The three Millennium problem solutions, RH, NSE, YME, and a Hilbert scale based quantum geometrodynamics

Preface

Klaus Braun
August 17, 2019
www.fuchs-braun.com

This homepage provides solutions to the following Millennium problems

- the Riemann Hypothesis
- well-posed 3D-nonlinear, non-stationary Navier-Stokes equations
- the mass gap problem of the Yang-Mills equations.

The Riemann Hypothesis states that the non-trivial zeros of the Zeta function all have real part one-half. The Hilbert-Polya conjecture states that the imaginary parts of the zeros of the Zeta function corresponds to eigenvalues of an unbounded self-adjoint operator. It is related to the Berry-Keating conjecture that the imaginary parts of the zeros of the Zeta function are eigenvalues of an „appropriate" Hermitian operator $H = \frac{1}{2}(xp + px)$ where x and p are the position and conjugate momentum operators, respectively, and multiplicity is noncommunative. The operator H is symmetric, but might have nontrivial deficiency indices (W. Bulla, F. Gesztesy, J. Math. Phys. 26 (1), October 1985), i.e. in a mathematical sense H is not Hermitian.

A common underlying distributional Hilbert space framework

- provides an answer to Derbyshine's question (in "Prime Obsession"): ... "*The non-trivial zeros of Riemann's zeta function arise from inquiries into the distribution of prime numbers. The eigenvalues of a random Hermitian matrix arise from inquiries into the behavior of systems of subatomic particles under the laws of quantum mechanics. What on earth does the distribution of prime numbers have to do with the behavior of subatomic particles*?"
- enables a quantum gravity theory based on an only Hamiltonian (*energy functional*) formalism. Due to reduced regularity assumptions to the domains of the concerned operators the "force" related Lagrange formalism is no longer valid; therefore the notion "force" plays no role anymore in the proposed quantum gravity theory.

The Bagchi Hilbert space reformulation of the Nyman, Beurling and Baez-Duarte RH criterion provides the link between the two solution areas above (BhB). The Zeta function on the critical line is an element of the distributional Hilbert space H_{-1}. Therefore, in order to verify the Hilbert-Polya conjecture any (weak) eigenfunction solution of a self-adjoint operator equation to verify the Hilbert-Polya conjecture needs to be elements of a $H_{-1/2}$.

The key ingredients of the Zeta function theory are the Mellin transforms of the Gaussian function and the fractional part function. To the author´s humble opinion, the main handicap to prove the RH is the not-vanishing constant Fourier term of both functions. The Hilbert transform of any function has a vanishing constant Fourier term.

Let H and M denote the Hilbert and the Mellin transform operators. For the Gaussian function $f(x):=e^{-\pi x^2}$ it holds

$$M[f](s)=\frac{1}{2}\pi^{-s/2}\Gamma(\frac{s}{2}) \ , \ M[-xf'(x)](s)=\frac{s}{2}\pi^{-s/2}\Gamma(\frac{s}{2})=\frac{1}{2}\pi^{-s/2}\Pi(\frac{s}{2}) \ .$$

The related Theta function properties (based on the Poisson summation formula) of

$$G(x)\coloneqq\theta(x^2):=\sum_{-\infty}^{\infty}e^{-\pi n^2x^2}=1+2\sum_1^{\infty}e^{-\pi n^2x^2}=:1+2\psi(x^2)=\frac{1}{x}\sum_{-\infty}^{\infty}e^{-\pi\frac{n^2}{x^2}}=\frac{1}{x}G\frac{1}{x})$$

provides the foundation to derive the Riemann duality equation given by (EdH) 1.8)

$$\xi(s):=\frac{s}{2}\Gamma\left(\frac{s}{2}\right)(s-1)\pi^{-\frac{s}{2}}\zeta(s)=(1-s)\cdot\zeta(s)M[-xf'(x)](s)=\zeta(s)\cdot M[-x(xf'(x))'](s)=\xi(1-s).$$

It implies that the invariant operator $x^{-s}\rightarrow\int_0^{\infty}x^{-s}G(x)dx$ is formally self-adjoint, but this operator has no transform at all ((EdH) 10.3). The Mellin transform for $H(x)\coloneqq -\frac{d}{dx}(x^2\frac{d}{dx})G(x)$ is well defined and it holds

$$\int_0^{\infty}x^{1-s}H(x)\frac{dx}{x}=\int_0^{\infty}x^{s}H(x)\frac{dx}{x}$$

Formally it also holds

$$\int_0^{\infty}x^{-s}\left[(-\frac{d}{dx}x^2\frac{d}{dx})G(x)\right]dx=s(1-s)\int_0^{\infty}x^{-s}G(x)dx$$

which states formally that the function $2\xi(s)/(s(s-1))$ is only formally the transform of the invariant, self-adjoint operator $x^{-s}\rightarrow\int_0^{\infty}x^{-s}G(x)dx$. This operator has no transform at all as the integrals do not converge, due to the not vanishing constant Fourier term of the Poisson summation formula. A similar situation is valid, if the duality equation is built on the fractional part function ([TiE] 2.1).

Replacing the Gaussian function $f(x)$ and the fractional part function by its Hilbert transforms enables an alternative Zeta function theory.

The Hilbert transform of the Gaussian function is given by the Dawson function

$$F(x)\coloneqq e^{-x^2}\int_0^x e^{t^2}dt=\int_0^{\infty}e^{-t^2}sin(2xt)dt=x\ {}_1F_1\left(1,\frac{3}{2};-x^2\right)=xe^{-x^2}\ {}_1F_1\left(\frac{1}{2},\frac{3}{2};x^2\right)$$

It leads to an alternative entire Zeta function $\xi^*(s)$ in the form

$$\xi^*(s):=\frac{1}{2}(s-1)\pi^{\frac{1-s}{2}}\Gamma(\frac{s}{2})\,tan(\frac{\pi}{2}s)\cdot\zeta(s)=\zeta(s)\cdot M\left[\frac{d}{dx}[-x\cdot f_H(x)]\right](s)$$

with same zeros as $\xi(s)$, as it holds $s(1-s)\xi^*(s)\xi^*(1-s)=\pi\xi(s)\xi(1-s)$.

The appropriate related Mellin transforms are given by the formula (GrI) 7.612

$$\int_0^{\infty}x^s\ {}_1F_1(\alpha,\beta;-x)\frac{dx}{x}=\frac{\Gamma(\beta)}{\Gamma(\alpha)}\Gamma(s)\frac{\Gamma(\alpha-s)}{\Gamma(\beta-s)}\ ,\ 0<Re(s)<Re(\alpha)\ ,$$

$$\int_0^{\infty}x^{\mu-1}e^{-\beta x^2}sin(\gamma x)dx=\frac{\gamma e^{-\frac{\gamma^2}{4\beta}}}{2\beta^{\frac{\mu+1}{2}}}\ {}_1\Gamma(\frac{1+\mu}{2})F_1(1-\frac{\mu}{2};\frac{3}{2};\frac{\gamma^2}{4\beta})\ \ ,\ Re(\beta)>0\ ,\ Re(\mu)>-1$$

leading e.g. to

$$\frac{1}{2}\int_0^{\infty}x^{s/2}\ {}_1F_1\left(\frac{1}{2},\frac{3}{2};-x\right)\frac{dx}{x}=\frac{\frac{1}{2}\Gamma\left(\frac{s}{2}\right)}{1-s}=\frac{\Gamma\left(1+\frac{s}{2}\right)}{s(1-s)}=\frac{\Pi\left(\frac{s}{2}\right)}{s(1-s)}\ \ ,\ \ 0<Re(s)<1.$$

It indicates a replacement of the Gauss „Gamma" function definition ((EdH) p.8)

$$\Pi\left(\frac{s}{2}\right)\coloneqq\Gamma\left(1+\frac{s}{2}\right)=\frac{s}{2}\Gamma\left(\frac{s}{2}\right)\qquad\rightarrow\qquad\Gamma^*\left(1+\frac{s}{2}\right)\coloneqq\Gamma\left(\frac{s}{2}\right)tan\left(\frac{\pi}{2}s\right)=\frac{\Gamma\left(\frac{1+s}{2}\right)\Gamma\left(\frac{1-s}{2}\right)}{\Gamma\left(1-\frac{s}{2}\right)}=\frac{\Gamma\left(1+\frac{s-1}{2}\right)\Gamma\left(1-\frac{s+1}{2}\right)}{\Gamma\left(1-\frac{s}{2}\right)}=\frac{2}{\pi}\sum_{k=1}^{\infty}\frac{\Gamma\left(1+\frac{s}{2}\right)}{\left(k-\frac{1}{2}\right)^2-\left(\frac{s}{2}\right)^2}.$$

The Bagchi Hilbert space based RH criterion is dealing with the fractional part function. Its Hilbert transform is given by

$$g(x) := \ln\left(2\sin\left(\frac{x}{2}\right)\right) = -\sum_{n=1}^{\infty}\frac{\cos(nx)}{n},$$

which is an element of H_0. Therefore, its related Clausen integral ((AbM) 27.8) is an element of H_1, and its first derivative,

$$\frac{1}{2}\cot\left(\frac{x}{2}\right) \text{ resp. } \cot(\pi x)$$

joins the Zeta function on the critical line as an element of H_{-1}. The H_{-1} Hilbert space corresponds to the weighted l_2^{-1} −space as considered in (BhB).

As $g(x) \in H_0 = H_0^*$, it holds

$$(g,v)_0 \cong (g',v)_{-\frac{1}{2}} = (S^1[g],v)_{-\frac{1}{2}} = (Cot,v)_{-1/2} < \infty, \quad \forall v \in H_0$$

i.e. the formally derived Fourier series representation of

$$Cot(x) = \sum_{n=1}^{\infty}\sin(nx) \quad \text{resp.} \quad Cot^*(x) = 2\sum_{n=1}^{\infty}\sin(2\pi nx)$$

is defined in a distributional H_{-1} −sense (see also (BeB) (17.12) (17.13)).

For $a > 0$ and $0 < |Re(s)| < 1$ it holds ((GrI) 3.761)

$$\int_0^{\infty} x^s\sin(a)\frac{dx}{x} = \frac{\Gamma(s)}{a^s}\sin(\frac{\pi}{2}s) \quad , \quad \int_0^{\infty} x^s\cos(a)\frac{dx}{x} = \frac{\Gamma(s)}{a^s}\cos(\frac{\pi}{2}s).$$

Therefore the Mellin transforms of the H_{-1} − distributional Fourier series representation of the $Cot^{(*)}$-functions are given by

$$M[Cot](s) = \Gamma(s)\sin(\frac{\pi}{2}s)\zeta(s) \text{ resp. } M[Cot^*](s) = 2(2\pi)^{-s}\Gamma(s)\sin(\frac{\pi}{2}s)\zeta(s).$$

The function $\zeta(s)$ is regular for all values of s except $s = 1$, where there is a simple pole with residue 1. It satisfies the functional equation ((TiE) (2.1.1))

$$\zeta(s) = 2^s\pi^{s-1}\sin\left(\frac{\pi}{2}s\right)\Gamma(1-s)\zeta(1-s) \text{ resp. } \zeta(1-s) = 2^{1-s}\pi^{-s}\cos\left(\frac{\pi}{2}s\right)\Gamma(s)\zeta(s)$$

leading to

$$M[Cot^*](s) = \zeta(1-s)\cdot tan\left(\frac{\pi}{2}s\right) = \zeta(1-s)\cdot cot\left(\frac{\pi}{2}(1-s)\right)$$

resp.

$$M\left[\frac{1}{x}Cot^*(\frac{1}{x})\right](s) = M[Cot^*](1-s) = \zeta(s)\cdot cot\left(\frac{\pi}{2}s\right) = \zeta(s)\cdot tan\left(\frac{\pi}{2}(1-s)\right).$$

The relationship to the alternative entire Zeta function

$$\xi^*(s) := \frac{1}{2}(s-1)\pi^{\frac{1-s}{2}}\Gamma(\frac{s}{2})\,tan(\frac{\pi}{2}s)\cdot\zeta(s) = \zeta(s)\cdot M\left[\frac{d}{dx}[-x\cdot f_H(x)]\right](s)$$

in a H_{-1} distributional sense is given by

$$2M[\sum_1^{\infty} f_H(nx)](s) = 2\sqrt{\pi}\sum_1^{\infty}\int_0^{\infty}\int_0^{\infty} x^s e^{-\pi t^2}\sin(2\pi nxt)\,dt\frac{dx}{x} = \sqrt{\pi}\int_0^{\infty} e^{-\pi t^2}\int_0^{\infty} x^s Cot^*(\pi tx)\frac{dx}{x}dt$$

$$= \sqrt{\pi}\left[\int_0^{\infty} t^{1-s}e^{-\pi t^2}\frac{dt}{t}\right]\cdot\left[\int_0^{\infty} x^s Cot^*(x)\frac{dx}{x}\right] = \pi^{s/2}\Gamma(\frac{1-s}{2})\cdot M[Cot^*](s)$$

i.e.

$$M[G_H(x)](s) := M\left[\sum\nolimits_1^{\infty} f_H(nx)\right](s) = 2M\left[\sum\nolimits_1^{\infty} f_H(nx)\right](s) = \pi^{\frac{s}{2}}\Gamma(\frac{1-s}{2})M[Cot^*](s)$$

resp.

$$M[-xG_H'(x)](s) = s[G_H(x)](s) \ , \ M[(xG_H)'(x)](s) = (1-s)[G_H(x)](s).$$

Some related formulas for complex values on the critical line $s=\frac{1}{2}+it$ are

i) $\sin\left(\frac{\pi}{2}s\right)=\frac{1}{\sqrt{2}}\left[\cosh\left(\frac{\pi}{2}t\right)+i\cdot\sinh\left(\frac{\pi}{2}t\right)\right]$ and $|\Gamma(s)|^2=\frac{\pi}{\cosh(\pi t)}$

ii) $\cot\left(\frac{\pi}{2}s\right)=\tan\left(\frac{\pi}{2}(1-s)\right)=1-i\cdot\tanh(\pi t)=1-2i\cdot\sum_{k=1}^{\infty}(-1)^k e^{-2kt}$ ($t>0$)

iii) $\cot\left(\frac{\pi}{2}(1-s)\right)=\tan\left(\frac{\pi}{2}s\right)=1+i\cdot\tanh(\pi t)=1+2i\cdot\sum_{k=1}^{\infty}(-1)^k e^{-2kt}$ ($t>0$)

and therefore,

$$M[Cot^*](s)\cdot M[Cot^*](1-s)=\zeta(s)\cdot\zeta(1-s)\cdot[1+tanh^2(\pi t)].$$

From (TiE) 4.14)) resp. (ObF)$^{(*)}$ we recall the formulas

$$\zeta(s)-\sum_{n<x}n^{-s}=\sum_{n>x}n^{-s}=-\frac{1}{2i}\int_{x-i\infty}^{x+i\infty}z^{1-s}\cot(\pi z)\frac{dz}{z}\ ,\ \ Re(s)>1.$$

$$M\left[\frac{1}{\pi}\frac{x^n}{1-x}\right](s)=\cot(\pi s)\ \text{(principle value)}\ \ -n<Re(s)<1-n\ ,\ n=0,\pm1,\pm2,\dots$$

From (EsR) p. 139 we recall the formula

$$F.p.(P.v.\int_0^{\infty}\frac{x^{\alpha}}{1-x}dx=\begin{cases}0 & ,\alpha\in Z\\ \pi\cot(\pi\alpha) & ,else\end{cases}.$$

For the second solution area the current quantum state Hilbert space $L_2=H_0$ is extended to the Hilbert space $H_{-1/2}$, which will include also "plasma" states, as the fourth state of matter. The dual space $H_{1/2}$ of $H_{-1/2}$ provides the corresponding quantum energy space. Its compactly embedded H_1 Hilbert space is proposed to be the fermions (kinetic) energy space governed by Fourier waves, while the H_1-complementary closed subspace of $H_{1/2}$ is proposed to be the bosons (potential) energy space (which could be also interpreted as dark energy space) governed by wavelets. As the quantum state Hilbert space $H_{-1/2}$ includes also "plasma", the $H_{1/2}=H_1\otimes H_1^{\perp}$ (fermions + bosons) energy space decomposition also provides an alternative model for cold and hot plasma.

The current "*symmetry break down*" model to generate matter is replaced by a "*self-adjointness break down*" effect defined by the orthogonal projection from $H_{1/2}$ onto H_1.

The selfadjoint Friedrichs extension of the Laplacian operator defined on H_1 is bounded. Therefore, the operator induces a decomposition of H_1 into the direct sum of subspaces, enabling the definition of a potential and a corresponding "grad" potential operator. Then a potential (barrier) criterion defines a manifold, which represents a hyperboloid in the Hilbert space $H_1=H_1^{repulsive}\otimes H_1^{attrac}$ with corresponding hyperbolic and conical "fermions type" regions. The "attractive fermions" region might be interpreted as hyperspace. We note that a vector space and any linear subspace are convex cones, i.e. the tool convex analysis and general vector spaces can be applied.

We note that the exterior Neumann problem admits one and only one generalized solution in case the related Prandtl operator of order one $P: H_r\rightarrow H_{r-1}$ is defined for domains with $1/2\leq r<1$.

The classical Yang-Mills theory is the generalization of the Maxwell theory of electromagnetism where chromo-electromagnetic field itself carries charges. As a classical field theory it has solutions which travel at the speed of light so that its quantum version should describe massless particles (gluons). However, the postulated phenomenon of color confinement permits only bound states of gluons, forming massive particles. This is the mass gap. Another aspect of confinement is asymptotic freedom which makes it conceivable that quantum Yang-Mills theory exists without restriction to low energy scales. A variational Maxwell equations representation in a $H_{-1/2}$ Hilbert space framework includes also "gluon" bosons and corresponding "*self-adjointness break downs*", i.e. there is no mass gap anymore.

$^{(*)}$ (ObF) Oberhettinger F., Tables of Mellin Transforms, p. 182

The central part to prove the well-posedness of the 2D non-linear, non-stationary Navier-Stokes equations is a proper energy norm inequality estimate. It do not lead to blow-up effects for $t = T$ and do not show a Serrin gap with respect to the corresponding Sobolev norm estimates. We note that the energy norm of the non-linear terms of the 2D-NSE vanishes, which is appreciated from a mathematical point of view, but seems to be questionable from a physical point of view. The corresponding analysis for the 3D-NSE fails due to not appropriate Sobolev norm estimates. The analog analysis in a $H_{-1/2}$ variational framework (including a not-vanishing non-linear energy term) works out well, due to the appropriate Sobolewski estimates.

The quantum gravity model also addresses the dilemma, as pointed out by E. Schrödinger: "*Since in the Bose case we seem to be faced, mathematically, with simple oscillator of Planck type, we may ask whether we ought not to adopt for half-odd integers quantum numbers rather than integers. Once must, I think, call that an open dilemma. From the point of view of analogy one would very much prefer to do so. For, the „zero-point energy*" of a Planck oscillator is not only borne out by direct observation in the case of crystal lattices, it is also so intimately linked up with the Heisenberg uncertainty relation that one hates to dispense with it.

The formalism of 2-"*spinors*" as an alternative to the standard vector-tensor calculus (Penrose R., Rindler W.) is proposed to be physically re-interpreted and mathematically applied in the context of a H_1-space decomposition into repulsive and attractive fermions subspaces, whereby it holds $spin(4) = SU(2)xSU(2)$.

„The two-component „spinor" calculus is a very specific calculus for studying the structure of space-time manifolds…. Space-time point themselves cannot be regarded as derived objects from spinor algebra, but a certain extension of it, namely the twistor algebra, can indeed be taken as more primitive than space-time itself. ... The programme of twistor theory, in fact, is to reformulate the whole of basic physics in twistor terms" (Penrose R., Rindler W. Volume II).

The point of departure for the twistor theory is the (classical) twistor equation (with a similar form as the continuity equation). Its corresponding weak variational represention with respect to the proposed $H_{-1/2}$ quantum state inner product leads to the Friedrichs extension of the classical Dirac spinor operator with domain H(1/2), which is about the square root operator of order one of the Laplacian operator. The corresponding singular integral operator representation is about the Calderón-Zygmund integrodifferential operator (G. I. Eskin, Boundary Value Problems for Elliptic Pseudodifferential Equations, example 3.5).

The journey started in 2010 resulting in the papers

Braun K., *The three Millennium problem solutions (RH, NSE, YME) and a Hilbert scale based quantum geometrodynamics*, July 2019

The journey is still ongoing: in order to support an appropriate change traceability this paper is split into

PART I:
Braun K., *A Kummer function based alternative Zeta function theory to solve the Riemann Hypothesis and the binary Goldbach conjecture*

PART II:
Braun K., *3D-NSE and YME mass gap solutions in a distributional Hilbert scale frame enabling a quantum gravity theory*

The three Millennium problem solutions, RH, NSE, YME, and a Hilbert scale based quantum geometrodynamics

Preface

Klaus Braun
April 8, 2019
www.fuchs-braun.com

This preface has been edited after a long journey, which started in 2010 until today. During this period of time this homepage has been developed. The journey is still going on, but the main pillars and their underlying mathematical challenges and delivered solutions are established yet.

The term "journey" already indicates, why the results are "published" in this form and not following standard processes in academic world per each of the identified problem areas, respectively per underlying specific problems within those areas.

The considered problem areas are currently seen as either purely mathematical (e.g. RH) or physical problems (quantum field vs. gravity field theory and its union) or, in case of the RH, there is also a mix between mathematical and physical problems (e.g. Berry conjecture). We claim that all those problems are purely mathematical modelling problems: there are the three views on the considered problems, which are the physical, the mathematical and the philosophical views.

Kant's critique of pure reason gives the rational for the interface and boundaries of those three areas, which is governed by the term "transcendence". We emphasis that the term "transcendental" in mathematics (number theory) is even beyond Kant's definition of the term "transcendental": the transcendental numbers are a subset of the set of the irrational numbers (from a mathematical (definition) point of view), but already the irrational numbers are transcendental in the sense of Kant. The mathematical terms "continuity" and "Riemann integral" are building on the concept of irrational numbers, i.e. they are also transcendental terms. The Lebesgue integral is defined as a generalization of the Riemann integral. In the framework of the Lebesgue integral concept the set of rational numbers is a so-called zero-set, only (!), i.e. the probability to pick a rational number out of the set of the real numbers is zero.

Our proposed mathematical model is building on the (Leibniz) mathematical transcendental term "*differential*". It is interesting to be mentioned that also Schrödinger in (ScE1) uses the term "differential" (in German version) to explain "perception process between "subconscious" and "awareness" of human mind. The Leibniz transcendental concept of "differential = monad" means that there is no additional transcendence level added (which would be anyway a contradiction by itself), but the mathematical model becomes now applicable to all considered problem areas. The physical-mathematical modelling requirements (measurement/ observation/ test results validation) is still building on the L_2- test space:

we "just" propose and show evidence of a consistent mathematical language (definitions, axioms) in an unusual distributional Hilbert space framework, which is less regular than the L_2- test space, but still more regular than the domain of the Dirac function, while still applying standard functional analysis/spectral analysis/variational theory.
There are multiple handicaps regarding the usage of the Dirac "function" as a central concept in the quantum theory: let e denote an arbitrarily small positive real number and n denote the space dimension. The Dirac "function" is a distribution which is not an element of the quantum state Hilbert space $L_2 = H_0$ Its regularity depends from the space dimension n, i.e. the Dirac "function" is an element of the Hilbert space $H_{-\frac{n}{2}-\varepsilon}$.

Our approach builds on an alternative quantum state Hilbert space $\mathrm{H}_{-1/2}$. Its definition is enabled by the Riesz and Calderon-Zygmund integrodifferential operators. We note that in case of space dimension $n = 1$ the Riesz operators are identical to the Hilbert transform operator.

The considered (distributional) Hilbert space framework enables a truly infinitesimal geometry (WeH); as one first consequence the manifold concept of Einstein's field equations with its handicap of *differentiable* manifolds (which is a purely mathematical requirement without any physical meaning/justification) can be omitted. The approach also omits concepts like exterior tensor & exterior algebras and exterior differential forms, as well as corresponding gauge theories. It leads to a modified Einstein-Hilbert action functional newly based on a Stieltjes integral representation replacing the Lebesgue measure $x^{(4)}$ by the corresponding Stieltjes integral measure $dg[x^{(4)}]$. Complementary variational principles can be derived from this, whereby the corresponding (classical) PDEs representation could be well defined without any boundary conditions (A. M. Arthurs, L. B. Rall).

A common Hilbert space framework for PDE field equations and quantum dynamics enables an integrated mathematical quantum and gravity field theory model, including a gravitational collapse and space-time singularity theory (R. Penrose).
The Berry-Keating (Hilbert-Polya refinement) conjecture is verified by a convolution representation of the Zeta function, enabled by the distributional Fourier series representation of the $\cot(x)$ –function (S. Ramanujan). This provides an answer to Derbyshine's question (in "Prime Obsession"):

... "*The non-trivial zeros of Riemann's zeta function arise from inquiries into the distribution of prime numbers. The eigenvalues of a random Hermitian matrix arise from inquiries into the behavior of systems of subatomic particles under the laws of quantum mechanics. What on earth does the distribution of prime numbers have to do with the behavior of subatomic particles?*"

The common distributional Hilbert space framework of classical field (PD) equations and quantum field equations and its corresponding classical and variational (weak) mathematical models require a change of a current paradigm: now the classical models become the mathematical approximations to the weak (Pseudo-) Differential Equations models and not the other way around.

Another consequence is that the term "force" is only valid for classical PDE, when the Lagrange formalism is equivalent to the Hamiltonian formalism due to a defined Legendre transform. Another consequence is the fact that the energy inequality (with respect to the newly proposed $\mathrm{H}_{1/2}$ energy space) of the non-linear, non-stationary NSE now also anticipates a contribution of the non-linear term, while, at the same time, enabling a global bounded energy inequality for the non-linear, non-stationary NSE in case of space dimension $n = 3$.

With respect to the NSE topic we also refer to

http://www.navier-stokes-equations.com/

With respect to the ground state energy and quantum gravity topic we also refer to

https://quantum-gravitation.de/

Leibniz's monad concept is an extension of the real numbers to ideal/hyper-real numbers. Those are nothing more than another set of "transcendental numbers" in the sense of Kant (whereby the term "real" for the real numbers is already miss-leading); the properties of the set of the ideal numbers are identical to those of the real numbers (which are (in a physical sense) not "real" at all with 100% probability), except only one missing valid axiom, the Archimedean axiom: this is related to *physical* measurement capabilities of a length by a given standard measurement length (!). The set of real numbers provides the baseline for standard analysis with the concepts of the Riemann and the Lebesgue integrals. The latter one is the fundamental concept to define the inner product of the test (Hilbert) space $L_2 = H_0$ resp. the Dirichlet (energy) integral with its underlying domain, the (Sobolev) Hilbert space H_1, which is a subset of the test space $L_2 = H_0$.

The set of Leibniz's ideal numbers provides the baseline of the non-standard analysis. In this framework the Stieltjes integral can be interpreted as the counterpart of the Lebesgue integral going along with a reduced regularity requirements of the corresponding domain, which becomes the newly proposed energy Hilbert space $H_{1/2}$. The corresponding quantum state Hilbert space framework changes from the current test space $L_2 = H_0$ to the distributions Hilbert space $H_{-1/2}$ whereby the test space $L_2 = H_0$ is compactly embedded. The complementary subspace $H_{-1/2} - H_0$ is closed enabling the definition of an orthogonal projection operator from $H_{-1/2}$ onto the test space H_0. It therefore provides the framework to model also an additional continuous spectrum of the (energy) Hamiltonian quantum operator (Berry conjecture), as well as superconductivity, super-fluids and condensates.

The fascination, motivation and energy to walk through this journey was and is primarily to contribute as much as possible to all those subject areas at that moment in time, when the one or the other idea popped up. The main drivers are “amazement” and “pursuit of new”, and not to follow academicals career paths. In this sense

"prosit" (lat. "may it be useful") :

there is a common baseline of all considered conceptual problem areas, which can be overcome by one single common mathematical model. From a mathematical-philosophical point of view this is not surprising at all ((KnA), (ScE), (ScE1), (WeH), (WeH1)): already an irrational number is an "universe" by itself, i.e. an "object" defined as an "existing" (i.e. mathematical defined) limit of an infinite sequence of "rational" "objects". In other words, the existence of any "irrational number" "object" "exists" per (mathematical) completeness axiom, only. The mathematical concepts of "continuity" and "differentiability" are then building on the concept of "real" numbers (the extension of the set of rational numbers). The (physical) body-contact problem, *(mathematically, not physically required) differentiable* (not continuous) manifolds concepts to model space-time structure in Einstein's field theory (WeH1), a well-defined, but still mysterious Dirac function with space-dimension depending distributional Hilbert space domain (!) and other concepts are built on top of it.

From a mathematical perspective there are two axioms essentially:

- the completeness axiom
  to enable the "building" of "real" numbers, including irrational numbers

- the Archimedean axiom
  originally formulated for segments, which states that if the smaller one of two given segments is marked off sufficient number of times, it will always produce a segment larger then the larger one of the original two segments or, in simple words, to enable the measurement of a given (potentially very large) number per n times a given "unit" of measure length.

Putting the journey as the objective and not the gathering and “owning” a list of deliverables/papers is a purely personal thing. Nevertheless, there is a famous book related to this different way looking at it, which might be interested to the one or other visitor of this homepage (FrE).

The philosophical questions regarding the relationship of current and the newly proposed “ideal” (transcendental) mathematical objects to describe very large and very small physical phenomena (R. Penrose) are still open, e.g. (RuB).

J. Gaarder addresses some of those philosophical questions in relation to a depiction of life in "Maya" (GaJ). It is about the universe, its related evolutionary theory and corresponding questions that tangle us here on earth: one protagonist's states that "*the universe seeks to understand itself and the eye that looks into the universe is the eye of the universe itself*". Anyway, and in any case reading the book of J. Gaarder is just fun; ... another related book, again just for fun, is ((HoJ). The underlying conceptual idea is going back to (NaT). .... or from another perspective: Yoda: "*the power* ('the vacuum energy') *be* ('is') *with you* ('with us')" ...:).

There are two basic assumptions to Th. Nagel's conceptial thoughts (NaT):

- awareness ("Bewusstsein") is an essential element/part of the evolution of the cosmos
- values are lens ("objektiv") and independent from the point of view of a judgmental (evaluating) person.

Based on those assumptions he concluded that "*the Materialist Neo-Darwinian Conception of Nature is Almost Certainly False*". From a mathematical point of view the most easiest counter argument to his thesis is just to not accept at least one of those assumptions. All other kind of reasoning which are not based on "assumption - conclusion" format is anyway out of scope of the descriptive science, mathematics. And here's the overall challenge to all of those kind of philosophical discussions, whenever the word "cosmos" is part of it: Mathematics is THE only language of theoretical physics to describe the "cosmos" on the one hand side, while on the other hand side mathematics is a descriptive science, only. If it is possible that mathematics provides a "field/mind" description which can be consistently embedded into existing mathematical-physical models (including "initial value" "functions"), already this kind of formal descriptions, only (based on commonsense, mathematical logic principles) go beyond human observation's horizon (and Kant's "critique of pure reason" boundaries of human understanding, as well). Already this would be another wonderful thing, what human beings are able to build. With respect to Nagel's second assumption then it would be only a small next step to its acceptance, as well.

There is a similar view possible on Th. Nagel's "*View from Nowhere*" (NaT1): human beings only observe parts of a "total", which shows several (or even infinite) aspects of all kinds of "infinities". (This is already the case when the baselines for all mathematical concepts are defined, which is about the "real" numbers. Already each irrational number is a very simple example of this kind of infinity (it is already an universe by itself) and the set of all irrational number, as well). Any human being observer "realizes" just from his individual perspective. There is no (objective) observer *perspective from somewhere* possible. Again, the descriptive science, the mathematics, is able to deal with different kinds and cardinalities of infiniteness. The concept of "continuous" "function" and "differentiable" "functions" is already a next dimension on infiniteness on top of "real" numbers or "ideal" numbers.

E. Schrödinger was also concerned with "*Mind and Matter*" questions (ScE):

"*The objective world has only been constructed at the prize of taking the self, that is, mind, out of it remaking it; mind is not part of it; obviously, therefore, it can neither act on it nor be acted on by any of its parts. If this problem of the action of mind on matter cannot be solved within the framework of our scientific representation of the objective world, where and how can it be solved?*" ... "*No single man can make a distinction between the realm of his perceptions and the realm of things that cause it, since however detailed the knowledge he may have acquired about the whole world, the story is occurring only once and not twice. The duplication is an allegory suggested mainly by communication with other beings.*"

(ScE) *A TENTATIVE ANSWER:'"A single experience that is never to repeat itself is biologically irrelevant. Biologic value lies only in learning the suitable reaction to a situation that offers itself again and again, in many cases periodically, and always requires the same response if the organism is to hold its ground." ... But whenever the situation exhibits a relevant* differential *- let us say the road is up at the place where we used to cross it, so that we have to make a detour - this* differential *and our response to it intrude into consciousness, from which, however, they soon fade below the threshold, if the* differential *becomes a constantly repeating feature. .... Now in those fashion differentials, variants of response, bifurcations, etc., are piled up one upon the other in unsurvey able abundance, but only the mose recent ones remain in the domain of consciousness, only those with regard to which the living substance is still in the stage of learning or practicing.*

... *I would summarize my general hypothesis thus: consciousness is associated with the learning of living substance; its knowing how (Können) is unconscious.*

Mind has erected the objective outside world of the natural philosopher out of its own stuff. Mind could note cope with this gigantic task otherwise than by the simplifying device of excluding itself - withdrawing from its conceptual creation. Hence the latter does not contain its creator.

Physical science ... faces us with the impasse that mind per se cannot play piano - mind per se cannot move a finger of a hand. Then the impasse meets us, the blank of the "how" of mind's leverage on matter. The inconsequence staggers us. It is a misunderstanding?

Neither can the body determine the mind to think, nor the mind determine the body to motion or rest or anything else (if such there be)."

(ScE1) 'THE VEDANTIC VISION':
"F*or philosophy, then, the real difficulty lies in the spatial and temporal multiplicity of observing and thinking individuals. If all events took place in one consciousness, the whole situation would be extremely simple. There would then be something given, a simple datum, and this, however otherwise constituted, could scarily present us with a difficulty of such magnitude as the one we do in fact have on our hands.*

I do not think that this difficulty can be logically resolved, by consistent thought, within intellects. But it is quite easy to express the solution in words, thus: the plurality that we perceive is only appearance; it is not real. Vedantic philosophy, ..."

(ScE2) 'FORM, NOT SUBSTANCE, THE FUNDAMENTAL CONCEPT':
"*It is clearly the peculiar form or shape (German: Gestalt) that raises the identity beyond doubt, not the material content. Had the material been melted and cast into the shape of a man, the identity would be much more difficult to establish. And what is more: even if the material identity were established beyond doubt, it would be of very restricted interest. I should probably not care very much about identity or not of that mass of iron, and should declare that my souvenir had been destroyed."*

(ScE2) 'THE NATURE OF OUR MODELS':
"*In this we must, of course, take shape (or Gestalt) in a much wider sense than as geometrical shape. Indeed there is no observation concerned with the geometrical shape of a particle or even of an atom. It is true that in thinking about the atom, in drafting theories to meet the observed facts, we do very often draw geometrical pictures on the black-board, or on a piece of paper, or more often just only in our mind, the details of the picture being given by a mathematical formula with much greater precision and in a much handier fashion then pencil or pen could ever give. That is true. ...."*

*(ScE2) 'THE ALLEGED BREAK-DOWN OF THE BARRIER BETWEEN SUBJECT AND OBJECT':
"For physical action always is inter-action, it always is mutual. What remains doubtful to me is only just this: whether it is adequate to term one of the two physically interacting systems the 'subject'. For the observing mind is not a physical system, it cannot interact with any physical system. And it might be better to reserve the term 'subject' for the observing mind."*

References

(FrE) Fromm E., To Have or to Be, Harper & Row, New York, 1976

(GaJ) Gaarder J., Maya, Phoenix House, London, 2000

(HoJ) Hold J., Why does the World Exists? An Existential Detective Story, Liveright Publishing Corporation, New York, 2012

(KnA) Kneser A., Das Prinzip der kleinsten Wirkung von Leibniz bis zur Gegenwart, B. G. Teubner Verlag, Leipzig, Berlin, 1928

(NaT) Nagel T., Mind and Cosmos: Why the Materialist Neo-Darwinian Conception of Nature is Almost Certainly False, Oxford University Press, New York, 2012

(NaT1) Nagel T., The View from Nowhere, Oxford University Press, New York, 2012

(RuB) Russel B., The Problems of Philosophy, Oxford university Press, Oxford, 1912

(ScE) Schrödinger E., Mind and Matter, Cambridge University Press, 1958

(ScE1) Schrödinger E., My View of the world, Cambridge University Press, 1964

(ScE2) Schrödinger E., Science and Humanism, Cambridge University Press, 1951

(WeH) Weyl H. The Continuum, A Critical Examination of the Foundation of Analysis, Dover Publications, Inc. New York, 1994

(WeH1) Weyl H. Philosophy of Mathematics and Natural Science, Princeton University Press, Princeton, 1949, 2009