with particular emphasis, Schuppe, express themselves in the same sense. Thus Schuppe says (in Avenarius, Der menschliche Weltbegriff, 3rd ed., p. 155): „What I am most anxious to emphasise continually is that, while a good deal of the content of consciousness is in this sense subjective, not all of it is; rather, a part of the contents of consciousness of various selves is not merely qualitatively similar but is and must be their common content, being numerically one and the same, being in the strict sense common to them.""*

(ScE2) *V, The Vedântic vision*
p. 18: *„For philosophy, then, the real difficulty lies in the spatial and temporal multiplicity of observing and thinking individuals. If all events took place in* one *consciousness, the whole situation would be extremely simple. There would then be something given, a simple datum, and this, however otherwise constituted, could scarly present us with a difficulty of such magnitude as the one we do in fact have on our hands.*
I do not think that this difficulty can be logically resolved, by consistent thought, within our intellects: the plurality that we perceive is only an appearance; it is not real*. Vedantic philosophy, in which this is a fundamental dogma, has sought to clarify it by a number of analogies, one of the most attractive being the many-faceted crystal which, while showing hundreds of little pictures of what is in reality a single existing object, does not really multiply that object."*

pp. 21-22: *„Looking and thinking in that manner you may suddenly come to see, in a flash, the profound rightness of the basic conviction in Vedanta: it is not possible that this unity of knowledge, feeling and choice which you call* your own *should have sprung into being from nothingness at a given moment not so long ago;*

rather this knowledge, feeling and choice are essentially eternal and unchangeable and numerically one *in all men, nay in all sensitive beings. But not in* this *sense – that you are a part, a piece, of eternal, infinite being, an aspect or modification of it, as in Spinoza's pantheism. For we should then have the same baffling question: which part, which aspect are* you*? What, objectively, differentiates it from the others? No, but, inconceivable as it seems to ordinary reason, you – and all other conscious beings as such – are all in all. Hence this life of yours which you are living is not merely a piece of the entire existence, but is in a certain sense the* whole*; only this whole is not so constituted that it can be surveyed in one single glance. This, as we know, is what the Brahmins express in that scared, mystic formula which is yet really so simple and so clear: „Tat tvam asi, this is you. Or, again, in such words as I am in the east and in the west, I am below and above,* I am this whole world*".*

(ScE2) *VIII, Consciousness, organic, inorganic, mneme*
pp. 40-41: *„There is something for grander, for more in accord with a clear recognition of what it is all about, in the ideas of Spinoza or Fechner. For Spinoza, the human body is „a modification of the infinite substance (God), in so far as it is expressed in the attribute of extension" and the human mind is* that same *modification, but expressed in the attribute of thought. But since according to him every material thing is a modification of God in this way and, as such, expressed both these attributes, this, when translated into our language, means nothing else than: that something corresponds to every material event in the way that our consciousness corresponds to the vital processes of our body. And Fechner's fertile mind went on to imagine not only plants but also the planet Earth and the stars as possessed of souls. I do not agree with these phatasies, but I would prefer not to have to pass judgement on the question of which came nearer to the ultimate truth, Fechner or the bankrupts of modern rationalism."*

p. 42: *„The question then certainly arises: What does „organic" mean? – that is, in a wider sense here supposed, naturally excluding such simple answers as „protein" or „protoplasm". Fixing our attention on a somewhat wider concpet than this, we arrive at the criterion of* metabolism*. Thus Schopenhauer's line of demarcation may be regarded as highly suitable, when he says that in inorganic being 'the essential and permanent element, the basis of identity and integrity, is the material, the matter, the inessential and mutable element being the form. In organic being the reverse is true; for its life, that is, its existence as an organic being, consists precisely in a constant change of matter while the form persists."*

p. 43-44: *„It does away with that recurrent doubt whether it is conceivable that organic being, which is „so utterly different", could have „gradually" emerged from the inorganic. In fact, though there is perfect continuity in the object, the transition is* not *gradual; because the mental focus can only change abruptly, even though the structure of the object exerts increasing pressure on it to change gradually. I can* either *focus my observation on the unchanging material with ist changing form,* or *on the unchanging form of this changing matter, but not very well on both at once. In the same way, I can express the equations of hydrodynamics either in Lagrange's form or Euler's; both forms have exactly the same content, yet cannot emerge from each other gradually but only by means of the single discontinuous step of changing the variables.*

Of course this realisation will not hinder us, but on the contrary spur us on to search for the mechanism which gives specialised organic tissue, in the narrower sense, its characteristic stamp. It is the peculiarity which Semon calls, by which a particular reaction, set in motion once, or more than once, by some stimulus-complex, gets „drilled in„ in such a way that in later, similar occurences, only a part, and often a very small part, of the original stimulus-complex is needed in order to achieve the same result. The mechanism of this process is still completely unknown; furthermore, there is as yet absolutely no mechanical model which would illustrate the process even in the quite general sense in which Boltzmann's bicycle model illustrates electro-magnetic processes; whereas we do have, in the physical action of relay*, a very effective illustration, at least in this sense, for the peculiar character of* stimulation *itself. Of course, no one has yet given very serious thought to the possibility of constructing a model of this sort for mneme, important though it would be for the advancement of our knowledge."*

(ScE2) *IX, On becoming conscious*
p. 45: *„Not all brain-processes are accompanied by consciousness. There are nerve-processes which, while exactly rssembling the „conscious" processes of the brain both in their whole centripetal-centrifugal pattern and in their biological significance as reaction-regulators, nevertheless are not assiciated with consciousness. They include not only the regulatory reflex processes in the ganglia of the spinal cord and that aprt of the nervous system which they control, but also a considerable number of reflex events which involve the brain itself but do not enter into consciousness.*

So here we have verious specimens of very similar nerve-processes taking place within our soma, some of which are accompanied by consciousness and some not; moreover – and this is something extremely valuable for our analysis – they include intermediate forms at every level. Surely, then, it should not be too difficult to work out the distinguishing characteristic conditions of each by a process of observation and thought!

It seems to me that the key to this lies in the well-known fact that any particular series of phenomena in which we consciously or even actively participate, if it is repeated over and over again in exactly the same way, gradually sinks out of the sphere of consciousness*; and it is only, so to speak, dragged up into it again if, on a fresh repetition, the event initiating the process, or the conditions affecting its continuance, are slightly different, in which case the reactions happen slightly differently too. But even then it is not the process as a whole, but only (primarily at least) the modifications or differentials, by which the new series is distinguished from the earlier ones, which enter into consciousness."*

p. 51: „Briefly summarising, we can express the proposed law thus: consciousness is bound up with learning *in organic substance; organic* competence *is unconscious. Still more briefly, and put in a form which is admittedly rather obscure and open to miss-understanding: Becoming is conscious, being unconscious."*

WHAT IS REAL? Reason for abandoning the dualism of thought and and existence, or mind and matter
(ScE2) p. 66: *„It seems to me that this brings us to a somewhat paradoxical conclusion: if, without involving ourselves in obvious nonsense, we are going to be able to think in a natural way about what goes on in a living, feeling, thinking being (that is, to see it in the same way as we are see what takes place in inanimate bodies) – without any directing demons, without offending against, say, the principle of increase of entropy, without entelechy or* vis viva *or any other such rubbish – then the condition for our doing so is that we think of* everything *that happens as taking place in our* experience *of the world, without ascribing to it any material substratum as the object* of which it is an experience*; a substratum which, as the rest of this investigation will show, would in fact be wholly and entirely superfluous."*

Schrödinger E.
Mind and Matter
(ScE1) pp. 93-164

(ScE1) chapter 1, The Physical Basis of Consciousness
pp. 95-97: *„To my mind the key is to be found in the following well-known facts. Any succession of events in which we take part with sensations, perceptions and possibly with actions gradually drops out of the domain of consciousness when the same string of events repeats itself in the same way very often. But it immediately shot up into the conscious region, if at such a repetition either the occasion or the environmental conditions met with on its pursuit differ from what they were on all the previous incidences. Even so, at first anyhow, only those modifications or „differentials" intrude into the conscious sphere that distinguish the new incidence from previous ones and thereby usually can for „new considerations". Of all this each of us supply dozens of examples out of personal experience, so that I may forgot enumerating any at the moment.*

The gradual fading from consciousness is of outstanding importance to the entire structure of our mental life, which is wholly based on the process of acquiring practice by repetition, a process which Richard Semon has generalized to the concepts of Mneme*, about which we shall have more to say later. A single experience that is never to repeat itself is biologically irrelevant. Biological value lies only in learning the suitable reaction to situation that offers itself again and again, in many cases periodically, and always requires the same response if the organism is to hold its ground. Now from our own inner experience we know the following. On the first few repetitions a new element turns up in the mind, the „already met with" or „notal" as Richard Avenarius has called it. On frequent repetition the whole string of events becomes more and more of a routine, it becomes more and more uninteresting, the responses becomes even more reliable according as they fade from consciousness. The boy recites his poem, , the girl plays her piano sonata „well-nigh in their sleep". We follow the habitual path to our workshop, cross the road at the customary places, turn into side-streets, etc., whilst our thoughts are occupied with entirely different things. But whenever the situation exhibits a relevant differential – let us say the road is up at the place where we used to cross it, so that we have to make a detour – this differential and our response to it intrude into consciousness, from which, however, they soon fade below the threshold, if the differential becomes a constantly repeated feature. Faced with changing alternatives,*

bifurcations develop and may be fixed in the same way. We branch off to the University Lecture Rooms or to the Physics Laboratory at the right point without much thinking, provided that both are frequently occuring destinations.

Now this fashion differentials, variants of response, bifurcations, etc., are piled up one upon the other in unsurveyable abundance, but only the most recent ones remain in the domain of consciuosness, only the most recent ones remain in the domain of consciousness, only the most recent ones remain in the domain of consciousness, oonly those with regard to which the living substance is still in the stage of learning or practising. One might say, metaphorically, that consciousness is the tutor who supervises the education of the living substances, but leaves his pupil alone to deal with all those tasks for which he is already sufficently trained. But I wish to underline three times in red ink that I mean this only as a metaphor. The fact is only this, that new situations and the new responses they prompt are kept in the light of consciousness; old and well prectised ones are no longer so."

(ScE1) chapter 3, The Principle of Objectivation
pp. 117-127: *„Nine years ago I put forward two general principles that form the basis of the scientific method, the principle of the understandability of nature, and the principle of objectivation. Since then I have touched on this matter now and again, last time in my little book* Nature and the Greeks*. I wish to deal here in detail with the second one, the objectivation. Before I say what I mean by that, let me remove a possible misunderstanding which might arise, as I came to realize from several reviews of that book, though I thought I had prevented it from the outset. I t is simply this: some people seemed to think that my intention was to lay down the fundamental principles which ought to be at the basis of scientific method or at least which justly and rightly are at the basis of science and ought to be kept at all cost. Far from this, I only maintained and maintain that they are - and, by the way, as an inheritance from the ancient Greeks, from whom all our Western science and scientific thought has originated.*

The misunderstanding is not very astonishing. If you hear a scientist pronounce basic principles of science, stressing two of them as particularly fundamental and of old standing, it is natural to think that he is at least strongly in favour of them and wishes to impose them. But on the other hand, you see, science never imposes anything, science states*. Science aims at nothing but making true and adequate statements about its object. The scientist only imposes two things, namely truth and sincerity, imposes them upon himself and upon other scientists. In the present case the object is science itself, as it has developed and has become and at present is, not as it ought to be or ought to develop in future.*

Now let us turn to these two principles themselves. As regards the first, 'that nature can be understood', I will say here only a few words. The most astonishing thing about it is that it had to be invented, that it was at all necessary to invent it. I t stems from the Milesian School, the physiologoi*. Since then it has remained untouched, though perhaps not always uncontaminated. The present line in physics is possibly a quite serious contamination. The uncertainty principle, the alleged lack of strict causal connection in nature, may represent a step away from it, a partial abandonment. It would be interesting to discuss this, but I set my heart here on discussing the other principle, that which I called objectivation.*

By this I mean the thing that is also frequently called the 'hypothesis of the real world' *around us. I maintain that it amounts to a certain simplification which we adopt in order to master the infinitely intricate problem of nature. Without being aware of it and without being rigorously systematic about it, we exclude the Subject of Cognizance from the domain of nature that we endeavour to understand. We step with our own person back into the part of an onlooker who does not belong to the world, which by this very procedure becomes an objective world. This device is veiled by the following two circumstances. First, my own body (to which my mental activity is so very directly and intimately linked) forms part of the object (the real world around me) that I construct out of my sensations, perceptions and memories. Secondly, the bodies of other people form part of this objective world. Now I have very good reasons for believing that these other bodies are also linked up with, or are, as it were, the seats of spheres of consciousness. I can have no reasonable doubt about the existence or some kind of actualness of these foreign spheres of consciousness, yet I have absolutely no direct subjective access to any of them. Hence I am inclined to take them as something objective, as forming part of the real world around me. Moreover, since there is no distinction between myself and others, but on the contrary full symmetry for all intents and purposes, I conclude that I myself also form part of this real material world around me. I so to speak put my own sentient self (which had constructed this world as a mental product) back into it - with the pandemonium of disastrous logical consequences that flow from the aforesaid chain of faulty*

conclusions. We shall point them out one by one; for the moment let me just mention the two most blatant antinomies due to our awareness of the fact that a moderately satisfying picture of the world has only been reached at the high price of taking ourselves out of the picture, stepping back into the role of a non-concerned observer.

The first of these antinomies is the astonishment at finding our world picture 'colourless, cold, mute'. Colour and sound, hot and cold are our immediate sensations; small wonder that they are lacking in a world model from which we have removed our own mental person.

The second is our fruitless quest for the place where mind acts on matter or vice-versa, so well known from Sir Charles Sherrington's honest search, magnificently expounded in Man on his Nature*. The material world has only been constructed at the price of taking the self, that is, mind, out of it, removing it; mind is not part of it; obviously, therefore, it can neither act on it nor be acted on by any of its parts. (This was stated in a very brief and clear sentence by Spinoza, see p. 122.)*

I wish to go into more detail about some of the points I have made. First let me quote a passage from a paper of C.G. Jung which has gratified me because it stresses the same point in quite a different context, albeit in a strongly vituperative fashion. While I continue to regard the removal of the Subject of Cognizance from the objective world picture as the high price paid for a fairly satisfactory picture, for the time being, Jung goes further and blames us for paying this ransom from an inextricably difficult situation. He says:

> All science (*Wissenschaft*) however is a function of the soul, in which all knowledge is rooted. The soul is the greatest of all cosmic miracles, it is the *conditio sine qua* non of the world as an object. It is exceedingly astonishing that the Western world (apart from very rare exceptions) seems to have so little appreciation of this being so. The flood of external objects of cognizance has made the subject of all cognizance withdraw to the background, often to apparent non-existence.

Of course Jung is quite right. It is also clear that he, being engaged in the science of psychology, is much more sensitive to the initial gambit in question, much more so than a physicist or a physiologist. Yet I would say that a rapid withdrawal from the position held for over 2,000 years is dangerous. We may lose everything without gaining more than some freedom in a special - though very important - domain. But here the problem is set. The relatively new science of psychology imperatively demands living-space, it makes it unavoidable to reconsider the initial gambit. This is a hard task, we shall not settle it here and now, we must be content at having pointed it out.

While here we found the psychologist Jung complaining about the exclusion of the mind, the neglect of the soul, as he terms it, in our world picture, I should now like to adduce in contrast, or perhaps rather as a supplement, some quotations of eminent representatives of the older and humbler sciences of physics and physiology, just stating the fact that 'the world of science' *has become so horribly objective as to leave no room for the mind and its immediate sensations.*

Some readers may remember A.S. Eddington's 'two writing desks'; one is the familiar old piece of furniture at which he is seated, resting his arms on it, the other is the scientific physical body which not only lacks all and every sensual qualities but in addition is riddled with holes; by far the greatest part of it is empty space, just nothingness, interspersed with innumerable tiny specks of something, the electrons and the nuclei whirling around, but always separated by distances at least 100,000 times their own size. After having contrasted the two in his wonderfully plastic style he summarizes thus:

> In the world of physics we watch a shadowgraph performance of familiar life. The shadow of my elbow rests on the shadow table as the shadow ink flows over the shadow paper ... The frank realization that physical science is concerned with a world of shadows is one ofthe most significant of recent advances.

Please note that the very recent advance does not lie in the world of physics itself having acquired this shadowy character; it had it ever since Democritus of Abdera and even before, but we were not aware of it; we thought we were dealing with the world itself; expressions like model or picture for the conceptual constructs ofscience canle up in the second half of the nineteenth century, and not earlier, as far as I know.
Not much later Sir Charles Sherrington published his momentous Man on his Nature*. The book is pervaded by the honest search for objective evidence ofthe interaction between matter and mind. I stress the epithet 'honest', because it does need a very serious and sincere endeavour to look for something which one is deeply*

convinced in advance cannot be found, because (in the teeth of popular belief) it does not exist. A brief summary of the result of this search is found on p. 357:

> Mind, the anything perception can compass, goes therefore in our spatial world more ghostly than a ghost. Invisible, intangible, it is a thing not even of outline; it is not a 'thing'. It remains without sensual confirmation and remains without it forever.

In my own words I would express this by saying: Mind has erected the objective outside world of the natural philosopher out of its own stuff. Mind could not cope with this gigantic task otherwise than by the simplifying device of excluding itself - withdrawing from its conceptual creation. Hence the latter does not contain its creator.
I cannot convey the grandeur of Sherrington's immortal book by quoting sentences; one has to read it oneself. Still, I will mention a few of the more particularly characteristic.

> Physical science ... faces us with the impasse that mind per se cannot play the piano - mind per se cannot move a finger of a hand (p.222).
>
> Then the impasse meets us. The blank of the 'how' of mind's leverage on matter. The inconsequence staggers us. Is it a misunderstanding? (p. 232).

*Hold these conclusions drawn by an experimental physiologist of the twentieth century against the simple statement of the greatest philosopher of the seventeenth century: B. Spinoza (*Ethics*, Pt III, Prop. 2):*

> Nec corpus mentem ad cogitandum, nec mens corpus ad motum, neque ad quietem, nec ad aliquid (si quid est) aliud determinare potest.
>
> [Neither can the body determine the mind to think, nor the mind determine the body to motion or rest or anything else (if such there be).]

The impasse is *an impasse. Are we thus not the doers of our deeds? Yet we feel responsible for them, we are punished or praised for them, as the case may be. It is a horrible antinomy. I maintain that it cannot be solved on the level of present-day science which is still entirely engulfed in the 'exclusion principle' - without knowing it - hence the antinomy. To realize this is valuable, but it does not solve the problem. You cannot remove the 'exclusion principle' by act of parliament as it were. Scientific attitude would have to be rebuilt, science must be made a new. Care is needed.*

*So we are faced with the following remarkable situation. While the stuff from which our world picture is built is yielded exclusively from the sense organs as organs of the mind, so that every man's world picture is and always remains a construct of his mind and cannot be proved to have any other existence, yet the conscious mind itself remains a stranger within that construct, it has no living space in it, you can spot it nowhere in space. We do not usually realize this fact, because we have entirely taken to thinking of the personality of a human being, or for that matter also that of an animal, as located in the interior of its body. To learn that it cannot really be found there is so amazing that it meets with doubt and hesitation, we are very loath to admit it. We have got used to localizing the conscious personality inside a person's head – I should sayan inch or two behind the midpoint of the eyes. From there it gives us, as the case may be, understanding or loving or tender - or suspicious or angry looks. I wonder has it ever been noted that the eye is the only sense organ whose purely receptive character we fail to recognize in naIve thought. Reversing the actual state of affairs, we are much more inclined to think of '*rays of vision*', issuing from the eye, than of the '*rays of light*' that hit the eyes from outside. You quite frequently find such a 'ray of vision' represented in a drawing in a comic paper, or even in some older schematic sketch intended to illustrate an optic instrunlent or law, a dotted line emerging from the eye and pointing to the object, the direction being indicated by an arrowhead at the far end. –*

Dear reader or, or better still, dear lady reader, recall the bright, joyful eyes with which your child beams upon you when you bring him a new toy, and then let the physicist tell you that in reality nothing emerges from these eyes; in reality their only objectively detectable function is, continually to be hit by and to receive light quanta. In reality! A strange reality! Something seems to be missing in it.

*It is very difficult for us to take stock of the fact that the localization of the personality, of the conscious mind, inside the body is only symbolic, just an aid for practical use. Let us, with all the knowledge we have about it, follow such a '*tender look*' inside the body. We do hit there on a supremely interesting bustle or, if you like, machinery. We find millions of cells of very specialized build in an arrangement that is unsurveyably intricate*

but quite obviously serves a very far-reaching and highly consummate mutual communication and collaboration; a ceaseless hammering of regular electrochemical pulses which, however, change rapidly in their configuration, being conducted from nerve cell to nerve cell, tens of thousands of contacts being opened and blocked within every split second, chemical transformations being induced and may be other changes as yet undiscovered. All this we meet and, as the science of physiology advances, we may trust that we shall come to know more and more about it. But now let us assume that in a particular case you eventually observe several efferent bundles of pulsating currents, which issue from the brain and through long cellular protrusions (motor nerve fibres), are conducted to certain muscles of the arm, which, as a consequence, tends a hesitating, trembling hand to bid you farewell - for a long, heart-rending separation; at the same time you may find that some other pulsating bundles produce a certain glandular secretion so as to veil the poor sad eye with a crape of tears. But nowhere along this way from the eye through the central organ to the arm muscles and the tear glands - nowhere, you may be sure, however far physiology advances, will you ever meet the personality, will you ever meet the dire pain, the bewildered worry within this soul, though their reality is to you so certain as though you suffered them yourself - as in actual fact you do!

The picture that physiological analysis vouchsafes to us of any other human being, be it our most intimate friend, strikingly recalls to me Edgar Allan Poe's masterly story, which I am sure many a reader remembers well; I mean The Masque of the Red Death*. A princeling and his retinue have withdrawn to an isolated castle to escape the pestilence of the red death that rages in the land. After a week or so of retirement they arrange a great dancing feast in fancy dress and mask. One of the masks, tall, entirely veiled, clad all in red and obviously intended to represent the pestilence allegorically, makes everybody shudder, both for the wantonness of the choice and for the suspicion that it might be an intruder. At last a bold young man approaches the red mask and with a sudden jolt tears off veil and head-gear. It is found empty.*

Now our skulls are not empty. But what we find there, in spite of the keen interest it arouses, is truly nothing when held against the life and the emotions of the soul.

To become aware of this may in the first moment upset one. To me it seems, on deeper thought, rather a consolation. If you have to face the body of a deceased friend whom you sorely miss, is it not soothing to realize that this body was never really the seat of his personality but only symbolically 'for practical reference'?
As an appendix to these considerations, those strongly interested in the physical sciences might wish to hear me pronounce on a line of ideas, concerning subject and object, that has been given great prominence by the prevailing school of thought in quantum physics, the protagonists being Niels Bohr, Werner Heisenberg, Max Born and others. Let me first give you a very brief description of their ideas. It runs as follows:

We cannot make any factual statement about a given natural object (or physical system) without 'getting in touch' with it. This 'touch' is a real physical interaction. Even if it consists only in our 'looking at the object' the latter must be hit by light-rays and reflect them into the eye, or into some instrument of observation. This means that the object is affected by our observation. You cannot obtain any knowledge about an object while leaving it strictly isolated. The theory goes on to assert that this disturbance is neither irrelevant nor completely surveyable. Thus after any number of painstaking observations the object is left in a state of which some features (the last observed) are known, but others (those interfered with by the last observation) are not known, or not accurately known. This state of affairs is offered as an explanation why no complete, gapless description of any physical object is ever possible.

If this has to be granted - and possibly it has to be granted - then it flies in the face of the principle of understandability of nature. This in itself is no opprobrium. I told you at the outset that my two principles are not meant to be binding on science, that they only express what we had actually kept to in physical science for many, many centuries and what cannot easily be changed. Personally I do not feel sure that our present knowledge as yet vindicates the change. I consider it possible that our models can be modified in such a fashion that they do not exhibit at any mornent properties that cannot in principle be observed simultaneously - models poorer in simultaneous properties but richer in adaptability to changes in the environment. However, this is an internal question of physics, not to be decided here and now. But from the theory as explained before, from the unavoidable and unsurveyable interference of the measuring devices with the object under observation, lofty consequences of an epistemological nature have been drawn and brought to the fore, concerning the relation between subject and object. It is maintained that recent discoveries in physics have pushed forward to the mysterious boundary between the subject and the object. This boundary, so we are told, is not a sharp boundary at all. We are given to understand that we never observe an object without its being modified or

tinged by our own activity in observing it. We are given to understand that under the impact of our refined methods of observation and of thinking about the results of our experiments that mysterious boundary between the subject and the object has broken down.

In order to criticize these contentions let me at first accept the time-hallowed distinction or discrimination between object and subject, as many thinkers both in olden times have accepted it and in recent times still accept it. Among the philosophers who accepted it - from Democritus of Abdera down to the 'Old Man of Konigsberg' - *there were few, if any who did not emphasize that all our sensations, perceptions and observations have a strong, personal, subjective tinge and do not convey the nature of the 'thing-in-itself, to use Kant's term. While some of these thinkers might have in mind only a more or less strong or slight distortion, Kant landed us with a complete resignation: never to know anything at all about his* 'thing-in-itself'. *Thus the idea of subjectivity in all appearance is very old and familiar. What is new in the present setting is this: that not only would the impressions we get from our environment largely depend on the nature and the contingent state of our sensorium, but inversely the very environment that we wish to take in is modified by us, notably by the devices we set up in order to observe it.*

Maybe this is so - to some extent it certainly is. May be that from the newly discovered laws of quantum physics this modification cannot be reduced below certain well ascertained limits. Still I would not like to call this a direct influence of the subject on the object. For the subject, if anything, is the thing that senses and thinks. Sensations and thoughts do not belong to the 'world of energy'*, they cannot produce any change in this world of energy as we know from Spinoza and Sir Charles Sherrington.*

All this was said from the point of view that we accept the time-hallowed discrimination between subject and object. Though we have to accept it in everyday life 'for practical reference'*, we ought, so I believe, to abandon it in philosophical thought. Its rigid logical consequence has been revealed by Kant: the sublime, but empty, idea of the 'thing-in-itself' about which we forever know nothing.*

It is the same elements that go to compose my mind and the world. This situation is the same for every mind and its world, in spite of the unfathomable abundance of 'cross-references' *between them. The world is given to me only once, not one existing and one perceived. Subject and object are only one. The barrier between them cannot be said to have broken down as a result of recent experience in the physical sciences, for this barrier does not exist."*

(ScE1) chapter 4, The Arithmetical Paradox: The Oneness of Mind
p. 129: *„There is obviously only one alternative, namely the unification of minds or consciousnesses. Their multiplicity is only apparent, in truth there is only one mind. This is the doctrine of the Upanishads. And not only of the Upanishads. The mystically experienced union with God regularly entails this attitude unless it is opposed by strong existing prejudices: and this means that it is less easily accepted in the West than in the East.*

(ScE1) chapter 5, Science and Religion
p. 144-145: *„Let us now turn to Kant. It has become a commonplace that he taught the ideality of space and time and that this was a fundamental, if not the most fundamental part of his teaching. Like most of it, it can be neither verified nor falsified, but it does not lose interest on this account (rather it gains; if it could be proved or disproved it would be trivial). The meaning is that, to be spread out in space and to happen in well-defined temporal order of „before and after" is not a quality of the world that we perceive, but pertains to be perceiving mind which, in its present situation anyhow, cannot help registering anything that is offered to it according to these two card-indexes, space and time. It does not mean that the mind comprehends these order-schemes irrespective of, and before, any experience, but that it cannot help developing them and applying them to experience when this comes along, and particularly that this fact does not prove or suggest space and time to be an order-scheme inherent in that „thing-in-itself" which, as some believe, causes our experience. … It is not difficult to make a case that this is humbug."*

p. 145-146: *„However, the supreme importance of Kant's statement does not consist in justly distributing the roles of the mind and its object – the world – between them in the process of „mind forming an idea of the world", because, as I just pointed out, it is hardly possible to discriminate the two. The great thing was to form the idea that this* one thing *– mind or world – may well be capable of other forms of appearance that we cannot grasp and that do not imply the notions of space and time. This means an imposing liberation from our*

inveterate prejudice. There probably are other orders of appearance than the space-time-like. It was, so I believe, Schopenhauer who first read this from Kant."

p. 152: *„To my view the ‚statistical theory of time' has an even stronger bearing on the philosophy of time than the theory of relativity. The latter, however revolutionary, leaves untouched the undirectional flow of time, which is presupposes, while the statistical theory constructs it from the order of the events. This means a liberation from the tyranny of old Chronos."*

Schrödinger E.
What is life?
(ScE1) pp. 1-90

(ScE1) chapter 7, Is Life Based on the Laws of Physics?
p. 76: *New laws to be expected in the organism*
„What I wish to make clear in this chapter is, in short, that from all we have learnt about the structure of living matter, we must be prepared to find it working in a manner that cannot be reduced to the ordinary laws of physics. And that not on the ground that there is any „new force" or what not, directing the behaviour of the single atoms within a living organism, but because the construction is different from anything we have yet tested in the physical laboratory. To put it crudely, an engineer, familar with heat engines only, will, after inspection the construction of an electric motor, be prepared to find it workling along principles which he does not yet understand. He finds the copper familar to him in kettles used here in the form of long, long wires wound in coils; the iron familar to hum in levers and bars and steam cylinders is here filling the interior of those coils of copper wire. He will be convinced that it is the same copper and the same iron, subject to the same laws of Nature, and he is right in that. The difference in construction is enough to prepare him for an entirely different way of functioning. He will not suspect that an electric motor is driven by a ghost because it is spinning by the turn of a switch, without boiler and steam."

p. 77: *Reviewing the biological situation*
„The unfolding of events in the life cycle of an organism exhibits an admirable regularity and orderliness, unrivalled by anything we meet with in inanimate matter. We find it controlled by a supremely well-ordered group of atoms, which represent only a very small fraction of the sum total in every cell. Moreover, from the view we have formed of the mechanism of mutation we conclude that dislocation of just a few atoms within the group of „governing atoms" of the germ cell suffices to bring about a well-defined change in the large-scale hereditary characteristics of the organism.

These facts are easily the most interesting that science has revealed in our days. We may be inclined to find them, after all, not wholly unacceptable. An organism's astonishing gift of concentration of a „stream of order" on itself and thus escaping the decay into atomic chaos – of „drinking orderliness" from a suitable environment – seems to be connected with the presence of the „aperiodic solids", the chromosome molecules, which doubtless represent the highest degree of well-ordered atomic association we know of – much higher than the ordinary periodic crystal – in virtue of the individual role every atom and every radical is playing here.
To put it briefly, we witness the event that existing order displays the power of maintaining itself and of producing orderly events. That sounds plausible enough, though in finding it plausible we, no doubt, draw on experience concerning social organization and other events which involve the activity of organisms. And so it might seem that something like a vicious circle is implied."

pp. 77-78: *Summarizing the physical situation*
„However that may be, the point to emphasize again and again is that to the physicist the state of affairs is not only not plausible but most common exciting, because it is unprecedented. Contrary to the common belief, the regular course of events, governed by the laws of physics, is never the consequence of one well-ordered configuration of atoms – not unless that configuration of atoms repeats itself a great number of times, either as in the period crystal or as in a liquid or in a gas composed of a great number of identical molecules.
Even when the chemist handles a very complicated molecule in vitro he always faced with an enormous of like molecules. To them his laws apply. He might tell us, for example, that one minute after he has started some particular reaction half of the molecules will have reacted, and after a second minute three-quarters of them will have done so. But whether any particular molecule, supposing you could follow its course, will be among

those which have reacted or among those which are still untouched, he could not preduct. That is a matter of pure chance.

This is not a purely theoretical conjecture. It is not that we can never observe the fate of a single small group of atoms or even of a single atom. We can, occasionally. But whenever we do, we find complete irregularity, co-operating to product regularity only on the average. …. The Brownian movement of a small particle suspended in a liquid is completely irregular. But if there are many similar particles, they will by their irregular movement give rise to the regular phenomenon of diffusion.

The disintegration of a single radiactive atom is observable (it emits a projectile which causes a visible scintillation on a fluorescent screen). But if you are given a single radioactive atom, its probable lifetime is much less certain that that of a healthy sparrow. Indeed, nothing more can be said about it than this: as long as it lives (and that may be for thousands of years) the chance of its blowing up within the next second, whether large or small, remains the same. This patent lack of individual determination nevertheless results in the exact exponential law of decay of a large number of radioactive atoms of the same kind."

pp. 79-80: *The striking contrast*
„In biology we are faced with entirely different situation. A single group of atoms existing only in one copy produces orderly event, marvellously tuned in with each other and with the environment according to most subtle laws. I said, existing only in one copy, for after all we have the example of the egg and of the unicellular organism. In the following stages of higher organism the copies are muliplied, that is true. But to what extent? Something like 10^{14} *in a grown mammal, I understand. What is that! Only a millionth of the number of molecules in one cubic inch of air. Though comparatively bulky, by coalescing they would form but a tiny drop of liquid. And look at the way they are actually distributed. Every cell harbours just one of them (or two, if we bear in mind diploidy). Since we know the power this tiny central office has in the isolated cell, do they not resemble stations of local government dispersed through the body, communicating with each other with great ease, thanks to the code that is common to all of them?*

Well, this is a phantastic description, perhaps less becoming a scientist that a poet. However, it needs no poetical imagination but only clear and sober scientific reflection to recognize that we are here obviously faced with events whose regular and lawful unfolding is guided by a „mechanism" entirely different from the „probability mechanism" of physics. For it is simply a fact of observation that the guiding principle in every cell is embodied in a single atomic association existing only in one copy (or sometime two) – and in fact of observation that it results in producing events which are a paragon of orderliness. Whether we find it astonishing or whether we find it quite plausible that a small but highly organized group of atoms be capable of acting in this manner, the situation is unprecedented, it is unknown anywhere else ecept in living matter. The physicist and the chemist, investigating inanimate matter, have never witnessed phenomena which they had to interpret in this way. The case did not arise and so our theory does not cover it – our beautiful statistical theory of which we were so justly proud because it allowed us to look behind the curtain, to watch the magnificent order of exect physical law coming forth from atomic and molecular disorder; because it revealed that the most important, the most general, the all-embracing law of entropy increase could be understood without a special assumption ad hoc, for it is nothing but molecular disorder itself.

p. 80*: Two ways of producing orderlines*
„The orderliness encountered in the unfolding of life springs from a different source. It appears that there are two different „mechanisms" by which orderly events can be produced: the „statistical mechanism" which produces „order from disorder" and the new one, producing „order from order". To the unprejudiced mind the second principle appears to be much simpler, much more plausible. No doubt it is. That is where physicists were so proud to have fallen in with the other one, the „order-from-disorder" principle, which is actually followed in Nature and which alone conveys an understanding of the great line of natural events, in the first place of their irreversibility. But we cannot expect that the „laws of physics" derived from it suffice straightaway to explain the behaviour of living matter, whose most striking features are visible based to a large extent on the „order-from-order" principle. You would not expect two entirely different mechanisms to bring about the same type of law – you would not expect your latch-key to open your neighbour's door as well.

We must therefore not be discouraged by the difficulty of interpreting life by the ordinary laws of physics. For that is just what is to be expected from the knowledge we have gained of the structure of living matter. We

must be prepared to find a new type of physical law prevailing in it. Or are we to term it a non-physical, not to say a super-physical law?"

pp. 81-82: *The new principle is not alien to physics*
„No, I do not think that. For the new principle that is involved is a genuinely physical one: it is, in my opinion, nothing else that the principle of quantum theory over again. To explain this, we have to go to some length, including a refinement, not to say an amendment, of the assertion previously made, namely, that all physical laws are based on statistics.

This assertion, made again and again, could not fail to arouse contraction. For, indeed, there are phenomena whose conspicuous features are visible based directly on the „order-from-order" principle and appear to have nothing to do with statistics or molecular disorder.

The order of the solar system, the motion of the planets, is maintained for an almost indefinite time. The constellation of this moment is directly connected with the constellation at any particular moment in the times of the Pyramids; it can be traced back to it, or vice versa. Historical eclipses have been calculated and have found in close agreement with historical records or have even in some cases served to correct the accepted chronology. These calculations do not imply any statistics, they are based solely on Newton's law of universal attraction.

Nor does the regular motion of a good clock or of any similar mechanism appear to have anything to do with statistics. In short, all purely mechanical events seem to follow distinctly and directly the „order-from-order" principle. And if we say „mechanical", the term must be taken in a wide sense. A very useful kind of clock is, as you know, based on the regular transmission of electric pulses from the power station.

I remember an interesting little paper by Max Planck on the topic „The Dynamical and the Statistical Type of Law" („Dynamische und Statistische Gesetzmässigkeit"), (PlM). The distinction is precisely the one we have here labbelled as „order from order" and „order from disorder". The object of that paper was to show how the interesting statistical type of law, controlling large-scale events, is constituted from the „dynamical" laws supposed to govern the small-scale events, the interaction of the single atoms and molecules. The latter type is illustrated by large-scale mechanical phenomena, as the motion of the planets or of a clock, etc.

Thus it would appear that the „new" principle, the order-from-order principle, to which we have pointed with great solemnity as being the real clue to the understanding of life, is not at all new to physics. Planck's attitude even vindicates priority to it. We seem to arrive at the ridiculous conclusion that the clue to the understanding of life is that it is based on a pure mechanism, a „clock-work" in the sense of Planck's paper. The conclusion is not ridiculous and is, in my opinion, not entirely wrong, but it has to be taken „with a very big grain of salt".

(PlM) Planck M., Dynamische und Statistische Gesetzmässigkeit, (the Dynamical and the Statistical Type of Law). In: Roos, H., Hermann, A. (eds) Vorträge Reden Erinnerungen. Springer, Berlin, Heidelberg, (2001) 87-102.

Schrödinger E.
Science and humanism

(ScE3) Form, not substance, the fundamental concept
p. 122: „*The situation is rather disconcerting. You will ask: What are these particles then, if they are not individuals? And you may point to another kind of gradual transition, namely that between an ultimate particle and a palpable body in our environment, to which we do attribue individual sameness. A number of particles constitute an atom. Several atoms compose a molecule. Molecules there are of various sizes, small ones and big ones, but without there being any limit beyond which we call it a big molecule. In fact there is no upper limit to the size of a molecule, it may contain hundreds of thousands of atoms. It may be a virus or a gene, visible under the microscope. Finally we may observe that any palpable object in our environment is composed of molecules, which are composed of ultimate particles … and if the latter lack individuality, how does, say, my wrist-watch come by individuality? Where is the limit? How does individuality arise at all in objects composed of non-individuals?"*

p. 125: *„„The* new *idea is that what is permanent in these ultimate particles or small aggregates is their shape and organization. The habit of everyday language deceives us and seems to require, whenever we hear the*

word „shape“ or „form“ pronounced, that it must be the shape or form of something*, that a material substratum is required to take on a shape. Scientifically this habit goes back to Aristotle, his* causa materialis *and* causa formalis*. But when you come to the ultimate particles constituting matter, there seems to be no point in thinking of them again consisting of some material. They are, as it were,* pure shape*, nothing but shape; what turns up again and again in successive observations is this shape, not an individual speck of material.“*

Shaw B.
Zurück zu Methusalem

(ShB) p. 40: *"Die metaphysische Seite der Evolution war also nicht neu, als Darwin auftauchte. Hätte Oken niemals gelebt, würde es noch Millionen von Menschen gegeben haben, die von Kindheit an in dem Glauben gedrillt waren, daß wir ständig weitergeführt werden durch eine Kraft, die Gottes Wille heißt. Im Jahre 1819 veröffentlichte Schopenhauer seine Schrift „Die Welt als Wille und Vorstellung“, die metaphysische Ergänzung zu Lamarcks Naturgeschichte, da sie den Beweis führte, daß die treibende Kraft aller Evolution der Wille zum Leben ist, und zwar zu einem volleren Leben, wie Christus schon lange vorher gesagt hat. Und die frühen Philosophen, von Plato bis Leibniz, hatten den menschlichen Geist dem Gedanken erschlossen, daß das Weltall hinter all seinen physikalischen, faßlichen Veränderungen eine einzige Idee sei.“*

(ShB) p. 41: *"“Was denken Sie über das große Ereignis?“ fragte Goethe. …. „Unsinn!“ sagte Goethe, „ich denke gar nicht an diese Leute; ich meine den offenen Bruch zwischen Cuvier und St. Hilaire in der französischen Akademie. Der ist für die Wissenschaft von höchster Bedeutung“. Der Bruch, auf den Goethe hinzielte, betraf die Evolution: Cuvier behauptete, es gäbe vier Spezies, St. Hilaire dagegen, es gäbe nur eine.“*

(ShB) p. 42: *"Heutzutage, wenn wir angewidert und enttäuscht vom Neo-Darwisismus und Mechanismus zum Vitalismus und zur schöpferischen Evolution zurückkehren, kann man sich schwer vorstellen, wie diese neue Richtung Darwins seinen Zeitgenossen als belebend, angenehm und vor allem als hoffnungsvoll erscheinen konnte. Ich will deshalb versuchen, etwas von der Atmosphäre jener Zeit heraufzubeschwören, indem ich eine für ihren Aberglauben sehr charakterische Szene beschreibe, in der ich eine unaussprechlich schreckliche Rolle spiele.“*

Shu F. H.
The Physics of Astrophysics, Gas Dynamics
The capability of stars to organize themselves in a stable arrangement

(ShF) p. 402: *"In its purest form, Landau damping represents a phase-space behavior peculiar to collisionless systems. Analogs to Landau damping exist, for example, in the interactions of stars in a galaxy at the Lindblad resonances of a spiral downsity wave. Such resonances in an inhomogeneous medium can produce wave absorption (in space rather than in time), which does not usually happen in fluid systems in the absence of dissipative forces (an exception in the behavior of corotation resonances for density waves in a gaseous medium)."*

Smolin L.
The Trouble with Physics
The Unfinished Revolution

(SmL1) p.3 ff.*:*

„Problem 1: Combine general relativity and quantum theory into a single theory that can claim to be the complete theory of nature.

Problem 2: Resolve the problems in the foundations of quantum mechanics, either by making sense of the theory as it stands or by inventing a new theory that does make sense.

Problem 3: Determine whether or not the various particles and forces can be unified in a theory that explains them as manifestations of a single, fundamental entity.

Problem 4: Explain how the values of the three constants in the standard model of particle physics are chosen in nature.

Problem 5: Explain dark matter and dark energy. Or, if they don't exist, determine how and why gravity is modified on large scales. More generally, explain why the constants of the standard model of cosmology, including dark matter, have the values they do"

Smolin L.
Time Reborn
From the Crisis in Physics to the Future of the Universe

(SmL) p. 154: „*Can the demand for sufficient reason be satisfied even in quantum physics? This depends on whether quantum mechanics can be extended to the universe as a whole and give the most fundamental description of nature possible or is only an approximation to a very different cosmological theory. If we can extend quantum theory to the universe as a whole, then the free-will theorem applies at the cosmological scale. Since we assume there is no theory more fundamental, it implies that nature is truly free. The freedom of quantum systems at the cosmological scale would imply a limit to the principle of sufficient reason, because no rational or sufficient reason could be given for the myriad of free choises quantum systems make*."

Spatschek K. H.
Theoretische Plasmaphysik

(SpK) S. 1: „*Ein System von Teilchen oder Quasiteilchen (Ionen, Elektronen, Moleküle, Quarks, Gluonen, Löcher etc.) wird unter recht unterschiedlichen Bedingungen* Plasma *genannt. Bei der Formulierung der Bedingungen treten in der Literatur Unterschiede auf, je nachdem ob man an ionisierten Gasen, Festkörpern, an voll- oder teilionisierten Systemen, oder an makroskopisch neutralen oder nicht-neutralen Anordnungen interessiert ist. Wie so oft werden die Unterschiede und ihre Auswirkungen erst deutlich, wenn allgemeine Kenntnisse vorhanden sind, die einen Einblick in die grundsätzlich neuen Phänomene zulassen. Wir werden deshalb zunächst von einer einfachen und nicht allzu strengen Definition ausgehen und die Systeme weitgehend vereinfachen, um dann im weiteren Verlauf zu verallgemeinern und zu vertiefen.*

Bei diesem Vorgehen lassen wir uns von zwei Gesichtspunkten leiten: Wir müssen enerseits die enorm wichtigen – aber einem Themenkreis für sich darstellenden – Fragen der Struktur der einzelnen „Teilchen" ausgrenzen und wollen andererseits die charakteristischen Erscheinungen eines Vielteilchensystems mit langreichweitiger Wechselwirkung in möglichst einfacher Form herauskristallisieren. Wir starten deshalb mit der Arbeitshypothese, nach der ein Plasma ein makroskopisch neutrales Gas aus vielen elektrisch geladenen (und gegebenenfalls neutralen) Teilchen ist, dessen Verhalten wesentliche durch kollektive Freiheitsgrade bestimmt wird."

(SpK) S. 8*: „An dieser Stelle wird bereits deutlich, warum ein Plasma nicht lediglich ein – wenn auch komplizierteres – Übungsbeispiel für die klassische Elektrodynamik ist. So wie die Elektrodynamik im Rahmen von Kursvorlesungen behandelt wird, handelt es sich bei ihr um einen Theorie der elektromagnetischen Felder und der Bewegung von Teilchen in äußeren vorgegebenen Feldern. Die kollektiven Effekte, die bei der Bewegung vieler Teilchen unter Berücksichtigung der langreichweitigen Wechselwirkung auftreten, stellen demgegenüber neue Erscheinungen da, die spezifische Eigenschaften des Plasmas ausmachen. Die elektrischen Ladungen im Plasmen erzeugen elektromagnetische Felder, die ihrerseits wieder Kräfte auf die Ladungen ausüben und deren Dynamik beeinflussen. Die Beschreibung eines Plasmas muß daher bereits im einfachsten Fall in selbstkonsistenter Weise durch die mechanischen und elektromagnetischen Grundgleichungen gemeinsam erfolgen. Es ist zu beachten, daß nicht notwendig in allen „Plasmen" die Coulomb-Kräfte die einzige bzw. wesentliche Form der Wechselwirkung darstellen. Generell sollen kollektive Prozesse in Plasmen immer Vorgänge sein, an denen eine große Zahl von Teilchen in geordneter Weise teilnimmt."*

(SpK) S. 9*: „Eine detailiertere Behandlung von Plasmen erfordert offensichtlich wegen des Vielteilchencharakters Methoden der statistischen Physik. Nur wenige Erscheinungen lassen sich bereits im Rahmen sehr einfacher Modelle, z.B. des Einteilchenmodells für die Bewegung einzelner geladener Teilchen in vorgebenen eletromagnetischen Feldern, berechnen. Im Rahmen der Magnetohydrodynamik wird das Plasma als leitfähiges*

kontinuierliches Medium angesehen, das mit den Gleichungen der Hydro- und Elektrodynamik beschrieben werden kann. Das Zweiflüssigkeitenmodell erlaubt die getrennte Behandlung von Ionen und Elektronen. Im allgemeinen ist jedoch eine kinetische Beschreibung angebracht, die die verschiedenen neuen Phänomene, z.B. auch die Welle-Teilchen-Wechselwirkung, erfassen kann."

(SpK) S. 12*: „Eine wesentliche Eigenschaft fast aller Plasmen ist die Quasineutralität. Darunter versteht man die elektrische Neutralität bis in Teilvolumina, die klein im Vergleich zu dem gesamten Plasmavolumen sind. Die Quasineutralität beruht darauf, daß jeder Ladungsüberschuß aufgrund der starken elektrischen Felder, die er hervorruft, schnell wieder ausgeglichen wird. Neutrale Plasma sind solche, die makroskopisch neutral sind. In jüngster Zeit haben aber auch nichtneutrale Plasmen erheblich an Bedeutung gewonnen. Es zeigt sich, daß ein Ensemble von Elektronen oder Ionen in einer elektromagnetischen Falle ziemlich gut eine Materieform verkörpert, die als Ein-Komponenten-Plasma bezeichnet werden kann. Die neuesten Experimente in Mikroplasmen, die aus wenigen in einer Paul-Falle eingeschlossenen geladenen Teilchen bestehen, erlauben nichtideales Verhalten in (stark gekoppelten) Systemen systematisch zu studieren."*

(SpK) S. 47*: „Gleichgewichtsstatistik eines Plasmas: Das Vielteilchensystem Plasma ist im thermodynamischen Gleichgewicht mit den bekannten Methoden der Gleichgewichtsstatistik und Thermodynamik beschreibbar. Insofern stellen die Rechnungen dieses Kapitels „nur" eine Anwendung der in der entsprechenden Kursvorlesung entwickelten Prinzipien dar. Allerdings sind die Auswertungen keinesfalls trivial; im Gegenteil: in Systemen mit innerer Wechselwirkung stößt man schnell auf sehr große mathematische Schwierigkeiten, deren Auflösung bis heute Gegenstand intensiver Forschung sind."*

Treder H.-J.
Einstein-Raum
Gravitation ohne Quellen und Geometrodynamik

(TrH1) S. 42: „*Gegen die Gravitationsgleichungen*

$$(*) \quad E_{ik} = R_{ik} - \frac{1}{2} g_{ik} R = \kappa T_{ik} \,, \quad T_{ik}\text{: Materietensor,}$$

hat Einstein selbst den Einwand erhoben, daß hier auf an sich unverständliche Weise geometrische Größen mit den nichtgeometrischen Größen

$$\frac{\delta L_{mat}}{\delta g_{ik}} = \frac{1}{2}\sqrt{-g}\, T_{ik} \,, \quad L_{mat}\text{: kovariant verallgemeinerte Wirkungsfunktionen,}$$

verknüpft werden und so ein Dualismus in die Physik hineingetragen wird. So erschien es Einstein konsequenter, die geometrische Struktur der Materie völlig miteinander zu identifizieren, wie dies in der allgemeinen Relativitätstheorie für Geometrie und Gravitationsfeld gelungen war.

Während jedes nichtgravische Feld über den Materietensor T_{ik} zum Gravitationsfeld beiträgt, also notwendig mit einem Gravitationsfeld gekoppelt ist, zeigt die Einsteinsche Gravitationstheorie die Existenz von freien Gravitationsfeldern. In der Tat bedeutet der Grenzübergang $\kappa \to 0$ in den Gravitationsgleichungen (*) nicht etwa gravitationsfreie Felder, sondern Gravitation ohne Quellen. Bei Verschwinden der rechten Seite gehen die Einsteinschen Feldgleichungen (*) in die Vacuumgleichungen $R_{ik} = 0$ über, die einen Einstein-Raum definieren, der für $R^{\sigma}_{ik\mu} \neq 0$ nicht mit dem speziell relativistischen Minkowski-Raum identisch ist. Solche Einstein-Räume existieren auch dann, wenn die rechte Seite von (*) überall verschwindet."

(UnA) p. 10: „*Cosmology's „concordance model" uses six numbers, which are called „free parameters" because they cannot be explained within the model but rather are fitted to the measurements. The standard model of particle physics needs not only six of them, but impressive 17*."

(UnA) p. 11: *In his book „*The Trouble with Physics*", Lee Smolin comments on the 17 free parameters (of the SMEP). „The fact that there are that many freely specifiable constants in what is supposed to be a fundamental theory is a tremendous embarrassment*."

(UnA) p. 48: Mach vs. Newton: Space without matter doesn't matter
„*We can see already that space and time are not such simple concepts, especially when we are considering the cosmos as a whole. It is one thing to have increasingly sophisticated technology for ever more precise clocks to measure time and spacecraft to measure distance. But it is the very nature of this basis for our perception that is still puzzling. What is time? What is space? … When we say that one second today is the same as one second yesterday, what does that mean? This is not a play on words, since all we have as measures of time is the observation of Nature's periodicities. Envisioning an absolute time, with flows without any relation to matter, might be completely false, as false as Newton's notion that absolute space without matter exists.*"

(UnA) p. 49: „*He (Mach) suggested that in such a case (in a rotating bucket filled with water, the centrifugal force will make the water level rise at the inside wall of the bucket), when the bucket's wall became increasingly thicker and more massive, the centrifugal force may vanish. He argued that there is no absolute space but rather that it is distant celestial bodies that tell us what means to be at rest. In other words, all that matters in our motion relative to other masses out in the universe, without presupposing an inertial framework of absolute space*.

J*. Barbour has written books and organized conferences on Mach's principle. Barbour's central idea, portrayed in his book The End of Time, is that time is defined through the various periodicities we observe in Nature. It is a profound generalization of Mach's principle. Barbour is a truly unconventional thinker. His theory, which even calls into question the expansion of the universe, is so far off the mainstream that cosmologists must fear for their jobs if it turns out to be right*."

(UnA) p. 50: „*What can we learn from Barbour? For one thing, we can recognize that it is probably much too naive to think that time is something „objective" that runs independently from what happes in the rest of the universe. Imagine a wristwatch ticking away from the beginning of the universe, telling us when the Big Bang took place, when the atomic nuclei formed, and a little later, when cosmic background raditation emerged. But unless atoms exist, there is nothing to tick. There really is no way to count time from the precise moment of the Big Bang.*

For simple reason, we should remain skeptical about the fairy-tale stories about what happened in the 10^{-35} *seconds after the Big Bang. No clock can measure such tiny intervals, and although this is evident, many cling to this all-too-simplistic picture of time.*

Unfortunately, once you discard the idea of an imaginary wristwatch ticking away time from the moment of the Big Bang, trying to define time is not trying to nail Jell-O to the wall.The cosmologist John Barrow has noted „The question if there is a unique absolute standard of time which globally is defined by the inner geometry of the universe, is a big unresolved problem of cosmology,*" And it is not an unimportant one*."

(UnA) p. 53: „*However, classical electrodynamics has its own problems. One is that steadily accelerated electrical charges radiate energy. But remember, that, due to the equivalence of inertia and weight, acceleration and gravity are fundamentally the same thing; thus, charges should radiate energy in a gravitational field even when they just sit there. This remains an unresolved puzzle.*

Actually there are far worse problems arising from fundamental law that accelerated charges radiate light (electromagnetic radiation of any wavelength). You may think that once the acceleration is known, physics has a formula for calculating the amount of radiation. Unfortunately, it doesn't, as Richard Feynman explains in his

Lectures. Feynman's books refreshingly differ from many others in that they address unsolved problems, rather than camouflaging them under a bunch of brilliant mathematical formulae.

The deeper reason for the mystery of the inability to calculate radiation is that classical electrodynamics is inconsistent. If you combine the formula for energy density with that of force field, a single electron has an infinite amount of energy, and due to Einstein's $E = mc^2$*, it also has an infinitely great mass. Something has got to be wrong! And if people tell you that quantum electrodynamics fixed the problem, don't believe it. Feynman, who got the Nobel Prize in 1965 for his role in developing quantum electrodynamics, says it does not*."

(UnA) p. 132: A briefer history of quantum gravity
„*Since the Planck length contains the gravitational constant G and Planck's quantum h, it is the scale at which „quantum effects of gravity" are supposed to become important. Dear reader, this is all. No theory of quantum gravity exists, let alone any evidence of an observable effect.*"

(UnA) p. 133: Does the gravitational constant cement the failure of quantum gravity?
„*While Niels Bohr's quantum theory marvelously derives energy levels for the atomic shell out of the constants of nature, nuclear physics has not achieved anything comparable yet*."
(UnA) p. 135: Quantum of solace: how to escape from black holes
„*Thus, Hawking concluded, there may be a net escape of particles from black holes through this quantum effect, which is forbidden by the classical laws of gravity.*

As neat as this thought might be, it is far cry from every observation, for a black hole with a solar mass would then need 10^{66} *years to evaporate by ejecting particles*"

(UnA) p. 144: Symmetries all over the place: where is this journey taking us?
„*The beta decay process, not fitting into common scheme of a force, is called „weak interaction". Why it occurs on average after 10 minutes but not to say, after 20 minutes is unknown. Even the very reason why neutrons don't live foreever is a mystery.*"

(UnA) p. 145: The dance of electrons and light:
„*Long before the symmetry fashion took over, Richard Feynman became famous for his intriguing interpretation of the interactions of electrons, positrons, and light. The basic idea is fairly easy to grasp. Thanks to Heisenberg's uncertainty principle, a traveling electron can borrow for a little time* t *an amount of energy* $E = h/t$*. Electrons may use this energy for juggling with photons. Like two people sitting on wheeled office chairs who are throwing heavy medicine balls to one another and rolling backward every time they pitch or catch the ball, two electrons that exchange photons knock each other back, too. Feynman managed to reformulate the laws of electrodynamics—two electrons feel a repulsive force—in these funny terms. The calculations based on this have led to predictions that have been precisely tested and are considered the best-measured results of all physics (The magnetic moment of an electron (its inherent magnetism) and the so-called Lamb shift in the spectral lines of a hydrogen atom). Richard Feynman, Julian Schwinger, and Sin-Itiro Tomonaga were justifiably awarded the Nobel Prize for this in 1965. The big insight of the theory is that light and the most basic particles, electrons and positrons, show such a puzzling similarity. Yet nobody knows the reason for it*."

(UnA) p. 146: The colorful ornamentation of quantum electrodynamics
„*In former times, classical physics upheld the picture that it was the electric and gravitational fields in space that caused the accelerations of charged particles. Quantum electrodynamics completely abandons this idea in favor of the exchange of borrowed photons. Feynman's theory worked so well that particle physicists decided to use it as a blueprint for all other interactions. Though the old wave-particle quantum theory of Bohr, Heisenberg, and Schrödinger should be a caveat against describing everything with particles, the idea entered the back door and seized hold of modern physics.*

But unlike quantum electrodynamics, the results of its extension to nuclear physics, called quantum chromodynamics, are anything but precise [(*)]*. It is therefore utter speculation that imposing the concept of quantum electrodynamics on atomic nuclei is the right way to go. Nevertheless, theorists almost exclusively walk on this we—trodden path.*"

[(*)] The magnetic moment of an electron (its inherent magnetism) and the so-called Lamb shift in the spectral lines of a hydrogen atom.

(UnA) p. 146*: „Richard Feynman became famous for his intriguing interpretation of the interactions of electrons, positrons, and light.*

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(UnA) p. 146*: „Feyman's theory worked so well that particle physicists decided to use it as a blue print for all other interactions."*

(UnA) p. 151: „*The standard model of particle physics is unable to predict the observed masses of its particles. This is really quite embarrassing, given that mass is such a basic property of particles*."

(UnA) p. 212: „R. D. Precht: „*The sum of obvious little steps is not seldom a way in the wrong direction*."

Unzicker A.
Einstein's Lost Key
E. Schrödinger's estimate of the gravitational potential
A. Einstein & R. Dicke's idea of a variable speed of light
P. Dirac's Large Number Hypothesis

(UnA1) p. 117: Schrödinger's hour of glory
„*There is a real gem of physical reasoning in a completely unknown article on cosmology published in 1925 by Erwin Schrödinger, who was later awarded the Nobel Prize. Today he is best known for his essential contribution to quantum mechanics; the wave equation that bears his name, which he fould incidentally, also in 1925 (during a skiing holiday in Switzerland with a lover who remained unidentified). Schrödinger's thoughts on cosmology are perhaps no less important, even though they are entirely forgotten. He, in fact, was the first to suspect the coincidence $G \approx c^2 \frac{R_U}{M_U}$, ($R_U$ radius of the universe; M_U mass of the universe).*

Whereas the relation $G \approx c^2 \frac{R_U}{M_U}$ as such is only numerical, Schrödinger went a step further and realized that the concept of the gravitational potential φ was concealed in the formula. Potential is simply energy per mass, for which Newton had derived an expression in his theory of gravitation: $\varphi = -\frac{GM}{r}$, when a mass is at a distance r from the Sun (with mass M).

Let us point out for the moment the subtle difference from gravitational force $F = \frac{GMm}{r^2}$, where the distance is squared in the denominator. This means that the gravitational force for distant celestial bodies strongly decreases, and the gravitational force the Sun exerts on the Earth is thus hardly noticeable (apart from the tides, to which it contributes). The gravitational potential is quite a different matter: the value of the solar potential in which we find ourselves exceeds the effect of the Earth by a factor of ten – which is easy to see of one considers the two quotients $\frac{M}{r}$ (mass divided by distance).

Schrödinger noticed that too. It looked plausible to him that the influence of the even more distant masses in the Milky Way had to be larger, even though it was impossible to perceive a force. Schrödinger tried to estimate this potential and noticed, of course, that it had the same unit as the square of the speed of light, c^2. With amazing intuition he suspected that all the potentials in the universe might just add up to c^2. In Schrödinger's own words:

„This remarkable relationship states that the (negative) potential of all masses at the point of observation, calculated with the gravitational constant valid at the observation point, must be equal to half the square of the speed of light.“

In spite of the rudimentary astronomical data back then, he concluded that this indicated a far bigger universe that it was known at the time:

„Thus only a vanishingly small fraction of the inertial effect observed on Earth and in the solar system can originate from their interaction with the masses of the Milky Way.“

In a way Schrödinger had thus anticipated the discovery of the size of the cosmos in the 1930s. He further insisted hat Mach's principle had to be incorporated into the theory of relativity. In this respect, Schrödinger's intuition went beyond Einstein's. This makes it all the more bizarre that Schrödinger's work on cosmology is completely unknown even among physicists.“

(UnA1) p. 138 ff: „*There are four so-called classical tests of the general relativity theory, called light defection, gravitational redshift, radar echo delay, and the perihelion advance of the planet Mercury. ….*

These results were obtained quite naturally by Dicke, unlike the case of the perihelion advance.
A particularly comprehensible presentation deserves to be mentioned here: …. Yet the article (DeH) does no less than explain all known tests of the theory with variable speed of light“.

(UnA1) p. 150: „*P. Dirac's* Large Number Hypothesis *concerns a connection between cosmology and particle physics.*“

(UnA1) p. 151: „*In the 1930s, he started to think about the biggest structures in the universe, and this led him to the large number hypothesis.*

Dirac had pondered for many years the question of why the electric force in the universe is so much stronger than the gravitational force, despite the fact that the laws of these forces are so similar in structure.“ ….

(UnA1) p. 152: „*If we consider a hydrogen atom in which both forces are at work when a proton and an electron (with masses m_p and m_e) orbit one another, how big is the ratio of the two forces? … we get the value*

$$\frac{F_e}{F_G} = \frac{e^2}{4\pi\varepsilon_0 m_p m_e} \approx 2{,}29 \cdot 10^{39}.“$$

(UnA1) p. 154: „*Dirac now wondered how many particles there were in the universe. He divided Hubble's mass estimate by the mass of the proton and got about 10^{78}. The number of particles were obviously the square of that number 10^{39}.“*

(UnA1) p. 156: „*Why are coincidences such as Dirac's considered exotics? Assuming that the number of hydrogen atoms in the universe is proportional to the square of its size indeed appears grotesque: as if the amount of matter in the universe had to do with its surface, rather then with its volume.*

To round off the value of Dirac's observation, however, one should mention that it is in complete harmony with Ernst Mach's thoughts on gravity, though Dirac apparently never dealt with Mach. But probably he was convinced as well that the relation $\frac{M_U}{R_U} \approx \frac{c^2}{G}$ had a meaning. The fact that Dirac considered the size and the mass of the universe, the two quantities that Mach also related to the origin of gravity, constitutes another piece in this fascinating puzzle.

MACH'S PRINCIPLE 2.0

However, Dirac's observation goes beyond Mach's principle. Imagine the number of particles in the universe was a billion times larger, while simultaneously their mass was a billion times smaller. This would change nothing about Mach' s principle (or „flatness“). But it would alter Dirac's observation. In other words, Dirac was the first to insinuate that the size and the mass of elementary particles had a meaning, and that it is no coincidence that they are as large and heavy as they are. Who thought soothe same? You've guessed it – Albert Einstein“:

„The real laws of nature are much more restrictive than the ones we know. For instance would it not violate our known laws, if we found electrons of any size or iron of any specific weigth. Nature however only realizes electrons of a particular size and iron of very specific weigth."

(UnA1) p. 157: „*Considering general relativity, i.e. gravity, in the most elementary quantum system, the hydrogen atom, yields the easily measurable yet enigmatic number* $2{,}29 \cdot 10^{39}$*. It is therefore cristal clear that any theory that hopes to unify quantum theory with relativity must calculate this number and explain it, if it does not want to end up in futile verbiage. …*

(UnA1) p. 158: „*Dirac took a risk and claimed that his hypothesis would force the gravitational constant to decrease with time.*"

(UnA1) p. 159: „*Dirac's Large Number Hypothesis, which was touched only tentatively by his former colleagues, was forgotten over the years. He may even have moved away from it himself (from the second coincidence regarding mass). This was what Pascual Jordan claimed at least when, admiringly, he wrote in 1952:*

„I consider Dirac's ideas for one of the greatest insights of our time; the further study of these ideas has to be one of our principal tasks".

Unzicker A.
The Mathematical Reality
(UnA2)

(UnA) ix: *„Applied physics has been terrific success to date, and the fundamental findings of theoretical physics in the early twentieth century were among the greatest accomplishments of humankind. But that was then. Today, the major part of theoretical physics has instead gotten lost in bizzare constructs that are completely disconnected from reality, in a mockery of the methods that grounded the success of physics for 400 years."*

(UnA2) p. 4*: „This book is about fundamental physics. It aspires to form a consistent picture of reality by observing nature from the cosmos to elementary particles. The new approach I present here is based on investigating constans of nature and questioning their origin. … From this analysis it also follows that current ideas in physics, especially the standard models of particle physics and cosmology, offer very little help for real understanding. … Consequently, tihs book is also aimed specifically at mathematicians. Although their activities are often misguided by current theoretical fashions, they nevertheless have a crucial contribution to make to the understanding of nature, especially by studying the three-dimensional unit sphere that plays an essential role in those considerations. … To get an even clearer picture, it will also be helpful to have a look at the cognitive mechanisms with which the species* Homo sapiens *has struggled so far to fathom the laws of nature."*

(UnA2) p. 85 ff.: The paramount role of the proton in fundamental physics
„*Planck's constant* h *is approximately equal to the product of the speed of light, the mass* m_p *of the proton and its radius* r_p

$$h \sim \frac{\pi}{2} c \cdot m_p \cdot r_p \, .$$

The formula $h = \frac{\pi}{2} m_p r_p$ *is even valid within the current measuring limits of about one percent (!). Of course, this formula displays the definition of the Compton wavelength*

$$\lambda_C = \frac{h}{c \cdot m_p} \, .$$

However, according to current wisdom, the wavelength λ_C *calculated from the mass alone does not reflect the actual size of a particle. … Accordingly, the proton is not given a prominent role among elementary particles. In reality, however, it is the only particle in the universe, that is massive and stable at the same time. The fact that its Compton wavelength approximately matches its real extension measured by experiments is a clear indication of the paramount role of the proton in fundamental physics. …. Since the formula contains fundamental constants of nature only, it would be important to derive it from a theory.*

Dirac's observation regarding the size and mass of of particles in the universe is

$$\frac{M_U}{m_p} \sim \frac{R_U^2}{r_p^2}.$$

… without Dirac's conjecture, there cannot be no further progress at all in understanding elementrary particles. A thorough understanding would require a calculation of their masses, which is literally unthinkable in the current paradigm, because the (available nature) constants … cannot be combined in a way that the unit of a mass, kg, emerges. … Dirac's observed large numbers would automatically appear, a consequence of the fact that the very nature of mass can only be understood cosmologically, as E. Mach had suspected."

(UnA2) p. 96: Big simplicity at the big flash
„*The hydrogen atom would then be similar to an object now called positrinium, consisting of an electron and its antiparticle positron that orbit each other. The definition of the fine structure constant implies that* $\frac{1}{\alpha} \sim 137$ *is the ratio of speed of light* c *to the electron's velocity on the innermost orbit of the hydrogen atom.*"

(UnA2) p. 183: „*All in all, there are many indications that electrons, including their strange spin behavior, are described more simple by* S^3*. In any case, despite the elegant representation Dirac had developed, it cannot be claimed that this sheds light on the reason for the existence of spin*,[(*)]"

[(*)] The spin matrices introduced by Pauli 1927 are also isomorphic to the unit quaternions and the simplest non-abelian Lie-group $SU(2)$

Vagt C.
Henri Bergson's Dauer und Gleichzeitigkeit,
Über Einsteins Relativitätstheorie, (BeH)

(BeH): Einführung
Beschäftigt sich Philosophie mit Physik, gilt ihr Engagement selten den Formeln, Diagrammen oder Experimentalapparaturen der Naturwissenschaft. Worauf sie sich in der Regel konzentriert, ist die Interpretation physikalischer Aussagen und Begriffe. Dauer und Gleichzeitigkeit verfolgt die genau gegenteilige Strategie: Das Buch, das 1922 in Paris erscheint, erhebt die Physikalischen Instrumente und mathematischen Verfahren der Relativitätstheorie sowohl zum Ausgangspunkt als auch zum Argument philosophischer Reflexion. … Größtenteils (…) spielen die mathematischen Ausdrücke mögliche Aussagen der immer gleichen Formeln und Diagramme durch, die das Gerüst der speziellen Relativitätstheorie bilden.

Vielleicht liegt (….) im Explizieren und Hinterfragen der Interpretationen mathematischer und experiementeller Verfahren durch den Philosophen die Möglichkeit eine viel allgemeinere Hürde zu nehmen, nämlich jene, die Gaston Bachelard ein „epistemologisches Hindernis" nennt; etwas, das als unbewusste Hemmung immer dort entsteht, wo die gewohnte Sicht der Dinge oder die tradierten Wege der wissenschaftlichen Erkenntnis nicht mehr in Frage gestellt werden.

Weinberg S.
The First Three Minutes

„*The first One-hundredth Second: Our account of the first three minutes in Chapter 5 did not begin at the beginning. Instead, we started at a „first frame" („ein erstes Bild") when the cosmic temperature has already cooled to* 100000 *million degrees of Kelvin, and the only particles present in large numbers were photons, electrons, neutrinos and their corresponding antiparticles. If these really were the only types of particles in nature, we could perhaps extrapolate the expansion of the universe backward in time and infer that there must have been real beginning, a state of infinite temperature and density, which occurred* $0{,}0108$ *seconds bevor our first frame (our „erstes Bild")*"

Weizsäcker C. F. v.
Der begriffliche Aufbau der theoretischen Physik
a. Der Aufbau der Physik
(WeC2) S. 7-9

„*Unsere Wissenschaft ist stark beeinflusst durch die deduktiven Methoden der Mathematik. Hier werden wenige Sätze, die Axiome, vorausgesetzt, alle anderen sollen aus ihnen folgen. Die Axiome sah man früher als*

evident an, in jüngster Zeit behandelt man sie oft als Voraussetzungen, über deren Wahrscheinlichkeit nichts angenommen wird, das ganze System dann als ein Gebilde der logischen Struktur „wenn-so".

Die Physik entsteht aber offensichtlich nicht so. Näher kommt ihrem Wesen der Begriff der induktiven Wissenschaft. Das unmittelbar Gegebene sind Einzelaussagen der Erfahrung, aus denen die wenigen einfachen Grundsätze durch systematische Verallgemeinerung gewonnen werden. Der vollzogene induktive Aufbau könnte dann etwa am Ende in deduktive Form umgegossen werden.

Dieses Bild kommt der Wirklichkeit unserer Wissenschaft näher, aber es enthält entscheidende Züge nicht. Die Worte Deduktion und Induktion lassen beide für die Wissenschaft das Bild einer Pyramide entstehen, die entweder auf einer Spitze ruht, oder in einer Spitze mündet. Erinnern Sie sich demgegenüber an unsere Disposition mit der Dreiteilung: Elementare Gegebenheiten, Regionale Disziplinen, Elementare Gegenstände. In diesem Bild hat die Wissenschaft zwei Spitzen. Die Physik lässt in der Tat einen doppelten Aufbau zu.

Man kann von elementaren Gegenbenheiten ausgehen, von Begriffen wie Zahl, Zeit, Raum, Ding, Ursache, Bewegung. Dieser Aufbau führt schließlich zum Atom wie zu einem äußersten Zweig eines verästelten Baumes. Man mag dies den phänomenologischen Aufbau der Physik nennen.

Man entdeckt aber, das Begriffe wie Atome, Feld, Wellenfunktion eine neue sachliche Einheit geben, von der aus die phänomenologischen Begriffe sogar eine Kritik erfahren. Der wahre Zusammenhang der Phänomene enthüllt sich erst, wenn man hinter die Phänomene vordringt. Es deutet sich ein andersartiger gegenständlicher Aufbau der Physik an.

Welcher Aufbau ist der wahre? Wir können keinen von beiden entbehren. Der einzige Weg zu den Gegenständen führt über die Phänomene, das Verständnis der Phänomene erschließt sich erst durch Gegenstände. Es besteht eine gegenseitige Abhängigkeit beider Aufbauweisen.

Dazu kommt, dass die beiden Spitzen nicht der gewisseste, sondern der ungewisseste Teil des Systems sind; sie sind wie Berggipfel, die in die Wolken stechen. Für die gegenständliche Spitze ist das klar. Sie ist nur ein gedachter und erhoffter Punkt; in Wirklichkeit gibt es nach der gegenständlichen Seite nur eine Front der Forschung, ja man hat ausgesprochen, dass die gegenständliche Spitze unvollendbar sein könnte. Aber um die phänomenologische Seite steht es nicht besser. Ihre einfachsten Begriffe wie Raum, Ding, Kausalität, ragen ins Gebiet der Philosophie hinein, und die Philosophie ist, welches auch sonst ihrer Verdienste sein mögen, berühmt als die Wissenschaft mit den ausdauerndsten und unlösbaren Streitigkeiten. Man wäre froh, wenn man in ihr auch nur eine allgemein anerkannte Front der Forschung vorfände. Frei vom Streit ist nur gerade die Mitte der Doppelpyramide, der Bauch der Wissenschaft, die klassische Mathematik und Physik: Euklidische Geometrie, Arithmetik und Analyis, Mechanik, Thermodynamik, Elektrik, Optik usf.

Auf ähnliche Schwierigkeiten stößt jeder Versuch der Erkenntnis von Wirklichem. Wie stellt man sich zu ihnen ein? Die philosophische Wissenschaftstheorie hat bislang keine Begriffe zur Verfügung gestellt, mit deren Hilfe wir diese Lage adäquate denken könnten. Fixieren wir sie dher lieber zunächst in Gleichnissen. Heisenberg sagt, dass die abgeschlossenen Disziplinen der exakten Wissenschaft gleichsam über einer allseits unergründlichen Tiefe schweben. Man könnte die Wissenschaft mit einem Schiff vergleichen, dass zwischen der unerforschten Höhe des Himmels und der unergründlichen Tiefe des Meeres „in der Mitte ist". Und wenn wir den Fortschritt der Forschung noch in das Bild aufnehmen wollen, so können wir ein weniger poetisches Gleichniss wählen: Die Wissenschaft gleicht der Aufgabe, ein Garnknäuel zu entwirren, von dem nur in der Mitte einige Fäden freiliegen, während wir keins der Enden in der Hand halten."

Weizsäcker C. F. v.
Philosophy of Science and the Nature of Time

(DrM) p. 189: „*Aristotle derives time from motion in general; motion does not have to be cyclic. Motion, in turn, he derives from the pair of concpets potential and actual, fundamental for his philosophy. He defines motion thus: „The actuality of that which potentially is, as such, is motion." This formulation has often been missunderstood, still today some English translations (and most German ones!) give, instead of „actuality", e.g.: „the progress of its realization" or „realization of their potentiality". This translations look more plausible at the first sight, but it is of no use as a definition since the concept of „realization" presupposes the very* process *that*

is to be defined. – The definition by Aristotle, read correctly, is especially interesting because it associates time *with* potentiality*, as we will do below as well."*

(DrM) p. 190: „*It is C.F. von Weizsäcker who, on the contrary, proposed his idea of a „logic of temporal propositions" a proper status for temporality even in logic, especially for the logic of future. Up to now, though, Weizsäcker only gives programmatic sketches. It would be worthwhile developing those sketches into a system."*

(DrM) p. 193: „*C.F. von Weizsäcker picks up this thread when he gives a refutation of the „reversal" objection in his paper of 1939: The difference between past and future, which is characteristic for thermodynamics, does not mysteriously come into the theory by an approximate description. It is rather ourselves who introduce this difference from outside, just in applying probability only to future. This appears so self-evident that nobody made it explicit before 1939. In 1971, when his paper was printed again, Weizsäcker himself writes: „When I wrote it I felt that I have set forth something rather trivial". He calls his text nothing but an attempt at explaining Gibbs' word."*

(DrM) p. 195: „ „*Probability is a predicted relative frequency." – Here the relation to the structure of time becomes apparent: A probability statement always refers to future events. Even if its propositional content refers to the past, as in our example of Napoleon's birthdate, probability refrs to the future possibility that the assertion about the past fact* will *prove true.*"

(DrM) p. 197: „*Quantum mechanics can be interpreted as a generalized probability theory. We can understand it much better, again, in considering the structure of time, as intoduced by C.F. von Weizsäcker into the interpretation of quantum mechanics.*

Kolmogorov's axioms of (classical) probability calculus allow a generalization to a quantum mechanical probability theory. Kolmogorov bases his axioms on the set F of random events, where every random event is represented by a set of elementary random events. His first axiom reads:

„I. F is a field of sets."

A field of sets is what is today called a Boolean lattice (of sets). For quantum mechanics we instead use a first axiom:

„I'. F is a lattice of closed subspaces of Hilbert space."

The difference between these two axioms contains all differences between classical physics and quantum mechanics; Kolmogorov's other axioms remain the same. The differences become clearer, again, from considering the structure of time. In fact, basing the theory on a lattice of subspaces instead of a field of sets entails a fundamental indeterminism."

Weizsäcker C. F. v.
Information and Evolution

(WeC) p. 298: „*Permanency or essence -this characterizes the approach of Plato's philosophy. It is about what is, what neither becomes nor passes away, the Eidos, the form or Gestalt, just the* Wesen*, to use the term from the German language tradition. The most important examples for mathematical natural science are mathematical structures. Circles drawn in the sand appear and disappear and are not truly circles; however, about the circle itself, the mathematical circle, we have insight into its eternal structure. But Eidos is also the Just, in contrast to the never ending ambiguities of our human actions. Eidos is the model of human society, of the Politeia, as the philosopher depicts it. Eidos, in the mythical language of Timaios, is the eternal model in whose imagine Heaven and Earth are created in mathematical order.The mythical language still seems to assert a separation of the here and now from the hereafter.But this only appears to be so from our ignorance which is still caught in the appearances, the shadows on the cave wall. Neoplatonists denote the unpronounceable One, the spirit eternally contemplating the One, and the soul of the world, moving itself and all things, as the Hypothases, the substances. He who has seen the Hypothases recognizes that all appearances are in truth agitated substance."*

(WeC) p. 298: „At the height of Aristotelian abstraction matter denotes potentialilty. Potentiality exists in time; due to it there is change, kinesis, what we usually and narrowly translate with motion. substance in the sense of of Aristotle is thus form in matter. Concrete things of course come into being and decay as matter assumes form and loses it again. The form is eternal as ever new things assume it. The classic example is a biological species whose individuals always recreate their kind. „Species", appearance, is the Latin translation of Eidos. The matter does not last forever. The material in question (e.g., this wood from which a cabinet is made) is itself a concretum of the form „wood" and the elements as matter. But the elements also have form. A „first matter" without form is a more abstraction."

(WeC) p. 299: „Aristotelian physics, as can be seen, is comprehensive. On the one hand, it is quite close to the phenomena. It can be expressed in everyday language. On the other hand, with concepts of form and potentiality, it reaches a very high level of abstraction. The mechanistic world of view of early modern physics is in both aspects more narrow. It shies away from the phenomena as well as the highest abstractions. It postulates concrete models of reality beyond the phenomena: extended bodies or point masses having only geometrical or kinematic attributes, while the sensory qualities are only created as „subjective impressions" in the consciousness of the observer".

(WeC) p. 299: „Its twofold retreat, however, creates a twofold uncertainty. As substance it knows matter in space, later on perhaps force fields; as „entities" (which only linguistically is a more abstract version of „substances") also space and time. Sensory phenomena are shoved aside into the subjective. Descartes is consistent when he then introduces consciousness as a special substance. Thereby, however, the unresolvable mind body problem is created. Material substance in this model is robbed of its sensory qualities. Modern natural science has neither a model for the interaction nor for the identity of both substances. The uncertainty is indeed twofold. The successful mechanical model, on the one hand, rules out the world of the phenomena as something merely subjective. On the other hand, it also avoids a more abstract and thus more comprehensive concept of substance."

Weizsäcker C. F. v.
Die Einheit der Natur, Materie und Energie

(WeC3) S. 344: „Das Seiende der Physik ist, so scheint es, die Materie".

Weizsäcker C. F. v.
Aufbau der Physik

(WeC1) S. 48: „Physik kann begrifflich nicht klar ausgesprochen werden ohne eine gewisse Artikulation unseres Wissens von der Zeit. Daß Zeit ihr methodisch schon deshalb zugrunde liegt, weil sie eine Erfahrungswissenschaft ist, haben wir soeben gesehen. Aber auch der Inhalt physikalischer Sätze ist immer auf Zeit bezogen. Die einzelne Beobachtung findet jeweils zu einer bestimmten Zeit statt, und ein konkretes Versuchsprotokoll enthält die Angabe der Zeit des Versuches. Die Gesetze der Physik geben, wie oben lose formuliert wurde, an, welche Erscheinungen auf welche Erscheinungen folgen. Mathematisch formuliert man physikalische Gesetze meist als (hyperbolische) Differentialgleichungen nach der Zeit. Hierbei wird der sogenannte Zeitpunkt des jeweiligen Zustands oder Geschehens durch den Wert eines reellen Parameters t beschrieben. Extremalprinzipien sind andere Formulierungen desselben mathematischen Sachverhalts wie die Differentialgleichungen, die dann als ihre Eulerschen Gleichungen erscheinen; sie enthalten t als Integrationsvariable. Erhaltungssätze schließlich besagen, daß sich gewisse Größen nicht mit der Zeit ändern; sie setzen die Ableitung der betreffenden Größen nach der Zeit gleich Null."

(WeC1) S. 79: Eine Aussage soll futuristische genannt heißen, wenn sie einen zukünftigen Sachverhalt aussagt. Ein Beispiel ist: „Morgen früh wird schönes Wetter sein". Die Formulierung ist gegenwartsbezogen; mit der Zeitbestimmung „morgen" drückt der Satz das Gemeinte nur heute richtig aus. Wir werden statt dessen im allgemeinen mit futuristischen Aussagen arbeiten, in denen die Zeitangabe auf eine objektive Zeitskala bezogen ist, z.B. „am 29.6.63 früh wird in Prägraten schönes Wetter sein". Aussagen dieser Art sollen formal-perfektisch heißen, weil sie die Form der Zeitbestimmung mit den perfektischen teilen; man kann auch sagen, sie bestimmen die Zeit des Geschehenen, so wie man sie bestimmen wird, wenn das Vorhergesagte vergangen sein wird. Die hohe Präzision der Umgangssprache gestattet jedoch, wie schon bemerkt, auch keine formal-

perfektische futurische Aussage, die korrekt formuliert bleibt, wenn der in ihr bezeichnete Zeitpunkt vergangen ist. Man muß sie dann durch eine echte perfektische ersetzen, z.B. „am 29.6.63 früh war in Prägraten schönes Wetter". …. Eine futurische Aussage läßt als *futurische überhaupt keine phenomenale Rechtfertigung zu. …. Die Physik rechtfertigt sich durch den Erfolg ihrer Prophezeihungen. Der Begriff der Erfahrung wäre sinnlos, wenn Erfahrungsurteile keine Anwendung auf die jeweilige Zukunft zuließen; in diesem Sinne wurde am Anfang dieses Kapitels Erfahrung als Lernen aus der Vergangenheit für die Zukunft definiert. Die einzelne futurische Aussage, die ich heute mache, ist aber immer gerade nicht schon phänomenal gerechtfertigt. Futurische Aussagen sind demnach, im Sinne der oben eingeführten Terminologie, stets* epistemisch *begründet. Sie setzen ein doppeltes Wissen voraus: über allgemeine Gesetze, genannt Naturgesetze, und über gegenwärtige bzw. vergangene Tatbestände, aus denen der vorausgesagte Sachverhalt naturgesetzlich folgt oder folgen könnte."*

(WeC1) S. 83: Die Notwendigkeit des Rekurses auf Naturgesetze läßt sich auch aus der modalen Gestalt ablesen, die wir den futurischen Aussagen geben. An sich hat eine schlicht (also nicht modal) behauptete Aussage („morgend wird es regnen") eine Chance phenomenaler Rechtfertigung, die die entsprechende perfektische Aussage („gestern hat es geregnet") nicht hat. Die Zukunft wird Gegenwart, man muß nur warten; so wird sie sprachlich mit Recht als das auf uns Zukommende (Zu-kunft) bezeichnet. Die Vergangenheit aber wird nie Gegenwart; sie ist weggegangen, ver-gangen. Die Beschränkung auf schlicht behauptete futurische Aussagen, die sich nachher entweder bewährten oder nicht, wäre bloßes Raten; wir aber suchen Wissenschaft. In der Tat wäre sogar das Raten nicht möglich ohne den Leitfaden wenigstens einer unsystematischen Kenntnis der Regelmäßigkeiten des Geschehens. Deshalb drücken wir in der modalen Gestalt die Weise des Wissens mit aus, die in der futurischen Aussage steckt, solange *sie futurisch ist. Eben die modale Aussage läßt nun aber überhaupt keine Ja-Nein-Entscheidung durch phenomenalen Ausweis zu, so wie dies für die schlichte Aussage möglich ist, sobald sie sich auf die Gegenwart bezieht. Die Aussage „am 29.6.63 ist das Wetter schön" wird an diesem Tag durch Hinsehen* entschieden*; derselbe Blick lehrt, ob sie wahr oder falsch ist (dabei dürfen wir von der logisch irrelevanten Möglichkeit absehen, daß man sich bei gewissen Wetterlagen nicht entscheiden kann, ob man sie schön nennen will oder nicht)."*

(WeC1) S. 85: „Die Kompliziertheit des Geschehens gibt uns Anlaß zur Einführung zweier für das Folgende wichtiger Begriffe, des Objektes und der Frage. Streng genommen hängt in der Welt alles mit allem zusammen. Will man aber eine bestimmte Vorhersage Np_t oder Mp_t entscheiden, so kann man nicht alle auf das Ereignis einwirkenden Faktoren berücksichtigen. Man vernachlässigt in der Praxis gewisse Einflüsse und nimmt die entstehende Ungewißheit der Vorhersage in Kauf. Diese Einschränkung der Fragestellung schematisieren die beiden genannten Begriffe. Wir betrachten nicht Fragen des Allgemeinheitsgrades: „was wird zur Zeit t überhaupt geschehen?", sondern nur Fragen, für die ein Katalog möglicher Antworten *schon vorgelegt ist; diese wollen wir im terminologisch engen Sinn als „Fragen" bezeichnen. …. Besonders interessieren uns* zeitüberbrückende *Fragen. …. Ein solcher* zeitüberbrückender *Antwortenkatalog heißt dann oft eine* Größe*, die möglichen Antworten heißen die möglichen* Werte *dieser Größe."*

Welzer H.
Nachruf auf mich selbst

(WeH4) S. 221:

3. Ich möchte, dass in meinem Nachruf steht:
Er hat sich stets bemüht, gute Fehler zu machen.

(WeH4) S. 235:

6. Ich möchte, dass in meinem Nachruf steht:
Er hat einen Unterschied gemacht.

(WeH4) S. 238:

7. Ich möchte, dass in meinem Nachruf steht:
Er hat Menschen Handlungsspielräume eröffnet.

(WeH4) S. 260: „Und es war die Rede davon, dass Zeit eigentlich keine Kategorie für die Bemessung des Lebens ist: Unter Gesichtspunkten des Sinns des Lebens ist es egal, wie lange es dauert. Man muss sich von dem Gedanken emanzipieren, dass ein Leben „zu kurz" sei und jemand „zu früh" gehe. Der Sinn eines Lebens, das sich – in den Worten von Johannes Heimraths – gewagt hat, hängt nicht von seiner Dauer ab. Wir denken das nur ersatzweise, wie in einer Ausweichbewegung, weil uns die Moderne mit dem wilden, privaten Tod konfrontiert, den wir jeder für uns allein sterben müssen – und das wollen wir logischerweise so lange wie möglich hinauszögern. So kommt die Kategorie der Dauer überhaupt erst ins Spiel, als logische Folge der Angst. Wenn, wie Wittgenstein sagt, die „Lösung des Rätsels des Lebens in Raum und Zeit (…) außerhalb von Raum und Zeit" liegt, hat das etwas zutiefst Versöhnliches, denn wir werden dieses Rätsel nie lösen können. Nicht, solange man lebt. Alles andere ist alles andere.

15. Ich möchte, dass in meinem Nachruf steht:
Er hatte gelernt, keine Angst vor dem Tod zu haben.

Na ja, fast keine.

Weyl H.
Philosophy of Mathematics and Natural Science

Matter and Fields
Ether

(WeH) p. 171: *„Just as the velocity of a water wave is not a substantial but a phase velocity, so the velocity with which an electron moves is only the velocity of an ideal „center of energy", constructed out of the field distribution. According to this view, there exists but one kind of natural laws, namely, field laws of the same transparent nature as Maxwell had established for the electromagnetic field. The obscure problem of laws of interaction between matter and field does not arise. This conception of the world can hardly be described as dynamical any more, since the field is neither generated nor acting upon an agent separate from the field, but following its own laws is in a quiet continuous flow. It is of the essence of the continuum. Even the atomic nuclei and the electrons are not ultimate unchangeable elements that are pushed back and forth by natural forces acting upon them, but they are themselves spread out continuously and are subject to fine fluent changes.*

On the basis of rather convincing general considerations G. Mie in 1912 pointed out a way of modifying the Maxwell equations in such a manner that they might possibly solve the problem of matter, by explaining why the field possesses a granular structure and why the knots of energy remain intact in spite of the back-and-forth flux of energy and momentum. The Maxwell equations will not do because they imply that negative charges compressed in an electron explode; to guarantee their coherence in spite of Coulomb's repulsive forces was the only service still required of the substance by H. A. Lorentz's theory of electrons. The preservation of the energy knots must result from the fact that the modified field laws admit only of one state of field equilibrium. …"

Relationship of physics to chemistry & biology
Organic & inorganic matter

(WeH) p. 266: *„The current understanding of the relationship of physics and chemistry may be briefly sketched by the statement that*

> „the valence bonds are an abbreviated symbol for the actual quantum-physical forces acting between the atoms, which themselves are complex dynamical system""

(WeH) p. 276: *„The current understanding of the relationship of physics and biology may be briefly sketched by the statement that*

> „One of the profoundest enigmas of nature is the contrast of dead and living matter. ….
> Incidentally, the gap between organic and inorganic matter has been bridged to a certain extent by the discovery of virusses. Virusses are submiroscopic entities that behave like dead inert matter unless placed in certain living cells. …. Many virusses have the structure typical of inorganic matter; they are crystals."

(WeH) pp. 276-278: *„Incidentally, the gap between organic and inorganic matter has been bridged to a certain extent by the discovery of viruses. Viruses are submicroscopic entities that behave like dead inert matter unless placed in certain living cells. As parasites in these cells, however, they show the fundamental chracteristics of life – self-duplication and mutation. On the other hand many viruses have the structure typical of inorganic matter; they are crystals. In size they range from the more complex protein molecules tot he smaller bacteria. Chemically they consist of nucleo-protein, as the genus do. A virus is clearly something like a naked gene. The best studied virus, that of tobacco mosaic disease, is a nucleo-protein of high molecular weight consisting of 95 per cent protein and 5 per cent nucleic acid; it cristallizes in long thin needles*. …

The specific properties of living matter will have to be studied within the general laws valid for all matter; the viewpoint of holism that the theory of life comes first and that one descends from there sort of deprivation to inorganic matter must be rejected. It is therefore significant that certain simple and clearcut traits of wholeness, organization, acausality, are ascribed by quantum mechanics to the elementary constituents of all matter. ….

The quantum physics of atomic processes will become relevant for biology wherever in the life cycle of an organism a moderate number of atoms exercises a steering effect upon the large scale happenings. …. On a broad empirical foundation, genetics furnishes the most convincing proof that organisms are controlled by processes of atomic range, where the acausality of quantum mechanics may make itself felt. … The mere fact of such X-rays induced mutations proves that the genes are physical structures. …

By ingenious methods H. J. Muller, N. W. Timoféeff-Ressowsky, and others have succeeded in establishing simple quantitive laws concerning the rate of induced mutations. These results indicate that the mutation is brought about by a single hit, not by the concerted action of several hits, and that this hit consists of an ionization, and is not, as one might have thought, a process directly released by the X-ray photon or absorbing the whole energy of the secondary electron.

These facts suggest the hypothesis that a gene is a (nucleo-protein) molecule of highly complicated structure, that a mutation consists in a chemical change of this molecule brought about by the effect of an ionization on the bonding electrons, and that thus allele genes are essentially isometric molecules."

Weyl H.
Space, Time, Matter
The Mie Theory

(WeH1) pp. 206-208: „*The theory of Maxwell and Lorentz cannot hold for the interior of the electron; therefore, from the point of view of ordinary theory of electrons we must treat the electron as something given* a priori*, as a foreign body in the field. A more general theory of electrodynamics has been proposed by Mie, by which it seems possible to derive the matter from the field.*

We shall sketch its outline briefly here – as an example of a physical theory fully conforming with the new idea of matter, and one that will be of good service later. It will give us an opportunity of formulating the problem of matter a little bit more clearly.

We shall retain the view that the following phase-quantities are of account: (1) the four-dimensional current vector s, the „electricity"; (2) the linear tensor of the second order F, the „field". Their properties are expressed in the equations

$$(1)\quad \frac{\partial s^i}{\partial x_i} = 0$$
$$(2)\quad \frac{\partial F_{kl}}{\partial x_i} + \frac{\partial F_{li}}{\partial x_k} + \frac{\partial F_{ik}}{\partial x_l} = 0.$$

Equations (2) hold if F is derivable from a vector Φ_i according to the formula

$$(3)\quad F_{ik} = \frac{\partial \Phi_i}{\partial x_k} - \frac{\partial \Phi_k}{\partial x_i}.$$

Conversely, it follows from (2) that a vector Φ must exist such that equations (3) hold.
In the same way (1) is fulfilled if s is derivable from a skew-symmetrical tensor H of the second order according to

$$(4)\quad s^i = \frac{\partial H^{ik}}{\partial x_k}.$$

Conversely, it follows from (1) that a tensor H satisfying these condititons must exist. Lorentz assumed generally, not only for the ether, but also for the domain of electrons, that $H = F$. Following Mie, we shall make the more general assumption that H is not a mere number of calculation but has a real significance, and that its components are, therefore, universal functions of the primary phase-quantities s and F. To be logical we must then make the same assumptions about Φ. The resultant scheme of quantities

Φ	F
s	H

contains the quantities of intensity in the first row; they are connected with one another by the differential equations (3). In the second row we have the quantities of magnitude, for which the differential quantities (4) hold. If we perform the resolution into space and time and use the same terms as in §20 we arrive at the well-known equations

$$(1)\ \frac{d\rho}{dt} + div(s) = 0$$
$$(2)\ \frac{dB}{dt} + curlE = 0 \qquad (divB = 0),$$
$$(3)\ \frac{df}{dt} + grad\Phi = E \qquad (-curlf = B),$$
$$(4)\ \frac{dD}{dt} - curlH = -s \qquad (divD = \rho).$$

If we know the universal functions, which express Φ and H in terms of s and F, then, excluding the equations in the brackets, and counting each component separately, we have ten „principal equations" before us, in which the derivatives of the ten phase-quantities with respect to the time are expressed in relation to themselves and their spacial derivatives; that is, we have physical laws in the form that is demanded by the principle of causality. The principle of relativity that here appears as an antithesis, in a certain sense, to the principle of causality, demands that the principle equations be accompanied by the bracketed „subsidiary equations," in which no time derivatives occur. The conflict is avoided by noticing that the subsidiary equations are superfluous. For it follows from the principle equations (2) and (3) that

$$\frac{\partial}{\partial t}(B + curlf) = 0,$$

and from (1) and (4) that

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial t}(divD).$$

It is instructive to compare Mie's Theory with Lorentz's fundamental equations of the theory of electrons.

In the latter, (1), (2), and (4) occur, whilst the law by which H is determined from the primary phase-quantitites is simply expressed by $D = E$, $H = B$. On the other hand, in Mie's theory, Φ and f are defined in (3) as the result of a process of calculation*, and there is no law that determines how these potentials depend on the phase-quantitities of the field and on the electricity.*

In place of this we find the formula giving the density of the mechanical force and the law of mechanics, which governs the motion of electrons under the influence of this force.

Since, however, according to the new view which we have put forward, the mechanical law must follow from the field-equations, an addendum becomes necessary; for this purpose, Mie makes the assumption that, Φ and f aquire a physical meaning in the sense indicated.

We may, however, enunciate Mie's equation (3) in a form fully analogous to that of the fundamental law of mechanics. We contrast the ponderomotive force occurring in it with the „electrical force" E in this case.

In the statistical case (3) states that

$$(*)\quad E - grad(\Phi) = 0$$

that is, the electric force E is counterbalanced in the ether by an „electrical pressure" Φ. In general, however, a resulting electrical force arises which, by (3), now belongs to the magnitude f as the „electrical momentum". It inspired us with wonder to see how, in Mie's Theory, the fundamental equation of electrostatics (*) which stands at the commencement of electrical theory, suddenly acquires a much more vivid meaning by the appearance of potential as an electrical pressure; this is the required cohesive pressure that keeps the electron together."

Weyl H.
Philosophy of Mathematics and Natural Science
Was ist Materie?

(WeH2) S. 18: *„Ich bin fest davon überzeugt, daß die Substanz heute ihre Rolle in der Physik ausgespielt hat. …Die Physik muß sich ebenso der ausgedehnten Substanz entledigen."*

(WeH2) p. 51: *„The classical philosopher of a dynamic world presentation is Leibniz. … For him the real of movement does not lie in a pure change of the location, but in a moving force „La substance est un etre capable d'action – une force primitive – overspatial, immaterial. … The last element is the dynamic point, from which the force erupts as an otherworldly power, an indecomposable strechless unit: the monade."*

Wheeler J. A.
The boundary of the boundary principle and geometrodynamics

(CiI) p. 49: *Einstein's "general relativity" or ""geometric theory of gravitation" or "geometrodynamics", has two central ideas:*

(1) Spacetime geometry "tells" mass-energy how to move; and
(2) mass-energy "tells" spacetime geometry how to curve.

ad (1): We have just seen that the way spacetime tells mass-energy how to move, is automatically obtained from the Einstein field equations by using the identity of Riemannian geometry, known as the Bianchi identity, which tells us that the covariant divergence of the Einstein tensor is zero.

In other words, Einstein geometrodynamics has the important and beautiful property that the equations of motion are a direct mathematical consequence of the Bianchi identities

ad (2): According to an idea of extreme simplicity of the laws at the foundations of physics, what one of us has called „the principle of austerity" or „law without law at the basis of physics", in geometrodynamics it is possible to derive the dynamical equations for matter and fields from the extremely simple but central identity of algebraic topology: the principle that the boundary of the boundary of a manifold is zero."

Whitehead A. N.
Process and Reality
An Essay in Cosmology

(WhA1) p. 166: *„The reformed subjectivist principle adopted by the philosophy of organism is merely an alternative statement of the principle of relativity. This principle states that it belongs to the nature of a „being" that it is a potential for everything „becoming."*

(PfM) S. 84: *„Whitehead nennt sein Werk „Prozess und Realität". Ein Prozess ist das Werden eines wirklichen Einzelwesens (actual entities). Realität besteht aus Prozessen. Realität ist ein gewordenes wirkliches Einzelwesens bzw. was viele gewordene Einzelwesen ergeben. Man kann auch sagen: Die Wirlichkeit ist die Innenseite der actual identities, ihr Werden, ihr Prozess. Die Realität ist die Außenseite, die gewordenen actual identities von außen erfahren. Eine Konsequenz daraus ist, dass in der Prozessphilosophie Zeit und Raum aus den Prozessen der actual identities entsteht. „Zeit" ist somit eine spezielle Verknüpfung der Realität; sie ist metaphysisch fundiert vom Prozess."*

Wigner E.

(WiE): „The Unreasonable Effectiveness of Mathematics in the Natural Sciences: „We now have, in physics, two theories of great power and interest: the theory of quantum phenomena and the theory of relativity. These two theories have their roots in mutually exclusive groups of phenomena. Relativity theory applies to macroscopic bodies, such as stars. The event of coincidence, that is, in ultimate analysis of collision, is the primitive event in the theory of relativity and defines a point in space-time, or at least would define a point if the colliding panicles were infinitely small. Quantum theory has its roots in the microscopic world and, from its point of view, the event of coincidence, or of collision, even if it takes place between particles of no spatial extent, is not primitive and not at all sharply isolated in space-time. The two theories operate with different mathematical concepts -

the four dimensional Riemann space and the infinite dimensional Hilbert space, respectively. So far, the two theories could not be united, that is, no mathematical formulation exists to which both of these theories are approximations. All physicists believe that a union of the two theories is inherently possible and that we shall find it. Nevertheless, it is possible also to imagine that no union of the two theories can be found. This example illustrates the two possibilities, of union and of conflict, mentioned before, both of which are conceivable.

In order to obtain an indication as to which alternative to expect ultimately, we can pretend to be a little more ignorant than we are and place ourselves at a lower level of knowledge than we actually possess. If we can find a fusion of our theories on this lower level of intelligence, we can confidently expect that we will find a fusion of our theories also at our real level of intelligence. On the other hand, if we would arrive at mutually contradictory theories at a somewhat lower level of knowledge, the possibility of the permanence of conflicting theories cannot be excluded for ourselves either. The level of knowledge and ingenuity is a continuous variable and it is unlikely that a relatively small variation of this continuous variable changes the attainable picture of the world from inconsistent to consistent. [This passage was written after a great deal of hesitation. The writer is convinced that it is useful, in epistemological discussions, to abandon the idealization that the level of human intelligence has a singular position on an absolute scale. In some cases it may even be useful to consider the attainment which is possible at the level of the intelligence of some other species. However, the writer also realizes that his thinking along the lines indicated in the text was too brief and not subject to sufficient critical appraisal to be reliable.]"

Considered from this point of view, the fact that some of the theories which we know to be false give such amazingly accurate results is an adverse factor. Had we somewhat less knowledge, the group of phenomena which these "false" theories explain would appear to us to be large enough to "prove" these theories. However, these theories are considered to be "false" by us just for the reason that they are, in ultimate analysis, incompatible with more encompassing pictures and, if sufficiently many such false theories are discovered, they are bound to prove also to be in conflict with each other. Similarly, it is possible that the theories, which we consider to be "proved" by a number of numerical agreements which appears to be large enough for us, are false because they are in conflict with a possible more encompassing theory which is beyond our means of discovery. If this were true, we would have to expect conflicts between our theories as soon as their number grows beyond a certain point and as soon as they cover a sufficiently large number of groups ofphenomena. In contrast to the article of faith of the theoretical physicist mentioned before, this is the nightmare of the theorist. Let us consider a few examples of "false" theories which give, in view of their falseness, alarmingly accurate descriptions of groups of phenomena. With some goodwill, one can dismiss some of the evidence which these examples provide. The success of Bohr's early and pioneering ideas on the atom was always a rather narrow one and the same applies to Ptolemy's epicycles. Our present vantage point gives an accurate description of all phenomena which these more primitive theories can describe. The same is not true any longer of the so-called free-electron theory, which gives a marvelously accurate picture of many, if not most, properties of metals, semiconductors, and insulators. In particular, it explains the fact, never properly understood on the basis of the "real theory," that insulators show a specific resistance to electricity which may be 10 26 times greater than that of metals. In fact, there is no experimental evidence to show that the resistance is not infinite under the conditions under which the free-electron theory would lead us to expect an infinite resistance. Nevertheless, we are convinced that the free-electron theory is a crude approximation which should be replaced, in the description of all phenomena concerning solids, by a more accurate picture.

If viewed from our real vantage point, the situation presented by the free-electron theory is irritating but is not likely to forebode any inconsistencies which are unsurmountable for us. The free-electron theory raises doubts as to how much we should trust numerical agreement between theory and experiment as evidence for the correctness of the theory. We are used to such doubts.

Wigner E.

(WiE1): „Gauge invariance as a dynamical principle: In addition, the concept of symmetry and invariance has been extended into a new area – an area where its roots are much less close to direct experience and observation than in the classical area of space-time symmetry … This relation is not quite the same for the classical invariance principles, which will be called geometrical, and the new ones, which will be called dynamical."

7. References and related literature

(AbM) Abramowitz M., Stegen I. A., Handbook of Mathematical Functions, Dover Publications, Inc., New York, 1965

(AcM) Acevedo M., López-Bonilla J., Sánchez-Meraz M., Quaternions, Maxwell Equations and Lorentz Transformations, Apeiron Vol. 12, No. 4, (2005) p. 371-384

(AlO) Alexandersson O., Lebendes Wasser, Viktor Schauberger und das Geheimnis natürlicher Energie, Ennsthaler Verlag Steyr, 1998

(AnE) Anderson E., The Problem of Time, Quantum Mechanics Versus General Relativity, Springer, International Publishing AG, 2017

(AnJ) Annett J. F., Superconductivity, Superfluids and Condensates, Oxford University Press, Oxford, 2004

(AnM) Anderson M. T., Geometrization of 3-manifolds via the Ricci flow, Notices Amer. Math. Soc. 51, (2004) 184-193

(ArA) Arthurs A. M., Complementary Variational Principles, Clarendon Press, Oxford, 1970

(AsP) Asmus P., Das Ich und das Ding an sich, Geschichte ihrer begrifflichen Entwicklung in der neuesten Philosophie, reprint, FlU-Verlag, 2004

(AvR) Avenarius R., Philosophie als Denken der Welt gemäss dem Prinzip des kleinsten Kraftmasses; Prolegomena zu einer Kritik der reinen Erfahrung, J. Guttentag, Verlagsbuchhandlung, Berlin, 1903

(AzA) Aziz A. K., Kellogg R. B., Finite Element Analysis of a Scattering Problem, Math. Comp. Vol. 37, No 156, (1981) 261-272

(AzT) Azizov T. Ya., Iokhvidov I. S., Linear Operators in Spaces with an Indefinite Metric, John Wiley & Sons, Chichester, New York, Brisbane, Toronto, Singapore, 1989

(AtT1) Azizov T. Ya.,Ginzburg Yu. P., Langer H., On M. Krein's papers in the theory of spaces with an indefinite metric, Ukrainian Mathematical Journal, Vol. 46, Nos. 1-2, 1994

(BaA) Bartholomew A., Hidden Nature, The Startling Insights of Viktor Schauberger, Floris Books, Edinburgh, 2003

(BaJ) Barbour J. B., Einstein and Mach's Principle, In: Janssen M., Norton J. D., Renn J., Sauer T., Stachel J. (eds), The Genesis of General Relativity, BSPS, Vol. 250, Springer, Dordrecht, 2007

(BaJ1) Barbour J.,B., The End of Time, The Next Revolution in Our Understanding of the Universe, Phoenix, London, 2000

(BeH) Bethe H. A., Morrison P., Elementary Nuclear Theory, Dover Publications, Inc., Mineola, New York, 2006

(BeH) Bergson H., Dauer und Gleichzeitigkeit, Über Einsteins Relativitätstheorie, Philo Fine Arts, Hamburg, 2014

(BeH1) Bergson H., Schöpferische Entwicklung, Elibron Classics, Adamant Media Corporation, 2006

(BeH2) Bergson H., Creative Evolution, Zinc Read, 2023

(BeM) Berry M., Kosmologie und Gravitation, Teubner Studienbücher, Stuttgart, 1990

(BiJ) Biitencourt J. A., Fundamentals of Plasma Physics, Springer, New York, Berlin, Heidelberg, 2004

(BiJ1) Binney J., Tremaine S., Galactic Dynamics, Princeton University Press, Princeton, Oxford, 2008

(BlD) Bleecker D., Gauge Theory and Variational Principles, Dover Publications, Inc., Mineola, New York, 1981

(BlS) Blundell S., Magnetism in Condensed Matter, Oxford University Press, Oxford, 2001

(BoD) Bohm D., The Special Theory of Relativity, Routledge Classics edition, London, New York, 1996

(BoD1) Bohm D., Wholeness and the Implicate Order, Routledge & Kegan Paul, London, 1980

(BoD2) Bohm D., Quantum Theory, Dover Publications, Inc., New York, 1989

(BoH) Bondi H., Gold T., The Steady-State Theory of the Expanding Universe, Monthly Notices, Royal Astronomical Society, No 3, Vol. 108, 1948, 252- 270

(BoH1) Bondi H., The steady-state theory of cosmology and relativity, Helvetica Physical Acta, Vol. 29, 1956, 152-154

(BöG) Böhme G., Idee und Kosmos, Platons Zeitlehre – Eine Einführung in seine theoretische Philosophie, Philosophische Abhandlungen Band 66, Vittorio Klostermann, Frankfurt a. M., 1996

(BöG1) Böhme G., Olschanski R., Licht und Zeit, Wilhelm Fink Verlag, München, 2004

(BöG2) Böhme G., Böhme H., Das Andere der Vernunft, Zur Entwicklung von Realitätsstrukturen am Beispiel Kants, Suhrkamp, Frankfurt a. M., 1996

(BoJ) Bognar J., Indefinite Inner Product Spaces, Springer-Verlag, Berlin, Heidelberg, New York, 1974

(BrK) Braun K., Interior Error Estimates of the Ritz Method for Pseudo-Differential Equations, Jap. Journal of Applied Mathematics, Vol. 3, No. 1, (1986) 59-72

(BrK1) Braun K., An alternative Schrödinger (-Calderón) momentum operator, 2017, unified-field-theory.de

(BrK2) Braun K., Global existence and uniqueness of the non-stationary 3D-Navier-Stokes initial-boundary value problem, navier-stokes-equations.com

(BrK3) Braun K., Extracts from the book „Kant's Theory of Natural Science" from Peter Plaass

(BrJ) Braunbeck J., Der andere Physiker, Das Leben von Felix Ehrenhaft, Leykam, Technisches Museum München, Wien, 2003

(CaC) Callender C., The Oxford Handbook of Philosophy of Time, Oxford University Press, Oxford, 2011

(CaE) Cassirer E., Kants Leben und Lehre, Berlin, 1918; Copyright 1975 by Yale University Press, New Haven

(CaF) Cap F., Lehrbuch der Plasmaphysik und Magnetohydrodynamik, Springer-Verlag, Wien, New York, 1994

(CaF1) Capra F., The Tao of Physics, An Exploration of the Parallels Between Modern Physics and Eastern Mysticism, Shambhala Publictions, Inc., Colorado, 1975

(CaF2) Capra F., Das Tao oder Physik, Die Kovergenz von westlicher Wissenschaft und östlicher Philosophie, O. W. Barth Verlag, München, 2012

(CaJ) Cao J., DeTurck D., The Ricci Curvature with Rotational Symmetry, American Journal of Mathematics 116, (1994), 219-241

(CaJ1) Canales J., The physicist & the philosopher, Einstein, Bergson, and the debate that changed our understanding of time, Princeton University Press, Princeton, Oxford, 2015

(CeC) Cercignani C., Theory and application of the Boltzmann Equation, Scottish Academic Press, Edingburgh, London, 1975

(ChD) Christodoulou D., Klainerman, The Global Nonlinear Stability of the Minkowski Space, Princeton University Press, New Jersey, 1993

(ChF) Chen F. F., Introduction to Plasma Physics and Controlled Fusion, Volume I: Plasma Physics, Plenum Press, New York, 1984

(ChJ) Chabrowski J., Variational Methods for Potential Theory, Walter de Gruyter, Berlin, New York, 1997

(ChS) Choroszavin S. A., A Reducing of the Invariant Semidefinite Subspace Problem for Krein Noncontraction to such a Problem for Krein Isometry, arXiv:math/9909101

(ChS1) Choroszavin S. A., An Example of J-unitary Operator. Solving a Problem, Stated by M.G. Krein, arXiv:1304.6144v4

(ChS2) Choroszavin S. A., Models of Z-Orbits of Unitary in Indefinite Inner Product Spaces Operators, arXiv:1305.3505

(CiI) Ciufolini I., Wheeler J. A., Gravitation and Inertia, Princeton University Press, Princeton, New Jersey, 1995

(ClR) Clark R. C., Derrick G. H., Mathematical Methods in Solid State and Superfluid Theory, Oliver & Boyd, Edingburgh, 1967

(CoP) Constantin P., Gallavotti G., Kazhikhov A. V., Meyer Y., Ukai S., Mathematical Foundation of turbulent Viscous Flows, Lectures given at the C.I.M.E. Summer School held in Martina Franca, Italy, September 1-5, 2003

(CoR) Courant R., Hilbert D., Methods of Mathematical Physics Volume II, J. Wiley & Sons, New York, 1989

(CuS) Cui S., Global well-posedness of the 3-dimensional Navier-Stokes initial value problem in $L^p \cap L^2$ with $3 < p < \infty$, April 2012, arXiv.org > math > arXiv:1204.5040

(DaJ) Davidson J., The Secrete Of The Creative Vacuum – Man And The Energy Dance, Daniel Company Limited, Essex, 1989

(DaJ1) Davidson J., Das Geheimnis des Vakuums, Omega-Verlag, Düsseldorf, 1996

(DeG) Deleuze G., Differenz und Wiederholung, Wilhelm Fink Verlag, München, 2007

(DeG1) Deleuze G., Die Falte, Leibniz und der Barock, Suhrkamp Verlag, Frankfurt a. M., 2023

(DeG2) Deleuze G., Nietzsche, ein Lesebuch von Gilles Deleuze, Merve Verlag, Berlin, 1979

(DeH) Dehnen H., Hönl H., Westphal K., Ein heuristischer Zugang zur allgemeinen Relativitätstheorie, Annalen der Physik, Vol. 461, No. 7-8, (1960), 370-406

(DeJ) Derbyshire J., Prime Obsession, Joseph Henry Press, Washington D.C., 2003

(DeK) Dee K., The Implosion Theory of Universe Creation, Khun Dee's Story, Blue Star, 2018

(DeP) Deligne P. et. al., Quantum Fields and Strings: A Course for Mathematicians, Volume 1, AMS, Institute of Advanced Study, 2000

(DeP1) Deligne P. et. al., Quantum Fields and Strings: A Course for Mathematicians, Volume 2, AMS, Institute of Advanced Study, 2000

(DeR) Dendy R. O., Plasma Dynamics, Oxford Science Publications, Oxford, 1990

(DeR1) Descartes R., Abhandlung über die Methode, richtig zu denken und Wahrheit in den Wissenschaften zu suchen, Holzinger, Berliner Ausgabe, 2016

(DeR2) Descartes R., Discourse on the method, Translated by John Veitch, printed in Poland by Amazon Fulfillment, Wrodaw

(DiP) Dirac P. A. M., The physical interpretation of quantum mechanics, Proc. Roy. Soc. London Ser. A 180, (1942) 1-40

(DiP1) Dirac P. A. M., Classical Theory of Radiating Electrons, Proc. Roy. Soc. London Ser. A 167, (1938) 148-169

(DiP2) Dirac P. A. M., A new basis for cosmology, Proc. Roy. Soc. London Ser. A 165, (1938) 199-208

(DiP3) Dirac P. A. M., The physical interpretation of quantum mechanics, Proc. Roy. Soc. London Ser. A 180, (1942) 1-40

(DrM) Drieschner M., Weizsaecker‘s philosophy of science and the nature of time, in: Castell L., Ischebeck O., Time, Quantum and Information, Springer, Berlin, Heidelberg, 2003, 187-202

(DüH) Dürr, H.-P., Heisenberg, W., Mitter, H., Schlieder, S., Yamazaki, K., Zur Theorie der Elementarteilchen, Zeitschrift für Naturforschung, 14a, 1959, S. 441-485

(DüH1) Dürr H.-P., Geist, Kosmos und Physik, Gedanken über die Einheit des Lebens, Crotona Verlag, 2016

(EbH) Ebbinghaus H.-D., et. al., Numbers, Springer Science + Business Media, New York, 1991

(EcU) Eco U., Kant und das Schnabeltier, Wissenschaftliche Buchgesellschaft, Darmstadt, 2000

(EcU1) Eco U., Die Geschichte der Schönheit, Carl Hanser Verlag, München, Wien, 2004

(EcU2) Eco U., Einführung in die Semiotik, Wilhelm Fink Verlag, München, 1994

(EhF) Ehrenhaft, F., Photophoresis and Its Interpretation by Electric and Magnetic Ions, Journal of the Franklin Institute, 233, (1942), 235-256

(EhF1) Ehrenhaft F., Banet L., Magnetization of Matter by Light, Nature, No. 3723, March 8, 1941

(EiA) Einstein A., Mein Weltbild, Ullstein, 2019; „Zur Methodik der theoretischen Physik“, Vortrag in der Royal Society, London 1929

(EiA1) Einstein A., The World as I See it, eBook

(EiA2) Einstein A., Lichtgeschwindigkeit und die Statik des Gravitationsfeldes, Annalen der Physik, Vol. 38, (1912), 355-369

(EiA3) Einstein A., Podolsky B., Rosen N., Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? Phsical Review, Vol. 47, 1935

(EiA4) Einstein A., The meaning of relativity, Princeton University Press, New Jersey, 1921

(EiA5) Einstein A., Ether and the theory of relativity, An Address delivered on May 5th, 1920, in the University of Leyden

(EiA6) Einstein A., Grundzüge der Relativitätstheorie, Friedr. Vieweg & Sohn, Braunschweig, 1969

(EsG) Èskin G. I., Boundary Value Problems for Elliptic Pseudodifferential Equations, American Mathematical Society, Providence, Rhode Island, 1980

(EyG) Eyink G. L., Stochastic Line-Motion and Stochastic Conservation Laws for Non-Ideal Hydrodynamic Models. I. Incompressible Fluids and Isotropic Transport Coefficients, arXiv:0812.0153v1, 30 Nov 2008

(FeE) Fermi E., Quantum Theory for Radiation, Reviews of Modern Physics, Vol. 4, 1932

(FeF) Ferzak F., Die Tesla-Turbine, FFWASP München, 2010

(FeR) Feynman R. P., The Pleasure of Finding Things Out: The Best Short Works of Richard P. Feynman, Basic Books, Perseus Books Group, New York, 1999

(FeR1) Feynman R. P., The Character of Physical Laws, Penguin Books, London, 1992

(FeR2) Feynman R. P., QED – The Strange Theory of Light, Princeton University Press, Princeton, 1985

(FiW) Finkelnburg W., Einführung in die Atomphysik, Berlin Göttingen, Heidelberg, Springer-Verlag, 1948

(FlT) Fließbach T., Elektrodynamik, Springer Spektrum, Berlin, 2021

(GaG) Galdi G. P., The Navier-Stokes Equations: A Mathematical Analysis, Birkhäuser Verlag, Monographs in Mathematics, ISBN 978-3-0348-0484-4

(GiJ) Ginzberg J., On J-contractive operator functions, Dokl. Akad. Nauk, SSSR, Vol. 117, (1957) 171-173

(GiY) Giga Y., Weak and strong solutions of the Navier-Stokes initial value problem, Publ. RIMS, Kyoto Univ. 19 (1983) 887-910

(GlJ) Gleick J., Genius, The life and science of Richard Feynman, Vintage Books, New York, 1991

(GoJ) Goethe J. W. v., Zur Farbenlehre, Jazzybee-Verlag Jürgen Beck, Loeschberg, 2015

(GöJ) Göken J., Hermann Hesses Glasperlenspiel, Ein platonisches Vermächtnis im 20. Jahrhundert, Edition Immanente, Berlin, 2018

(GöK) Gödel, K., An Example of a New Type of Cosmological Solutions of Einstein‘s Field Equations of Gravitation, Review of Modern Physics, Vol. 21, No. 3, 1949

(GöK1) Gödel K., What Is Cantor’s Continuum Problem?, in: Philosophy of Mathematics, Paul Benacerraf/Hilary Putnam (Hg.), Cambridge, Cambridge University Press, 1983

(GöT) Görnitz Th., Görnitz B., Von der Quantenphysik zum Bewusstsein, Kosmos, Geist und Materie, Springer-Verlag, Berlin, Heidelberg, 2016

(GrI) Gradshteyn I. S., Ryzhik I. M., Table of Integrals Series and Products, Fourth Edition, Academic Press, New York, San Francisco, London, 1965

(GuR) Gundersen R. M., Linearized Analysis of One-Dimensional Magnetohydrodynamic Flows, Springer Tracts in Natural Philosophy, Vol 1, Berlin, Göttingen, Heidelberg, New York, 1964

(HaP) Halmos P. R., Lectures on Ergotic Theory, Martino Publishing, Mansfield Centre, CT, 2013

(HaS) Hawking S. W., The Theory Of Everything, Jaico Publishing House, Ahmedabad, Bangladore, Chennai, Delhi, Mumbai, 2006

(HeH) Helfrich H.-P., Simultaneous approximation in negative norms of arbitrary order, RAIRO Numerical Analysis, Vol. 15, No. 3, 1981, 231-235

(HeH1) Hesse H., Das Glasperlenspiel, Suhrkamp Taschenbuch Verlag, 1971

(HeH2) Hesse H., Die Morgenlandfahrt, Suhrkamp, 20. Auflage, 2021

(HeJ) Heywood J. G., Walsh O. D., A counter-example concerning the pressure in the Navier-Stokes equations, as $t \to 0^+$, Pacific Journal of Mathematics, Vol. 164, No. 2, 1994, 351-359

(HeM) Heidegger M., The Age of the World Picture, Cambridge University Press

(HeM1) Heidegger M., Zur Sache des Denken, Zeit und Sein, Tübingen, 1962/1969

(HeW) Heisenberg W., Introduction to the Unified Field Theory of Elementary Particles, John Wiley & Sons, London, New York, Sydney, 1966

(HiP) Higgs P. W., Spontaneous Symmetry Breakdown without Massless Bosons, Physical Review, Vol. 145, No. 4, (1966), 1156-1162

(HiS) Hildebrandt S., Tromba A., The Parsimonious Universe, shape and form in the natural world, Springer-Verlag, New York, Inc., 1996

(HiS1) Hildebrandt S., Wahrheit und Wert mathematischer Erkenntnis, Vortrag gehalten in der Carl Friedrich von Siemens Stiftung, Bonn, 6. Februar 1992

(HoM) Holschneider M., Wavelets, An Analysis Tool, Clarendon Press, Oxford, 1995

(HoE) Hopf E., Ergodentheorie, Springer-Verlag, Berlin, Heidelberg, New York, 1970

(HoJ) Holt J., Why Does the World Exist? An Existential Detective Story, Liveright Publishing Corporation, New York, 2012

(HoJ1) Holt J., When Einstein Walked With Gödel, Farrar, Strauss and Giroux, New York, 2018

(HuD) Hume D., Dialoge über die natürliche Religion, Reclam, Stuttgart, 2021

(HuE) Husserl E., The Phenomenology of Internal Time-Consciousness, Indiana University Press, Bloomington, 2019

(HuE1) Husserl E., Logische Untersuchungen, Verlag von Veit & Comp., Leipzig, 1900

(HüA) Hübscher A., Otto Volgers Brief an Schopenhauer zur Farbenlehre, 1856

(JeJ) Jeans J., The Universe Around Us, First Edition, Cambridge, Cambridge University Press, 1929

(JoF) John F., Formation of singularities in elastic waves, Lecture Notes in Phys., Springer-Verlag, 190-214, 1984

(JoS) Joffily S., The Riemann Zeta Function and Vacuum Spectrum, Proceedings of Science, Fourth International Winter Conference on Mathematical Methods in Physics, Brazil, Vol. 13, 2004

(JüF) Jüttner F., Beiträge zur Theorie der Materie, Mathematische Annalen, Band 87, Heft 3/4, Verlag von Julius Springer, 1922

(KaI) Kant I., The Critique of Judgement, Translation of: Kritik der Urteilskraft, Hackett Publishing Company,Inc., Indianapolis, 1987

(KaY) Kato Y., The coerciveness for integro-differential quadratic forms and Korn's inequality, Nagoya Math. J. 73, 7-28, 1979

(KiA) Kirsch A., Hettlich F., The Mathematical Theory of Time-Harmonic Maxwell's Equations, Expansion-, Integral-, and Variational Methods, Springer, Cham, Heidelberg, New York, Dordrecht, London, 2015

(KiC) Kittel C., Introduction To Solid State Physics, Wiley, New Delhi, 2005

(KiG) Kirchhoff G., Ueber die Vertheilung der Elektrizität auf zwei leitenden Kugeln, Journal für reine und angewandte Mathematik, Bd. 59, (1861), 89-110

(KiH) Kim H., Origin of the Universe: A Hint from Eddington-inspired Born-Infeld gravity, Journal of the Korean Physical Society, Vol. 65, No. 6, pp. 840-845, 2014

(KiM) Kirschner M., Gerhart J., Mitchison T., Molecular „Vitalism", Cell. Vol. 100, 79-88, 2000

(KlB) Kleiner B., Lott J., Notes on Perelman's papers, Mathematics ArXiv

(KlS) Klainerman S., Rodnianski, Regularity and geometric properties of solutions of the Einstein-Vacuum equations, Journées équations aux dérivées partielles, No. 14, p. 14, 2002

(KlS1) Klainerman S., Nicolò, The Evolution Problem in General Relativity, Birkhäuser, Boston, Basel, Berlin, 1950

(KlS2) Klainerman S., Remarks on the global Sobolev inequalities in Minkowski space, Comm. Pure. Appl. Math., 40, 111-117, 1987

(KlS3) Klainerman S., Uniform decay estimates and the Lorentz invariance of the classical wave equation, Comm. Pure Appl. Math., 38, 321-332, 1985

(KlH) Klüber v., H., Das Vorkommen der chemischen Elemente im Kosmos, Verlag Johann Ambrosius Barth, Leipzig, 1931

(KnA) Kneser A., Das Prinzip der kleinsten Wirkung von Leibniz bis zur Gegenwart, B. G. Teubner Verlag, Berlin, Leipzig, 1928

(KrH) Kramers H. A., Die Grundlagen der Quantentheorie, Theorien des Aufbaus der Materie, Akademische Verlagsgesellschaft, Leipzig, 1933

(KuJ) Kuipers J. B., Quaternions and Rotation Sequences, A Primer with Applications to Orbits, Aerospace, and Virtual Reality, Princeton University Press, Princeton and Oxford, 2002

(KuT) Kuhn T. S., The Structure of Scientific Revolutions, University of Chicago Press, 1962

(KuT1) Kuhn T. S., Die Struktur wissenschaftlicher Revolutionen, Suhrkamp, Fankfurt a. M., 1976

(LaC) Lanczos C., Mathematical methods in solid state physics and superfluid theory, Eds.R.C. Clark and G. H. Derik, Oliver & Boyd, Edinburgh, 1969

(LaK) Lanczos K., Über tensorielle Integralgleichungen, Mathematische Annalen, Band 95, Heft 1, Verlag von Julius Springer, 1925

(LaM) Lachièze-Rey M., Gunzig E., The Cosmological Background Radiation, Cambridge University Press, Cambridge, 1999

(LaS) Lattacher S., Viktor Schauberger, Ennsthaler Verlag, Stegr, 2015

(LeE) Leedskalnin E., Magnetic Current, Wilder Publications, Inc., Blacksburg, 2011

(LeE) Leedskalnin E., Magnetischer Strom, Hesper Verlag, 2018

(LeN) Lerner N., A Note on the Oseen Kernels, A. Bove et. Al. (eds.), Advances in Phase Space Analysis of Partial Differential Equations, DOI 10.1007/978-0-8161-9_10, Birkhäuser Verlag, Boston, 2009

(LeS) Leo De S., Quaternions and special relativity, American Institute of Physics, 1996

(LiI) Lifanov I. K., Poltavskii L. N., Vainikko G. M., Hypersingular integral equations and their applictions, ChapmAN 6 Hall/CRC, Boca Raton, London, New York, Washington D.C., 2004

(LiP) Lions P. L., Compactness in Boltzmann's equation via Fourier integral operators and applications. III, J. Math. Kyoto Univ., 34-3, 539-584, 1994

(LiP1) Lions P. L., On Boltzmann and Landau equations, Phil. Trans. R. Soc. Lond. A, Vol. 346 (1994) 191-204

(LiT) Li-xin T., The maximum dissipative extension of Schrödinger operator, Applied Mathematics and Mechanics, Vol. 15, No. 10, (1994) 973-980

(LoA) Louis A. K., Maass P., Rieder A., Wavelets, Theorie und Anwendungen, B. G. Teubner, Stuttgart, 1998

(LoA1) Louis A. K., Inverse und schlecht gestellte Probleme, B. G. Teubner, Stuttgart, 1989

(LuA) Luckner A., Martin Heidegger: „Sein und Zeit", Verlag Ferdinand Schöningh, Paderborn, 1997

(LuG) Lumer G., Phillips R. S., Dissipative Operators in a Banach spaces, Pacific J. Math., Vol. 11, No.2 (1961) 679-698

(MaE) Mach E., Die Mechanik in ihrer Entwicklung, XENOMOI, Verlag, Berlin, 2012

(MaW) Macke W., Quanten und Relativität, Akademische Verlagsgesellschaft Geest & Portig K.-G., Leipzig, 1965

(MaW1) Macke W., Quanten, Akademische Verlagsgesellschaft Geest & Portig K.-G., Leipzig, 1959

(MaW2) Marx W., Hegels „Phänomenologie des Geistes", Eine Interpretation von Vorrede, Einleitung und der Abschnitte A und B, Klostermann Rote Reihe, Vittorio Klostermann GmbH, Frankfurt, 2019

(MaW3) Marx W., Hegel's phenomenology of Spirit: A Commentary Based on the Preface and Introduction, Harper & Row Publisher Inc., The University of Chicago Press, Chicago, 1988

(McP) McLaughlin P., Kants Kritik der teleologischen Urteilskraft, Bouvier Verlag, Bonn, 1989, in Abhandlungen zur Philosophie und Pädagogik, Band 221

(MeM) Mehta M. L., Random Matrices and the Statistical Theory of Energy Levels, Academic Press, New York, 1967

(MeY) Meyer Y., Coifman R., Wavelets, Calderón-Zygmund and multilinear operators, Cambridge studies in advanced mathematics 48, Cambridge University Press, Cambridge, 1996

(MiD) Mihalas D., Weibel-Mihalas, Foundation of Radiation Hydrodynamics, Oxford University Press, Oxford, 1984

(MiH) Minkowski Hermann, Zwei Abhandlungen über die Grundgleichungen der Elektrodynamik, B. G. Teubner, Leipzig, Berlin, 1910

(MiK) Miyamoto K., Fundamentals of Plasma Physics and Controlled Fusion, NIFS-PROC-48, Oct. 2000

(MoH) Montgomery H. L., The pair correlation of the zeta function, Proc. Symp. Pure Math. Vol. 24 (1973) 181-193

(MoJ) Morgan J. W., Tian G., Ricci Flow and the Poincare Conjecture, Mathematics ArXiv

(MüE) Müller E., Nietzsche-Lexikon, Wilhelm Fink Verlag, 2020

(MüO) Müller O. L., Mehr Licht, Goethe mit Newton im Streit um die Farben, S. Fischer Verlag GmbH, Frankfurt a. M., 2015

(NaM) Nathanson M. B., Additive Number Theory, Springer, New York, Berlin, Heidelberg, 2010

(NaS) Nag S., Sullivan D., Teichmüller theory and the universal period mapping via quantum calculus and the $H^{1/2}$ space on the circle, Osaka J. Math., 32, 1-34, 1995

(NaT) Nagel Th., Mind & Cosmos, Why the Materialist Neo-Darwinian Conception of Nature Is Almost Certainly False, Oxford University Press, Oxford, 2012

(NaY) Nakagami Y., Spectral Analysis in Krein Spaces, Publ. Res. Inst. Sci. 24, no. 3, (1988) 361-378

(NeD) Neuenschwander D. E., Emmy Noether's Wonderful Theorem, The Johns Hopkins University Press, Baltimore, 2011

(NeJ) Neumann von, J., Mathematische Grundlagen der Quantenmechanik, Springer, Berlin, Heidelberg, New York, 1996

(NiJ) Nitsche J. A., lecture notes 3, Approximation Theory in Hilbert Scales

(NiJ1) Nitsche J. A., lecture notes 4, Extensions and Generalizations

(NiJ2) Nitsche J. A., A remark on the „inf-sup-condition" in the finite element method, AMS (MOS) subject classifications: 65 N 30, navier-stokes-equations.com

(NiJ3) Nitsche J. A., The regularity of piecewise defined functions with respect to scales of Sobolev spaces

(NiJ4) Nitsche J. A., Direct Proofs of Some Unusual Shift-Theorems, Anal. Math. Appl., Gauthier–Villars, Montrouge, 1988, pp. 383–400

(NiJ5) Nitsche J. A., Free boundary problems for Stokes´s flows and finite element methods, Equadiff 6, 2006, pp. 327-332

(NiJ6) Nitsche J. A., Finite-Element Methods For Conformal Mappings, SIAM J. Numer. Anal., Vol. 26, No. 6, (1989) 1525-1533

(NiJ7) Nitsche J. A., Wheeler M. F., L_∞ boundedness of the finite element method galerkin operator for parabolic problems, Numerical Functional Analysis and Optimization, 4:4, 325-353

(NiN) Nielsen N., Handbuch der Theorie der Gammafunktion, Chelsea Publishing Company, New York, 1965

(NuI) Nussbaumer I., Zur Farbenlehre, Entdeckung der unordentlichen Spektren, edition splitter, Wien, 2008

(OdA) Odlyzko A. M., On the distribution of spacings between zeros of the zeta function, Math. Comp. Vol. 48, No. 177 (1987), 273-308

(PaH) Pape H., Charles S. Peirce zur Einführung, Junius Verlag GmbH, Hamburg, 2004

(PaW) Pauli W., On Dirac's new method of field quantization, Rev. Modern Phys. 15, (1943) 175-207

(PeB) Introduction to the Fourier Transform and Pseudo-Differential-Operators, Pitman Advanced Publishing Program, Boston, London, Melbourne, 1983

(PeM) Peskin M. E., Schroeder D. V., An Introduction To Quantum Field Theory, Westview Press, 2016

(PeR) Penrose R., The Emperor's New Mind, Oxford University Press, Oxford, 1989

(PeR1) Penrose R., Cycles of Time, Vintage, London, 2011

(PeR2) Penrose R., Structure of space-time, Batelle Rencontre, C. M. DeWitt and J. M. Wheeler, 1967

(PeR3) Penrose R., Zero rest mass fields including gravitation: asymptotic behaviours, Proc. Toy. Soc. Lond., A284, 159-203, 1962

(PeR4) Penrose R., The Road To Reality, A complete guide to the laws of the universe, Alfred A. Knopf, New York, 2005

(PeB) Petersen B. E., Introduction to the Fourier Transform and Pseudo-Differential Operators, Pitman Advanced Publishing Program, Boston, London, Melbourne, 1983

(PfM) Pflaum M., Deleuze – seine philosophischen Welten für Einsteiger, 1. Band, Books on Demand, Norderstedt, 2023

(PfM1) Pflaum M., Deleuze – seine philosophischen Welten für Einsteiger, 2. Band, Books on Demand, Norderstedt, 2021

(PhR) Phillips R. S., Dissipative Operators and Hyperbolic Systems of Partial Differential Equations, Trans. Amer. Math. Soc., Vol. 90, No. 2, (1959), 193-254

(PlJ) J. Plemelj, Potentialtheoretische Untersuchungen, B.G. Teubner, Leipzig, 1911, navier-stokes-equations.com

(PlM) Planck M., Dynamische und Statistische Gesetzmässigkeit, (transl., the Dynamical and the Statistical Type of Law). In: Roos, H., Hermann, A. (eds) Vorträge Reden Erinnerungen, Springer, Berlin, Heidelberg, (2001) 87-102

(PlP) Plaass P., Kant's Theory of Natural Science Translation, Analytic Introduction and Commentary by Alfred E. and Maria G. Miller (Boston Studies in the Philosophy and History of Science, 159, Band 159)

(PoL) Pontryagin L. S., Hermitian operators in spaces with indefinite metric, Izv. Akad. Nauk SSSR Ser. Mat. 8, (1944) 243-280, (Russian)

(PoP) Poluyan P., Non-standard analysis of non-classical motion; do the hyperreal numbers exist in the quantum-relative universe?

(PoR) Pohl R. W., Optik und Atomphysik, Springer-Verlag, Berlin, Heidelberg, New York, 1975

(RaC) Radlberger C., Der hyperbolische Kegel, nach Walter Schauberger, PKS Eigenverlag, Bad Ischl, 2014

(ReN) Rescher N., The Riddle of Existence, Lanham, University Press of America, 1994

(ReT) Regehly T., Schubbe D., Schopenhauer und die Deutung des Existenz, Perspektiven auf Phänomenologie, Exsitenzphilosophie und Hermeneutik, J. B. Metzler Verlag, Stuttgart, 2016

(RiH) Risken H., The Fokker-Planck Equation, Methods and Solution and Applications, Springer, Berlin, Heidelberg, New York, 1996

(RiN) Ribe N., Steinle F., Exploratory Experimentation: Goethe, Land and Color Theory, Physics Today 55, (7) (2002) 43-49

(RiW) Ritz W., Einstein A., On the present status of the radiation problem, Physikalische Zeitschrift, 10 (1909) 323-324

(RoC) Rovelli C., Quantum Gravity, Cambridge University Press, Cambridge, 2004

(RoC1) Rovelli C., The Order of Time, Penguin Science, 2018

(RoH) Rollnick H., Quantentheorie 2, Springer, Berlin, Heidelberg, New York, 2003

(RoP) Robitaille P.-M., Fourty lines of evidence for condensed matter - The Sun on trial, Liquid metallic hydrogen as a solar building block, Progr. Phys., 2013, Vol. 4, 90–142

(RoP1) Robitaille P.-M., Blackbody radiation and the loss of universality, Implications for Planck's formulation and Boltzmann's constant. Progr. Phys., 2009, Vol. 4, 14–16

(RoP2) Robitaille P.-M., Water, Hydrogen Bonding, and the Microwave Background, Progr. Phys., 2009, Vol. 2, L5–L8

(RoP3) Robitaille P.-M., Blackbody radiation and the carbon particle. Progr. Phys., 2008, Vol. 3, 36–55

(RoP4) Robitaille P.-M., A critical analysis of universality and Kirchhoff's law, A return to Stewart's law of thermal emission. Progr. Phys., 2008, Vol. 3, 30–35

(RoR) Rorty R., Philosophy and the Mirror of Nature, Princeton University Press, Princeton, 1979

(RoR1) Rorty R., Der Spiegel der Natur, Suhrkamp, Frankfurt a. M., 1981

(RoW) Rother W., Vorlesung über Hegels Phänomenologie des Geistes, conexus, Zürich, 2020

(RuB) Russell B., The Philosophy of Leibniz, Routledge, London, New York, paperback edition, 1992

(RuB1) Russell B., History of western philosophy, Routledge, London, 1995

(SaR) Safranski R., Nietzsche, Biographie seines Denkens, Fischer Taschenbuch Verlag, Frankfurt a. M., 2002

(ScA) Schopenhauer A., Die Welt als Wille und Vorstellung, Haffmans Verlag, Zürich, 1988

(ScA1) Schopenhauer A., Welt und Mensch, Eine Auswahl aus dem Gesamtwerk von Arthur Hübscher, Philipp Reclam Jun., Stuttgart, 1960

(ScC) Schäfer C., Einführung in die Maxwellsche Theorie der Elektrizität und des Magnetismus, Walter de Gruyter & Co., Berlin, 1949

(ScD) Schubbe D., Koßler M., Schopenhauer Handbuch, Leben-Werk-Wirkung, J. B. Metzler Verlag, 2018

(ScD1) Schmicking D. A., Zu Schopenhauers Theorie der Kognition bei Mensch und Tier – Betrachtungen im Lichte aktueller kognitionswissenschaftlicher Entwicklungen' in Schopenhauer-Jahrbuch 2005, pp. 149-176

(ScE) Schrödinger E., Statistical Thermodynamics, Dover Publications Inc., New York, 1989

(ScE1) Schrödinger E., What is Life? The Physical Aspects of the Living Cell with Mind and Matter, Cambridge University Press, Cambridge, 1967

(ScE2) Schrödinger E., My View of the World, Ox Bow Press, Woodbridge, Connecticut, 1983

(ScE3) Schrödinger E., Nature and the Greeks and Science and Humanism, with a foreword by Roger Penrose, Cambridge University Press, Cambridge, 1996

(ScE4) Schrödinger E., Die Erfüllbarkeit der Relativitätsforderung in der klassischen Mechanik, Annalen der Physik, 382 (1925) pp. 325-336

(ScE5) Schrödinger E., Die gegenwärtige Situation in der Quantenmechanik, Naturwissenschaften 23 (1935) 807–812, 823-828, 844-849

(ScF) Schiller F., On the Aesthetic Education of Man, Angelico Press, reprint originally published by Yale University Press, New Haven, 1954

(ScJ) Schauberger J., Viktor Schauberger, Das Wesen des Wassers, atVerlag, Baden, München, 2019

(ScP) Scott P., The Geometries of 3-Manifolds, Bull. London Math. Soc., 15 (1983), 401-487

(ScW) Schpolski E. W., Atomphysik, Teil I, VEB Deutscher Verlag der Wissenschaften, Berlin, 1970

(ShB) Shaw B., Zurück zu Methusalem, Artemis-Verlag, Zürich, 1947

(ShF) Shu F. H., The Physics of Astrophysics, Vol. II, Gas Dynamics, University Science Books, Sausalito, California, 1992

(ShF1) Shu F. H., The Physics of Astrophysics, Vol. I, Radiation, University Science Books, Mill Valley, California, 1991

(SiT) Sideris T., Formation of singularities in 3-D compressible fluids, Comm. Math. Phys., 101, 47-485, 1985

(SlP) Sloterdijk P., Wer noch kein Grau gedacht hat, Eine Farbenlehre, Suhrkamp, Berlin 2022

(SmL) Smolin L., Time Reborn, From the Crisis in Physics to the Future of the Universe, Houghton Mifflin Harcourt, New York, 2013

(SmL1) Smolin L., The Trouble with Physics, The Rise of String Theory, the Fall of a Science, and What Comes Next, Houghton Mifflin Company, Boston, New York, 2007

(SoH) Sohr H., The Navier-Stokes Equations, An Elementary Functional Analytic Approach, Birkhäuser Verlag, Basel, Boston, Berlin, 2001

(SoP) Sobolevskii P. E., On non-stationary equations of hydrodynamics for viscous fluid. Dokl. Akad. Nauk SSSR 128 (1959) 45-48 (in Russian)

(SpK) Spatschek K. H., Theoretische Plasmaphysik, B. G. Teubner, Stuttgart, 1990

(SpV) Spierling V., Arthur Schopenhauer, Junius Verlag GmbH, Hamburg, 2002

(StD) Straassberg D., Der Wahnsinn der Philosophie, Cronos Verlag, Zürich, 2014

(StE) Stein E. M., Singular Integrals and Differentiability Properties of Functions, Princeton University Press, Princeton, New Jersey, 1970

(StR) Streater R. F., Wightman A. S., PCT, Spin & Statistics, and all that, W. A. Benjamin, Inc., New York, Amsterdam, 1964

(StR1) Steiner R., The Riddles of Philosophy, SteinerBooks, 2009

(StR2) Steiner R., Die Philosophie der Freiheit, Rudolf Steiner Ausgaben, Bad Liebenzell, 2024

(SuL) Susskind L., Friedman A., Special Relativity and Classical Field Theory, Basic Books, New York, 2017

(TaR) Taschner R., Das Unendliche, Mathematiker ringen um einen Begriff, Springer-Verlag, Berlin, Heidelberg, New York, 1995

(TeR) Teman R., Navier-Stokes Equations and Nonlinear Functional Analysis, SIAM, Philadelphia, 1983

(ThW) Thurston W. P., Three Dimensional Manifolds, Kleinian Groups and Hyperbolic Geometry, Bulletin American Mathematical Society, Vol 6, No 3, 1982

(TsV) Tsytovich V., Theory of Turbulent Plasma, Consultants Bureau, New York, 1977

(TrH) Treder H.-J., Singularitäten in der Allgemeinen Relativitätstheorie, Astron. Nachr. Bd. 301, H. 1, 9-12, 1980

(TrH1) Treder H.-J., Relativität und Kosmos, Akademie Verlag, Berlin, Pergamon Press, Oxford, Vieweg & Sohn, Braunschweig, 1968

(UnA) Unzicker A., Bankrupting Physics, Alexander Unzicker and Sheilla Jones, 2013

(UnA1) Unzicker A., Einstein's Lost Key: How We Overlooked the Best Idea of the 20th Century, 2015

(UnA2) Unzicker A., The Mathematical Reality, Why Space and Time are an Illusion, 2020

(UnA3) Unzicker A., Vom Urknall zum Durchknall, Springer Spektrum, Berlin, Heidelberg, 2010

(UnA4) Unzicker A., Die flüssige Sonne, Eine kommende Revolution in der Astrophysik, A. Unzicker, 2023

(UnA5) Unzicker A., Auf dem Holzweg durchs Universum, Hanser Verlag, München, 2012

(VaM) Vainberg M. M., Variational Methods for the Study of Nonlinear Operators, Holden-Day Inc., San Francisco, London, Amsterdam, 1964

(VeW) Velte W., Direkte Methoden der Variationsrechnung, B. G. Teubner, Stuttgart, 1976

(VoH) Vogt H., Atomenergie und Atomumwandlungen, Demmig Verlag, Darmstadt, 1948

(WaA) Wachter A., Relativistische Quantenmechanik, Springer, Berlin, Heidelberg, New York, 2005

(WaC) Waal de, C., A Guide For The Perplexed Peirce, Bloomsbury, London, New Delhi, New York, Sydney, 2013

(WeC) Weizsäcker C. F., The Structure of Physics, Springer, Dortrecht, 2006

(WeC1) Weizsäcker C. F., Aufbau der Physik, Deutscher Taschenbuch Verlag GmhH & Co. KG, München, 1988

(WeC2) Weizsäcker C. F., Der begriffliche Aufbau der theoretischen Physik, S. Hirzel Verlag, Stuttgart, 2004

(WeC3) Weizsäcker C. F., Die Einheit der Natur, Carl Hanser Verlag, München, 1971

(WeC4) Weizsäcker C. F., Zeit und Wissen, Carl Hanser Verlag, München, Wien, 1992

(WeD) Werner D., Funktionalanalysis, Springer Spektrum, Springer Verlag, Berlin, 2018

(WeH) Weyl H., Philosophy of Mathematics and Natural Science, Princeton University Press, Princeton, 1949

(WeH1) Weyl H., Space, Time, Matter, Cosimo Classics, New York, 2010

(WeH2) Weyl H., Was ist Materie, Verlag Julius Springer, Berlin, 1924

(WeH3) Weyl H., The Continuum, A critical examination of the foundation of analysis, Dover Publications, Inc., New York, 1987

(WeH4) Welzer H., Nachruf auf mich selbst, Fischer Taschenbuch, Frankfurt a. M., 2023

(WeP) Werner P., Self-Adjoint Extensions of the Laplace Operator with Respect to Electric and Magnetic Boundary Conditions, J. Math. Anal. 70 (1979) 131-160

(WeP1) Werner P., Spectral Properties of the Laplace Operator with respect to Electric and Magnetic Boundary Conditions, J. Math. Anal. 92 (1983) 1-65

(WeS) Weinberg S., The First Three Minutes, Basic Books Inc., New York, 1977

(WhA) Whitehead A. N., Prozeß und Realität, Entwurf einer Kosmologie, Suhrkamp Verlag, Frankfurt a. M., 2021

(WhA1) Whitehead A. N., Process and Reality, An Essay In Cosmology, The Free Press, New York, 1985

(WhA2) Whiting A. B., Hindsight and Popular Astronomy, World Scientific Publishing Co. Pte. Ltd., 2011

(WiE) Wigner E., The unreasonable effectiveness of mathematics in the natural sciences, Comm. Pure and Appl. Math., Vol. 13, No. 1, (1960) 1-14

(WiE1) Symmetries and Reflections, OX BOW PRESS, Woodbridge, Connecticut, 1979, original copyright by Indiana University Press, 1967

(WoM) Wolff M., Geschichte der Impetustheorie, Untersuchungen zum Ursprung der klassischen Mechanik, Suhrkamp, Frankfurt a. M., 1978

(XiK) Xiao K., The Fine Structure Constant and Interpretations of Quantum Mechanics, Electronic Journal of Theoretical Physics, Vol. 9, No. 26, (2012) 135-146

(ZaD) Zahavi D., Husserl's Phenomenology, Stanford University Press, 2003

(ZaD1) Zahavi D., Husserl's Legacy: Phenomenology, Metaphysics, and Transcendental Philosophy, Reprint Edition, Oxford University Press, Oxford, 2020

(ZiR) Zimmer R., Arthur Schopenhauer, Ein philosophischer Weltbürger, Deutscher Taschenbuch Verlag, München, 2012