

## The three Millennium problem solutions (RH, NSE, YME) and a Hilbert scale based geometrodynamics

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This homepage addresses the problem & solution areas

- A. The Riemann Hypothesis (RH)
- B. The 3D-Navier-Stokes equations (NSE)    [navier-stokes-equations.com](http://navier-stokes-equations.com)
- C. The Yang-Mills equations (YME)        [quantum-gravitation.de](http://quantum-gravitation.de)
- D. Plasma dynamics
- E. Geometrodynamics

building on a common Hilbert scale framework enabling a combined usage of spectral theory, variational methods for non-linear operators (VaM), Galerkin-Ritz approximation theory (VeW), and tools like Pseudo-Differential operators ((EsG), (LoA), (PeB)), degenerated hypergeometric functions (GrI), Hilbert (resp. Riesz) transform(s) and wavelets (HoM). The link between PDO and the Galerkin-Ritz approximation theory is given by the Garding inequality and the concept of hypoellipticity ((AzA), (GaL), (PeB)). The norms of the Hilbert scale  $H(a)$  can be enriched with an additional norm enjoying an "exponential decay" behavior. Each Hilbert space norm with  $a < 0$  is governed by the norm of the Hilbert space  $H(0)$  and this "exp-decay" norm (BrK5), (BrK6). Already in the context of the provided RH solutions the Hilbert space  $H(-1/2)$  turned out to be the appropriate alternative to its subspace  $H(0)$ . Its norm is governed by the norm of the "observation" space  $H(0)$  and this "exp-decay" norm. This property is proposed to be applied in the context of the decomposition of the Hilbert space  $H(-1/2) = H(0) + H(0,ortho)$  ((BrK), (BrK1), (BrK3), (BrK7)).

The Zeta function is an element of  $H(-1)$ , but not an element of  $H(-1/2)$ . Therefore, there is a characterization of the Zeta function on the critical line in the form  $((Z,v))$  for all  $v \in H(0)$ . As the "test space"  $H(0)$  is compactly embedded into  $H(-1/2)$  this shows that there is an extended Zeta function  $Z(*) = Z + Z(\#)$  (Friedrichs' extension) with the characterization  $((Z(*),v))$  for all  $v \in H(-1/2)$ , where  $Z$  can be interpreted as orthogonal approximation of  $Z(*)$  with discrete spectrum.

The story line of this page is structured as follows:

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  - b. The  $H(-1/2)$  Hilbert space and a new ground state energy model
  - c. The  $H(-1/2)$  space and a consistent plasma&geometrodynamics
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## 1. RH solutions enabling a $H(-1/2)$ based quantum geometrodynamics

an appropriate Hilbert scale framework is proposed to enable several RH criteria, while its correspondence to classical function spaces of the current Zeta function theory is ensured. One of the RH criteria is about the Hilbert-Polya criterion, which is about a convolution operator representation of the Zeta function requiring a properly defined domain. This Hilbert space based operator representation with its underlying Hilbert scale frame (defining also the regularity of the Zeta function) can be applied to answer the Berry-Keating conjecture.

### 1a. The Berry-Keating conjecture and the $H(-1/2)$ quantum state space

The Berry-Keating conjecture puts the zeros of the Zeta function (on the critical line, if the RH is true) in relationship to the (energy level) eigenvalues associated with the classical Hermitian operator  $H(x,p)=x*p \sim x*(d/dx)$ , where  $x$  denotes the position coordinate and  $p$  the conjugate momentum. The Friedrichs extension of the variational representation of the Zeta function (on the critical) with  $L(2)$ -test space indicated a  $H(-1/2)$  quantum state space with related  $H(1/2)$  energy space. The today's standard quantum state resp. energy spaces are  $H(0)=L(2)$  resp.  $H(1)$ , i.e. the latter Hilbert spaces are compactly embedded subspaces of the proposed new ones. Applying the physical quantum (fluid) Hilbert (state) space  $H(-1/2)$  to the 3-D non-linear, non-stationary NSE enables a well posed variational representation of the NSE with appropriate valid energy inequality, closing the Serrin gap problem. The correspondingly variational representation of the Maxwell equations enables a quantum field model (modified YME), enabling a differentiation of "elementary particles" with and w/o mass (modelled by the orthogonal decomposition of the Hilbert spaces  $H(-1/2) = H(0) + H(0,ortho)$  resp.  $H(1/2) = H(1) + H(1, ortho)$ ). It enables the concept of orthogonal projection, which can be interpreted as "mass generation process during observation". Purely energy interaction of the "EP" are "acting" in the orthogonal space (which might be interpreted as zero point energy pool, "wave package resp. eigen-differential space). The macroscopic and microscopic state of quanta relate to corresponding frequencies of its vibrations. The corresponding action variables of the system ((HeW) II.1.c) define the related kinematical (physical) and thermodynamical concept of "time" ((RoC), (SmL)).(RoC1), section 13, "the source of time": *"Our interaction with the world is partial, which is why we see it in blurred way. To this blurring is added quantum indeterminacy. The ignorance that follows from this determines the existence of a particular variable - thermal time - and of an entropy that quantifies our uncertainty. Perhaps we belong to a particular subset of the world that interacts with the rest of it in such a way that this entropy is lower in one direction of our thermal time."*

### **1b. The $H(-1/2)$ Hilbert space and a new ground quantum state ( $H(0,ortho)$ ) & ground quantum energy ( $H(1; ortho)$ ) model**

If in this context a successfully applied least action principle (being interpreted as a maxime of Kant's reflective judgment) results into appropriate consistent mathematical-physical models, those models can be declared as law of natures. The above is related to the three "forces of nature" as modelled by the SMEP. The nature of those elementary particles and the way they move, is described by quantum mechanics, but quantum mechanics cannot deal with the curvature of space-time. Space-time are manifestations of a physical field, the gravitational field. At the same time, physical fields have quantum character: granular, probabilistic, manifesting through interactions. The to be defined common mathematical solution framework needs to provide a quantum state of a gravitational field, i.e. a quantum state of space. The crucial difference between the photons characterized by the Maxwell equations (the quanta of the electromagnetic field) and the to be defined quanta of gravity is, that photons exists in space, whereas the quanta of gravity constitute space themselves ((RoC2) p. 148). The proposed mathematical framework provides a common baseline to integrate quantum mechanics & thermodynamics with gravity & thermodynamics. From a physical model problem perspective this is about a common mathematical framework for black body radiation ((BrK4) remark 2.6, Note O55, O71, O72) and black hole radiation ((RoC3) p. 56, 60 ff)). The thermodynamics is the common physical theory denominator with the Planck concept of zero point energy of the harmonic quantum oscillator (BrK), (BrK1), and the Boltzmann entropy concept. An integrated model needs to combine the underlying Bose-Einstein and the Dirac-Fermi statistic. In this context already Schrödinger suggested half-odd quantum numbers rather than integers. *"From the point of analogy one would very much prefer to do so. For, the "zero point energy" of a Planck oscillator is not only borne out by direct observation in the case of crystal lattices, it is also so intimately linked up with the Heisenberg uncertainty relation that one hates to dispense with it. On the other hand, if we adopt it straightaway, we get into serious trouble, especially on contemplating changes of the volume (e.g. adiabatic compression of a given volume of black-body radiation)"* ((ScE) p. 50).

### 1c. The $H(-1/2)$ quantum state space and a consistent plasma & geometrodynamics

For all of the considered physical problem/solution areas of this homepage the Hilbert space  $H(-1/2)$  is suggested as physical quantum (Hilbert) state space model accompanied by correspondingly defined variational (Differential, Pseudo Differential or singular integral operator) equations. Beside the NSE and the YME problem areas the following other related areas are considered: (1) plasma dynamics (Landau damping phenomenon, i.e. "*wave damping w/o energy dissipation by collision in plasma*") and (2) geometrodynamics. Concerning the problem area (1) we note that based on the classical Vlasov (partial differential) equation (describing the time evolution of the distribution function of plasma consisting of charged particles with long-range interaction) the non-linear Landau damping phenomenon has been proven in (MoC). The central element of the proof is about analytical (!) norm estimates in sync with the underlying Gaussian distribution function regularity. At the same point in time, the Vlasov equation in its classical PDE representation overlooks the important physical phenomenon of "*electrons travelling with exactly the material speed and the wave speed*" ((ShF) p. 392). The not physical problem adequate (analytical) norm estimates in combination with the physical modelling gap on plasma collision level show that the Vlasov equation is a not appropriate mathematical model for the non-linear Landau damping phenomenon. In (BrK6) an alternative model (based on the original Boltzmann-Landau (collision) equations; (LiP) (LiP1)) is proposed. It turns out that the non-linear Boltzmann-Landau collision operator can be approximated by a linear Pseudo Differential Operator (PDO) of order zero with symbol  $b(i,j)(z) := (z/abs(z)) * a(i,j)(z)$ , whereby  $a(i,j)(z)$  denotes the symbol of the Oseen kernel (LeN). Corresponding Hilbert space norm estimates are provided to build a problem adequate proof of the Landau damping phenomenon. An appropriate plasma collisions (dynamics) model is a central building block for the related geometrodynamics problem/solution area (2). The proposed framework is also suggested to be applied to build a unified quantum field and gravity field theory based on the conceptual thoughts of Wheeler/deWitt (CiI), and and the related Loop Quantum Theory (LQT), which is a modern version of the theory of Wheeler and deWitt, where "*the variables of the theory describe the fields that form matter, photons, electrons, other components of atoms and the gravitational field - all on the same level*" ((RoC1) section 8, "dynamics as relation").

(CiI) 2.8: *Einstein's "general relativity" or "geometric geometry of gravitation" or "geometrodynamics", has two central ideas: (1) Space-time geometry "tells" mass-energy how to move, (2) mass-energy "tells" space-time geometry how to curve. The concept (1) is automatically obtained by the Einstein field equations, (CiI) (2.3.14), basically as the covariant divergence of the Einstein tensor is zero. At the same point in time there are multiple tests of the geometrical structure and of the geodesic equation of motion, e.g. gravitational deflection and delay of electromagnetic waves, de Sitter and Lense-Thirring effect, perihelion advance of Mercury, Lunar Laser Ranging with its relativistic parameters: time dilation or gravitational redshift, periastron advance, time delay in propagation of pulse, and rate of change of orbital period, (CiI) 3.4.*

(CiI) 3.5: *"Hilbert used a variational principle and Einstein the requirement that the conservation laws for momentum and energy for both, gravitational field and mass-energy, be satisfied as a direct consequence of the field equations. ... Einstein geometrodynamics, ..., has the important and beautiful property the the equations of motion are a direct mathematical consequence of the Bianchi identities."*

With respect to the overall conceptual idea of this homepage a Hilbert space based geometrodynamics is proposed to be built on "space-time states", which are represented by elements of  $H(-1/2)$ , while their corresponding "space-time energy" elements are represented by the corresponding "dual" (wavelets) elements in  $H(1/2)$ . The Einstein field equations are proposed to be re-formulated as a weak (!) least action minimization problem by correspondingly defined variational equations representation. With respect to

the Bianchi identities we emphasize that if  $((u,v))$  denotes the inner product of  $H(-1/2)$  the following relationships hold true:  $((\text{div}(u),v)) \sim (u,v) \sim ((u,\text{grad}(v)))$ . The methods of functional analysis are basically the same as those in the elasticity theory (MaJ1). The building principles for an appropriately defined variational representation is about that the way, (1) how "*Space-time geometry "tells" mass-energy how to move*", can be obtained by those representation and that the multiple tests (observed phenomena) of the geometrical structure and of the geodesic equation of motion ((2) "*where mass-energy "tells" space-time geometry how to curve*") is modelled (as a kind of symmetry break down) as approximation solution in the compactly embedded sub-spaces  $H(0)$  resp.  $H(1)$  of  $H(-1/2)$  resp.  $H(1/2)$ .

#### **1d. The $H(-1/2)$ quantum state space replacing the Dirac distributions space**

With respect to the below we note that the Dirac theory with its underlying concept of a Dirac "function" is proposed to be replaced by (fluid/quantum/... state) "elements" of the distributional Hilbert space  $H(-1/2)$ . We note that the regularity of the Dirac distribution "function" depends from the space dimension, i.e. it is an element of  $H(-n/2-e)$  ( $e>0$ ,  $n =$  space dimension). Therefore, the alternative  $H(-1/2)$  quantum state concept avoids space dimension depending regularity assumptions for quantum mechanics "wave packages" / "eigen-functions" / "momentum functions" with corresponding continuous spectrum.

## 2. A Kummer function based alternative Zeta function theory

In order to prove the **Riemann Hypothesis** (RH) the Polya criterion can not be applied in combination with the Müntz formula ((TiE) 2.11). This is due to the divergence of the Müntz formula in the critical stripe due to the asymptotics behavior of the baseline function, which is the **Gaussian function**. The conceptual challenge (not only in this specific case) is about the not vanishing constant Fourier term of the Gaussian function and its related impact with respect to the Poisson summation formula. The latter formula applied to the Gaussian function leads to the Riemann duality equation ((EdH) 1.7). A proposed alternative "baseline" function than the Gaussian function, which is its related Hilbert transform, the **Dawson function**, addresses this issue in an alternative way as Riemann did. As the Hilbert transform is a convolution integral in a correspondingly defined distributional Hilbert space frame it enables the **Hilbert-Polya conjecture** (e.g. (CaD)). The corresponding distributional ("periodical") Hilbert space framework, where the Gaussian / Dawson functions are replaced by the fractional part /  $\log(2\sin)$ -functions enables the **Bagchi** reformulation of the **Nyman-Beurling RH criterion**.

The corresponding formulas, when replacing the Gaussian function by its Hilbert transform, are well known: the **Hilbert transform** of the **Gaussian** is given by the **Dawson integral** (GaW). Its properties are e.g. provided in ((AbM) chapter 7, (BrK4) lemma D1). The Dawson function is related to a special **Kummer function** in a similar form than the (error function)  $\text{erf}(x)$ -function resp. the  $\text{li}(x)$ -function ((AbM) (9.13.1), (9.13.3), (9.13.7), (LeN) 9.8, 9.13). A characterization of the Dawson function as an  $\sin$ -integral (over the positive  $x$ -axis) of the Gaussian function is given in ((GrI) 3.896 3.). Its Mellin transform is provided in ((GrI) 7.612, (BrK4) lemma S2). The **asymptotics of the zeros** of those degenerated hypergeometric functions are given in (SeA) resp. ((BrK4) lemma A4). The **fractional part function** related Zeta function theory is provided in ((TiE) II).

With respect to the considered distributional Hilbert spaces  $H(-1/2)$  and  $H(-1)$  we note that the Zeta function is an integral function of order 1 and an element of the distributional Hilbert space  $H(-1)$ . This property is an outcome of the relationship between the Hilbert spaces above, the **Dirichlet series** theory (HaG) and the Hardy space isometry as provided in e.g. ((LaE), §227, Satz 40). With respect to the physical aspects below we refer to (NaS), where the  $H(1/2)$  dual space of  $H(-1/2)$  on the circle (with its inner product defined by a **Stieltjes integral**) is considered in the context of **Teichmüller theory** and the universal period mapping via **quantum calculus**. For the corresponding Fourier series analysis we refer to ((ZyA) XIII, 11). The approximation by polynomials in a complex domain leads to several notions and theorems of convergence related to **Newton-Gaussian** and **cardinal series**. The latter one are closely connected with certain aspects of the theory of Fourier series and integrals. Under sufficiently strong conditions the cardinal function can be resolved by Fourier's integral. Those conditions can be considerably relaxed by introducing Stieltjes integrals resulting in **(C,1) summable series** ((WhJ1) theorems 16 & 17, (BrK4) remarks 3.6/3.7).

The RH is connected to the **quantum theory** via the Hilbert-Polya conjecture resp. the **Berry-Keating conjecture**. It is about the hypothesis, that the imaginary parts  $t$  of the **zeros**  $1/2+it$  of the **Zeta function**  $Z(t)$  corresponds to **eigenvalues** of an unbounded self-adjoint operator, which is an appropriate **Hermitian operator** basically defined by  $QP+PQ$ , whereby  $Q$  denotes the location, and  $P$  denotes the (Schrödinger) momentum operator. In (BrK3) the corresponding model (convolution integral) operator  $S(1)$  (of order 1 with "density"  $d(\cot x)$ ) for the one-dimensional harmonic quantum oscillator model is provided.

The theory of spectral expansions of non-bounded self-adjoint operator is connected with the notions "Lebesgue-Stieltjes integral" and **"functional Hilbert equation for resolvents** ((LuL) (7.8)). The corresponding Hilbert scale framework plays also a key role

on the inverse problem for the double layer potential. The corresponding model problem (w/o any compact disturbance operator) with the Newton kernel enjoys a **double layer potential integral operator** with the **eigenvalue 1/2** (EbP).

The Riemann entire Zeta function  $Z(s)$  enjoys the functional equation in the form  $Z(s)=Z(1-s)$ . The alternatively proposed Dawson (baseline) function leads to an alternative entire Zeta function definition  $Z(*;s)$  with a corresponding **functional equation** in the form  $Z(*,1-s) = Q(s) * Z(*,s)$ , with  $Q(s):=P(s)/P(1-s)$ , whereby  $P(x):=cx*\cot(cx)$  and the constant  $c$  denotes the number "pi"/2. Therefore, the alternative entire Zeta function definition  $Z(*;s)$  have same nontrivial zeros as Riemann's entire Riemann Zeta function  $Z(s)$ .

The RH is equivalent to the **Li criterion** governing a sequence of real constants, that are certain logarithmic derivatives of  $Z(s)$  evaluated at unity (LiX). This equivalence results from a necessary and sufficient condition that the logarithmic of the function  $Z(1/(1-z))$  be analytic in the unit disk. The proof of the Li criterion is built on the two representations of the Zeta function, its (product) representation over all its nontrivial zeros ((HdE) 1.10) and Riemann's integral representation derived from the Riemann duality equation, based on the Jacobi theta function ((EdH) 1.8). Based on Riemann's integral representation involving Jacobi's theta function and its derivatives in (BiP) some particular probability laws governing sums of independent exponential variables are considered. In (KeJ) corresponding Li/Keiper constants are considered. The proposed **alternative** entire Zeta function  $Z(*,s)$  is suggested to derive an analogue **Li criterion**.

One proof of the Riemann functional equation is based on the **fractional part function**  $r(x)$ , whereby the zeta function  $zeta(s)$  in the critical stripe is given by the Mellin transform  $zeta(1-s) = M(-x*d/dx(r(x)))(s-1)$  ((TiE) (2.1.5)). The functional equation is given by  $zeta(s) = chi(s)*zeta(1-s)$ , whereby  $chi(s)$  is defined according to ((TiE) (2.1.12)). The Hilbert transform of the fractional part function is given by the  $\log(\sin(x))$ -function. After some calculations (see also (BrK4) lemma 1.4, lemma 3.1 (GrI) 1.441, 3.761 4./9., 8.334, 8.335) the corresponding alternative  $zeta(*,s)$  function is given by  **$zeta(*,1-s) * s = zeta(1-s) * \tan(c*s)$** .

The density function  $J(x)$  of the  $\log(zeta(s))$  Fourier inverse integral representation can be reformulated into a representation of the function  $\pi(x)$  (that is, for the "number of primes counting function" less than any given magnitude  $x$  ((EdH) 1.17)). Riemann's proof of the formula for  $J(x)$  results into the famous Riemann approximation error function ((HdE) 1.17 (3)) based on the product formula representation of the Gamma function  $\Gamma(1+s/2)$  ((HdE) 1.3 (4), (GrI) 8.322). The challenge to prove the corresponding **li(x) function approximation criterion** (i.e.  $li(x) - \pi(x) = O(\log(x)^2) = O(x*\exp(1/2+e))$ ,  $e>0$ , (BrK4) p.10) is about the (exponential) asymptotics of the Gaussian function ((EdH) 1.16, (BrK4) note S25). In this context we note that the Dawson function enjoys an only polynomial asymptotics in the form  $O(x*\exp(-1))$ . In summary, the alternatively proposed  $\Gamma(*,s/2) := \Gamma(s/2) * \tan(c*s)$  function leads to an **alternative Riemann approximation error function** with improved convergence behavior (at least with respect to the proposed Hilbert space norms). The appreciated asymptotics of the Dawson function suggested an **alternative li(\*,x) function** definition, whereby, of course, the result of Chebyshev about the proven relative error in the approximation of  $\pi(x)$  by Gauss'  $li(x)$  function needs to be taken into account ((EdH) 1.1 (3)). Alternatively to the **Gaussian density**  $dg=\log(1/t)dt$  the above indicates to consider the **Clausen density**  $dw$ , where  $w(t)$  denotes the periodical continuation of the Clausen integral ((AbM) 27.8). Obviously the Clausen integral is related to the Hilbert transform of the fractional part function.

The **Dawson function F(x)** (i.e. the Hilbert transform of the Gaussian function  $f(x):=\exp(-x*x)$ ) is related to the two special Kummer functions  $K(1,3/2;z)$  and  $K(1/2;z):=K(1/2,3/2,z)$  by  **$F(x) = x*K(1,3/2;-x*x)$**  ((LeN) (9.13.3)) resp.  **$F(x) = x * f(x) * K(1/2,x*x)$**  ((GrI), 9.212). It provides an option to replace the auxiliary

functions  $G(b)$  resp.  $E(b)$  in (EdH) 1.14, 1.16, to derive the formula for the Riemann density function  $J(x)$  ((EdH) 1.12 (2)). Both special Kummer functions enjoy appreciated **non-asymptotics** of its **zeros** (SeA): let  $c="pi"$  denote the unit circle constant, then the imaginary part of the zeros of both functions fulfill the inequality  **$(2n-1)*c < \text{abs}(\text{Im}(z(n))) < 2n*c$** , while the real parts fulfill  $\text{Re}(z) < -1/2$  resp.  $\text{Re}(z) > 1/2$  for  $K(1,3/2;z)$  resp.  $K(1/2;z)$ . In other words, there are no zeros of  $K(1/2;z)$  on the critical line  $s=1/2*it$  ( $t \in \mathbb{R}$ ), resp. there are no zeros of  $K(1,3/2;z)$  on the "dual" line  $(1-s)$  (see also (BrK4) Notes O5, O22, O23, (BrK7) Note 11).

The density of prime numbers appears to be the **Gaussian density**  $dg = \log(1/t)dt$  defining the corresponding prime number counting integral function ((EdH) 1.1 (3)). We mention the Kummer function based representation of the li-function in the form  $\text{li}(x) = -x*K(1,1;-\log x)$  ((LeN) (9.13.7)). The asymptotics of the special Kummer functions  **$K(a;x) := K(a,a+1;x)$**  are given by  $K(a;x) \sim e*\exp(x+\log x) / \Gamma(a)$  ((OIF), 7 §10.1, (AbM) 13.5.1.). Let  $G(x)$  denote the first derivative of  $K(1/2;z)$ , i.e.  $(d/x) K(1/2;x) = (1/3)*K(3/2;x)$  with  $K(3/2;x) := K(3/2,5/2;x)$ , then it holds  $K(1/2,x) + 2xG(x) = e*\exp(x)$  ((BrK4), lemma K2). For the related equations with respect to the incomplete Gamma function we refer to (OIF1) 7.2.2, 8.4.15). The asymptotics of the Kummer functions are given by  $K(a,c;x) \sim e*\exp(x+(a-c)\log x) / \Gamma(a)$  ((OIF), 7 §10.1, (AbM) 13.5.1.) Therefore the functions  $e*\exp(x)/x$ ,  $K(1/2,x)$  and  $K(3/2,x)$  are asymptotically identical. By substitution of the integration variable by  $t \rightarrow \exp(y)$  of the li-function integral this results into an alternative prime number approximation function in the form  **$K(1/2,\log x) = x - \log x * (2/3) * K(3/2,\log x)$** . We also note the relationship of  $K(a;-x)$  to the incomplete Gamma function ((AbM) 13.6.10). The **incomplete Gamma function** play a key role to compute the action of the **Leray projection operator** on the **Gaussian functions** (LeN1). Those action formulas can be applied to derive in the context of the **well-posedness** topic of the **NSE** and related (based on tempered distribution and a Carleson measure characterization of the BMO space) estimates ((LeN1), (KoH), theorems 1 and 2, see also (BrK4) pp. 26, 58, 64, 99, 121).

The asymptotics of the special Kummer functions  $K(a;x) := K(a,a+1;x) \sim e*\exp(x+\log x) / \Gamma(a)$  ((OIF), 7 §10.1, (AbM) 13.5.1.) is proposed as alternative tool for the additive number theory. **Landau** predicted the proof of the **binary Goldbach conjecture** (with high probability) based on the Stäckel approximation formula in combination with his own corresponding additions (LaE1). With the notation of (LaE1) the prime pair  $(p,q)$  counting function  $H(x)$  with the condition  $p+q \leq x$  corresponds asymptotically  $H(x) \sim (1/2)*(x/\log x)*(x/\log x)$ . The **Stäckel formula** shows the corresponding asymptotics with respect to the (number theoretical) Euler  $\phi(n)$ -function in the form  $(n/\log n)*(n/\log n)/\phi(n)$ . We suggest to apply a modified "density" function in the form  **$H(*,x) \sim c(a,b) * K(a;\log x) * K(b;\log x)$** . The structure of the alternative prime number approximation function  $K(1/2,\log x)$  indicates a correspondingly **modified Landau density function**  $\theta(x) = x - c*\log x - \dots$  resp.  $T(x) := \theta(e*\exp(x))$  (as defined and applied e.g. in ((BrK4) pp. 8-10, 23, 104, Notes S29/S30/S56/O51, (KoJ), (LaE) §50), (OsH) Kap. 8)) in the form  **$\theta(*;x) := K(1/2,\log x) - \dots = x - \log x * (2/3) * K(3/2;\log x) - \dots$**

The relationship of the considered Kummer functions to the incomplete Gamma function is provided in (AbM) 6.5.12. We further note, that the **generalized asymptotic (Poincaré) expansion** admits expansions that have no conceivable value, in an analytical or numerical sense, concerning the functions they represent. In (OIF) §10, the expansion of  $\sin(x)/x$  is provided with first summand term  $\exp(-x)/\log x$ .

Additionally, the above alternative  $Z(s)$  resp.  $\zetaeta(s)$  function representations indicate an alternative Gamma (auxiliary) function definition in the form  $G(*,s/2) := G(s/2)*\tan(cs)/s$  with identical asymptotics for  $x \rightarrow 0$ . Corresponding formulas for the  $\tan(x)$ - resp. the **log(tan)-function** are provided in ((GrI) 1.421,1.518). In (EIL) the **Fourier expansion** of the  $\log(\tan)$  function is provided, giving a note to its related Hilbert space  $H(a)$

regularity. In (EL1) log-tangent integrals are evaluated by series involving  $\zeta(2n+1)$ . Its graph looks like a beautiful white noise diagram. In (EsO), formulas (6.3), (6.4), the Fourier expansion of  $\log(\Gamma(x))$  function is provided with coefficients  $a(n)=1/(2n)$ ,  $b(n)=(A+\log n)/(2cn)$  and  $a(0)=\log(\sqrt{4c})$ . For a corresponding **Hilbert transform** evaluation we refer to (MaJ).

For other related application areas of  $\Gamma(*,s/2)$  we refer to Ramanujan's chapter "Analogues of the Gamma Function" ((BeB) chapter 8).

In (TiE) theorem 4.11, an approximation to the *zeta* function series in the critical stripe by a partial sum of its **Dirichlet series** is given ((BrK4) remark 3.8). One proof of this theorem is built on a simple application of the theorem of residues, where the *zeta* series is expressed as a (Mellin transform type) **contour integral** of the **cot(cz)-function** ((TiE) 4.14). As the cot and the *zeta* function are both elements of the distributional Hilbert space  $H(-1)$  the contour integral above with a properly chosen contour provides a contour integral representation for the *zeta* in a weak  $H(-1)$  sense. In (ChK) VI, §2, two expansions of  $\cot(z)$  are compared to prove that all coefficients of one of this expansion ( $\zeta(2n)/\pi(\exp(2n))$ ) are rational. Corresponding formulas for odd integers are unknown. In (EsR), example 78, a "**finite part**"-"**principle value**" **integral representation** of the  **$c \cdot \cot(cx)$**  is given (which is zero also for positive or negative integers). It is used as enabler to obtain the asymptotic expansion of the p.v. integral, defined by the "restricted" Hilbert transform integral of a function  $u(x)$  over the positive  $x$ -axis, only. In case  $u(x)$  has a structure  $u(x)=v(x) \cdot \text{squar}(x)$  the representation enjoys a remarkable form, where the numbers  $n+1/2$  play a key role. In (OIF1) 25.6.6, an integral value representation for  $\zeta(2n+1)$  is provided with  $\cot(2cx)$  "density" function.

In ((BrK4) lemma 3.4, lemma A12/19) the function  $P(x)$  is considered in the context of (appreciated) quasi-asymptotics of (corresponding) distributions ((ViV) p. 56/57) and the Riemann mapping theorem resp. the Schwarz lemma. The considered "function"  $g(x) := -d/dx(\cot(x))$  (whereby the cot-"function" is an element of  $H(-1)$ ) is auto-model (or regular varying) of order  $-1$ . This condition and its corresponding asymptotics property ((BrK) lemma 3.4) provide the prerequisites of the **RH Polya criterion** ((PoG), (BrK5) theorem 6). The above quasi-asymptotics indicates a replacement of the differential  $d(\log x)$  by  $d(\log(\sin x))$ . The  $\cot(z)$  function expansions (ChK) VI, §2) in combination with **Ramanujan's formula** ((EdH) 10.10) resp. its generalization theorem ((EdH) p.220) is proposed to be applied to define an appropriate analytical (Mellin transform) function in the stripe  $1/2 < \text{Re}(s) < 1$ .

In (GrI) 8.334, the relationship between the cot- and the Gamma function is provided. From (BeB) 8. Entry17(iii)) we quote: "*the **indefinite Fourier series of the cot(cx)-function** may be formally established by differentiating the corresponding Fourier series equation for (the  $L(2)=H(0)$ -function)  **$-\log(2\sin(cx))$** " ((BrK4) remark 3.8). The proposed distributional Hilbert scales provide the proper framework to justify Ramanujan's related parenthetical remark "for the same limit" (in a  $H(-1)$ -sense).*

The **RH** is connected to the **quantum theory** via the Hilbert-Polya conjecture resp. the **Berry-Keating conjecture**. The latter one is about a physical reason, why the RH should be true. This would be the case if the imaginary parts  $t$  of the zeros  $1/2+it$  of the **Zeta function**  $Z(t)$  corresponds to eigenvalues of an unbounded self-adjoint operator, which is an appropriate **Hermitian operator** basically defined by  $QP+PQ$ , whereby  $Q$  denotes the location, and  $P$  denotes the (Schrödinger) momentum operator.

The **Zeta function** is an **element of  $H(-1)$** , but not an element of  $H(-1/2)$ . Therefore, there is a characterization of the Zeta function on the critical line in the form  $((Z,v))$  for all  $v \in H(0)$ . As the "test space"  $H(0)$  is compactly embedded into  $H(-1/2)$  this shows that there is an extended Zeta function  $Z(*)=Z+Z(\#)$  (Friedrichs' extension) with the

characterization  $((Z(*), v))$  for all  $v \in H(-1/2)$ , where  $Z$  can be interpreted as orthogonal approximation of  $Z(*)$  with discrete spectrum.

Riemann's "workaround" function  $h(x) = -x \frac{d}{dx}(f(x))$  do have an obvious linkage to the "**commutator**" concept in quantum theory. In this context the Gaussian function  $f(x)$  can be characterized as "minimal function" for the Heisenberg uncertainty inequality. Applying the same solution concept as above then leads to an alternative Hilbert operator based representation in  $H(-1/2)$ , resp. to a  $H(-1)$  based definition of the commutator operator with extended domain. The common denominator of the alternatively proposed Hilbert space framework  $H(-1/2)$  goes along with the definition of a correspondingly defined "momentum" operator (of order 1)  $P: H(1/2) \rightarrow H(-1/2)$  defined in a variational form. In the one-dimensional case the Hilbert transform  $H$  (in the  $n > 1$  case the **Riesz operators  $R$** ) is linked to such an operator given by  $((Pu, v)) = (Hu, v)$ . With respect to quantum theory this indicates an **alternative Schrödinger momentum operator** (where the gradient operator "grad" is basically replaced by the Hilbert transformed gradient, i.e.  $\mathbf{P} = -i \mathbf{R}(\text{grad})$ ) and a corresponding alternative commutator representation  $QP - PQ$  in a weak  $H(-1/2)$  form. We note that the **Riesz operators  $R$**  commute with **translations** and **homothesis** and enjoy nice properties relative to **rotations**.

### 3. Overview: NSE, YME and plasma/geometrodynamic problem/solution areas

The common Hilbert scale is about the Hilbert spaces  $H(a)$  with  $a=1, 1/2, 0, -1/2, -1$  with its corresponding inner products  $((u,v)), (u,v), (u,v), ((u,v)), (((u,v)))$ . The proposed mathematical concepts and tools are especially correlated to the names of **Plemelj**, **Stieltjes** and **Calderón**.

The **Dirac theory** with its underlying concept of a "Dirac function" (where the regularity of the Dirac distribution "function" depends from the space dimension) is **omitted** and replaced by a distributional Hilbert space (domain) concept. This alternative concept avoids space dimension depending regularity assumptions for (quantum) physical variational model (wave package) states and solutions (defined e.g. by energy or operator norm minimization problems) and physical problem specific "force" types. We note that for signals on  $\mathbb{R}$  the spectrum of the Hilbert transform is (up to a constant) given by the distribution  $v.p.(1/x)$ , whereby the symbol "v.p." denotes the Cauchy principal value of the integral over  $\mathbb{R}$ . Its corresponding Fourier series is given by  $-i \cdot \text{sgn}(k)$  with its relationship to "positive" and "negative" Dirac "functions" and the unit step function  $Y(x)$ . In a  $H(-1/2)$  framework the Dirac "function" concept can be avoided, which enables a generalization to dimensions  $n > 1$  without any corresponding additional regularity requirements (the **Dirac/Delta "function"** is **an element of  $H(-n/2-e)$** ,  $e > 0$ ).

The newly proposed "fluid/quantum state" Hilbert space  $H(-1/2)$  with its closed orthogonal subspace of  $H(0)$  goes also along with a combined usage of  $L(2)$  waves governing the  $H(0)$  Hilbert space and "orthogonal" wavelets governing the  $H(-1/2)$ - $H(0)$  space. The wavelet "reproducing" ("duality") formula provides an additional degree of freedom to apply wavelet analysis with appropriately (problem specific) defined wavelets, where the "*microscope observations*" of **two wavelet (optics) functions** can be compared with each other (LoA). The prize to be paid is about additional efforts, when re-building the reconstruction wavelet.

We propose modified Maxwell equations with correspondingly extended domains according to the above. This model is proposed as alternative to SMEP, i.e. the modified Maxwell equation are proposed to be a "Non-standard Model of Elementary Particles (NMEP)", i.e. an alternative to the Yang-Mills (field) equations. The conceptual approach is also applicable for the Einstein field equations. Mathematical speaking this is about potential functions built on corresponding "density" functions. The source density is the most prominent one. Physical speaking the source is the root cause of the corresponding source field. Another example is the invertebrate density (=rotation) with its corresponding rotation field. The Poincare lemma in a 3-D framework states that source fields are rotation-free and rotation fields are source-free. The physical interpretation of the rotation field in the modified Maxwell equations is about rotating "mass elements w/o mass" (in the sense of Plemelj) with corresponding potential function. In a certain sense this concept can be seen as a generalization of the Helmholtz decomposition (which is about a representation of a vector field as a sum of an irrotational (curl-free) and a solenoidal (divergence-free) vector field): it is derived applying the delta "function" concept. In the context of the proposed distributional Hilbert space framework, the Dirac function concept (where the regularity of those "function" depends from the space dimension) is replaced by the quantum state Hilbert space  $H(-1/2)$ . The solution  $u$  (ex  $H(1/2)$ ) of the Helmholtz equation in terms of the double layer potential is provided in ((LiI), 7.3.4). From the Sobolev embedding theorem it follows, that for any space dimension  $n > 0$  the modified Helmholtz equation is valid for not continuous vector fields.

### 3b. The related NSE problem/solution area (BrK2)

The Navier-Stokes Equations (NSE) describes a flow of incompressible, viscous fluid. The three key foundational questions of every PDE is existence, and uniqueness of solutions, as well as whether solutions corresponding to smooth initial data can develop singularities in finite time, and what these might mean. For the NSE satisfactory answers to those questions are available in two dimensions, i.e. 2D-NSE with smooth initial data possesses unique solutions which stay smooth forever. In three dimensions, those questions are still open. Only local existence and uniqueness results are known. Global existence of strong solutions has been proven only, when initial and external forces data are sufficiently smooth. Uniqueness and regularity of non-local Leray-Hopf solutions are still open problems.

Basically the existence of 3D solutions is proven only for "large" Banach spaces. The uniqueness is proven only in "small" Banach spaces. The question of global existence of smooth solutions vs. finite time blow up is one of the Clay Institute millennium problems.

The existence of weak solutions can be provided, essentially by the energy inequality. If solutions would be classical ones, it is possible to prove their uniqueness. On the other side for existing weak solutions it is not clear that the derivatives appearing in the inequalities have any meaning. Basically all existence proofs of weak solutions of the Navier-Stokes equations are given as limit (in the corresponding weak topology) of existing approximation solutions built on finite dimensional approximation spaces. The approximations are basically built by the Galerkin-Ritz method, whereby the approximation spaces are e.g. built on eigenfunctions of the Stokes operator or generalized Fourier series approximations. It has been questioned whether the NSE really describes general flows: The difficulty with ideal fluids, and the source of the d'Alembert paradox, is that in such fluids there are no frictional forces. Two neighboring portions of an ideal fluid can move at different velocities without rubbing on each other, provided they are separated by a streamline. It is clear that such a phenomenon can never occur in a real fluid, and the question is how frictional forces can be introduced into a model of a fluid.

The question intimately related to the uniqueness problem is the regularity of the solution. Do the solutions to the NSE blow-up in finite time? The solution is initially regular and unique, but at the instant  $T$  when it ceases to be unique (if such an instant exists), the regularity could also be lost. Given a smooth datum at time zero, will the solution of the NSE continue to be smooth and unique for all time?

The **NSE** are derived from the (Cauchy) stress tensor (resp. the shear viscosity tensor) leading to liquid pressure force. In electrodynamics & kinetic plasma physics the linear resp. the angular momentum laws are linked to the electrostatic (mass "particles", collision, static, quantum mechanics, displacement related; "fermions") Coulomb potential resp. to the magnetic (mass-less "particles", collision-less, dynamic, quantum dynamics, rotation related; "bosons") Lorentz potential.

We note that the solution of the Navier-Stokes equation are related to the considered degenerated hypergeometric functions by its corresponding integral function representation (PeR1).

With respect to the open Millenium **3D non-stationary, non-linear NSE** problem we note that the alternatively proposed "**fluid state**" Hilbert space  $H(-1/2)$  with corresponding alternative energy ("**velocity**") space  $H(1/2)$  enables a (currently missing) energy inequality based on existing contribution of the non-linear term. In the standard weak NSE representation this term is zero, which is a great thing from a mathematical perspective, avoiding sophisticated estimating techniques, but a doubtful thing from a physical modelling perspective, as this term is the critical one, which jepordized all

attempts to extend the 3D problem based on existing results from the 2D case into the 3D case. The corresponding estimates are based on Sobolev embedding theorems; the **Sobolevskii estimate** provides the appropriate estimate given that the "fluid state" space is  $H(-1/2)$  in a corresponding weak variational representation.

A "3D challenge" like the NSE above is also valid, when solving the monochromatic scattering problem on surfaces of arbitrary shape applying electric field integral equations. From (IVV) we recall that the (integral) operators  $A$  and  $A(t)$ :  $H(-1/2) \rightarrow H(1/2)$  are bounded Fredholm operators with index zero. The underlying framework is still the standard one, as the domains are surfaces, only. An analog approach as above with correspondingly defined surface domain regularity is proposed.

With respect to the relationship to the considered Hilbert space  $H(-1/2)$  we note that (BrK2):

- the NSE initial boundary value problem determines the initial pressure by the Neumann problem. Applying formally the div-operator to the classical NSE the pressure field must satisfy the Neumann problem
- the Prandtl operator is bounded and coercive and the exterior Neumann problem admit one and only on generalized solution.

As a consequence the prescription of the pressure at the boundary walls or at the initial time independently of  $u$ , could be incompatible with and, therefore, could render the NSE problem ill-posed.

A  $H(-1/2)$  (fluid state) Hilbert space framework is also applied to derive optimal finite element approximation estimates for non-linear parabolic problems with not regular initial value data (BrK2).

Kolmogorov's turbulence theory is a purely statistical model (based on the  $H(0)$  (observation/test) Hilbert space), which describes (only!) the qualitative behavior of turbulent flows. There is no linkage to the quantitative fluid behavior as it is described by the Euler or the Navier-Stokes equations. The physical counterpart of his low- and high-pass filtering Fourier coefficients analysis is a "local Fourier spectrum", which is a contradiction in itself, as, either it is non-Fourier, or it is nonlocal ((FaM)). WE propose to combine the wavelet based solution concept of (FaM) with a revisited CLM equation model in a physical  $H(-1/2)$  Hilbert space framework to enable a turbulent  $H(-1/2)$  signal which can be split into two components: coherent bursts and incoherent noise. The model enables a localized Heisenberg uncertainty inequality in the closed (noise) subspace  $L(2;ortho) = H(0;ortho) = H(-1/2)-H(0)$ , while the momentum-location commutator vanishes in the (coherent bursts) test space  $H(0)$ .

### 3b. The related YME problem/solution area

We propose an alternative mathematical framework for the Standard Model of Elementary Particles (SMEP), which replaces gauge theory and variational principles: The underlying concepts of exterior derivatives and tensor algebra are replaced by (distributional) Hilbert scales and (purely Hamiltonian) variational principles. As a consequence, the vacuum energy becomes an intrinsic part of the variational principles, i.e. it is identical for all considered Lagrange resp. Hamiltonian mechanisms of all related differential equations, while the corresponding "force" becomes an observable of the considered (Hamiltonian) minimization problem.

In some problem statements of the YME there are basically two assumptions made:

1. the energy of the vacuum energy is zero
2. all energy states can be thought of as particles in plane-waves.

As a consequence the mass gap is the mass of the lightest particle.

Our challenge of proposition 1 is about the measure of the vacuum energy, which gives the value "zero". While the energy norm in the standard  $H(1)$  Hilbert space might be zero, the value of the quantum state with respect to the energy norm of the sub-space  $H(1/2)$  still can be  $>0$ .

Our challenge of proposition 2 is going the same way: a particle with mass can be measured (condensed energy), i.e. it is an element of the test space  $H(0)$ , while there still can be "wavelets" in the closed complementary space  $H(-1/2)-H(0)$ , where the test space is "just" compactly embedded. Those "wavelets" might be interpreted as all kinds of today's massless "particles" (neutrinos and photons) with related "dark energy". As a consequence there is no YME mass gap anymore, but there is a new concept of vacuum energy (wave packages, eigen-differentials, rotation differential) governed by the Heisenberg uncertainty principle. This is about an alternative harmonic quantum energy model enabling a finite "quantum fluctuation = total energy", while replacing Dirac's Delta function by  $H(-1/2)$  distributions enabling and an alternative Schrödinger's momentum operator (BrK7).

A physical interpretation could be about "rotating differentials" ("quantum fluctuations"), which corresponds mathematically to Leibniz's concept of monads. The mathematical counterparts are the ideal points (or hyper-real numbers). This leads to non-standard analysis, whereby the number field has same cardinality than the real numbers. It is "just" the Archimedean principle which is no longer valid.

The proposed mathematical concepts and tools are especially correlated to the names of **Schrödinger** and **Weyl** (e.g. in the context of "half-odd integers quantum numbers for the Bose statistics" and resp. Weyl's contributions on the concepts of matter, the structure of the world and the principle of action (WeH), (WeH1), (WeH2)). It enables an **alternative** (quantum) **ground state energy model** embedded in the proposed distributional Hilbert scale frame of this homepage covering all variational physical-mathematical PDE and Pseudo Differential Operator (PDO) equations (e.g. also the Maxwell equations).

The electromagnetic interaction has gauge invariance for the probability density and for the Dirac equation. The wave equation for the gauge bosons, i.e. the **generalization** of the **Maxwell equations**, can be derived by forming a gauge-invariant field tensor using generalized derivative. There is a parallel to the definition of the covariant derivative in general relativity. With respect to the above there is an alternative approach indicated, where the fermions are modelled as elements of the Hilbert space  $H(0)$ , while the

complementary closed subspace  $H(-1/2)-H(0)$  is a model for the "**interaction particles, bosons**". For gauge symmetries the fundamental equations are symmetric, but e.g. the ground state wave function breaks the symmetry. When a gauge symmetry is broken the gauge bosons are able to acquire an effective mass, even though gauge symmetry does not allow a boson mass in the fundamental equations. Following the above alternative concept the "symmetry state space" is modelled by  $H(0)$ , while the the ground state wave function is an element of the closed subspace  $H(-1/2)-H(0)$  of  $H(-1/2)$  (BrK).

When one wants to treat the time-harmonic Maxwell equations with variational methods, one has to face the problem that the natural bilinear form is not coercive on the whole Sobolev space. One can, however, make it coercive by adding a certain bilinear form on the boundary of the domain (vanishing on a subspace of  $H(1)$ ), which causes a change in the natural boundary conditions.

In SMEP (**Standard Model of Elementary Particles**) symmetry plays a key role. Conceptually, the SMEP starts with a set of fermions (e.g. the electron in quantum electrodynamics). If a theory is invariant under transformations by a symmetry group one obtains a conservation law and quantum numbers. Gauge symmetries are local symmetries that act differently at each space-time point. They automatically determine the interaction between particles by introducing bosons that mediate the interaction.  $U(1)$  (where probability of the wave function (i.e. the complex unit circle numbers) is conserved) describes the electromagnetic interaction with 1 boson (photon) and 1 quantum number (charge  $Q$ ). The group  $SU(2)$  of complex, unitary  $(2 \times 2)$  matrices with determinant 1 describes the weak force interaction with 3 bosons ( $W(+)$ ,  $W(-)$ ,  $Z$ ), while the group  $SU(3)$  of complex, unitary  $(3 \times 3)$  matrices describes the strong force interaction with 8 gluon bosons.

Reformulated Maxwell or gravitation field equations in a weak  $H(-1/2)$ -sense leads to the same effect, as dealing with an isometric mapping  $g \rightarrow H(g)$  in a weak  $H(0)$ -sense ( $H$  denotes the Hilbert transform) alternatively to a second order operator in the form  $x * P(g(x))$  in a weak  $H(-1/2)$ . This results into some opportunities as

- the solutions of the Maxwell equations in a vacuum do not need any calibration transforms to ensure wave equation character; therefore, the arbitrarily chosen Lorentz condition for the electromagnetic potential (to ensure Lorentz invariance in wave equations) and its corresponding scalar function ((FeR), 7th lecture) can be avoided
- enabling alternative concepts in GRT to e.g. current ("flexible") metrical affinity, affine connexions and local isometric 3D unit spheres dealing with rigid infinitesimal pieces, being replaced by **geometrical manifolds**, enabling isometrical stitching of rigid infinitesimal pieces ((CiI), (ScP)).

### 3c. The related plasma/ geometrodynamics problem/solution area

Replacing the affine connexion and the underlying covariant derivative concept by a geometric structure with corresponding inner product puts the spot on the

**Thurston conjecture:** *The interior of every compact 3-manifold has a canonical decomposition into pieces which have geometric structure (ThW).*

This conjecture asserts that any compact 3-manifold can be cut in a reasonably canonical way into a union of geometric pieces. In fact, the decomposition does exist. The point of the conjecture is that the pieces should all be geometric. There are precisely eight homogeneous spaces  $(X, G)$  which are needed for geometric structures on 3-manifolds. The symmetry group  $SU(2)$  of quaternions of absolute value one (the model for the weak nuclear force interaction between an electron and a neutrino) is diffeomorph to  $S^3$ , the unit sphere in  $R(4)$ . The latter one is one of the eight geometric manifolds above (ScP). We mention the two other relevant geometries, the Euclidean space  $E^3$  and the hyperbolic space  $H^3$ . It might be that our universe is not an either... or ..., but a combined one, where then the "connection" dots would become some physical interpretation. Looking from an Einstein field equation perspective the Ricci tensor is a second order tensor, which is very much linked to the Poincare conjecture, its solution by Perelman and to  $S^3$  (AnM). The **geometrodynamics** provides alternative (pseudo) tensor operators to the Weyl tensor related to  $H^3$  (CiI). In (CaJ) the concept of a Ricci potential is provided in the context of the Ricci curvature equation with rotational symmetry. The single scalar equation for the Ricci potential is equivalent to the original Ricci system in the rotationally symmetric case when the Ricci candidate is nonsingular. For an overview of the Ricci flow regarding e.g. entropy formula, finite extinction time for solutions on certain 3-manifolds in the context of Perelman's proof of the Poincare conjecture we refer to (KIB), (MoJ).

The single scalar equation for the Ricci potential (CaJ) might be interpreted as the counterpart of the **CLM vorticity equation** as a simple one-dimensional turbulent flow model in the context of the NSE.

The link back to a Hilbert space based theory might be provided by the theory of spaces with an indefinite metric ((DrM), (AzT), (DrM), (VaM)). In case of the  $L(2)$  Hilbert space  $H$ , this is about a decomposition of  $H$  into an orthogonal sum of two spaces  $H_1$  and  $H_2$  with corresponding projection operators  $P_1$  and  $P_2$  relates to the concepts which appear in the problem of S. L. Sobolev concerning **Hermitean operators** in spaces with **indefinite metric** ((VaM) IV). For  $x$  being an element of  $H$  this is about a defined "**potential**"  $p(x) := \langle x, x \rangle^* \langle x, x \rangle$  ((VaM) (11.1)) and a corresponding "**grad**" potential operator **W**(x), given by

$$\mathbf{grad}(p(x)) := 2\mathbf{W}(x) := P_1(x) - P_2(x) \quad (\text{VaM}) (11.4).$$

The potential criterion  $p(x) = c > 0$  defines a manifold, which represents a **hyperboloid in the Hilbert space H** with corresponding hyperbolic and conical regions. The tool set for an appropriate **generalization** of the above "grad" definition is about the (homogeneous, not always non-linear in  $h$ ) **Gateaux differential** (or weak differential)  $\mathbf{VF}(x, h)$  of a functional  $F$  at a point  $x$  in the direction  $h$  ((VaM) §3)). The appropriate weak inner product might be the inner product of the "velocity" space  $H(1/2)$ . We note the Sobolev embedding theorem, i.e.  $H(k)$  is a sub-space of  $C(0)$  (continuous functions) for  $k > n/2$ , i.e. there is no concept of "continuous velocity/momentum" in the proposed Hilbert space framework, i.e. there is no Frechet differential existing ((VaM) 3.3). This refers to one of the several proposals, which have been made to drop some of the common sense notions about the universe ((KaM) 1.1), which is about continuity, i.e. space-time must be granular. The size of these grains would provide a natural cutoff for the Feynman integrals, allowing to have a finite S-matrix.

A selfadjoint operator  $B$  defined on all of the Hilbert space  $H$  is bounded. Thus, the operator  $B$  induces a decomposition of  $H$  into the direct sum of the subspaces, and therefore generates related hyperboloids ((VaM) 11.2). Following the investigations of Pontrjagin and Iohvidov on linear operators in a Hilbert space with an indefinite inner product, M. G. Krein proved the **Pontrjagin-Iohvidov-Krein theorem** (FaK).

In an universe model with appropriately connected geometric manifolds the corresponding symmetries breakdowns at those "connection dots" would govern corresponding different conservation laws in both of the two connected manifolds. The Noether theorem provides the corresponding mathematical concept (symmetry  $\rightarrow$  conservation laws; energy conservation in GT, symmetries in particle physics, global and gauge symmetries, exact and broken). Those symmetries are associated with "non-observables". Currently applied symmetries are described by finite- (rotation group, Lorentz group, ...) and by infinite-dimensional (gauged  $U(1)$ , gauged  $SU(3)$ , diffeomorphisms of GR, general coordinate invariance...) Lie groups.

A manifold geometry is defined as a pair  $(X,G)$ , where  $X$  is a manifold and  $G$  acts transitively on  $X$  with compact point stabilisers (ScP). Related to the key tool "Hilbert transform" resp. "conjugate functions" of this page we recall from (ScP), that Kulkarni (unpublished) has carried out a finer classification in which one considers pairs  $(G,H)$  where  $G$  is a Lie group,  $H$  is a compact subgroup and  $G/H$  is a simple connected 3-manifold and pairs  $(G_1,H_1)$  and  $(G_2,H_2)$  are equivalent if there is an isomorphism  $G_1 \rightarrow G_2$  sending  $H_1$  to a conjugate of  $H_2$ . Thus for example, the geometry  $S^3$  arises from three distinct such pairs,  $(S^3,e)$ ,  $(U(2),SO(2))$ ,  $(SO(4),SO(3))$ . Another example is given by the Bianchi classification consisting of all simply connected 3-dimensional Lie groups up to an isomorphism.

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