TACMB-1: The Theory of Anisotropies in the Cosmic Microwave Background:

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Abstract

This Resource Letter provides a guide to the literature on the theory of anisotropies in the cosmic microwave background. Journal articles, web pages, and books are cited for the following topics: discovery, cosmological origin, early work, recombination, general CMB anisotropy references, primary CMB anisotropies (numerical, analytical work), secondary effects, Sunyaev-Zel'dovich effect(s), lensing, reionization, polarization, gravity waves, defects, topology, origin of fluctuations, development of fluctuations, inflation and other ties to particle physics, parameter estimation, recent constraints, web resources, foregrounds, observations and observational issues, and gaussianity.

Introduction

The cosmic microwave background (CMB) radiation is a relic of a time when the universe was hot and dense, and as such it encodes a wealth of information about the early universe and the formation of the large-scale structure we see in the universe today. The very existence of the CMB is one of the four pillars of the hot big bang cosmology. That the spectrum [7, 8, 9] is the best measured black body spectrum in nature provides stringent constraints on its origin and on any injection of energy at early times [14]. Perhaps the most exciting and active area of CMB research, however, is the study of its *anisotropies*: the small fluctuations in intensity from point to point across the sky. As we shall discuss below, these anisotropies provide us with a snapshot of the conditions in the universe about $300,000$ years after the big bang, when the universe was a simpler place. This snapshot is both our earliest picture of the universe and an encoding of the initial conditions for structure formation.

As a consequence of the hot big bang model, the CMB was predicted for a long time [20, 21]. The history and drama of its discovery [5, 6] is a full story in and of itself, starting points are the "historical" references [3, 4]. For the anisotropies, although several fundamental calculations were done before 1992, it was with the COBE [10] detection in 1992 that interest and activity exploded.

Origin of the CMB

If we run the expansion of the universe backwards in time, the universe becomes hotter and denser. Beyond a point when distances in the universe were only 0.1% of their current size, the temperature was high enough to ionize the universe, and it was filled with a plasma of protons, electrons, and photons (plus a few He nuclei and traces of other species). This transition from a neutral to an ionized medium is especially important. Before this time the universe could be modelled as a smooth gas of photons, baryons (the protons and electrons) and dark matter. Since the number density of free electrons was so high, the universe was opaque to the microwave background photons: the mean free path for photons to Thomson scatter off electrons was extremely short. Consequently, the photons and baryons could be considered as a single "tightly coupled" fluid [22]. In this fluid, the baryons provide the weight, while the photons provide the pressure (for photons, pressure and density obey $p_\gamma = \rho_\gamma/3$). As the universe expands, the wavelengths of photons are stretched out, lowering their energy. Eventually, when the universe had cooled to $T \sim 4,000$K, the photon energies became too small to ionize hydrogen [30] ($kT \sim 0.25$ eV, smaller than the ionization energy of hydrogen, 13.6 eV, because the photon to baryon ratio, $10^9 : 1$, is so large that the high-energy tail of the thermal distribution is significant). At this point the protons and electrons (re)combined to form neutral hydrogen and the photon mean free path increased to essentially the size of the observable universe.[1] The photons were set free, and have since travelled almost unhindered through the universe.

[1] The mean free path of photons through the universe must be huge or we would not see galaxies and quasars out to distances of thousands of Mpc (1 Mpc = 3.3×10^6 light years [lyr]).

Once it started, the recombination of hydrogen was a phase transition, completing very rapidly. We refer to this time as the epoch of recombination. When we observe the universe in the microwave bands we see the photons which last interacted with matter at this epoch. These photons have travelled to us from a sphere, centered on the observer and known as the surface of last scattering, whose radius is essentially the entire observable universe $\sim 10^4$Mpc or 10^{10} light years. The photons have continued to lose energy with the expansion of the universe, and now form a black body with a temperature of 2.73K. One can think of the temperature of the cosmic microwave background photons as the temperature of the universe.

Describing CMB anisotropies

Numerous observations of the cosmic microwave background photons support this assumption of cosmological origin [11]: the background is isotropic [5, 12, 13] and a black body [7, 8, 9] and has no correlations with local structures in the universe [10, 15, 16, 17]. Upon closer examination the CMB temperature is not uniform across the sky, but has slight fluctuations from place to place. We shall be interested in the fluctuations of the temperature about the mean: $\Delta T(\hat{n})$ where $\hat{n}$ is a unit vector pointing in a particular direction on the sphere.

The largest anisotropy is a fluctuation of about 1 part in 1000 that forms a dipole pattern across the sky. The reason for this dipole is that the earth is not at rest with respect to the CMB, and we see a Doppler shift in the CMB temperature owing to our relative motion. Since this changes as the earth orbits the sun, this dipole is modulated throughout the year. One of the great triumphs of modern cosmology is that if we take the mass distribution observed around us and compute from this a gravitational acceleration, then multiply this acceleration by the age of the universe, we obtain a good match to both the direction and the amplitude of our velocity vector in the CMB rest frame [18]. However this dipole is clearly of (relatively) local rather than primordial origin, and so we generally subtract it (plus the mean or "monopole") before dealing with the CMB anisotropy.

After this dipole is taken out, the size of the fluctuations is about 1 part in 100,000. Mathematically we describe these anisotropies by expanding the temperature field on the sphere using a complete set of basis functions, the spherical harmonics

$$\frac{\Delta T}{T}(\hat{n}) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\hat{n}) \ . \tag{1}$$

The $a_{\ell m}$ are a curved-sky version of a Fourier transform of the temperature field. By definition the mean value of the $a_{\ell m}$ is zero.

As there are no preferred directions cosmologically, theories predict only statistical information about the sky, not that the temperature in a certain direction should have a particular value. For this reason the quantities of interest are statistics of the observed temperature pattern. The most common and useful statistic is known as the correlation function (or 2−point function) of the temperature field $C(\theta)$. We form this by calculating the average of $\Delta T/T(\hat{n}_1)\Delta T/T(\hat{n}_2)$ across all pairs of points in the sky $(\hat{n}_1, \hat{n}_2)$ separated by an angle θ (*i.e.* $\cos\theta = \hat{n}_1 \cdot \hat{n}_2$). Under the assumptions that our theory has no preferred direction in the sky (statistical isotropy) and that the fluctuations in temperature have Gaussian statistics, the correlation function encodes all of the physical information in the CMB anisotropies. (For non-Gaussian fluctuations there will be additional information in higher order, *e.g.* 3-point, correlations.)

Original theoretical calculations and observations were performed almost entirely on the correlation function. However, Wilson and Silk, [60] introduced the "multipole expansion," which isolates the physics much more robustly and simplifies the calculations:

$$C(\theta) = \frac{1}{4\pi} \sum_{\ell} (2\ell + 1) C_\ell P_\ell(\cos\theta) \tag{2}$$

where $P_\ell(\cos\theta)$ are the Legendre polynomials and the C_ℓ are the quantities of interest known as the multipole moments. In terms of the $a_{\ell m}$ defined above

$$C_\ell = \sum_{m=-\ell}^{\ell} a^*_{\ell m} a_{\ell m} \tag{3}$$

One can think of ℓ as the variable "Fourier" conjugate to angle, $\ell \sim \theta^{-1}$.

From this description one of the fundamental limitations to the study of CMB anisotropies becomes evident. We are trying to estimate these quantities C_ℓ statistically from a finite number of samples, hence our estimates will be uncertain by an amount proportional to the square root of the number of samples (often called "cosmic variance" since we would need more universes to get a better determination) [27, 28]. Each C_ℓ comes from averaging over $2\ell+1$ modes, and thus the sample variance error on C_ℓ is

$$\frac{\delta C_\ell}{C_\ell} = \sqrt{\frac{2}{2\ell+1}} \qquad (4)$$

where the 2 in the numerator arises because C_ℓ is the square of a Gaussian random variable ($a_{\ell m}$) and not the variable itself.[2]. If only a fraction $f_{\rm sky}$ of the sky is observed then the error is increased by $f_{\rm sky}^{-1/2}$ [265, 266, 267, 219]

The physics of CMB anisotropies I: The simplest picture

Current CMB anisotropy measurements are improving rapidly [252] and a broad outline of C_ℓ as a function of ℓ is taking shape. At large angular scales (small ℓ) there is a flat plateau that rises into a narrow peak at about one degree. On arcminute scales the power has fallen once more and on even smaller scales currently there are only upper limits. In this section we shall describe how we now understand this structure.

Historically the key ingredients were the recognition that in the early universe there was a tightly coupled photon-baryon plasma [22] that decoupled suddenly [30, 31, 32, 33, 250] and that in the presence of perturbations the fundamental modes of excitation were sound waves [26, 22, 23, 60, 61, 63, 64]. However the language has changed significantly since those early papers, and the sophistication with which the calculations are performed has improved radically [38, 248, 249], so below we outline this more modern description, developed in [34, 74, 36, 75, 76].

This description falls within the current paradigm of cosmological structure formation [19]. When we observe the distribution of galaxies about us we find that they are not arranged at random, but rather cluster together in coherent patterns that can stretch for up to 100Mpc. The distribution is characterized by large voids and a network of filamentary structures meeting in large overdense regions. A great deal of evidence suggests that this large-scale structure arose through the action of gravity on initially small amplitude perturbations in density. As inflation [210, 211, 212, 213] is the most promising theory for the origin of these primordial density perturbations, we will use its properties for illustrative purposes. Generically inflation predicts that at very early times there were small, almost scale-invariant adiabatic[3] fluctuations in the density of the universe on a wide range of physical scales. A region of space that was initially overdense would give rise to a larger than usual gravitational potential. Surrounding matter would fall into this potential, increasing the overdensity. Similarly matter would flow out of regions of underdensity, increasing the density contrast further. In this way gravity can amplify any already existing density perturbations. Eventually the density contrasts would become so large that we could ignite nuclear fusion and form stars, galaxies, etc.

The CMB anisotropies that we see are a snapshot of the conditions when the universe was 300,000 years old, that is on the surface of last scattering, plus some (small) processing that occurred en route to us. After last-scattering the CMB photons stream essentially freely to us and the density fluctuations are seen as CMB temperature differences (anisotropy) across the sky ($\delta T/T = \frac{1}{4}\delta\rho_\gamma/\rho_\gamma$, since $\rho_\gamma \propto T_\gamma^4$). The key concept is that anisotropy on a given angular scale is related to density perturbations on the last scattering surface of a given wavelength. The relevant wavelengths correspond to the length projected by that angle on the last-scattering surface: $\lambda \sim 200{\rm Mpc}\,(\theta/{\rm deg})$. Phrased another way, multipole moment ℓ receives its dominant contribution from Fourier mode k, where $\ell = kr$ and r is the (comoving angular diameter) distance to last scattering.

We show in Fig. 1 a theoretical prediction for the anisotropy spectrum (i.e. $\ell(\ell+1)C_\ell$ vs. ℓ)[4] of a cosmological model with cold dark matter and an initial spectrum as given by inflation. Similar to the current observations

[2]The variance of x^2 is twice the square of that of x if x is a Gaussian random variable of zero mean.

[3]The term adiabatic implies that a positive fluctuation in the number density of one species is also a positive fluctuation in all of the other species (*i.e.* more photons means more baryons and more dark matter). A spectrum of fluctuations is scale-invariant if the gravitational potential fluctuation it produces has the same amount of power per logarithmic interval in wavelength.

[4]It is conventional to plot $\ell(\ell+1)C_\ell$ rather than C_ℓ because this is approximately the power per logarithmic interval in ℓ (or angle). Also, in the simplest possible model of scale-invariant fluctuations from the Sachs-Wolfe effect (see below) $\ell(\ell+1)C_\ell$ is constant.

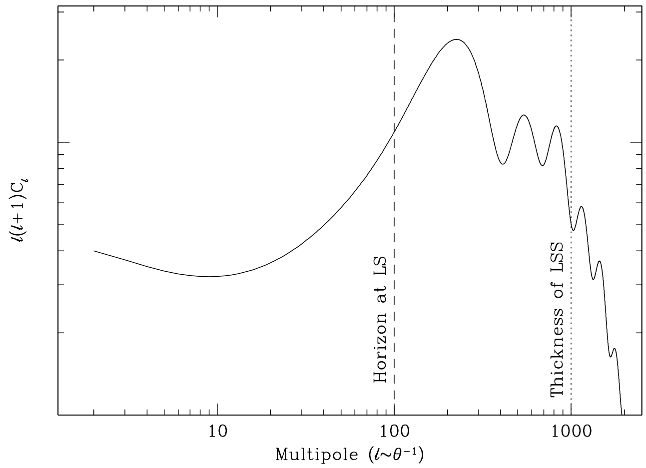

Figure 1: The CMB angular power spectrum (Eq. 2) as a function of multipole moment $\ell \sim 1/\theta$. Roughly, one degree on the sky today corresponds to $\ell \sim 10^2$, one arcminute to $\ell \sim 10^3$.

mentioned earlier, the spectrum clearly has 3 distinct pieces: at low-ℓ (large angular scales) there is a flat plateau that rises into a series of bumps and wiggles that then damp quasi-exponentially on small angular scales. These 3 regimes are separated by 2 angular scales, the first at about 1 degree and the second at a few arcminutes.

To understand the origin of these features let us go back in time to just before recombination. At this time the universe contained the tightly coupled photon-baryon fluid and dark matter, with perturbations in the densities and thus gravitational potentials on a wide range of scales. While perturbations in the dark matter grow continuously as the universe ages, the gravity-driven collapse of a perturbation in the baryon-photon fluid is resisted by the pressure restoring force of the photons. For example, as an overdensity falls into a gravitational potential it becomes more and more compressed. Eventually photon pressure halts the collapse and the mode rebounds, becoming increasingly rarefied. The expansion is slowed and halted owing to the weight of the fluid and the gravitational potential, causing the mode to recollapse once more. In short, an acoustic wave is set up, with gravity the driving force and pressure the restoring force. Mathematically, the Fourier mode k of the temperature fluctuation is governed by a harmonic-oscillator-like equation [22, 75, 94]

$$[m_{\rm eff}\Delta T_k']' + \frac{k^2}{3}\Delta T_k = -F_k \tag{5}$$

where F is the gravitational forcing term owing to the dark-matter potentials, $m_{\rm eff}$ describes the inertia of the fluid, and primes denote derivatives with respect to (conformal) time ($\eta = \int dt/a(t)$ where $a(t)$ is the scale-factor of the universe). The solutions are acoustic waves.

We are now in a position to understand the features in Fig. 1.

The large-angular scale (Sachs-Wolfe) plateau ($\ell < 100$) in the angular power spectrum arises from perturbations with periods longer than the age of the universe at last scattering, *i.e.* $\sim$ larger than the horizon, scales that can be affected by causal physics at that time. These waves are essentially frozen in their initial configuration and provide us with a probe of the physics that created them, unspoiled by cosmological evolution. Since CMB photons lose energy climbing out of the potential wells associated with these long-wavelength density perturbations, the temperature differences seen on the sky reflect the gravitational potential differences on the last-scattering surface [24, 89, 90]. If the density fluctuations are approximately scale-invariant the plateau in the angular power spectrum is flat.

At scales smaller than the horizon, the baryon–photon fluctuations that produce anisotropy on sub-degree angular scales ($10^2 < \ell < 10^3$) have sufficient time to undergo oscillation. At maximum compression (rarefaction) the CMB temperature is higher (lower) than average. Neutral compression corresponds to velocity maxima of the fluid, which leads to a Doppler-shifted CMB temperature. The Doppler effect is subdominant because we see only the line-of-sight component of the velocity and the speed of sound is less than the speed of light. Since last-scattering is nearly instantaneous, the CMB provides a snapshot of these acoustic oscillations, with different wavelength modes being caught in different phases of oscillation. Because a given multipole ℓ is dominated by the effects of a narrow band of Fourier modes, this leads to peaks and valleys in the angular power spectrum. The peaks are modes that were maximally under or overdense at last-scattering (since the power spectrum is the amplitude *squared*), and the troughs are velocity maxima, which are $\pi/2$ out of phase with the density maxima.

On even shorter scales ($\ell \gtrsim 10^3$) the finite duration of recombination has an observable effect [26, 65]. During this time the photons can random walk a distance given by the mean free path (which is increasing during recombination) times the square root of the number of scatterings [75, 76, 131]. Thus photons can diffuse out of any overdensity on smaller scales than this. This leads to an exponential damping of the spectrum on small scales (known as Silk damping). If we approximate last scattering as extremely rapid, the damping is exponential with e-folding scale the geometric mean of the horizon and the photon mean free path. The finite duration of last scattering changes this somewhat, and the damping is closer to an exponential of a power of scale [75, 76].

The physics of CMB anisotropies II: Beyond the simplest picture

While the above picture explains the gross features of Fig. 1, a number of other effects have received detailed study. Here we discuss these effects in the order in which they occur in the evolution of the universe, which is not the historical order in which they were discovered.

At last scattering and since, the photons not only respond to the gravitational potentials caused by dark matter density perturbations, but also to any other perturbations in the space-time metric. Technically, gravitational potentials owing to density perturbations are often referred to as scalar, corresponding to their Lorentz transformation properties (properties under boosts and rotations). Since the metric has more complicated transformation properties (specifically it is a spin-2 tensor), vector and tensor fluctuations are also possible. Vector perturbations, also called vortex perturbations, decay as the universe expands unless they are constantly generated. Tensor perturbations (also called gravity waves) can be generated by quantum fluctuations of the spacetime. These persist and can have an effect in many cases. Tensor perturbations of spacetime do not create the same baryon-photon oscillations, but can contribute a Sachs-Wolfe plateau [24, 168, 169, 170, 171, 27, 173, 174, 175, 176]. Inflationary theories usually produce no vector perturbations, and small tensor perturbations.

As the photons travel through the universe from the surface of last scattering they can interact gravitationally with the matter. If the gravitational potentials are still evolving, additional temperature perturbations are generated by the "integrated Sachs-Wolfe effect" [24, 172, 173]. Schematically a photon falling into a gravitational potential will gain energy. If the potential evolves during the photons' traverse, the energy lost climbing back out will be different from that gained falling in, leading to a net anisotropy. To linear order in the perturbations the gravitational potential ϕ is constant when matter dominates the energy budget of the universe and this phase gives no contribution. However, right after recombination photons still contribute enough to the energy density of the universe that the change in time of the potential, $\dot{\phi}$, is non-zero (the "early ISW effect") and at very late times if either curvature or a cosmological constant dominate $\dot{\phi} \neq 0$ (the "late ISW effect"). Additionally when non-linear structures form the potential can change with time owing to both the growth and movement of bound halos leading to anisotropies through the Rees-Sciama effect [25]. In modern theories this effect is very small, and is not the dominant source of anisotropy on any scale [101, 102, 103].

In addition to the energy gained and lost by photons, the path a photon takes is altered by non-zero potentials. This gravitational lensing causes the spectrum to be slightly "blurred" [119, 120, 121, 122, 123, 124, 125], smoothing the third acoustic peak by a few percent and slightly altering the shape of the damping tail. The signature of gravitational lensing may be used to reconstruct the projected gravitational potential along the line-of-sight [126].

Observations of the spectra of high redshift QSOs indicate that the universe is highly ionized out to redshift $z \sim 5$. Thus photons can again scatter off free electrons in a second "scattering surface." Unlike the $z \sim 10^3$ surface, however, the electron density today is quite low, and the baryons and photons do not become tightly coupled. Because of this the two fluids can have a large relative velocity, which enhances the power of the Doppler effect. Reionization, as this is called, damps power on angular scales smaller than the horizon subtended by the epoch of reionization while generating extra power owing to Doppler scattering [127, 65, 128, 129, 130, 134, 131]. There is also a second order effect known as the Ostriker-Vishniac effect [132, 133, 134, 135, 136] that affects only the small scale.

It is unlikely that the reionization of the universe will occur uniformly throughout space, so anisotropies will be generated owing to the "patchiness" of reionization. Depending on the redshift of reionization and whether the ionizing sources are quasars or stars, the angular scale of this anisotropy could be quite different. Early analytic attempts to discuss patchy reionization [138, 139, 137, 136] used crude models to estimate the required correlation functions. Recent numerical simulations [140, 141, 142, 143] have improved upon these results, but this remains an area of active research at present. Current calculations suggest the patchy reionization will not dominate except on extremely small angular scales.

Finally, once structure formation is well underway, the photons can interact with hot gas in the intergalactic medium [105, 106]. The CMB photons can either be upscattered in energy when interacting with hot gas (the "thermal" Sunyaev-Zel'dovich effect) or have their temperature altered by Doppler scattering from moving gas (the "kinetic" S-Z effect). For recent reviews see [107, 108]. The thermal SZ effect is probably the largest source of anisotropy on angular scales of a few arcminutes and has been calculated both analytically [109, 110, 111, 112, 113] and numerically [114, 115, 116, 117, 118]

On top of these effects are "foregrounds" (as the signal is a background) that mask the CMB physics and are the source of many headaches and much work. These include dust, free-free emission, and synchrotron radiation, all of which have estimated dependencies on frequency and angular scale (a summary and comparison of these can be found in [254, 255], some references are [256, 257, 258, 259]). Many experiments measure the CMB in many different frequency bands to account for these foregrounds. Some foregrounds, such as point sources, will produce non-gaussian anisotropies. As the simplest inflationary models produce gaussian anisotropies, this also can be used to distinguish them from the desired signal.

Polarization

Not only do the CMB photons have temperatures, as described above, they also are expected to have polarization [152, 153, 66]. The Thomson scattering cross section σ as a function of solid angle Ω depends on polarization

$$\frac{d\sigma}{d\Omega} \propto |\varepsilon_i \cdot \varepsilon_f|^2 \tag{6}$$

where $\varepsilon_{i,f}$ are the incident and final polarization directions. The scattered radiation intensity peaks normal to, and with polarization parallel to, the incident polarization. If the incoming radiation field is isotropic then orthogonal polarization states balance and the outgoing radiation remains unpolarized. In the presence of a quadrupole anisotropy, however, a linear polarization is generated by scattering.

Since we have observational evidence for anisotropies at last scattering, we expect that the CMB be linearly polarized. The degree of polarization is directly related to the quadrupole anisotropy at last scattering. While the exact properties of the polarization depend on the mechanism for producing the anisotropy, several general properties arise. The polarization peaks at angular scales smaller than the horizon at last scattering (*i.e.* smaller scales than the first temperature peak) owing to causality. Since only those photons that scattered in an optically thin region near last scattering could have had a quadrupole anisotropy, the polarization fraction is small and dependent on the duration of last scattering. For the standard thermal history it is a few percent of the temperature anisotropy. An additional change in polarization can occur during subsequent interaction with ionized matter (e.g. during reionization as mentioned above [29]). Gravitational interactions do not generate or destroy polarization.

The formalism for the description of polarized radiation on the sphere has been developed in [154, 155, 156, 88]. In analogy with the temperature, the polarization is expanded in a series of spin-weighted spherical harmonics whose coefficients can be used to define "E-mode" and "B-mode" polarization power spectra[5] that transform into one another under a 45-degree rotation of the polarization. There is additionally a cross-power spectrum between T and E. Density (or scalar) perturbations have no "handedness" and so generate only E mode polarization. Vector and tensor modes create both E and B mode polarization.

[5]The modes are called "E" and "B" to denote their parity transformation properties; they should not be confused with the electric and magnetic fields of the CMB signal itself. Some authors also refer to these as the "gradient" and "curl" components in analogy with the decomposition of a vector field.

Information from polarization is complementary to information from temperature anisotropies. Different sources of anisotropy (scalar, vector, tensor) generate different patterns of polarization [157, 158, 88] and adiabatic and isocurvature modes generate different polarization spectra [159, 160, 88]. The presence of polarization increases the number of spectra that can be measured from 1 to 4 (temperature, the two polarizations, and the T-E cross spectrum), which allows better constraints on cosmological models [161, 220]. More beneficially, polarization depends on some of the cosmological parameters differently than the temperature anisotropy, allowing degeneracies in the fitted parameters to be removed and improving parameter constraints by a large factor [161, 220, 224, 227].

What can we learn from CMB anisotropies?

CMB anisotropies represent one of the cleanest astrophysical systems known: the anisotropies arise from electron-photon interactions and weak gravitational fields. Thus the predictions can be calculated accurately and reliably, while at the same time providing us with valuable information about the early universe, the formation of large-scale structure, and the cosmological parameters. Here we discuss what we have already learned and what we hope to learn soon [37] from a comparison of these calculations with high-precision observations.

To begin with generalities, because the large angle anisotropies are 1 part in 10^5, this constrains the amplitude of the fluctuations in matter densities on the scale of the horizon. Any theory of fluctuation generation and evolution must be normalized to agree with this value [202, 203, 204, 205, 206]. Within the limited statistics currently available the fluctuations appear to be Gaussian [280, 281, 282, 283], as predicted by the simplest models of inflation.

Comparing the size of these early fluctuations to the size of density perturbations today provides more circumstantial evidence for nonbaryonic dark matter. (Nonbaryonic dark matter was first introduced in other contexts for other reasons.) While a model-independent statement is difficult to make, if there were only baryons, the level of inhomogeneity required to produce the observed large-scale structure through gravitational infall would generally lead to CMB anisotropy that is about ten times larger than that observed [60].[6] A model based on gravitational amplification of initially small adiabatic perturbations in a universe whose dominant matter component is cold and dark manages to reproduce the amplitude of the fluctuations required over many decades in linear scale.

A first acoustic peak has been detected in the CMB temperature anisotropies [229, 230, 231, 232]. The position of this peak is related to the size of the horizon at last scattering and the distance travelled by the photons since this time. This angular scale is sensitive to spatial curvature [23, 62, 92, 93, 77], appearing smaller if the universe is open (negatively curved) and larger if the universe is closed (positively curved, similar to a sphere). The currently observed position is consistent with the universe being spatially flat.

The amplitude of the peak indicates that either the baryon density is high, or the the matter density of the universe is below critical (or both) [233, 234, 232, 235]. Because we see any CMB signal at all on degree scales means that the photons were able to travel unhindered to us for some time before reionization occurred, *i.e.* the universe was neutral for a while between $z \sim 10^3$ and $z \sim 5$ [35, 236, 234].

Many of these above features cause difficulties for non-inflationary theories of structure formation. Of the dozens of theories proposed before 1990, only inflation and cosmological defects survived after the COBE announcement, and only inflation is currently regarded as viable by the majority of cosmologists. Cosmological defects are configurations in spacetime of some field, *e.g.* domain walls, strings, monopoles, or textures, which can be produced as the universe cools through several phase transitions [182]. In contrast to inflation, perturbations caused by defects form continuously, as larger and larger regions come into causal contact and feel the influence of the defects moving around. Calculations with defects are extremely challenging technically, making it difficult to draw robust conclusions. However, several trends emerged early on: defect theories fail to reproduce the observed power in the matter fluctuations when normalized to the CMB [202, 183], generically produce non-Gaussian fluctuations on degree scales [184, 185] that are not observed [280, 281, 282, 283], a high redshift of reionization [186], and even in the absence of reionization give a very low (or absent) broad peak [95, 96] around degree scales. In the many cases where detailed temperature anisotropy calculations have been carried out [188, 187], defect models strongly disagree with observations.

Because the surface of last scattering is a sphere of radius $\sim$ 10Gpc, it is sensitive to any non-trivial topology in the universe. Current measurements indicate that the large-scale structure of space-time appears topologically

[6] To get enough large-scale structure power but suppress the large-angle anisotropies there are models with a power-law initial spectrum whose power grows with decreasing length scale. However, then reionization is required to flatten the spectrum at COBE scales. This simultaneously damps the power at degree scales, leading to conflict with observations [91].

simple [190, 191, 192, 193].

More information is hinted at in the current data but not yet as precisely measured. For example, the narrowness of the first peak means perturbations were created a long time ago, for instance laid down early on by inflation [233]. The spectrum of initial perturbations is close to scale-invariant. The position of the damping tail provides a feature in the power spectrum that is almost independent of the source of the fluctuations, depending only on the properties of the fluid at last scattering (*e.g.* the baryon-to-photon ratio) and the angular diameter distance to last scattering [134, 131].

Future determinations of the temperature spectrum and detections of the polarization spectrum can provide even more information. The low-ℓ shape and relative amplitudes of the polarization power spectra indicate which modes (scalar, vector or tensor) are populated by the source of fluctuations [88]. The relative positions and heights of the peaks can provide a test of inflation [94, 189, 159] or more generally of an apparently acausal generation of curvature perturbations on super-horizon scales [195].

If all of our modelling assumptions are borne out, and the angular power spectrum is well fit by an inflationary CDM model, then we can expect to constrain on the order of 10 cosmological parameters to the few percent level from high resolution anisotropy observations, *e.g.*, [218, 219, 220, 221, 222, 223, 224, 225, 226]. Writing the initial spectrum as a power law, more precise constraints on both the power and deviations from power law behavior will be possible [196, 197].

Generally the CMB constrains quite well the angular diameter distance to last scattering, the physical matter ($\Omega_{\rm mat}h^2$) and baryon densities ($\Omega_B h^2$), and the spectral index of the fluctuations. Here we have written the Hubble constant as $H_0 = 100h$km s^{-1} Mpc^{-1} and the densities as a fraction of the critical density ($\rho_{\rm crit} \equiv 3H_0^2/(8\pi G)$), $\Omega_i = \rho_i/\rho_{\rm crit}$. Typically some combinations of the other parameters are well constrained while some are very poorly constrained [227]. To break these parameter degeneracies one needs to include measurements that complement the CMB constraints. For example, the degeneracy between $\Omega_{\rm mat}$ and Ω_Λ that enters into the angular diameter distance to last scattering can be broken with low redshift measurements [222, 224]. An accurate measurement of the Hubble constant H_0, combined with the accurate determination of $\Omega_{\rm mat}h^2$ mentioned above, also breaks many degeneracies, and this is what large-scale structure surveys or direct H_0 measurements can provide.

If we are lucky enough that inflation takes place near the GUT scale, then a measurably large gravitational wave component to the anisotropy is predicted [168, 169, 170, 171, 27, 173, 174, 175, 176, 177]. Using both temperature and polarization information, tensor signals as small as $0.1 - 1\%$ [162, 178, 179] of the total anisotropy can be detected, corresponding to $E_{\rm inf} > 10^{15}$GeV. Should our luck hold out, and inflation be dominated by a single scalar field, it may even be possible to reconstruct the inflationary potential to some extent from detailed measurements of the scale-dependence of the signals [214].

Nuts and bolts: Calculating CMB anisotropies

While the CMB anisotropy description above is physically clear, it is very heuristic in comparison to how the calculations are done in practice. One begins with the coupled Einstein, fluid, and radiative transfer equations, expanding about an exact solution and truncating the expansion at linear order [22, 62, 78, 79, 66, 80, 69, 70, 88]. This is consistent as the observed fluctuations are small; in addition, the higher order terms have been calculated and shown to be small as expected [129, 130]. This results in a set of coupled ODEs that describe the evolution of each independent Fourier mode (or its curved-space generalization). While in some cases an analytic solution is possible, the equations are usually numerically integrated from early times until the present [60, 62, 64, 66, 68, 67, 69, 70, 71, 248].

The formalism for computing the C_ℓ (or the higher-order moments) for any FRW space-time and any model of structure formation exists [81, 83] and for many cases of interest can be done with publicly available codes such as CMBFAST [248, 71] and CAMBfast [249, 82]. These codes incorporate many refinements [38] and have become quite complex. However, in addition to calculating self-consistency within a given code, calculations have been done (mostly for CDM models) using several independently developed codes, with an agreement found of $\mathcal{O}(1\%)$. [81].

Observational Outlook

Since COBE first detected anisotropies, there has been a flurry of observational "firsts" in CMB research. We now have observational evidence for a nearly scale-invariant low-ℓ plateau, a peak in power on degree scales and a subsequent fall in power ("damping") on arcminute scales. As of this writing (Fall 2000), polarization has not yet been detected. Improved ground-based, balloon, and satellite CMB experiments are underway or under construction that will measure a range of properties, from small scale anisotropies on small regions of sky to full sky maps, with and without polarization. The most current information in this rapidly progressing area can be found on the experimental web pages [252]. Recent experiments span a larger range of angular scales than ever before, allowing features in the spectrum to be identified from individual experiments rather than statistical compilations. This minimizes the effect of calibration uncertainties that can offset different experimental results by of order 10-20% in amplitude. Increased sky coverage (to allow calibration off the dipole) and better control of systematics are reducing this uncertainty in the next generation of experiments. Although many early measurements were statistical detections, in current experiments the signal-to-noise on each resolution element is larger than one. The flood of new, higher-resolution, higher signal-to-noise data has required the development of specialized analysis tools to extract the maximum cosmological information. CMB analysis is a flourishing sub-field that we have not attempted to address here.

Summary

Anisotropies in the CMB are one of the premier probes of cosmology and the early universe. Theoretically the CMB involves well-understood physics, in the "linear regime" and is thus under good calculational control. Model independent constraints on the cosmology and the model of structure formation exist. Within any given model parameter extraction can be made very precisely, especially when CMB data is combined with other data ("complementarity").

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[251] related web sites

- http://background.uchicago.edu/∼whu/metaanim.html
- http://astron.berkeley.edu/*sim*/mwhite/movies.html

[252] Pages with lists of cmb experiments:

- http://background.uchicago.edu/∼whu/cmbex.html

- http://astron.berkeley.edu/*sim*/mwhite/cmbexptlist.html
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