

A single truly geometric gravity and quantum field theory

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Some royal road markers

After having passed the two milestones, Einstein's Special Relativity Theory, (BoD1), and Pauli's spin concept (accompanied by the spin statistics CPT theorem, (StR)), the General Relativity Theory (GRT) and the quantum theory of fields became two „dead end road“ theories towards a common gravity and quantum field theory. The physical waymarking labels directing into those dead end roads may be read as

dead end road label (1):

"towards space-time regions with not constant gravitational potentials governed by a globally constant speed of light", (UnA)

dead end road label (2):

"towards Yang-Mills mass gap".

The waymarker labels of the royal road towards a geometric gravity and quantum field theory may be

royal road label 1:

towards mathematical concepts of „potential“, „potential operator“, and „potential barrier“ as intrinsic elements of a geometric mathematical model beyond a metric space (*)

royal road label 2:

towards a Hilbert space based hyperboloid manifold with hyperbolic and conical regions governed by a „half-odd-integer“ & „half-even integer“ spin concept

royal road label 3:

towards the Lorentz-invariant, CPT theorem supporting weak Maxwell equations model of „proton potentials“ and „electron potentials“ as intrinsic elements of a geometric mathematical model beyond a metric space

royal road label 4:

towards „the understanding of physical units“, (UnA) p. 78, modelled as „potential barrier“ constants, (*),(**), (***), (****), (*****)

(*) Einstein quote, (UnA) p. 78: „The principle of the constancy of the speed of light only can be maintained by restricting to space-time regions with a constant gravitational potential.“

(**) The Planck action constant may mark the "potential barrier" between the measurable action of an electron and the action of a proton, which "is acting" beyond the Planck action constant barrier.

(***) The „potential barrier“ for the validity of the Mach principle determines the fine structure constant and the mass ratio constant of a proton and an electron: Dirac's large number hypothesis is about the fact that for a hydrogen atom with two masses, a proton and an electron mass, the ratio of corresponding electric and gravitational force, orbiting one another, coincides to the ratio of the size of a proton and the size of the universe (as measured by Hubble), (UnA) p. 150. In the proposed geometric model the hydrogen atom mass is governed by the Mach principle, while the Mach principle is no longer valid for the electron mass, governed by the CPT spin statistics.

(****) The norm quadrat representation of the proposed "potential" definition indicates a representation of the fine structure constant in the form $256/137 \sim (\pi^2 \pi) - 8$. In (GaB) there is an interesting approach (key words: "Margolus-Levitin theorem", "optimal packaged information in micro quantum world and macro universe") to „decrypt“ the fine structure constant as the borderline multiplication factor between the range of the total information volume size (calculated from the quantum energy densities) of all quantum-electromagnetic effects in the universe (including those in the absence of real electrodynamic fields in a vacuum; Lamb shift) and the range of the total information volume size of all matter in the four dimensional universe (calculated from the matter density of the universe).

(*****) The vacuum is a homogeneous, dielectric medium, where no charge distributions and no external currents exist. It is governed by the dielectric and the permeability constants, which together build the speed of light; the fine structure constant can be interpreted as the ratio of the circulation speed of the electron of a hydrogen atom in its ground state and the speed of light. This puts the spot on the Maxwell equations and the "still missing underlying laws governing the "currents" and "charges" of electromagnetic particles. ...The energetical factors are unknown, which determine the arrangement of electricity in bodies of a given size and charge", (EiA), p. 52

The proposed Hilbert space based model ...

... overcomes the (mathematical model caused) Yang-Mills-Equations mass gap problem

... builds on the (mathematically) proven (physical) PCT theorem

... overcomes the main gap of Dirac's quantum theory of radiation, i.e. the small term representing the coupling energy of the atom and the radiation field becomes part of the $H(1)$ -complementary (truly bosons) sub-space of the overall energy Hilbert space $H(1/2)$; the new concept replaces Dirac's $H(-n/2-e)$ -based point charge model by a $H(-1/2)$ -based quantum element model

... acknowledges the primacy of micro quantum world against the macro (classical field) cosmology world, where the Mach principle governs the gravity of masses and masses govern the variable speed of light, (DeH)

... allows to revisit Einstein's thoughts on ETHER AND THE THEORY OF RELATIVITY in the context of the space-time theory and the kinematics of the special theory of relativity modelled on the Maxwell-Lorentz theory of the electromagnetic field

... acknowledges the Mach principle as a selecting principle to select the appropriate classical cosmology field model out of the few current physical relevant ones, (DeH): the to be selected classical cosmology field equation model may be modelled as the Ritz approximation equation (= orthogonal projection onto the coarse-grained (energy) Hilbert sub-space $H(1)$ of the overall variational representation in the overall $H(1/2)$ (energy Hilbert) space) of an extended Newton model, accompanied with the Dirichlet integral based inner product, the and the three dimensional unit sphere $S(3)$ based on the field of quaternions in sync with the Lorentz transformation

... acknowledges Bohm's property of a "particle" in case of quantum fluctuation, (BoD), chapter 4, section 9, (Sml).

The GRT describes a mechanical system characterized by the (inert = heavy) mass of the considered „particles“ and their related interacting „gravity force“; the mathematical model framework is about a real number based (purely) metric space. This is about the distance measurement of real number points, which are per definition without any „mass density“, (as long as the field of real numbers is not extended to the field of hyper-real number, i.e. as long as the Archimedian axiom is still valid). For a mechanical system every real-valued function of the location and momentum (real number) coordinates represents an observable of the physical system. The underlying metric space concept (equipped with an (only) "exterior" product of differential forms, because a metric space has no geometric structure at all) accompanied by a global nonlinear stable, Minkowski space, (ChD), is replaced by a $H(1/2)$ -quantum energy Hilbert space concept equipped, where the $H(1/2)$ -inner product is equivalent to a corresponding inner product of differential forms.

The Hilbert space framework supports also the solution to related challenges, e.g. regarding the „first mover“ question (inflation, as a prerequisite) of the „Big Bang“ theory, the symmetrical time arrow of the (hyperbolic) wave (and radiation) equation, governing the light speed and derived from the Maxwell equations by differentiation, no long term stable and well-posed 3D-NSE, no allowed standing (stationary) waves in the Maxwell equation, the mystery of the initial generation of an uplift force in a ideal fluid environment of aircraft wings, i.e. no fluids collisions with the wings surfaces, and a Landau equation based proof of the Landau damping phenomenon.

Some more details

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Both quantum theories, the quantum mechanics and the quantum dynamics, are physical systems consisting of measurable variables and observable data; the mathematical model framework is about the same complex number based $L(2)$ Hilbert space for both theories; its elements have wave character, which are built on Fourier series representation based on Hermite polynomials to model observables like location, momentum, energy density and energy radiation. To establish the states of different observables in the same Hilbert space framework only simultaneously measurable observables may be considered.

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The Yang-Mills-, Maxwell- and Helmholtz-Equations

The Yang-Mills theory is the generalization of the Maxwell theory of electromagnetism, where the chromo-electromagnetic field itself carries charges. As a classical field theory it has solutions which travel at the speed of light so that its quantum version should describe massless particles (gluons). However, the postulated phenomenon of color confinement permits only bound states of gluons, forming massive particles. This is the mass gap. The physical ("color") confinement challenge is that the phenomenon that "color-charged" particles (such as quarks and gluons) have not been isolated until today. Another challenge of „confinement“ is asymptotic freedom which makes it conceivable that quantum Yang-Mills theory exists without restriction to low energy scales. Therefore, before going for hunting massless bosons, we should either intensify to capture quarks, or revisit the triple, "electron", "neutron", "proton", becoming the only physically observable fermions in a SRT & CPT aligned quantum theory of fields, based on the theory of Hilbert spaces with indefinite inner product, ((DrM), (AzT), (DrM), (VaM)) accompanied with the calculus of variations in a distributional Hilbert scale framework.

The integral form of the Maxwell equations is more closely connected to the physical situation, which is basically the combined Gauss' Electric Law and Ampere's Law, where electromagnetic waves propagate in the interior of a region in $R(3)$ enclosed by a related region boundary. The corresponding two variational representations of the standard „electric“ and „magnetic“ boundary problem is enabled by the Green formulae accompanied with corresponding „natural“ boundary conditions. When applying variational methods, the natural bilinear form of time-harmonic Maxwell equations is not coercive on the whole Sobolev space $H(1)$. One can, however, make it coercive by adding a certain bilinear form on the boundary of the domain (CoM). The extended „electric“ and „magnetic“ (well-posed) boundary value problem is accompanied by the extended Green formulae, replacing the standard normal derivative concept by a „mass element“ and „flux“ concept, which is purely defined on the surface only, omitting the unit vector tangent concept, (PIJ). The proposed physical governing concept is the least action principle, (KnA).

We note that Maxwell equations hold only in regions with smooth parameter functions, modelling conductivity, dielectricity and permeability. Vacuum (i.e. the conductivity is zero) is a homogeneous, dielectric medium, where no charge distributions and no external currents exist. Therefore, the dielectric and the permeability tensors reduce to the dielectric and the permeability constants, which together build the speed of light. In order to consider situations in which a surface separates two media from each other, the constitutive parameters (modelling conductivity, dielectricity and permeability) are no longer continuous (i.e. also not differentiable). The concepts „conductivity“, „dielectricity“ and „permeability“ are proposed to become intrinsic elements of the Hilbert space model, whereby the „transmission boundary conditions“, ((KiA), p. 12) are governed by a least action principle. In scattering theory the solutions live in the unbounded exterior of a bounded domain. In standard theory the behavior of electromagnetic fields at infinity has to be taken into account (example: Hertz dipole).

The solution of time-harmonic Maxwell equations in a vacuum leads to the Helmholtz equation. The fundamental solution of the Helmholtz equation at the origin is given by spherical wave fronts. The time-dependent magnetic field has the form of the Hertz dipole centered at the origin, (KiA) p. 14. In this example the considered solution functions tends to infinity when absolute(x) tends to infinity. In order to overcome this challenge the wave number k is replaced by the wave number $-k$. It leads to „outgoing“ and „ingoing“ wave fronts. For the scattering of electromagnetic waves the scattering waves have to be "outgoing" waves. Thus, it is required to exclude the second ones by additional conditions which are called radiation conditions (KiA) p. 15. The concept of „ingoing“ wave fronts (accompanied by (the Sommerfeld or the Silver-Müller) radiation conditions) are proposed to become intrinsic elements of the Hilbert space model, whereby the „transmission boundary conditions“ are governed by a least action principle.

The proposed Hilbert space based model ...

... builds on the (mathematically) proven

PCT theorem

The (mathematically proven) spin-statistics PCT theorem says that "any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have PCT (Charge-Parity-Time) symmetry". Mathematically speaking, the PCT theorem states, that the product of the transformations „P“ („Parity transformation“ (resp. a "half integer" and "integer" spin concept), „C“ („Charge conjugation“ (resp. confinement of spin property), „T“ („Time reversal“ (resp. SRT (hyperbolic PDE) scope)), in one order or the other, is always a symmetry in local fields, whether or not the individual factors are. It is one of the only few mathematically proven quantum physical "theorems". The two essential assumptions in proving the spin/statistics relation are "relativity", i.e. the physical laws do not change under Lorentz transformation, and "microcausality", i.e. space-like separated fields either commute or anticommute. It can be made only for relativistic theories with a time direction.

Some further royal road markings towards a single, geometric gravity and quantum field theory

- the kinematical energy Hilbert space of the fermions is the standard weak Hilbert space $H(1)$, equipped with the "Dirichlet integral" inner product
- the Hilbert space $H(1)$ allows the split into two complementary spaces, providing a model for the physical concepts of "half-odd-integer" and "even-odd-integer" spins with related repulsive and attractive effects, mathematically governed by Riesz bases coming along with the non-harmonic Fourier series theory
- the Friedrichs extension of the Laplacian operator is a selfadjoint, bounded operator B with domain $H(1)$. Thus, also the operator B induces a decomposition of $H(1)$ into the direct sum of two subspaces, enabling the definition of a „potential" and a corresponding „grad" potential operator. Then a potential criterion defines a manifold, which represents a hyperboloid in the Hilbert space $H(1)$ with corresponding hyperbolic and conical regions ((VaM) 11.2)
- the kinematical Hilbert space $H(1)$ is compactly (coarse-grained) embedded into an overall energy Hilbert space $H(1/2)$, whereby the closed complementary subspace of $H(1)$ governs all non-kinematical energies, i.e. the closed sub-space $H(1,ortho)$ replaces all current bosons-type elementary particles
- theoretical physics models are defined as weak (distributional Hilbert scale based) variational (PDE) representations of the considered mechanical system, which is compactly (coarse-grained) embedded into an overall dynamical system. A first proof of concept is given by the Millennium problem of a well-posed initial-boundary value problem of the 3D non-linear, non-stationary Navier -Stokes Equations, where the corresponding variational representation accompanied with an extended $H(1/2)$ energy Hilbert space norm is well-posed
- the physical concept of "potential" is an intrinsic element of the geometric quantum field model, and is no longer an appropriately interpreted physical phenomenon of the considered physical situation; mathematically speaking, the concept of "potential" is an intrinsic element of the geometric Hilbert scale framework, and do not need to be chosen properly as a given functions of the considered PDE
- the "electron potential" (:= electric potential energy/unit charge) in the geometric quantum field ($H(1/2)$ energy related Hilbert space) model corresponds to the electric potential as defined by the weak representation of the Maxwell equations; the hyperboloid manifold in the Hilbert space $H(1)$ with corresponding hyperbolic and conical regions enables a corresponding "proton potential" governed by the related "spin type"; in other words, also the concept of repulsive electrons and attractive positrons are intrinsic parts of the geometric model; the differentiator from a physical perspective is about the fact, that the Planck action constant marks the barrier between the measurable action of an electron and the action of a proton, which is beyond the Planck action constant
- Fourier transforms do not allow localization in the phase space, leading to the concept of windowed Fourier transforms. From a group theoretical perspective windowed Fourier transforms are identical to the wavelet transforms. The wavelet admissibility condition puts the spot on the Hilbert space $H(1/2)$, (LoA). From a „mathematization" perspective „wavelet analysis may be considered as a mathematical microscope to look by arbitrary (optics) functions over the real line \mathbb{R} on different length scales at the details at position b that are added if one goes from scale „ a " to scale „ $a-da$ " with $da>0$ but infinitesimal small", (HoM) 1.2
- the one-dimensional counterpart of the below considered $S(3)$ unit sphere is the unit circle. For the continuous wavelet transform of a function over the unit circle with respect to a wavelet g we recall from (HoM): „Geometrically, the wavelet transform of a function over the circle T is a function over the half-cylinder $\mathbb{R}(+) \times T$ The Poisson summation formula shows a vanishing constant Fourier term and the positive and negative frequencies do not mix, enabling a corresponding (+/-) split of the $L(2)$ space. ... the wavelet transform over the circle conserves energy. .. The wavelet transform with respect to a progressive, admissible wavelet is an isometry. In case where $g=h$ it is now an orthogonal projector on the image of the wavelet transform
- the Hilbert space $L(2)$ is compactly embedded into the larger generalized Hilbert space $H(-1/2)$, (with its dual (energy) Hilbert space $H(1/2)$ with respect to the $L(2)$ -norm = $L(2)$ metric)
- the Hilbert space $H(-1/2)$ governs specific variables like (the not measurable) ground state energy
- weak variational (PDE) representations of real valued functions in a Hilbert scale framework, where a „coarse-grained" (Lebesgue integral based) Hilbert space $L(2)$ governs the modelling requirements of mechanical systems
- the field equations may be represented by the Einstein-Hilbert action functional, which is already a baseline concept in the variational calculus
- the simultaneous measurement of the variables in the $H(-1/2)$ framework is modelled as the orthogonal projections onto the (thermodynamics) statistical („coarse-grained") Hilbert space $L(2)$, which is compactly embedded into $H(-1/2)$

- complex numbers play an important role in quantum mechanics; the extension to quantum dynamics needs a somehow richer object than $R(3)$ or $C(3)$ leading to Hamilton's quaternions as a fundamental building block of the universe accompanied by the three dimension unit sphere (UnA1)

- the Hilbert scale structure conserves the mechanical measurements in the standard $L(2)$ Hilbert space framework, while the underlying energy dynamics are governed by the (much larger) orthogonal, closed subspace of $L(2)$ in $H(-1/2)$ with respect to the overall "quantum element" Hilbert space $H(-1/2)$

- a Hilbert space is THE mathematical role model building geometric spaces, i.e. the proposed „coarse-grained“ Hilbert scale structure is the best possible mathematical option to model the universe, which is nearly all about vacuum, whereby most of the rest of the vacuum is nearly all about the sophisticated notions of „dark matter“ and „dark energy“

- the metric space comes along with the need to choose between three so-called "geometries" (spherical, flat, hyperbolic) in combination with multiple possible, only locally relevant metrics (e.g. Schwarzschild, Robertson-Walker, Gödel), while indefinite inner product spaces provide an all-in-one package, which includes the concepts of a "potential", (independently defined from any coupling constant (!)) and a "potential operator", enabling the definition of a corresponding potential criterion (barrier) defining a manifold, which represents a hyperboloid with underlying hyperbolic and conical regions

- in a 4D-field based Hilbert scale framework the metric space based so-called "geometry" types are replaced by (problem specifically considered) PDE types, which are elliptic (mathematical object behind the naming convention: ellipse), parabolic (mathematical object behind the naming convention: straight (not a parabola)) and hyperbolic (mathematical object behind the naming convention: hyperbole) PDE; only the parabolic (time arrow) and the hyperbolic (time symmetry) PDEs require the concept of "time". The model problems for the three PDE types are (1) the Newton potential equation resp. the Helmholtz equation (elliptic, steady state), (2) the heat equation (parabolic) and (3) the wave (or radiation) equation (hyperbolic). In the context of the concept "wave" we note that "distortion-free progressing wave" (alternatively to "standing waves"), which are key for the theory of transmission of signals, are indeed a central subject in the theory of hyperbolic PDE. Relatively undistorted spherical waves relate spherical waves to the problem of transmitting with perfect fidelity signals in all directions. For the related Courant-Hilbert conjecture, putting spherical waves for arbitrary time-like lines in case of two or four variables into a one-to-one relationship to the wave equation, we refer to (CoR) VI. §18. In order to prove this conjecture one might consider the fact that the only possible domain fields of the considered functions are the sets of the 1D real numbers, 2D complex numbers, and the 4D quaternions

- without the parabolic or hyperbolic space-time concept in the purely elliptic world, there is no theoretical model from which the numerical values of the velocity of light and the Planck constant can be derived, as the underlying units „meter“ and „second“ are out of scope, (UnA1) p. 138 ff: Anticipating the Courant-Hilbert conjecture there is the need for an alternative 4D field model, and further anticipating Hamilton's quaternions. Quaternions are a four dimensional number system enabling a tricky multiplication of complex numbers, leading to a four dimension unit ball with its underlying three-dimensional unit sphere, which is equivalent with rotations in three dimensional space. We note that Plemelj's concepts of „mass element“ and „flow“ in the context of the Green formulae are intrinsic elements of a unit sphere without the need of a „normal vector“ accompanied with the concept of a „mass density“. With respect to the physical universe we recall the following two quotes, (UnA1) p. 155:

R. Feynman: „Now you can look back and say that Pauli's spin matrices and operators were nothing but Hamilton's quaternions“

W. Hamilton: „Somehow quaternions are a fundamental building block of physical universe“

- a coarse-grained mechanical Hilbert space $L(2)$ embedded into the Hilbert space $H(-1/2)$ allows a re-interpretation of the notion "entropy" in the context of the "discrete" Shannon entropy vs. the $\log(x)$ -function based "continuous entropy"; we note that the (still best known) relationship between the notions "volume V", "number of particles N", "temperature T", "pressure P", "entropy S" and the related total energy E, as a function of the parameters S, V, N, is given by the total differential $dE=TdS-PdV+c*dN$

- the coarse-grained mechanical (statistical) Hilbert space $L(2)$ embedded into the overall Hilbert space $H(-1/2)$ puts the spot on the Hawking-Hartle (probability theory related) "Interpretation of "The Wave Function of the Universe"" in (DrW): "the ground state is the amplitude for the Universe to appear from nothing"; ... "Physical probabilities, as exemplified by radioactive decay, start with something, a first situation (particle in space and time) becoming another situation (other particles in space and time). The probability is the chance that the transition from situation one to situation two happens during a certain interval of time, or that a particle is found in a certain volume of space, or something like that". In this context we note that the embeddedness of $L(2)$ into $H(-1/2)$ relates to the embeddedness of the set of rational numbers into the set of real numbers, whereby the countable set Q is a „zero quantity set“ with respect to the $L(2)$ inner product (i.e. the probability to pick rational numbers out of the set real numbers is zero). We note that the set of the (infinite) solutions of the field equations has the same cardinality as the set of real numbers

- the model supports also Einstein's composition model of radiation (Physikalische Zeitschrift 10 (1909), 185–193), while at the same point in time indicates to revisit the related discourse note with W. Ritz (Physikalische Zeitschrift, 10, (1909), 323-324)

- the extended $H(1/2)$ (energy) Hilbert space framework replacing the Dirichlet integral based inner product space $H(1)$ (derived from the classical Newton potential equation accompanied with Green's identities) leads to a well-posed 3D-non-linear, non-stationary Navier-Stokes boundary-initial value problem accompanied with Plemelj's extensions of Green's identities based on intrinsic surface notions "mass element" and "flux". At the same time, it omits the Yang-Mills system (the expansion of the Maxwell equations in the context of the SMEP) accompanied with the corresponding (physical) Yang-Mills mass gap problem

- the SMEP (with its underlying separation into the categories fermions (EPs with spin $1/2$ (electron, neutrino, quarks) and bosons ("EPs" with spin 0 (Higgs), spin 1 (gluons, W/Z-boson, spin 2 (graviton)) is replaced by a purely fermion based EP model defined in the coarse-grained Hilbert scale pair $((H(0), H(1))$, governed by the complementary closed sub-space with respect to the overall $((H(-1/2), H(1/2))$ Hilbert scale framework. As a consequence of the ("color") confinement problem of quarks (the phenomenon that "color-charged" particles (such as quarks and gluons) have not been isolated until today, i.e. they have not been observed until today), the "quark" concept is omitted by the original proton

- fermions are governed the Fermi statistics, whereby two identical fermions (EP with mass) cannot occupy the same quantum state (in a corresponding mathematical world this means that a mathematical „fermion“ object behaves like a rational number). The Fermi oscillator is „a particularly simple system. It is a thing capable only for two levels, zero and „epsilon““, (ScE) p.20. The Fermi oscillator is basically the mathematical model for his famous „spin-concept“. „The so-called „spin statistics theorem“ is one of the few theorems in theoretical physics, which are proven based on very very little assumptions. .. it states that EPs with integer spin are represented by symmetric wave functions, while EPs with half integer spin $(k+1/2, k=0,1,2,...)$ are represented as anti-symmetric wave functions“ (HeW) p. 103. All bosons are governed by the Bose-Einstein statistics, which is concerned with "photon gases". A characteristic of the Bose-Einstein „photon gases“ statistics is, that the concerned particles do not restrict the number of them that occupy the same (continuous) quantum state . All „photon gas“ particles can be brought into the energetically lowest quantum state (below the critical temperature of "normal gas"), where they show the same "collective" behavior. They occupy a single (continuous) quantum state of zero momentum, while „normal gas“ particles all have finite momentum. The statistical thermodynamics model for "normal gases" is the Planck oscillator; in the proposed model the related Planck-Schrödinger statistics of the n-particle problem starts with $k=1$, while in case $k=0$, the ground state energy case, is interpreted as physical condition for an model adequate ground state energy value approximation, from which the reduced Planck action variable can be derived, (ScE)

- Pauli's spin concept and its related spin statistics theorem is about amathematical object with the 2-value attributes, "half integer" and "integer" multipliers of the reduced Planck action quantum. The "connection" model of those spins is mathematically given by „complex number multiplication“. The proposed fermions model is about the three observable EPs (electron, neutrino, positron) considered in a Hilbert space framework. The underlying field is the three-dimensional unit sphere $S(3)$ enjoying the following properties

i) $S(3)$ is isometric to "rotations in the three dimensional space"

ii) $S(3)$ relates to the multiplication of triplets, Hamilton's concept of quaternions, (UnA1) p. 148

iii) quaternionic multiplication of a spatiotemporal derivative with „electromagnetic potential“ leads to two terms precisely matching the known expressions for the electric and the magnetic fields of the Maxwell equations, (UnA1) p.152

iv) the governing mathematical concept of the Maxwell equations is the Lorentz transformation, which is a linear transformation mapping space-time onto space-time, preserving the scalar product

- the $S(3)$ model in combination with a complex Lorentz group puts the spot on the famous Spin-Statistics-PCT theorem, (StR), pp. 9, 13: "The PCT theorem is a fundamental symmetry of physical laws. It is the only combination of C, P, and T transformations (see below) that is observed to be an exact symmetry of nature at the fundamental level. In terms of Einstein's SRT, "the CPT theorem says that CPT symmetry holds for all physical phenomena, or more precisely, that any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian must have CPT symmetry"

- regarding general irreducible spinor fields and assumed "wrong" connections of spins with statistics of integer and half-odd integer spins, in which all fields either commute or anti-commute, we refer to the Spin-Statistics Theorem for General Spin, (StR) Theorem 4-10

- „Hubble's observations of a shift to the red in the spectra of the spiral nebulae—the farthest parts of the universe—indicated that they are receding from us with velocities proportional to their distances from us“ (DiP). In the context of the spherical wave conjecture regarding observable the space-time hyperbolic world) Dirac's „new basis for cosmology“ ((DiP) 1937), suggests „a model of the (physical) universe in which there is a natural velocity (requiring the concept of time) for the matter at any point, varying continuously from one point to a neighbouring point. Referred to a four-dimensional space-time picture, this natural velocity provides us with a preferred time-axis at each point, namely, the time-axis with respect to which the matter in the neighbourhood of the point is at rest. By measuring along this preferred time-axis we get an absolute measure of time, called the epoch.“

- wave-mechanical vibrations correspond to the motion of particles of a gas resp. the eigenvalues and eigenfunctions of the harmonic (Planck) quantum oscillator modelled in the Hilbert scales $L(2)/H(1)$. The alternatively proposed energy space $H(1/2) = H(1) + H(1, \text{ortho})$ indicates to revisit Schrödinger's "purely quantum wave" vision, which is about half-odd integer wave numbers, rather than integer quantum numbers. "The wave point of view in both cases (Bose-Einstein and Fermi-Dirac statistics) or at least in the Bose case (mathematically a simple oscillator of the Planck type) raises another interesting question: we may ask whether we ought not to adopt for the quantum numbers half-odd integers. On the other side, if we adopt straightaway, we get into serious trouble, especially on contemplating changes of the volume (e.g. adiabatic compression of a given volume of black body radiation), because in this process the (infinite) zero-point energy seems to change by infinite amounts!" (ScE) p. 50. We emphasize that the proposed model overcomes this challenge, as there is no n-particle problem anymore in the purely continuous spectrum world of the considered complementary sub-space of the statistical Hilbert space $L(2)$

- An elliptic mathematical „reality“ model of the universe governs two complementary subsets. There is the parabolic/hyperbolic "physical reality world", (which is a very „small“ (in the sense of a zero quantity set) countable subset of the overall mathematical „reality“ model), accompanied with physical-kinematical notions like „time“, „velocity“, „action“ and e.g. the second law of thermodynamics. Then there is the dominant closed, complementary subset of the "physical reality" model, accompanied with purely mathematical-not measurable notions like „ground state energy“. In this mathematical reality, (UnA1), concepts like Dirac's „epoch“ or Penrose's „cycles of time“ (for an alternative steady state model to describe, what came before the big bang), may be revisited to describe how elementary particles with mass are generated out of the purely mathematical, non-kinematical „world“ and how a kind of "annihilation force" occurs measurable by the second law of thermodynamics. The guiding principle may be, that "time" (resp. only "time duration") as perceived by human consciousness aggregates and reflects the "actions" (accompanied with related "action variables", (HeW)) of observed physical systems, which are detectable and understandable (e.g. the scattering theory based methods or Einstein's composition of radiation model) only by statistical ($L(2)$ -Hilbert space based) measurement methods. Regarding the prospects success measuring the generation of an elementary particles with mass we note that the sum of countable zero quantity sets is again a zero quantity set