

Origin of the Blueshift in Signals from Pioneer 10 and 11

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June 21, 2005

Abstract

A previous paper [1] introduced a quantum-mechanical theory of gravity, and showed it agrees with the standard experimental tests of general relativity. Doppler tracking signals returned by the Pioneer 10 and 11 space probes offer an additional test. Analysis by Anderson, Laing, Lau, Liu, Neito and Turyshev [2] finds a persistent blueshift, equivalent to an extra acceleration of the probes toward the Sun. While unexplained by general relativity or prevailing cosmology, it's shown this effect is predicted by the quantum-mechanical alternative.

1 Introduction

NASA's Pioneer 10 probe was launched in 1972, and sent the first close-up pictures of Jupiter. Pioneer 11, launched the following year, showed us Saturn. In addition to those revealing images, they sent a puzzle: Doppler tracking signals returned by both probes indicated an anomalous acceleration toward the Sun [2]. Pioneer 10 crossed Pluto's orbit in 1983 and continued sending data until two years ago, when it was 82 AU from the Sun. Pioneer 11 relayed tracking signals until an electronics unit failed in 1990, at a distance of 30 AU.

Microwave signals were sent from Earth stations to the probes, which transmitted phase-locked signals back. Each station's signal was derived from a hydrogen maser frequency reference with an accuracy exceeding 1 part in 10^{12} , and the Doppler-shifted frequency of the returning signal was compared continuously. Unlike the subsequent Voyager missions to the outer planets, the Pioneers used spin stabilization, which maintains a spacecraft's orientation without frequent use of thrusters. Consequently, Doppler data was accumulated over long periods during which the motions of the craft were undisturbed.

The Doppler data was checked against models of the probes' motions by separate groups at NASA's Jet Propulsion Laboratory and The Aerospace Corporation. An unmodeled blueshift was found in each case, equivalent to an acceleration a_p of $\sim 8 \times 10^{-8} \text{ cm/s}^2$. Extensive further analysis of the Pioneer 10 data by Anderson, Laing, Lau, Liu, Neito and Turyshev [2] arrived at $(8.74 \pm 1.33) \times 10^{-8} \text{ cm/s}^2$, in the approximate direction of the Sun and Earth. This effect hasn't been reconciled with general relativity.

According to general relativity, space-time is curved by mass-energy. Quantum mechanics says space is filled with vacuum energy. Yet measurements of the universe's large-scale curvature show none. Wilczek [3] writes:

Any theory of gravity that fails to explain why our richly structured vacuum, full of symmetry-breaking condensates and virtual particles, does not weigh much more than it does is a profoundly incomplete theory.

And [4]

Since gravity is sensitive to all forms of energy it really ought to see this stuff. . . . A straightforward estimation suggests empty space should weigh several orders of magnitude (no misprint here!) more than it does. It "should" be much denser than a neutron star, for example. The expected energy of empty space acts like dark energy, with negative pressure, but there's much too much of it. To me this discrepancy is the most mysterious fact in all of physical science, the fact with the greatest potential to rock the foundations.

A previous paper [1] introduced an alternative to general relativity, in which the gravitational motion of particles and bodies is based on the optics of de Broglie waves. This "weight problem" doesn't arise there, and inflation, strange dark matter, and strange dark energy aren't needed for a flat universe. The theory derives

from general principles, but different ones. Instead of introducing a geodesic principle, there is Huygens', which is already part of quantum mechanics.

Poincaré's principle of relativity is also extended to reference frames in uniform gravitational potentials; the observed laws of physics remain unchanged. There is no assumption of the equivalence principle, in any of its various forms. Nevertheless, that principle is effectively obeyed for weak fields, as shown for the lunar orbit [1]. And while neither Mach's principle nor manifest covariance is assumed, as in general relativity, special relativity is obeyed in uniform potentials.

While this quantum-mechanical theory agrees with the standard experimental tests of general relativity, it makes different predictions for the second-order solar deflection of starlight, and for the pending NASA satellite experiment Gravity Probe B [1]. After summarizing the basic theory, here we'll derive its prediction for the Pioneer probes.

General relativity is often characterized as a "complete" theory of gravity, but provides no way to distinguish the future from the past. This acknowledged "problem of time" is targeted in theories of loop quantum gravity. Their intent is to make time as we experience it completely unnecessary – to make it go away.

Prigogine [5] argued we can't, that time with a direction is essential for describing thermodynamics and quantum mechanics. For that purpose, theories of parameterized quantum mechanics introduce a time parameter τ into Minkowski space-time, as a function of a Newtonian time t . However Hartle [6] has noted it may be impossible to do that to the curved space-time of general relativity.

This gravity theory is built on the preferred-frame special relativity advocated by Lorentz, Poincaré, and more recently by Bell [7]. There space and time are kept separate, time is already a Newtonian parameter, and nothing more is needed to describe time's direction. Bell saw this relativity as a likely necessity for a causal quantum mechanics. And it can be argued a preferred frame is precisely identified by the universal cosmic microwave background.

Instead of curving space-time, the fundamental effect of gravitational potentials in this theory is a slowing of quantum-mechanical waves. Where Einstein assumed an absolute speed of light, with space and time variable, the assumption here is the opposite. For an absolute space and time, of course the natural coordinates are isotropic, and are the type we'll be using.

Gravitational potentials are treated as attributes of elementary particles, having the same relativistic form as the electromagnetic scalar potential. In rectangular coordinates, the potential due to a particle at the origin moving in x is

$$\Phi = \frac{-G m_0}{\sqrt{x^2 + (y^2 + z^2)(1 - v^2/c^2)}} \quad (1)$$

where the role analogous to charge is played by its rest mass m_0 , and G is the gravitational constant. (One gravitational potential, vs. *ten* in general relativity.) There is also a relativistic wave equation corresponding to that for the electromagnetic scalar potential,

$$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = 4\pi G \rho \quad (2)$$

where ρ is the rest mass density.

Following are five basic transformations, whose derivations from general principles can be found in the previous paper [1]. The speed of light as a function of the gravitational potential varies as

$$c = c_0 e^{2\Phi/c_0^2} \quad (3)$$

where c_0 is the value in the absence of a gravitational potential. The velocity of de Broglie waves is similarly

$$V = V_0 e^{2\Phi/c_0^2} \quad (4)$$

where the 0 subscript again indicates the corresponding quantity with no potential.

The de Broglie frequency is

$$\nu = \nu_0 e^{\Phi/c_0^2} \quad (5)$$

and the wavelength is

$$\lambda = \lambda_0 e^{\Phi/c_0^2} \quad (6)$$

Since the rate of any clock is determined by the de Broglie frequency of its particles, clocks in gravitational potentials slow by the factor e^{Φ/c_0^2} . The dimensions of atoms and meter sticks are determined by the de Broglie wavelength, and shrink by the same factor.

A particle or body's rest mass m_0 varies as

$$m_0 = m_{00} e^{-3\Phi/c_0^2} \quad (7)$$

where m_{00} is the mass for a zero velocity and zero potential. Another equation derived gives the relativistic acceleration \mathbf{a} of a particle or body

$$\mathbf{a} = -\nabla\Phi \left(e^{4\Phi/c_0^2} + \frac{v^2}{c_0^2} \right) + \frac{4\mathbf{v}}{c_0^2} \left(\frac{d\Phi}{dt} \right) \quad (8)$$

where \mathbf{v} is its velocity.

Since there is no instability in the universe's geometry, there is no Big Bang. But inherent instability exists in the gravitational potential of the universe, and the overall speed of light [1]. The rate of change for the cosmological potential is taken to be

$$\frac{d}{dt} \left(\frac{\Phi}{c_0^2} \right) = -H \quad (9)$$

where H is the Hubble constant. From this and Eq. (5), clocks are slowing. And from Eq. (6), atoms and meter sticks are shrinking.

The term "relativity" was Poincaré's, and to illustrate the principle [8] he asked: What if you went to bed one night, and when you awoke the next day everything in the world was a thousand times bigger? Would you notice anything? As he pointed out, such effects aren't observed locally, since measuring devices change with the objects they measure. (According to Einstein [9], he also insisted the true geometry of the universe is Euclidean.)

Still, in this gradually evolving universe, an apparent expansion would be seen in distant galaxies. Suppose a galaxy's light takes time t to reach an observer here. During that time, the cosmological potential changes by $-Ht$, and from Eq. (3), the speed of light diminishes by e^{-2Ht} . Since no difference arises in the relative velocities of two successive wavefronts, the absolute wavelength of light from a remote source doesn't change after its emission.

But the wavelength of a local spectral reference shrinks, in accord with Eq. (6). We'll define the wavelength of an observer's spectral reference as λ_0 . The quantity in Eq. (6) corresponding to Φ/c_0^2 then is $+Ht$, and the relative wavelength λ seen by the observer is

$$\lambda = \lambda_0 e^{Ht} \cong \lambda_0 (1 + Ht) \quad (10)$$

When the galaxy's distance d is modest, this gives the familiar Hubble relation for low redshifts,

$$\lambda \cong \lambda_0 (1 + Hd/c) \quad (11)$$

usually interpreted as an expansion of the universe.

The diminishing speed of light does change the absolute frequency at which wavefronts arrive, and the apparent frequency of the source. With c slowing as e^{-2Ht} , the observed frequency ν is

$$\nu = \nu_0 e^{-Ht} \cong \nu_0 (1 - Ht) \quad (12)$$

where ν_0 is the frequency of a local reference. From Eq. (5), the frequency of light from a remote galaxy is e^{Ht} greater than ν_0 when emitted. But it's less by the same factor when observed, remaining proportional to c/λ .

2 Signal for a Stationary Earth and Non-gravitating Probe

To calculate the predicted Pioneer Doppler signal, we'll start with a simplified example, where there are no local gravitational fields. Earth will be treated as stationary and massless, with the probe moving away in a straight line. While the signal the probe returns is sent at a different frequency than the one it receives, the two are phase-locked. So we'll treat these as a simple reflection of the same signal by a moving body.

From the cosmological redshift described above, without a Doppler shift, the relative frequency of the returning signal would be

$$\nu \cong \nu_0 (1 - 2Ht) \quad (13)$$

where t is the signal's one-way travel time. After including the first-order Doppler effect, this becomes

$$\nu \cong \nu_0 \left(1 - \frac{2v}{c} - 2Ht \right) \quad (14)$$

where v is the probe's velocity.

From Eq. (9) and the final term in Eq. (8), the probe has a velocity-dependent acceleration

$$a = -4Hv \quad (15)$$

Where T is the time since its departure, v_i is its initial velocity, and \bar{v} the average, the probe's velocity diminishes as

$$\begin{aligned} v &= v_i - 4\bar{v}HT \\ &\cong v_i(1 - 4HT) \end{aligned} \quad (16)$$

From Eq. (3), the speed of light also changes as

$$\begin{aligned} c &= c_i e^{-2HT} \\ &\cong c_i(1 - 2HT) \end{aligned} \quad (17)$$

with c_i the initial speed of light when the probe departs Earth. Since the terms involving HT are small, from the last two equations, the ratio v/c can be approximated as

$$\frac{v}{c} \cong \frac{v_i}{c_i} (1 - 2HT) \quad (18)$$

The probe's travel time and that for the signal are related approximately by

$$T \cong \frac{c_i t}{v_i} \quad (19)$$

After substituting for T , the previous equation becomes

$$\frac{v}{c} \cong \frac{v_i}{c_i} - 2Ht \quad (20)$$

Then substituting for v/c in Eq. (14) gives

$$\nu \cong \nu_0 \left(1 - \frac{2v_i}{c_i} + 2Ht \right) \quad (21)$$

From the usual model of the probe's motion, where v and c remain equal to v_i and c_i respectively, the result is an unmodeled blueshift of $2\nu_o Ht$. (Note this approximation doesn't hold for large values of Ht , where v and c are changing substantially.)

Anderson *et al.* [2] discuss the possibility the "Pioneer effect" is due to an extra acceleration a_p of the probes toward the Sun or Earth. Equating the blueshift from such an acceleration to that from the last equation

$$\nu_0 \frac{2a_p t}{c} = \nu_0(2Ht) \quad (22)$$

where the direction of a_p is defined oppositely to the acceleration a above. Solving for a_p

$$a_p = Hc \quad (23)$$

Anderson *et al.* note that various workers have observed the value of a_p is close to c multiplied by the estimated Hubble constant. The predicted blueshift here is equivalent to such an acceleration.

3 Earth's Orbit

At around 29.8 km/s, Earth's orbital velocity with respect to the solar system barycenter is more than twice that of the Pioneer 10 and 11 probes (now moving at 12.2 and 11.6 km/s respectively). Are the Pioneer Doppler signals also influenced by change in Earth's motion? To answer that, we'll need a description of its orbit in an evolving universe.

If the Sun-Earth distance is regarded as a measuring rod, and the frequency at which Earth orbits as a clock, from Eqs. (6) and (5), these quantities should diminish relativistically as e^{-Ht} . Earth's average orbital speed is proportional to the product, and should slow as e^{-2Ht} , by the same factor as light. As seen in Eqs. (16) and (17), the velocity of the probe in the preceding example diminishes by about twice that factor.

Unlike that body, Earth is bound in a nearly circular orbit. Using the same acceleration equation that was applied to the probe, we'll show Earth's motion is approximately relativistic, slowing in proportion to light. To do that, we'll first determine what Earth's acceleration would be if it moves relativistically, and then compare that to the acceleration given by Eq. (8).

What we need are only small corrections to Earth's Newtonian motion. And since the orbital radius varies by only ± 1.7 percent, here we'll approximate the observed orbit as circular and centered on the Sun. In absolute coordinates, the radius required for a relativistic orbit then varies as

$$r = r_0 e^{-Ht} \quad (24)$$

Also, the corresponding orbital velocity changes as

$$v = v_0 e^{-2Ht} \quad (25)$$

Using x - y coordinates, we'll place the Sun at the origin and Earth at $(0, r)$, moving in approximately the $+x$ direction. Taking the derivative of Eq. (24), Earth's velocity v also has a component in the y direction

$$\begin{aligned} v_y &= \frac{dr}{dt} \\ &\cong -Hr \end{aligned} \quad (26)$$

And the trajectory spirals inward at a very small angle α

$$\alpha \cong \frac{v_y}{v_x} \cong -\frac{Hr}{v} \quad (27)$$

With Earth's centripetal acceleration perpendicular to this angled trajectory, it has a small component in the x direction. If Earth's speed were constant, its acceleration in that dimension would be

$$a_x \cong \frac{v^2}{r} \sin \alpha \cong -Hv \quad (28)$$

since α and $\sin \alpha$ are effectively the same. That deceleration corresponds to the orbit's increasing curvature. In addition, Earth's orbital motion is slowing. The derivative of Eq. (25) gives

$$\frac{dv}{dt} \cong -2Hv \quad (29)$$

This deceleration is almost entirely in the x dimension. Combining this and the x component of the centripetal acceleration, the total x deceleration is

$$a_x \cong -3Hv \quad (30)$$

Now we'll compare this to the acceleration given by Eq. (8). From the Sun's potential gradient, the first term of that equation gives an acceleration in the $-y$ direction. This approximately equals Earth's centripetal acceleration. For a spiral trajectory with the same inward angle α , it has a small component in the direction of Earth's velocity vector. That acceleration a_v can be approximated in terms of the centripetal acceleration as

$$a_v \cong -\frac{v^2}{r} \sin \alpha \cong Hv \quad (31)$$

again substituting α for $\sin \alpha$.

From the changing cosmological potential, the last term in Eq. (8) contributes a deceleration in that direction equal to $-4Hv$. The combined effect is then

$$a_v \cong -3Hv \quad (32)$$

Since a_v is almost entirely in the x direction, this also gives Eq. (30).

Again, that equation corresponds to an orbital velocity which decreases as e^{-2Ht} , in proportion to the speed of light. Earth's deceleration due to the changing cosmological potential is partly manifested as change in its direction of motion. And its deceleration is partially counteracted by the Sun's gravity, as the planet spirals inward and gains kinetic energy.

Earth's orbit shrinks with Earth itself, as in a diminishing Poincaré world. And its orbital frequency slows together with an atomic clock. So there is no measured drift in the length of a year.

4 Predicted Signal

To determine the contribution of Earth's motion to the Pioneer signal, we'll suppose Earth orbits while the probe is stationary at some large distance in the heliocentric reference frame. Eq. (14) applies again. However, for an apparently circular orbit, v and c diminish at the same rate and the ratio v/c doesn't change. Hence the decrease in Earth's tangential velocity doesn't contribute a Doppler shift.

The inward radial component of Earth's velocity described by Eq. (26) does give a Doppler shift. This velocity varies sinusoidally in the probe's direction, and the resulting shift can be expressed as

$$\Delta\nu \cong -2\bar{v} \frac{Hr \cos \theta}{c} \quad (33)$$

where $\bar{\nu}$ is the average observed frequency, r the radius of Earth's orbit and θ is the angle between the heliocentric position vectors for the probe and Earth.

The last term in Eq. (14) also contributes. Due to Earth's changing position, the signal's one-way travel time t varies by $(-r \cos \theta/c)$. The resulting shift is the opposite of that in the last equation, with the redshift arising when Earth is farther from the probe. Consequently, Earth's motion brings no net unmodeled frequency shift. We'll express that as

$$\Delta\nu_e \cong 0 \quad (34)$$

Now we'll determine the unmodeled shift due to the probe's motion on its actual trajectory, again in the heliocentric frame. First we'll generalize Eq. (21) to give the unmodeled shift for an arbitrary, short segment of the probe's trajectory. We rewrite that equation as

$$\nu \cong \nu_i \left(1 - \frac{2v_i}{c_i} + 2Ht \right) \quad (35)$$

where ν_i is the observed frequency when the probe is at the beginning of a segment, v_i and c_i are the quantities at that point, and t is the additional time a signal takes to reach the probe after it moves beyond that point. Since we're not calculating the Doppler shift due to the probe's modeled acceleration by local gravitational fields, those are neglected again.

The derivation of this equation is the same given in Section 2 for Eq. (21). In this case, T refers to the probe's travel time from the beginning of the trajectory segment. Since the Doppler and cosmological shifts are functions of its radial velocity, we define the quantities v and v_i as the radial velocity components. Likewise, a in Eq. (15) refers to its radial acceleration. After these definitions, the previous derivation holds when the probe's net motion is in any direction.

A sufficiently short segment of the probe's curved trajectory can be represented as a straight line, for which Eq. (35) gives an unmodeled shift of $2\nu_i Ht$. Summing the shifts for successive segments, in the limit where the segment length goes to zero, gives the total. The resulting unmodeled frequency shift for an arbitrary trajectory can again be expressed as

$$\Delta\nu_p \cong 2\nu_0 Ht \quad (36)$$

where ν_0 is the reference and t is the total one-way signal travel time.

Note this equation doesn't depend on the probe's velocity – since t is proportional to its radial distance traveled, not the time it takes to reach a given radius. As Anderson *et. al.* point out, rather than an acceleration of the probe, this effect could instead be attributed to a slowing frequency reference on Earth. (In that case it may be more apparent the resulting frequency shift is a function of the probe's distance.)

As shown previously, the blueshift given by the last equation is equivalent to the anomalous acceleration a_p of Anderson *et. al.*, when its value is Hc . Several effects are involved here: a diminishing speed of light, a slowing frequency reference on Earth, and a four-times-larger deceleration of the probe.

(Although a_p was taken to be directed toward the Sun, the $-4Hv$ acceleration here is opposite the probe’s motion. Its main component is toward the Sun, and contributes to a net frequency shift equivalent to a_p . It also has a small perpendicular component. That causes no additional Doppler effect, but shifts the probe’s angular position in the heliocentric reference frame. The contribution of Earth’s orbital motion to the Pioneer signal then changes, in both phase and amplitude. This may relate to a small annual oscillation found by Anderson *et. al.* [2].)

Equating the value of a_p to Hc , H is then 89.8 ± 13.6 km/s/Mpc. Measurements of the Hubble constant from astronomical observations have yielded widely varying results. With the goal of measuring H to ten percent accuracy, Freedman *et. al.* [10] have used the Hubble Space Telescope to calibrate the distances of Cepheid variable stars in nearby spiral galaxies. Their final estimate was 72 ± 8 km/s/Mpc.

Using very long baseline interferometry, Herrnstein *et. al.* [11] have made a precise, direct measurement of the distance to a water maser in one of the same galaxies. It puts the galaxy, NGC4258, twelve percent closer. And when the Cepheid yardstick is recalibrated accordingly, the result is a Hubble constant of 80 ± 9 km/s/Mpc. The value of H given by this theory and the Pioneer data is in agreement with both these estimates.

5 Conclusions

In a *New York Times* essay commemorating a century of quantum mechanics, John Wheeler [12] remarks:

It was 228 years later when Einstein, in his theory of general relativity, attributed gravity to the curvature of spacetime. . . . Even that may not be the final answer. After all, gravity and quantum mechanics have yet to be joined harmoniously.

General relativity fails to explain why the universe isn’t curved by the vacuum’s quantum-mechanical energy. Since this alternative is based on de Broglie waves instead of space-time curvature, there is no need to reconcile it with quantum mechanics or the observed flatness of the universe. There also is no “problem of time.” Further, unlike general relativity, this theory agrees with the signals from Pioneer 10 and 11.

Appendix: Elliptical Earth Orbit

Treating Earth’s observed orbit as circular, we found its motion contributes no unmodeled frequency shift to the Pioneer Doppler signal. Here we’ll explore the effects of its orbital eccentricity ϵ , which is 0.0167. (The terms “Earth” and “Sun” are used loosely to refer to the barycenters of the Earth-Moon and Solar systems.)

We’ll use x - y coordinates, where the origin corresponds to both the Sun’s position and a focus of Earth’s elliptical orbit. Initially, the aphelion lies on the positive x axis, and perihelion on the negative. For an ellipse of small eccentricity, the major

and minor axes are approximately equal. Consequently, where \bar{r} is Earth's mean orbital radius, its trajectory can be approximated as a circle with radius \bar{r} , whose center lies on the x axis, displaced a positive distance $\epsilon\bar{r}$ from the origin.

We'll also use polar coordinates with the same origin, where Earth's radial position is approximately

$$r \cong \bar{r} + \epsilon\bar{r} \cos \theta \quad (37)$$

Its velocity in terms of the mean is given by

$$v \cong \bar{v} - \epsilon\bar{v} \cos \theta \quad (38)$$

Substituting from

$$\theta \cong \frac{\bar{v}t}{\bar{r}} \quad (39)$$

and taking the time derivative of the velocity gives a tangential acceleration along the trajectory

$$a_t \cong \frac{\epsilon\bar{v}^2}{\bar{r}} \sin \theta \quad (40)$$

Our previous analysis omitted the varying velocity component ($-\epsilon\bar{v} \cos \theta$) in Eq. (38). From the changing cosmological potential given by Eq. (9) and the final term in Eq. (8), this velocity component produces an added tangential acceleration

$$a_t \cong 4H\epsilon\bar{v} \cos \theta \quad (41)$$

We'll find this alternating acceleration is offset by a y displacement of Earth's approximately circular trajectory. The circle's center has the y coordinate,

$$y_c \cong \frac{4H\epsilon\bar{r}^2}{\bar{v}} \quad (42)$$

Taking H equal to 89.8 km/s/Mpc, this gives a y shift of 14.6 cm. Eq. (37) for the orbital radius becomes

$$r \cong \bar{r} + \epsilon\bar{r} \cos \theta + \frac{4H\epsilon\bar{r}^2}{\bar{v}} \sin \theta \quad (43)$$

By itself, the final term of this would introduce another velocity component in Eq. (38), with the value ($-4H\epsilon\bar{r} \sin \theta$), and a tangential acceleration opposite that in Eq. (41). From the cancelation of those accelerations, Earth's net velocity and acceleration are unchanged, and still described by Eqs. (38) and (40).

Due to the shift of Earth's trajectory, its aphelion and perihelion no longer coincide with the points of minimum and maximum orbital velocity. Since the trajectory is effectively circular, the aphelion and perihelion lie on a line passing through its center and the Sun's position. The line's angle β with respect to the x axis is given approximately by its slope

$$\beta \cong \frac{y_c}{\epsilon\bar{r}} \cong \frac{4H\bar{r}}{\bar{v}} \quad (44)$$

where β is in radians.

This is the angular difference between the aphelion and the point of minimum velocity in the heliocentric reference frame. Over the entire orbit, the magnitude of Earth's velocity is shifted by this angle from its usual value. Using the same H , β is 5.8×10^{-11} radians, or $12 \mu\text{arcsec}$. Multiplying β by \bar{r} , this corresponds to a distance of 8.7 meters along Earth's trajectory. Dividing that by \bar{v} , the magnitude of its velocity precedes the usual in phase by about 0.29 millisecc.

This perturbation of Earth's orbit appears too small to be detected in the Pioneer Doppler signals. However, its effects are much larger for the outer planets, and may be observable in angular position measurements. For Pluto, β is 3 milliarcsec.

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