A Hilbert scale and MHD based unified plasma, quantum and gravity field theory

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In a nutshell

About 95% of the universe is about the phenomenon „vacuum“. The same proportion applies to the emptyness between a proton and an electron. The remaining 5% of universe's vacuum consists roughly of 5% matter, of 25% dark matter, and of 70% dark energy. Nearly all (about 99%) of the 5% matter in the universe is in "plasma state". A presumed physical concept of „dark matter“ „explains“ the phenomenon of the spiral shapes in the universe. A presumed physical concept of „dark energy“ expains the phenomenon of the cosmic microwave background (CMP).

The dominance of the vacuum (in the macro case, as well as in the micro case) in opposite to the small amount of matter is addressed by an appropriate Hilbert scale based definition of an „ether/ground state energy“ space, which is „orthogonal“ to a „kinematical energy“ Hilbert space.

In this model the phenomenon of spiral shapes is a consequence of the MHD-based balance law of angular momentum. The phenomenon of CMP is a consequence of the MHD-based balance law of linear momentum.

The tool box of the proposed unified plasma, quantum und gravity field theory consists of:

- Hilbert scale decompositions in the form $H = H^1 \otimes H^2$
- Potential operators defined by indefinite inner products
- Krein spaces in a quarternionic setting, (AlD)
- variational (approximation) methods in Hilbert scales
- the complex Lorentz group and its two *connected* components
- classical & quantum tensor theory
- quantum (two-component) spinor theory
- two-component Magnetohydrodynamics.

The proposed „two-component“ resp. „one-component“ Hilbert scale decompositions are given by

$$H_{1/2} = H_1 \otimes H_1^{\perp} = H_{(-)} \otimes H_{(+)} \otimes H_1^{\perp} \text{ resp. } H_{1/2} = H_1 \otimes H_1^{\perp} = H_2 \otimes H_2^{\perp} \otimes H_1^{\perp}.$$

The Hilbert space H_1 denotes the kinematical sub-Hilbert space and $H_1^{\perp}$ denotes the orthogonal closed "ground state" energy sub-space of $H_{1/2}$. The kinematical Hilbert space H_1 is governed by a countable orthogonal Riesz basis enabled by the eigen-pairs of a (kinematical) selfadjoint, positive definite operator with domain H_1. Geometrically speaking, the kinematical Hilbert space H_1 builds a coarse-grained embedded Hilbert sub-spaces of $H_{1/2}$. Its related closed (ground state energy) sub-space $H_1^{\perp}$ of $H_{1/2}$ is governed by a continuous spectrum. The „one-component“, classical kinematical Hilbert space decomposition of H_1 is given by $H_1 = H_2 \otimes H_2^{\perp}$. The „two-component-plasma“ kinematical Hilbert space decomposition of H_1 is given by $H_1 = H_{(-)} \otimes H_{(+)}$(*).

In CPT (Charge-Parity-Time) the PT transformations are geometric transformations, while the C transformation is not. The CPT theorem becomes a purely geometric PT theorem if one restricts to classical tensor theory, quantum tensor theory and quantum spinor theory. The fact that it holds for quantum field theories suggests that space-time has neither a temporal orientation nor a spatial handedness (**), (appendix).

(*) This proposed new „conceptual elements“ match to the "unconventional features of the mathematical formalism" mentioned in (HeW).
(**) The group of restricted real Lorentz transformations has the same structure as the group of $2x2$ complex matrices of determinant one, $SL(2,C)$. This restricted Lorentz group contains the 1-transformation. Its connected component contains the space-time inversion. The other pair of connected components of the Lorentz group contains the space inversion resp. the time inversion. The group of proper (complex) Lorentz transformations has the same structure as $SL(2,C) \times SL(2,C)$. The classical real Klein-Gordon field is a real field, whose field values are invariant under the restricted Lorentz transformation. The full group of (real) Lorentz transformations is the group of transformations that leaves the Minkowski metric invariant. In quantum field theory particle states correspond to „positive frequency“ solutions of the corresponding classical field theory, while anti-particle states correspond to „negative frequency“ solutions. Therefore, the PT transformation in quantum field theory turns particles into anti-particles and any quantum tensor Lagrangian is invariant under the PT transformation. In classical two-component spinor theory one can construct Lagragians that are not invariant under PT. Miraculously, this problem goes away, when one goes to spinor quantum field theory, which again is invariant under the PT transformation. In the context of the concepts “spin”, “dual turns”, line geometry & kinematic, and “quaternions” we refer to Study's “movement indicator”, (BlW) §58.

Conceptual design elements

The underlying indefinite inner product of a Hilbert space decomposition in the form $H = H^1 \otimes H^2$ enables the definition of projection operators P^1 and P^2, a „potential function" on H, and a related „potential operator" for any x being an element of H, (appendix).

This tool is applied to define appropriately potential functions and related potential operators accompanied with related potential barrier constants governing the „boundary layers" between

i) the kinematical & ground state energy spaces $H_{1/2} = H_1 \otimes H_1^{\perp}$
ii) either a „one-component" or a „two-component" kinematical Hilbert space H_1.

Nearly all of the matter in the universe consists of "plasma". The more specific two-component" kinematical Hilbert space decomposition supports the specifics of „plasma particle field interactions", (*). Therefore, the proposed „two-component" Hilbert scale decomposition is given by

$$H_{1/2} = H_1 \otimes H_1^{\perp} = H_{(-)} \otimes H_{(+)} \otimes H_1^{\perp}.$$

The requirements that the numbers of positively and negatively charged kinematical particles per considered volume element need to be approximately identical modelling of the more specific two-component" kinematical Hilbert space is supported by the fact that the Hilbert space H_1 is governed by a countable orthogonal Riesz basis. This enables a decomposition $H_1 = H^{(-)} \otimes H^{(+)}$ into two orthogonal sub-spaces $H^{(-)}$ and $H^{(+)}$ with same "cardinality" (**).

The underlying indefinite inner product of the decomposition $H_1 = H_{(-)} \otimes H_{(+)}$ enables the definition of projection operators P^1 and P^2, a „potential function" on H_1, and a related „potential operator" for any x being an element of H_1. More specifically, the "potential function" is about an indefinite metric $m(x)$ with a related potential operator $W(x)$. The concepts allow to build hyperbolids, conical regions V_0 and its boundary K, and ellipsoids, where the manifold K is an asymptotic conical manifold for the hyperboloid $m(x) = c$ (appendix).

The constants c defining the hyperboloids $m(x) = c$ of the several „boundary layers" between the proposed decomposed sub-spaces provides a modelling tool of related physical coupling constants. Their building principles are governed by the energy resp. the action minimization laws.

The closest „boundary layer" to the „Nature" world is between the two granular Hilbert spaces H_1, H_2, building the „one-component" decomposition $H_1 = H_2 \otimes H_2^{\perp}$. This decomposition joins the modelling stage, when the „two-components" fields of the plasma field interaction model $H_1 = H^{(-)} \otimes H^{(+)}$ collapses to a „one-component" field governed by a classical kinematical operator with domain $H_1 = H_2$. The constant c defining the corresponding hyperboloid is an obvious candidate for the fine structure constant, (***), (UnA1) pp. 200, 203, 212, 222, 280, 289.

The collapsing process may be considered as a model for the beta decay. It goes together with a collapse of the space-time structure into a temporal orientation („arrow of time") and the spatial handedness of the Euclian space (****). Physically speaking this is about the observation of stationary states over time, e.g. related to thermostatistics. Instead of asking „*what will the state of the system be at time t*?" one could ask, „*what is the probability that at a time t the state of the system will belong to a specified subset of phase space*?" and „*what will (probably) happen to the system as t tends to infinity*?" Those kind of questions are addressed by ergodic theory, (HaP), (HoE).

(*) (CaF) 1.1:plasma particle field interactions is about kinematical electromagnetic interactions between positively and negatively charged particles, whereby the number of the positively and negatively charged kinematical particles per considered volume element may be arbitrarily small oder arbitrarily large, but both numbers need to be approximately identical. Thereby the number of neutral particles (atomes or molecules) is irrelevant for kinematical two-type plasma particle field interactions.

(**) This is achieved in the following way: the set of the orthogonal basis elements of H_1 is first decomposed into odd and even basis elements, and, secondly, the even elements are replaced by the Riesz transforms of the odd basis elements. We note that the inner product of a basis element and its Riesz transform vanishes. We further note that the Snirelman density of of both sets is $1/2$.

(***) (UnA) pp. 94, 96: The definition of the fine structure constant implies that $1/\alpha \approx 137$ is the ratio of speed of light to the electron's velocity on the innermost orbit of the hydrogen atom.

(****) We also note the nonlinear gravitational stability of the geometric Minkowski space in classical gravity field theory, (ChD).

The MHD-based integrated plasma field model

Nearly all of the matter in the universe consists of "plasma". The proposed MHD-based integrated plasma field model deals with a two-type kinematical quantum element concept (*).

MHD is concerned with the motion of electrically conducting fluids in the presence of electric or magnetic fields. In MHD one does not consider velocity distributions. It is about notions like number density, flow velocity and pressure.

The proposed unified field theory is built on an extended MHD model anticipating the physical modelling requirement of „plasma particle field interaction", where two mathematical types of elementary/quantum particles/elements are considered.

The MHD equations are derived from continuum theory of non-polar fluids with three kinds of balance laws, (**):

1. conservation of mass
2. balance of angular momentum (Ampere law and Faraday law, Maxwell equations)
3. balance of linear momentum.

The additional conceptual ingredients of an enhanced MHD-based model for the integrated mathematical-physical plasma, gravity and quantum field model are...

1. ... there are two moving electric and magnetic (kinematical) quantum elements of H_0 in the presence of electric and magnetic fields
2. two types of plasma particles with complementary charges
3. ... their corresponding „wave" energies are element of $H_1 = H_{(-)} \otimes H_{(+)}$ (**).

The related variational PDE are governed by the two connected components $L_{(\pm)}$ of the complex Lorentz group, which plays the key role proving the CPT (Charge-Parity-Time) theorem, (StR) p. 13. As PT in quantum field theory turns particles into anti-particles we quote the following tentative conclusions as provided in (ArF) pp. 639, 646:

> *„Whether a particle has positive or negative charge is determined by the temporal direction in which the four-momentum of particle points. … The CPT theorem should be called the PT-theorem. … the fact that it holds for quantum field theories suggests that space-time has neither a temporal orientation than nor a spatial handedness".*

The mathematical plasma type classification, (***), is supported by two underlying fundamental physical theories, (CaF) p. 20,

1. Magnetohydrodynamics (MHD)
2. Hydrodynamics
   a. with (single type) particle collisions (e.g. Vlasov equation)
   b. w/o (single type) particle collisions (e.g. Fokker-Planck equation).

(*) Similar to the notion „elementary particle", there is no unique mathematical-physical definition of the notion „plasma particle". The key differentiator between plasma to neutral gas or neutral fluid is the fact that its electrically positively and negatively charged kinematical particles are strongly influenced by electric and magnetic fields, while neutral gas is not. Conceptually, „plasma particles" need to fulfill the following two pre-requisites, (CaF) p. 1:

(1) there must be electromagnetic interactions between charged particles
(2) the number of positively and negatively charged particles per considered volume element may be arbitrarily small oder arbitrarily large, but both numbers need to be approximately identical. The number of neutral particles (atomes or molecules) is irrelevant for the definition of a plasma.

(**) Regarding the balance laws 2. and 3. (angular and linear momentum) we quote from H. A. Lorentz, (1) and A. Einstein, (2), (EiA): (1): „It is only essential, that next to the observable objects there is another to be viewed as a real but not imperceptible object to accept the acceleration resp. the rotation as something real", (2): „light speed is caused by the movements of bodies through the ether".

(***) in the mathematical modelling world there are different plasma phenomena considered, where the related physical experimental situations anticipate appropriately defined critical parameters of the correspondingly defined PDE (e.g. Vlasov equation or Landau equation). Then, those model input parameters result in different kinds of plasma waves leading to notions like cold, warm, hot and ideal plasma or Vlasov- and MHD-plasma. Regarding nonlinear wave motions in the theory governing interaction of magnetic fields with conducting compressible fluids in (FrK1) it is shown that the basic equations governing the magneto-hydrodynamics have essentially the same mathematical character as those governing gas dynamics and that, consequently, essentially the same mathematical methods that have proved successful in gas dynamics can be employed. In (FrK), (FrK1) it is pointed out that it is possible to build up a theory of shock waves and simple waves to magnetodynamics flows parallel to that of conventional gas dynamics. Based on (FrK) in (GuR) a method is presented for discussing weakly non-isentropic quaisi-one-dimensional flows of an ideal, inviscid, perfectly conducting compressible fluid subjected to a transverse magnetic field.

The „two-component“ quantum element concept leads to extended variational representations of Maxwell-Mie equations and correspondingly adapted (SRT extended) Einstein field equations.

With respect to an appropriate modelling of Mie's "electric pressure" we note that the related pressure p of the NSE can be expressed in terms of the velocity u by the formula

$$p = \sum_{j,k=1}^{3} R_j R_k (u_j u_k),$$

where $\boldsymbol{R} \coloneqq (R_1, R_2, R_3)$ is the Riesz transform and $\boldsymbol{u} \otimes \boldsymbol{u} = (u_j u_k)$ is a $3x3$ matrix. It enables a representation of the sum of the non-linear NSE term and the negativ pressure in the form $\boldsymbol{P}\nabla \cdot (\boldsymbol{u} \otimes \boldsymbol{u})$, where $\boldsymbol{P}$ denotes the Helmholtz-Weyl projection operator and $\nabla \cdot$ represents the column vector with each component being the divergence of the row vectors of the matrix $\boldsymbol{u} \otimes \boldsymbol{u}$, e.g. (CuS). We further note that the Riesz transform $\boldsymbol{R}$ commutes with translations, dilations, rotations, and anticommutes with reflections. In terms of the corresponding modelling in a Hilbert scale framework we refer to (SoH) p. 153 ff. For the vorticity-stream formulation of the 3D-NSE flow we refer to (MaA).

In the context of correspondingly adapted Maxwell-Mie-Ehrenhaft equations and Einstein's field equations we also refer to (TaM), where it is found that an electromagnetic field can be coupled to a gravitoelectric and gravitomagnetic field.

The „two-component“ quantum element model puts the spot on:

(a) *Mie's* physical modelling concept of an *electric pressure*, (WeH):

"*G. Mie in 1912 pointed out a way of modifying the Maxwell equations in such a manner that they might possibly solve the problem of matter, by explaining why the field possesses a granular structure and why the knots of energy remain intact in spite of the back-and-forth flux of energy and momentum*."

(b) *Ehrenhaft's* experimentally validated concept of *photophoresis*, (EhF), (BrJ) p. 31:

"*I have discovered that in a ray of light physically and chemically similar matter particles move in direction of the propagation of light and in the opposite direction. In case the light direction is reversed, then those movements are reversed. I call this phenomenon photophoresis*".

(c) *Leedskalnin's* view of the *magnetic current* phenomenon, (LeE):

„*Magnetic current is not one current, they are two currents, one current is composed of North Pole individual magnets in concentrated streams and the other is composed of South Pole individual magnets in concentrated streams, and they are running one stream against the other stream in whirling, screwlike fashion, and with high speed*“.

The proposed three A-C world layer models

In the context of writing an equation for life R. Feynman stated (1964), (FrU) p. 1:

> „*Often, people in some unjustified fear of physics say you can't write an equation for life. Well, perhaps we can. As a matter of fact, we very possibly already have the equation to a sufficient approximation when we write the equation of quantum mechanics".*

From U. Frisch (FrU) we further quote:

> *„Of course, if we only had this equation, without detailed observation of biological phenomena, we would be unable to reconstruct them."*

The three kinds of balance laws of the MHD equations are re-interpreted as governing laws for the following a three layer world model A-C:

A. The overall mathematical A-world, governed by the principle of $H_{1/2}$ energy minimization modelling the overall „conservation of energy law" of the plasma, quantum and gravity field model
B. The mathematical-physical B-world, governed by two „angular, (B2-world) and linear, (B3-world) momentum balance laws" modelled by underlying kinematical mathematical-physical models and accompanied by parameters/variables like space-time, space and time variables, volume, area, force and pressure
C. The chemical-biological C-world accompanied by parameters like duration, distance, the concepts of "moving form/shape".

The mathematical-physical B-world provides the modelling framework for the above „reconstruction of specific physical observation" situations modelled in corresponding physical theories.

The B2-world is is governed by a (new) principle of (angular momentum based) „order potential of energy", concerned with ectropia and the physical notions of space-time and „time points". The B3-world is governed by the principle of action minimation and concerned with the physical concepts of entropy and Planck's related quantum action constant.

The mathematical-physical concepts of „potential functions" and „potential operators" are related to the physical concept of „potential" and the concept of „potential energy", measured by the norm of the corresponding potential operator of this PDE model. The new notion in C-world is a "moving form/shape", which can be interpreted as an approximation to the finer granulated B-world.

The several conceptual flavors of the B-world's notion „time" put the spot on the

(A) mathematical „action differential" variable, (Heisenberg)
(B) mathematical-physical „space-time" and „time" variable
(C) biological-chemical „time duration" and „space distance" concepts.

The distinction between the mathematical-physical world and the chemical-biological world puts the spot on

(a) Schrödinger's concepts of a "consciousness differential" and a "biological potential of living cells", (ScE1)

(b) Heidegger's distinction between mathematical-physical science and humanistic sciences, (HeM)

(c) Bergson's distinction between "time" and "duration", where the physical-mathematical time is not measurable by any objective standard, and his concept of an „élan vital".

According to H. Bergson the physical notion of "time" is accompanied with attribues like quantitative, mechanical, geometric, measurable, while „time of life" is essentially qualitative („living time"), (BeH); life is always and on each level development, renovation, change. Plants, animals and people are different kinds of manifestations of an „élan vital" (life swing), (BeH2). It does not overcome dead matter in the sense of a gradual advancement, but fits to each possible development opportunities.

We note that the (true) mathematical-statistical formulation of Bergson's claim is the following:

> the probability to pick a rational physical-mathematical "point in time" from the real "time arrow" axis is zero, as the set of rational numbers is a zero (sub-) set of the set of real numbers with respect to the Lebesgue integral concept.

Layer A-C model cases

Einstein's field equations

The "geometry" of space-time of the GRT in modelled as a pair (M, g), where M is a $(3+1)$-dimensional manifold and g is an Einstein metric on M, that is, a non-degenerated, 2-covariant tensor field with the property that at each point one can choose $(3+1)$ vectors e_0, e_1, e_2, e_3, such that $g(e_\alpha e_\beta) = E(\alpha,\beta)$; $\alpha, \beta = 0,1,2,3$, where $E(\alpha,\beta)$ is the diagonal matrix with entries $-1,1,1,1$. In other words, Einstein's "geometry" of space-time (M, g) has no intrinsic geometric structure. There are only the following eight 3-dimensional geometries: either the constant curvature spaces E^3, H^3, S^3, the products $H^2 \times R$, $S^2 \times R$, or the twistered products $\widetilde{SL}(2,R)$, Nil, or Sol.

With respect to the Sobolev space based Hilbert space scales we note that the Sobolev embedding theorems for R^n and for compact manifolds are basically identical, (HeE).

Einstein's equivalence principle is the following: "a pure local (infinitesimal) gravity force is equivalent to acceleration of the considered elementrary mass particle in the corresponding local Euclidian reference system".

In the Einstein field equations the Riemann tensor is decomposed into "Riemann = Ricci + Weyl". The Ricci tensor includes all data about energy, pressure, the momentum of matter particles, and the electromagnetic field. Specifically, the "pressure" energy part of the Ricci tensor is related to the second tensor of the Einstein field equation, the Weyl tensor.

We note that the energy-momentum $T_{i,j}$ in Einstein's theory is analogous to the charge-current vector $\vec{J_\iota}$ of Maxwell theory. The quantity of $T_{i,j}$ may be regarded as describing the source of gravitation, in the same way as $\vec{J_\iota}$, is the source of electromagnetism, (PeR) p. 464.

In the classical B-world the Ricci tensor describes the inwards directed acceleration of all mass particles inside a considered mass ball with a given 3D volume, (PeR1). In other words, in the B-world the Weyl (or conformal) tensor is the appropriate analogue of the Maxwell field tensor $F_{i,j}$ describing the gravitational degrees of freedom, in case of a not pure locally valid equivalence principle, (PeR1).

We note that the notion "not purely locally" is additionally required in the GRT (the crucial extension of the SRT) to model the equivalence principle; an only infinitesimal existing Minkowski-Lorentz geometry is not sufficient to model "acceleration". This means that the Weyl tensor is not relevant for layer A, but it is required to link the quantum tensor calculus with the classical tensor claculus reflecting the "tidal effect" phenomenon, (PeR1).

Hydrodynamics

Regarding the nine-teenth century view of the world in terms of „elementary particles of a fluid“ in the context of the physical layer B vs. the term „shape of a fluid“ and any change to it in the context of the biological/chemical layer C we recall from (AcJ) p. 43:

„In his paper Stokes (1845) obtains his equations by first examining the kinetic conditions governing the relative fluid motion about a point, from which he deduces that „… the most general instantaneous motion of an elementary portion of a fluid is compounded of a motion of translation, a motion of rotation, a motion of uniform dilatation, and two motions of shifting …,“ (these last motions later be identified as the rates of strain). In this respect Stokes here confirms the general conclusions stated by Cauchy (1843) to the effect that any change of shape of a material element can be decomposed into strains and rotation.“

The B2 related "angular" energy norm term of the variational representation of the non-linear, non-stationary 3D Navier-Stokes equations vanishes in the standard variational L_2 framework, (TeR) (2.34). Physically speaking, there exists no „order potential“ avoiding "time bomb" solutions in the standard variational L_2 framework case, while the (variational) 3D NSE in the $H_{-1/2}$ framework the corresponding term is bounded.

Ehrenhaft's photophoresis

We further refer to the obervations of F. Ehrenhaft, (reported in (AlO) S.223 with reference to (EhF1), (EhF2)), which might enable an explanation of the fine structure constant as the ratio of the linear momentum and the angular momentum. In other words, the fine structure constant might be show up as borderline marker between the physical B world and the chemical/biological C world.

Spiral phenomena, "big bang", and other related model cases

Regarding the several observable "spiral" phenomena in the Nature "C-world" including spiral nebula in the universe we note that a spriral movement is the overlay of a linear and a rotational movement. This indicates to revisit

(A) Gödel's type of cosmological solutions of Einstein's field equations
(B) Prandtl's rigid airfoil theory
(C) Schauberger's view of the world
(D) Heidegger's view of the world
(E) Khun Dee's view of the world.

(A) (GöK): "All cosmological solutions with non-vanishing density of matter ...have the common property, that… they contain an absolute time coordinate, owing to the fact that there exists a one-parametric system of three-spaces everywhere orthogonal on the world lines of matter. It is easily seen that the non-existence of such a system is equivalent with a rotation of matter relative to the compass of inertial. The four dimensional space S, which this solution defines, has further properties:

(a) S is homogeneous
(b) any two world lines in S are equivalent
(c) S has rotational symmetry
(d) a positive direction of time can consistently be introduced in the whole solution"

(B) The today's B-world aerodynamics model is based on Prandtl's rigid airfoil theory:

a. on the one hand side this theory simplifies the full Navier-Stokes equations by basically excluding the critical viscous flow ouside a critical layer in the context of the Reynolds number
b. on the other hand side the Prandtl model focus on the pressure (resp. the uplift force) throughout the boundary layer in the direction of the normal to the airfoil surface.

The underlying mathematical concepts of the Prandtl model are "single and double layer integrals", „circulation“ and „normal derivative“. The mathematical definition of „circulation“ is based on the Stokes theorem, where the circulation around a closed curve is the total sum of all enclosed vortex forces. In case of the Prandtl model this is about enclosed vortex forces within an airfoil, which is without any relevance for the B and C world. Fortunately the underlying mathematical baseline concepts are the 2D-Cauchy-Riemann differential equations (i.e. the correponding physical velocity fields are solutions of the potential equation) and the physical relevant fact, that the circulation in a non-viscous, barotropic fluid is constant, (HaW).

The essential concept integrating double layer integrals into a Hilbert space framework is about the Riesz transforms, which commutes with translations, dilations, rotations, and anticommutes with reflections.

In (RuC) appropriately modified 3D-Cauchy-Riemann differential equations are considered, which additional allow vortex streams. The direction of the related vortex lines is rectified to the direction of the velocity or opposite to it. For a related mathematical theory linking back to harmonic analysis we refer to (StE), (StE1), (StE2).

An appropriate interpretation of the Stokes theorem based proof of an "airfoil layer caused uplift force" is enabled by Plemelj's mathematical concepts of a mass element (replacing a mass density of a double or single layer integral) and a related purely surface domain based definition of a flow on/through a surface (replacing a normal derivative):

Physically speaking, the Plemelj flow on a surface is the proposed B world model of the Prandtl uplift force "flow vector" on a airfoil layer.

In (HaG) a 2D viscous and incompressible fluid dynamics PDE model is provided. Its solutions are satisfied by not physically relevant potential flow based movements (i.e. all solutions of the potential equation), as viscous fluids adhere to solid walls. One known other exact physical relevant solutionis the Poiseuille laminar movement, but this solution is disregarding the quadratic terms of the PDE. Exact solutions taking into account the quadratic terms of the PDE are movements in spiral streamlines. Regarding the considered two-plasma-particles model we note that in fluid mechanics when two liquid flows flow together the most natural phenomenon at the separation surfaces is the formation of vortices.

(C) Transforming Schauberger's ideas concerning Nature's basic principles (e.g. explaining vortex movements, energy storage and energy conversion) into a mathematical model lead to the concept of a hyperbolic cone, (RaC) S. 24.

In the context of the above A-C worlds we note the amazing similarity of the underlying C2 based action principles of Schauberger's "trout turbine", (AlO) S. 135, with the Tesla turbine, (FeF).

Schauberger's explanation how the "trout turbine" works is built on the appropriately construction morphology of the gills (the „layer") of a trout, drawing energy from the streaming water by giving the water a strong curling motion.

The C-world related counterpart of Schauberger's trout in aerodynamics is about insect and bird flights accompanied with a related instationary uplift force creation enabled by three kinds of construction morphologies: fast flyers, swiling flyers, and permanent flyers, (NaW). For the aerodynamics of insects and birds there are basically no related B-world models existing, (DuR). This may join in the most comprehensive definition of the quality of investigations in natural sciences (e.g. chemistry or biology), which is a "moving form"; it means a certain form actively notifies its surroundings, while a mathematical model can only investigate "frozen forms", (RaC) S. 10.

(D) From a philosophical view on the above hierarchical mathematical-physical-biological structure we quote from M. Heidegger, (HeM):

(72) *„Modern physics is called mathematical because, in a remarkable way, it makes use of a quite specific mathematics. But it can proceed mathematically in this way only because, in a deeper sense, it is already itself mathematical."*

(73) „The rigor of mathematical physical science is exactitude. Here all events, if they are to enter at all into representation as events of nature, must be defined beforehand as spatiotemporal magnitudes of motion. Such defining is accomplished through measuring, with the help of number and calculation. But mathematical research into nature is not exact because it calculates with precision; rather it must calculate in this way because its adherence to its object-sphere has the character of exactitude. The humanistic sciences, in contrast, indeed all the sciences concerned with life, must necessarily be inexact just in order to remain rigorous. A living thing can indeed also be grasped as spatiotemporal magnitude of motion, but then it is no longer apprehended as living. The inexactitude of the historical humanistic sciences is not a deficiency, but is only the fulfillment of a demand essential to this type of research. It is true, also, that the projecting and securing of the object-sphere of the historical sciences is not only of another kind, but is much more difficult of execution than is the achieving of rigor in the exact sciences."

(E) Khun Dee's view of the world is about an implosion theory, alternatively to the big bang explosion theory, (StB). The implosion concept also plays a key role in (C), Schauberger's view of the C world.

Appendix

Extracts

from

(StR) p. 9 ff., (UnA) p. 152 ff., (CoR) p. 763

with comments from the author *in italic*

The real Lorentz transform

(StR): A Lorentz transformation is a linear transformation Λ mapping space-time onto space-time which preserves the scalar product $(\Lambda\vec{x},\Lambda\vec{y}) = (\vec{x},\vec{y})$, where

$$(\vec{x},\vec{y}) := (x^0,y^0) - [(x^1,y^1) + (x^2,y^2) + (x^3,y^3)] = x^\mu g_{\mu\nu} x^\nu = x^\mu y_\mu .$$

If $(\Lambda x)^\mu = \Lambda^\mu{}_\nu x^\nu$, the (real) matrix $\Lambda^\mu{}_\nu$ of the transformation must satisfy

$$\Lambda^\kappa{}_\mu \Lambda_{\kappa\nu} = g_{\mu\nu} \text{ or } \Lambda^T G \Lambda = G, \qquad (1\text{-}5)$$

where the transpose Λ^T of Λ is defined by $(\Lambda^T)^\mu{}_\nu = \Lambda^\mu{}_\nu$ and indices on Λ are lowered according to

$$\Lambda_{\kappa\nu} = g_{\kappa\sigma}\Lambda^\sigma{}_\nu = (G\Lambda)_{\kappa\nu}.$$

If Λ and M satisfy (1-5), so do $\Lambda\mathrm{M}$ and Λ^{-1}. Here

$$(\Lambda\mathrm{M})^\mu{}_\nu = \Lambda^\mu{}_\nu \, \mathrm{M}^\kappa{}_\nu \qquad (\Lambda^{-1})^\mu{}_\nu \Lambda^\kappa{}_\nu = \mathrm{g}^\mu{}_\nu = \begin{cases} 0 & \mu\neq\nu \\ 1 & \mu=\nu \end{cases},$$

so the (real) Lorentz transformations form a group, the Lorentz group L.

Two Lorentz transformations Λ and M are defined to be close to one another if the numbers $\Lambda^\mu{}_\nu$ and $\mathrm{M}^\mu{}_\nu$ are close for all $\mu,\nu = 0,1,2,3$. Clearly, with this definition, Λ^{-1} and $\Lambda\mathrm{M}$ are continuous functions of Λ and M, respectively. Furthermore, it make sense to say that two (real) Lorentz transformations can be connected to one another by a continuous curve of Lorentz transformations.

The Lorentz group L has four components, each of which is connected in the sense that any point can be connected to any other, but no Lorentz transformation in one component can be connected to another in another component.

One of this components is the restricted Lorentz group, which is the group of $2x2$ complex matrices of determinant one, $SL(2,C)$. It is isomophic to the symmetry group $SU(2)$, containing as elements the complex-valued rotations, which can be written as a complex-valued matrix of type

$$\begin{pmatrix} a+ib & c+id \\ -c+id & a-ib \end{pmatrix} \text{ with determinant one.}$$

It is important in describing the transformation properties of spinors. In SMEP the group $SU(2)$ describes the weak force interaction with 3 bosons W^+, W^-, Z.

This restricted Lorentz group contains the 1-transformation. Its connected component contains the space-time inversion. The other pair of connected components of the Lorentz group contains the space inversion resp. the time inversion.

The Lorentz group has also three important subgroups, which are

- the orthochronous Lorentz group
- the proper Lorentz group
- the orthochorous Lorentz group.

The complex Lorentz transform

Another group associated with the Lorentz group L is the complex Lorentz group, which we shall denote by $L(C)$. It is essential in the proof of the PCT theorem as we shall see. It is composed of all complex matrices satisfying

$$\Lambda^{\kappa}{}_{\mu}\Lambda_{\kappa\nu} = g_{\mu\nu} \text{ or } \Lambda^{T}G\Lambda = G, \qquad (1\text{-}5).$$

It has just two connected components, $L_{+}(C)$ and $L_{-}(C)$ according to the sign of det(Λ). The transformations 1 and -1, which are disconnected in L are connected in $L(C)$. In other words, the complex Lorentz transformation connects

- the two components containing the 1-transformation and space-time inversion, i.e. the pair

$$\{\det(\Lambda) = +1\,, \det\,(\Lambda^{0}{}_{0} = +1)\}, \{\det(\Lambda) = +1\,, \det\,(\Lambda^{0}{}_{0} = -1)\},$$

- the two components containing the space inversion and the time inversion, i.e. the pair

$$\{\det(\Lambda) = -1\,, \det\,(\Lambda^{0}{}_{0} = +1)\}, \{\det(\Lambda) = -1\,, \det\,(\Lambda^{0}{}_{0} = -1)\}.$$

Summary:

While two (real) Lorentz transformations need to be connected to one another by an appropriately defined continuous curve of Lorentz transformations, there are two pairs of components of the complex Lorentz transform, which are both already connected by definition.

Just as the restricted Lorentz group is associated with $SL(2,C)$, the complex Lorentz group is associated with $SL(2,C) \otimes SL(2,C)$. The latter group is the set of all pairs of $2x2$ matrices of determinants one with the multiplication law

$$\{A_1, B_1\} \cdot \{A_2, B_2\} = \{A_1A_2, B_1B_2\}.$$

Is is easy to see that only matrix pairs which yield a given $\Lambda(\mathrm{A}, \mathrm{B})$ are $(\pm\mathrm{A}, \pm\mathrm{B})$. In particular,

$$\Lambda(-1,1) = \Lambda(1,-1) = -1.$$

The corresponding complex Poincare group admits complex translation but also the multiplication law

$$\{a_1, \Lambda_1\} \cdot \{a_2, \Lambda_2\} = \{a_1 + \Lambda_1 a_2, \Lambda_1\Lambda_2\}.$$

It has two components $\mathrm{P}_{\pm}(C)$, which are distinguished by $\det(\Lambda)$ and a corresponding inhomogeneous group to $SL(2,C)$.

Quaternions

(UnA): The multiplication rule for quarternions reads as follows:

$(a_1, b_1, c_1, d_1) \cdot (a_2, b_2, c_2, d_2) = (a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2, a_1b_2 + b_1a_2 + c_1d_2 + d_1c_2, a_1c_2 - b_1d_2 + c_1a_2 + d_1b_2, a_1d_2 + b_1c_2 - c_1b_2 + d_1a_2)$.

Quaternions constitute a skew field, that means $a \cdot b = -b \cdot a$. Unit quaternions are equivalent to the 3D unit sphere S^3.

The quaternions product can also be written using the scalar and cross products known from vector analysis. If unit quarternion (a, b, c, d) is decomposed into a real part a and a vector $\vec{u} = (b, c, d)$. Then we get:

$$(a_1, \vec{u}_1) \cdot (a_2, \vec{u}_2) = (a_2a_2 - \vec{u}_1 \cdot \vec{u}_2, a_1\vec{u}_2 + a_2\vec{u}_1 - \vec{u}_1 x \vec{u}_2).$$

In vector analysis, both the scalar product (also called „dot product“) and the vector product (also called „cross product“) are of significant importance and are widely used in most areas of modern physics. One may combine these products with the symbol of a spatial derivative ∇, and generate the differential operators divergence and curl, indicating the source and the vorticity of a vector field, respectively. Maxwell's equations represent the most prominent example of how fundamental laws can be formulated in an elegant vectorial form. … Just to illustrate the close relation between quarternion algebra and vector analysis, consider the quaternionic multiplication of a spatiotemporal derivative vector with electromagnetic potential:

$$\left(\frac{\partial}{\partial t}, \vec{\nabla}\right) \times \left(\varphi, \vec{A}\right) = \frac{\partial \varphi}{\partial t} - \vec{\nabla} \cdot \vec{A} \frac{\partial \vec{A}}{\partial t} + \vec{\nabla}\varphi + \vec{\nabla} \times \vec{A} \ .$$

The last two terms precisely match the known expressions for the electric and magnetic fields $\vec{E}$ and $\vec{B}$.

Comment:

The two connected components of the complex Lorentz transformations provide the link to the proposed two type plasma spinor quantum field model with the related symmetry group $SL(2, C) \otimes SL(2, C)$. We note that the space-time concept shows up on stage by linking the inner product of quaternions and the scalar product defining the Lorentz transformation property.

We further note, that the one type spinor field in current quantum field theory becomes already a first approximation to this finest degree of physical accuracy. In this context we also note, that from a mathematical perspective the Sobolev Hilbert spaces are the appropriate framework for elliptic PDE, e.g. providing „optimal“ shift theorems, while the appropriate framework for hyperbolic PDE is still an open question. A proof of a related „space-time“ problem, the Courant hypothesis, is still missing (CoR) p. 763:

Families of spherical waves for arbitrary time-like lines Λ exits only in the case of two and four variables, and then only if the differential equation is equivalent to the wave equation.

Extracts

from

(VaM) chapter IV

Let B be a self-djoint operator defined on all of the Hilbert space H. Then this operator induces a decomposition of H into a direct sum of two sub-spaces $H = H^1 \otimes H^2$. Both sub-spaces are no Hilbert spaces. However, the orthogonal projection operators P^1 and P^2 enable the definition of the indefinite metric

$$\varphi(x) \coloneqq \big((x)\big)^2 := \|P^1x\|^2 - \|P^2x\|^2$$

Thus, putting $x_1 \coloneqq P^1x$, $x_2 \coloneqq P^2x$ the operator B generates a hyperboloid and a related ellipsoid

i) Hyperboloid: $\varphi(x_1 + x_2) = \|x_1\|^2 - \|x_2\|^2 = c > 0$

ii) Ellipsoid: $\frac{\|x_1\|^2}{a_1^2} + \frac{\|x_2\|^2}{a_2^2} = 1$; elliptical region: $E_c \coloneqq \left\{x \in H \,|\, \frac{\|x_1\|^2}{a_1^2} + \frac{\|x_2\|^2}{a_2^2} \le c, c > 0\right\}$.

The indefinite metric $\varphi(x)$ can be interpreted as a „potential" accompanied with the gradient of the potential $\varphi(x)$ defined by

$$grad\varphi(x) = \mathrm{grad}\big((x)\big)^2 = 2P^1x - 2P^2x.$$

The corresponding potential operator is then given by

$$\boldsymbol{W}(x) := \frac{1}{2} grad\big((x)\big)^2 = P^1x - P^2x.$$

The fundamental properties of the potential operator $\boldsymbol{W}(x)$ are completeness, invertibility, ($\boldsymbol{W} = \boldsymbol{W}^{-1}$) isometry, and symmetry. Thus, the bilinear form $(\mathrm{x},\mathrm{y})_{\mathrm{W}} \coloneqq (\mathbf{W}(\mathrm{x}),\mathrm{y})$ defines an inner product, (BoJ) p. 52.

From physical modelling perspective we note that the model enables the definition of a PDE specific potential criterion (e.g. the famous coupling constants) accompanied with related hyperbolic and conical region V_c and V_0, whose points satisfy the corresponding potential barrier conditions. Evidently V_c is a subspace of V_0.

We remark that if x is an exterior point of the conical region V_0, i.e.

$$\sqrt{\|P^1x\|^2 - \|P^2x\|^2} = \alpha > 0,$$

then those points of the ray $tx, t \in [0, \infty)$ for which $t \ge c/a$ belong to the hyperbolic region V_c, and those for which $0 \le t < c/a$ do not belong to V_c. If x is not an element of V_0, then the ray $tx, t \in [0, \infty)$ does not have any point in common with V_c. Thus, every interior ray of the conical region V_0 intersects the hyperbolid $\big((x)\big) = c > 0$ in a single point. We denote by K the boundary of the conical region V_0. The manifold K is defined by the condition $\big((x)\big) = 0$. If we look at the unit sphere S^1 ($\|x\|^2 = 1$), then those points of S^1 for which $\|P^1x\| = \|P^2x\|$ belong to K, and those points of S^1 for which $\|P^1x\| > \|P^2x\|$ intersect the hyperboloid $\big((x)\big) = c > 0$ at the point whose distance from θ is given by

$$t = \mathrm{c}(\|P^1x\|^2 - \|P^2x\|^2)^{-1/2}.$$

From this it is seen that $t \to \infty$ if $\|P^1x\|^2 - \|P^2x\|^2 \to 0$, i.e. the manifold K is an asymptotic conical manifold for the hyperboloid $\big((x)\big) = c > 0$.

Further Extracts

from

(AnM), (CaM), (ChH), (HaR), (PeG), (PeR1), (ScP), (ThP)

for the Einstein-Hilbert functional and Hamilton's Ricci flow equation we refer to (MüR)
for further references we refer to (BeR), (ChJ)

Three dimensional geometries

Regarding the geometrization of the three manifolds the baseline result is the following, (ScP), (ThP):

„there are only the following eight 3-dimensional geometries: either the constant (Euclidian, hyperbolic, spherical curvature spaces E^3, H^3, S^3 with the curvatures $0, -1, +1$, the products $H^2 \times R$, $S^2 \times R$, or the twistered products $\widetilde{SL}(2, R)$, Nil, or Sol. Of those eight 3-dimensional geometries, seven are very well understood. In particular, one can classify all the 3-manifolds which possess a geometric structure of one of these seven types. For six of those geometries all the closed manifolds are Seifert fibre spaces".

In other words, *the most difficult geometry is that of hyperbolic space H^3.*

The constant curvature spaces E^3, H^3, S^3 are simply connected spaces.

The geometric structure of a simply connected space X is a homogenous space structure on X, that is, a transitive action of a Lie group G on X. Thus, X is given by G/H, for H a closed subgroup of G. In order to avoid redundancy, it is assumed that the identity component H_0 of the stabilizer H is a compact subgroup of G, and that G is maximal. Further, G is assumed to be unimodular; this is equivalent to the existence of compact quotients of X, (AnM).

„*The geometry S^3 arises from three distinct pairs (G, H) where G is a Lie group, H is a compact subgroup and G/H is a simply connected 3-manifold, namely (S^3, e), $(U(2), SO(2))$ and $(S(4), S(3))$*", (ScP).

The groups S^3 are $PSL(2, C)$, $R^3 \times SO(3)$, $SO(4)$. In all cases, $H_0 = SO(3)$, (AnM).

„*The three-sphere S^3 has the structure of a group: the group of unit quaternions. Therefore $S^3 \times S^3$ acts on S^3 by the formula $x \to gxh^{-1}$. The kernel of this action is $Z/2$, (where Z denotes the positive and negative integers) generated by $(-1,1)$ in quaternionic notation, and this produces an incredible isomorphism $S^3 x S^3 \; /Z/2 = SO(4)$*", (ThP1) p. 368.

Thurston's geometrization conjecture and the Ricci flow

"*If the Riemannian manifold M/G is a quotient of a Riemannian manifold M by a group of isometries G at the start, it will remain so under the Ricci Flow. This is because the Ricci Flow on M preserves the isometry group*", (HaR).

In his 2002 paper G. Perelman sketched his proof of Thurston's geometrization conjecture. The central tool is the theory of Ricci flows regarded as gradient flows, (PeG).

„*The Ricci flow has also been discussed in quantum field theory, as an approximation to the renormalization group flow for the two-dimensional non-linear sigma-model*", (GaK) §3.

In this context Perelman's speculation is on the Wilsonian piciture of the renormalization group flow, (PeG).

In the Einstein field equations the Riemann tensor is decomposed into, (PeR1)

"RIEMANN = RICCI + WEYL".

The Ricci tensor includes all data about energy, pressure, the momentum of matter particles, and the electromagnetic field. The Weyl (or conformal) tensor is the appropriate analogue of the Maxwell field tensor $F_{i,j}$ describing the gravitational degrees of freedom, in case of a not pure locally valid equivalence principle (see below). This "pressure" energy part of the Ricci tensor is related to the Weyl tensor. In this context we also recall the "electric pressure" concept of Mie's theory.

The (time-dependent) Ricci flow is an intrinsic geometric flow. It can be looked at a process which "deforms" the metric of a Riemannian manifold in a positive way, that it smooths out irregularities in this metric.

If the Riemannian manifold M/G is a quotient of a Riemannian manifold M by a group of isometries G at the start, it will remain so under the Ricci Flow. This is because the Ricci Flow on M preserves the isometry group, (HaR).

Homogeneous metrics

Since the Ricci Flow is invariant under the full diffeomorphism group, any isometry in the initial metric will persist as isometries in each subsequent metric. A metric is homogeneous when the isometry group is preserved; hence if we start with a homogeneous metric the metric will stay homogeneous. For a given isometry group there is only a finite dimensional space of homogeneous metrics, and the Ricci Flow can be written for these metrics as a system of a finite number of ordinary differential equations. In three dimensions there are eight distinct homogeneous geometries; in (CaM) the Ricci Flow has been worked out on each.

One example of the typical phenomena that occur is the Berger spheres, which are homogeneous metrics on S^2 which respect the Hopf fibration over S^2 with fibre S^1. Under the Ricci Flow the metrics on S^2 and on S^1 shrink to points in a finite time, but in such a way that the ratio of their radii goes to 1.

A symmetric bilinear form is called 2-positive if the sum of its two smallest eigenvalues is positive. In (ChH) it has observed that two-positive curvature operator is also preserved by the Ricci Flow.

From (CaM) we recall the:

> *Abstract*: Hamilton's Ricci flow convergence theorems generally deal with metrics whose Ricci curvature is positive semidefinite. Here, we exhibit a non-trivial class of three-dimensional Riemannian metrics with Ricci curvature of indefinite sign for which the Ricci flow converge.

(PeG): On the other hand, Hamilton (HaR1) discovered a remarkable property of solutions with nonnegative curvature operator in arbitrary dimension, called a differential Harnack inequality, which allows, in particular, to compare the curvatures of the solution at different points and different times.

From (PeG) 13.1, we recall the:

„Let $g_{ij}(t)$ *be a smooth solution to the Ricci flow on* $M \times [1, \infty[$*, where* M *is a closed oriented three-manifold.*

… for sufficiently large times t *the manifold* M *admits a thick-thin decomposition.*

… either M_{thick} *is empty for large* t*, or, for an appropriate sequence of* $t \to \infty$ *and* $w \to \infty$*, it converges to a (possibly, disconnected) complete hyperbolic manifold of finite volume, whose cusps (if there are any) are incompressible in* M*.*

On the other hand, collapsing with lower curvature bound in dimension three is understood well enough to claim that, for sufficiently small $w > 0$*,* M_{thin} *is homeomorphic to a graph manifold.*

The natural questions that remain open are whether the normalized curvatures must stay bounded as $t \to \infty$*, and whether reducible manifolds and manifolds with finite fundamental group can have metrics which evolve smoothly by the Ricci flow on the infinite time interval.*

Now suppose that $g_{ij}(t)$ *is defined on* $M \times [1, \infty[$*,* $T < \infty$*, and goes singular as* $t \to T$*. Then using 12.1 we see that, as* $t \to T$*, either the curvature goes to infinity everywhere, and then* M *is a quotient of either* S^3 *or* $S^2 \times R$*, or the region of high curvature in* $g_{ij}(t)$ *is the union of several necks and capped necks, which in the limit turn into horns (the horns most likely have finite diameter, but at the moment I don't have a proof of that). Then at the time* T *we can replace the tips of the horns by smooth caps and continue running the Ricci flow until the solution goes singular for the next time, e.t.c. It turns out that those tips can be chosen in such a way that the need for the surgery will arise only finite number of times on every finite time interval.*

Another differential-geometric approach to the geometrization concecture is being developed by Anderson, (AnM); he studies the elliptic equations, arising as Euler-Lagrange equations for certain functionals of the riemannian metric, perturbing the total scalar curvature functional, and one can observe certain parallelism between his work and that of Hamilton, especially taking into account that, as we have shown in 1.1, Ricci flow is the gradient flow for a functional, that closely resembles the total scalar curvature."

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