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Leibniz on Force and Absolute Motion*

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I elaborate and defend an interpretation of Leibniz on which he is committed to a stronger space-time structure than so-called Leibnizian space-time, with absolute speeds grounded in his concept of force rather than in substantival space and time. I argue that this interpretation is well-motivated by Leibniz's mature writings, that it renders his views on space, time, motion, and force consistent with his metaphysics, and that it makes better sense of his replies to Clarke than does the standard interpretation. Further, it illuminates the way in which Leibniz took his physics to be grounded in his metaphysics.

1. Introduction. Leibniz's views on space, time, and motion appear to be fraught with difficulties. His rejection of absolute space and time and his doctrine of the equivalence of hypotheses seem to commit him to the view that the only well-defined quantities of motion are relative ones. In other words, he seems to be committed to the view that the spatiotemporal structure of our world is that of what is now called *Leibnizian space-time.* But Leibnizian space-time seems to be inadequate for the purposes of physics, even the kind of physics Leibniz was doing (since his laws of motion can be consistently applied only on the understanding that they hold only in inertial frames of reference, that is, frames that are absolutely non-accelerating) (Earman 1989, 71–73).

Furthermore, in many texts Leibniz insists that there is a distinction between true motion and merely relative motion, and that this distinction can be drawn in terms of *force:* a body is truly moving when it is moving relatively to other bodies and it possesses the force that causes this relative motion. This move seems problematic for two reasons. First, it appears

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to be inconsistent with Leibniz's relationalism. This charge has been made by Clarke in his fifth letter to Leibniz (Alexander 1956, 105) and by Russell (1900, 86), Alexander (1956, xxvii), and Earman (1989, 131). Second, in order to apply the notion of Leibnizian force to the empirical world, one needs an empirical measure of it, which according to Leibniz is given by the quantity mv^2. The problem is that mv^2 itself is relative to a choice of reference frame, so it seems that any attempt to ground a distinction between absolute and relative motion upon it is circular: Before we can use mv^2 to distinguish absolute from relative motion, we must already know which reference frame to use in measuring mv^2. But that means we must know which reference frame to use in measuring a body's velocity, so we must already be able to distinguish absolute from relative motion (Earman 1989, 131).

I will argue that these difficulties are merely apparent, for there is a well-supported interpretation of Leibniz's claims about space, time, motion and force on which they are internally consistent and free of circularity. This interpretation also has the advantage of casting light on the way in which Leibniz's concept of force serves to provide a metaphysical foundation for his physics.

I will be concerned primarily with Leibniz's writings during the period between 1686 and 1695; that is, between the *Discourse on Metaphysics* and the "Specimen Dynamicum." This is the period in which Leibniz did most of his mature technical work in physics (Garber 1985, 1995). But I will also draw on the Leibniz-Clarke correspondence of 1715–1716. By the time of that correspondence, Leibniz's metaphysical views had undergone considerable development. In particular, between 1686 and 1695 he defended a metaphysics of individual substances, including corporeal substances, but in the later period this system gave way to one in which the only true created substances are incorporeal, soul-like monads (see Garber 1985 and Rutherford 1995). But these changes in Leibniz's view do not appear to be relevant to the correspondence with Clarke, since there he does not go into the metaphysics of substances in any detail, and is content to speak of physical bodies as if they were real existents. Further, I doubt that these changes in Leibniz's view are relevant to those of his views during the earlier period that I will discuss here.

2. What Leibniz's Relationalism Is and Is Not. On the standard interpretation, Leibniz's relationalism commits him to the view that the only well-defined quantities of motion are the relative ones, or in other words, that the spatiotemporal structure of the world is that of what is today called *Leibnizian space-time.* I will argue that this is mistaken. In fact, from 1686 on, Leibniz consistently maintained a family of views that imply that bodies have absolute speeds, though not absolute directions of velocity or absolute accelerations. Thus, Leibniz's own favored space-time structure is stronger than what is now called Leibnizian space-time. It is weaker than Full Newtonian space-time (in which absolute accelerations exist, as well as absolute speeds), but is neither strictly stronger nor strictly weaker than Neo-Newtonian space-time (in which absolute acceleration exists but absolute speed does not). As far as I know, no name has yet been coined for this space-time structure. If my interpretation is correct, then it ought to be called "Leibnizian space-time." But that name already has an established (if inappropriate) usage, which I will respect here. (See Earman 1989, Chapter 2 for discussion of the space-times mentioned above.)

If Leibniz is committed to absolute speed, then what becomes of his relationalism? What is his dispute with the Newtonians about absolute space and time all about? The answer is that this dispute is primarily concerned not with the question of space-time structure, i.e. the question of which quantities of motion are well-defined, but rather with the question of the ontology that supports space-time structure. In contemporary terms, Newton was a substantivalist about space and time, maintaining that space, time, and their parts exist and maintain their own identities independently of physical bodies and events. Thus, in the Scholium on space and time, Newton writes:

> Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable . . . Place is the part of space that a body occupies . . . Absolute motion is the change of position of a body from one place to another . . . Just as the order of the parts of time is unchangeable, so, too, is the order of the parts of space. Let the parts of space move from their places, and they will move (so to speak) from themselves. For times and spaces are, as it were, the places of themselves and of all things. (Netwon 1999, 409–410)

This view implies the existence of certain absolute quantities of motion, and hence it implies a certain space-time structure. But it is not equivalent to the view that space-time has a certain structure. The existence of Full Newtonian space-time is not sufficient to guarantee the existence of space and time as described by Newton.

As the quoted passage shows, for Newton the parts of space and time are individuated independently of their relations to material objects. Hence, it makes sense to ask what the absolute position of a body is at a given time, i.e. which part of absolute space that body occupies. Full Newtonian space-time does not afford any well-defined quantity that represents the answer to this question, for the symmetries of Full Newtonian space-time include rigid translations and rotations, which map positions onto other positions. Of course, in Full Newtonian space-time, it does make sense to ask whether one body at one time is at the same position as a second body (or the same body) at some later time (Friedman 1983, 77). That is, identity of positions over time is well-defined. But the identity of positions across physically possible situations is not well-defined—and it should be, if the parts of space and time have their identities independently of material objects and events. The Newtonian conception of absolute space implies that there is a genuine, non-actualized possibility in which things are in different places, but with the same spatial relations to one another as in the actual world (as Clarke and Leibniz both recognize in their dispute). But such a possible situation is related to the actual situation by a symmetry of Full Newtonian space-time. Thus the structure of Full Newtonian space-time does not suffice to draw a distinction among possibilities that Newton's conception of absolute space recognizes. Similarly, Newton's conception of absolute time implies a distinction among possibilities that are related by a time-translation (e.g., there is a possible world in which the universe was created ten minutes later than it actually was), another symmetry of Full Newtonian space-time. So Full Newtonian space-time is weaker than Newton's conception of absolute space and time, in that the latter implies the existence of a broader range of physical possibilities than is implied by the former. This means it is possible consistently to endorse Full Newtonian space-time, and yet deny the reality of space and time as Newton conceived of them. So Leibniz's dispute with the Newtonians is not simply a dispute about space-time structure.

To clarify Leibniz's view of space and time, we must examine both his view of the ontology of space-time relations and his view of the structure of those relations. On the ontological question, it is clear that he rejected substantivalism, but it is less clear what positive view he put in its place. One common interpretation attributes to him a two-level view of the world: first, there is the metaphysical level of simple substances and their modes, which alone exist in the fullest sense; second, there is the phenomenal level of things that do not truly exist in their own right, but are *well-founded* on the metaphysical level. The well-founding of the phenomena amounts, in Rescher's words, to their being "strictly *derivative*" (Rescher 1967, 81) of what goes on at the metaphysical level, in that all details of what exists and what goes on at the phenomenal level are determined by what exists at the monadic level. On this interpretation of Leibniz, space and time are well-founded phenomena. (Rescher 1967, Chapter 8). Over the past three decades, a number of authors (including McGuire (1976), Winterbourne (1982), and Hartz and Cover (1988)) have defended a competing interpretation, according to which Leibniz has not two but three levels in his ontology: the metaphysical, the phenomenal, and the ideal. The ideal level consists of things that are neither real substances nor supervenient on substances and their characteristics, but rather are fictions of the mind; they "form a conceptual grid that the mind imposes on phenomenal change" (McGuire 1976, 308). On this interpretation, bodies and their actions upon one another are on the phenomenal level, but space and time are on the ideal level. As will emerge below, my view on this question is intermediate between these interpretations. Before I can address this question, though, I need to examine the question of Leibniz's view of the geometric structure of space-time.

Many texts encourage the view that Leibniz believed that our world has the structure of (so-called) Leibnizian spacetime. A typical example is the opening passage of the 1689 paper "On Copernicanism and the Relativity of Motion":

> Since we have already proved through geometrical demonstrations the equivalence of all hypotheses with respect to the motions of any bodies whatsoever, however numerous, moved only by collisions with other bodies, it follows that not even an angel could determine with mathematical rigor which of the many bodies of that sort is at rest, and which is the center of motion for the others. And whether the others are moving freely or colliding with one another, it is a wondrous law of nature that no eye, wherever in matter it might be placed, has a sure criterion for telling from the phenomena where there is motion, how much motion there is, and of what sort it is, *or even whether God moves everything around it, or whether he moves that very eye itself.* (Ariew and Garber 1989, 91)

Although similar comments can be found in other writings, this and similar texts are misleading when considered in isolation. One's view on the structure of space-time is determined by one's view concerning which quantities of motion are well-defined. Leibniz's views on this matter are subtle. From at least 1686 until his death, Leibniz consistently distinguishes two very different senses of motion. Motion in the strictest sense is not merely a change of location with respect to other bodies, but rather such a change in conjunction with its cause. In the "Critical Thoughts on Descartes" of 1692, he writes:

> . . . in order to say that something is moving, we will require not only that it change its position with respect to other things but also that there be within itself a cause of change, a force, an action. (Loemker 1989, 393)

And in a letter to Arnauld of October 9, 1687, he writes:

> . . . I say that a corporeal substance gives itself its motion, or rather what is real in motion at each moment, namely, the derivative force from which motion follows. (Loemker 1989, 341)

It is also possible to define motion in purely geometric and kinematic terms, as mere change of spatial relations. Motion in this sense is what Leibniz thinks the equivalence of hypotheses covers, as he explains in a letter to Arnauld of July 14, 1686:

> [W]e are right in [attributing motion to a boat rather than the whole sea], even though abstractly speaking we could maintain another description of their motion, since *motion, abstracted from its cause,* is always a relative thing. (Woolhouse and Francks 1998, 112–113; italics added)

Motion in the strict sense, however, is not subject to this relativity, as Leibniz explains in a letter to Arnauld of Januray 14, 1688:

> [M]ovement in itself separated from force is something merely relative and its subject cannot be determined; force, however, being something real and absolute, and its calculations, as I clearly show, different from that of motion, we must not be surprised if nature preserves the same quantity of force but not the same quantity of motion. (Woolhouse and Francks 1998, 135)

In this passage, Leibniz alludes to his disagreement with the Cartesians over the conservation laws governing motion, but what is important for our purposes is that he clearly states that motion is relative only if it is considered in abstraction from force, whereas force is "real and absolute"; and that what Leibniz is referring to here by "force" is clearly the quantity mv^2 that he claims is conserved in all interactions.

In the passages quoted above, true motion is distinguished from merely relative motion by the presence of "force" (though at one point Leibniz uses "derivative force") which is "what is real in motion." "Force" is a term Leibniz uses in many different senses, which don't get fully sorted out in his published writings until the 1695 "Specimen Dynamicum". Derivative active force is there defined as a magnitude admitting of degrees, which results from the "limitations" of primitive active force, which Leibniz identifies with entelechy. Derivative active force is the feature of a substance that is causally responsible for what manifests itself in the phenomenal world as changes in the spatial relations of bodies. Derivative active force, "joined with actual motion" (Ariew and Garber 1989, 121), is called living force (or vis viva) by Leibniz. Thus, "living force" is a name for a conjunction of two things, existing at different levels in Leibniz's ontology: (i) a motion of bodies with respect to one another, which appears at the phenomenal level, and (ii) a derivative active force, which exists at the level of substances and their properties and provides the "well-founding" of (i). These two things are not wholly distinct, since one serves as the metaphysical underpinning of the other, but it still makes a difference for Leibniz whether we consider them separately or together.

Living force can be given an empirical, quantitative measure in terms of the degree of the effect that can be brought about by a corporeal substance possessing it. Famously, Leibniz argues that this measure is given by mv^2. Hence, mv^2 is a measure of that which is real in motion, that feature of a corporeal substance that is causally responsible for motion. It seems clear that it is this that Leibniz refers to by the names "force" and "derivative force" in the passages quoted above, though they were written before Leibniz standardized his use of terminology for the many varieties of "force".

So Leibniz holds that the features of the phenomenal world that are determined by the underlying real world of simple substances include both the changing spatial relations among bodies, and the degree of living force, i.e. vis viva, present in each body, which can be measured by the quantity mv^2. This determines an absolute speed for each body (but not an absolute direction of velocity, since v^2 depends only on the magnitude of the vector v). It does not follow that there is an absolute magnitude of acceleration, since rotational acceleration need not produce changes in speed (as Nick Huggett pointed out to me). Hence, Leibniz is committed to a space-time structure stronger than that of (so-called) Leibnizian space-time. Moreover, this space-time structure is fully determined by the individual substances and their attributes, so it is a well-founded phenomenon. Still, this doesn't underwrite a notion of absolute space in Newton's sense, since it provides no way of identifying places independently of spatial relations among actual bodies. It might underwrite a notion of same-place-at-different-times (see above), since any body whose absolute speed is zero during a time interval will occupy the same place at each moment in the interval. But, as argued above, this is not sufficient to underwrite absolute space in Newton's sense.

The idea I am attributing to Leibniz is similar to the proposal of Sklar (1974, 230), that absolute acceleration be taken as a primitive, monadic property. Like Sklar's proposal, this one funds absolute quantities of motion without defining them in terms of an independently existing spatiotemporal "background". But Leibniz's view differs from Sklar's proposal in that Leibniz takes absolute speed, rather magnitude of acceleration to be monadic, and he reduces absolute speed to more basic monadic properties, rather than taking it as a primitive.

What, then, are we to make of the opening passage of "On Copernicanism", which I quoted above? There, as we have seen, Leibniz argues that all frames of reference are equivalent for the purpose of describing the motions of bodies, and that we are justified in using the reference frame that makes the phenomena out to be most intelligible. But very early in this paper, Leibniz makes it explicit that in this context, by "motion" he means nothing more than "change of place" (Ariew and Garber 1989, 91). As we have seen, Leibniz makes it clear, in other writings spanning the time of publication of "On Copernicanism", that this is not the only way of regarding motion. In the essay on Copernicanism, Leibniz is working with a relatively thin conception of motion by his own standards, and what he says there shouldn't be taken to trump what he says elsewhere about motion in the fullest sense.

This interpretation of Leibniz's views on space, force, and motion is confirmed by looking at the Leibniz-Clarke correspondence. Section 47 of Leibniz's fifth letter contains what is perhaps the best-known statement of Leibniz's relationalism. In this passage, the notion of space is derived from the notion of place, space being "that which comprehends all those places," and the notion of a place is derived from considering the changing relations among bodies.

> And to give a kind of definition, *place* is that which we say is the same to A and to B, when the relation of the coexistence of B with C, E, F, G, etc., agrees perfectly with the relation of coexistence which A had with the same C, E, F, G, etc., supposing there has been no cause of change in C, E, F, G, etc. It may also be said, without entering into any further particularity, that place is that which is the same in different moments to different existent things when their relations of coexistence with certain other existents which are supposed to continue fixed from one of those moments to the other agree entirely together. And *fixed existents* are those in which there has been no cause of any change of the order of their coexistence with others, or (which is the same thing) in which there has been no motion. (Ariew and Garber 1989, 338)

If Leibniz were committed only to the structure of Leibnizian spacetime, then he ought to allow that we can legitimately take *any* body to be fixed.[1] But this is not what he does; he explicitly states that those bodies are fixed in which there is no cause of relative motion. For Leibniz, which bodies have velocity zero is not up to us; it is determined by the causal relations. It follows from this that we are given no choice in assigning speeds to any bodies, for the speeds of all bodies are mathematically determined from the relative speeds and positions of bodies, together with a specification of the bodies that are at rest.[2] Hence, even while stating his doctrine of

1. Wilson (1989, 218) appears to interpret the quoted passage as doing just this.

2. Of course, this holds only when the set of bodies at rest is non-empty. Leibniz's considered view is that there are no bodies that are truly at rest; for instance, in a letter to Arnauld of October 9, 1687, he writes: "It is true that a body which has no motion

the relational character of space, Leibniz commits himself to the existence of absolute speed.

Now I can return to the question of the ontological status of space and time for Leibniz. There is, for Leibniz, a space-time structure stronger than so-called Leibnizian space-time that is a well-founded phenomenon. But it doesn't follow that Leibniz is committed to the existence of Newtonian space and time. Newtonian space is an arrangement of positions that not only maintain their identities over time, but also possess identities in a stronger sense that allow for their identification across possible worlds. No structure like this supervenes on the level of individual substances (or even on the level of phenomenal bodies) for Leibniz, so no such structure could be a well-founded phenomenon for him. Similar remarks apply with respect to Newtonian time. Leibniz, of course, lacked contemporary space-time vocabulary, so for him "space" and "time", used as names for existing things, could only have meant something like what Newton meant by them. This makes sense of Leibniz's claim that space and time are merely "ideal" or "fictional", for "space" and "time" in Newton's usage denote things that do not supervene on the level of individual substances. The claim that a certain space-time structure really exists was not available for Leibniz to have made, although we can see now that this is just the way to express his view in contemporary vocabulary. Space and time as Newton conceived them are not well-founded phenomena for Leibniz; but a spacetime structure equipped with absolute speed is. The former are ideal, the latter are phenomenal.

3. Force and its Problems. The space-time structure to which Leibniz is committed is grounded in his concept of force. This concept is, in an important sense, the lynchpin of Leibniz's views, the link between his metaphysics and his physics (Gale 1988; Garber 1985, 1995). Two complications having to do with force must be addressed at once. The first is that Leibniz takes force to be a property of simple substances, whereas the living forces that are used to distinguish true from relative motion are properties of phenomenal bodies, which are not simple substances for Leibniz. This complication raises the issue of how bodies are grounded in the real world of simple substances, an issue that has been well-worked over in the literature and cannot be given a proper treatment here (for discussion, see e.g. Rutherford 1995, Gale 1988, and Garber 1985). Suffice

cannot give impart motion to itself, but I hold that there is no such body" (Loemker 1989, 341). This is one of the many metaphysical points that Leibniz lets slide for the purposes of the correspondence with Clarke, but on which he is elsewhere strict. But the attribution of absolute values of mv^2 to all bodies does suffice to determine absolute speeds, even if no bodies are truly at rest.

it to say that phenomenal, physical bodies are either constituted by collections of substances that are unified by a single dominant substance, or else aggregates of such collections, and the living force of a physical body is derivative of the force(s) of the dominant substance(s) in the unified collection (aggregate of collections) that constitute it.

The second complication comes from the fact that for Leibniz, nothing but God causally interacts with anything else, strictly speaking. Yet, the living force of a physical body is supposed to consist in a capacity of that body to cause relative motions. How can these two ideas be reconciled? The answer is that Leibniz admits a kind of "second-class" causal relation in which created substances can stand; following Sleigh (1990), I will call this relation *quasi-causality.* For Leibniz, created substances never directly interact with one another, but they are governed by a pre-established harmony that gives the impression of causal interaction. This is what underwrites what truth there is in vulgar notions of causality. More specifically, when two substances both undergo changes in which they are accommodated to one another by the pre-established harmony, it sometimes happens that one of them comes to perceive (or express) the other more distinctly, while the second comes to perceive the first less distinctly. When this happens, the first substance may be said to be active and the second passive. Thus, in section 15 of the *Discourse on Metaphysics,* Leibniz writes:

> The action of one finite substance on another consists only in the increase of degree of its expression together with the diminution of the expression of the other, insofar as God requires them to accommodate themselves to each other. (Ariew and Garber 1989, 48)

Just how Leibniz conceives of perception and degrees of its distinctness is a question beyond the scope of this paper. Living force, then, can be thought of as a measure of the degree to which (the substances that constitute) a given body is quasi-causally active in the changes that it and other bodies undergo which appear as relative motion.

Leibniz's appeal to force in order to ground the concept of absolute motion gives rise to several difficulties. First, mv^2 is an empirical quantity, an arithmetical combination of features of extended matter and its modes, yet Leibniz claims that it is not phenomenal, but metaphysical and real.[3] What is worse, not only is mv^2 relative to a choice of reference frame, but its value isn't uniquely determined even if we specify that it is to be mea-

3. Gale (1988, 63) argues that by defining force as he does, Leibniz guarantees that it is an absolute quantity, because it cannot take a negative value. But this doesn't solve the problem; although mv^2 never takes a negative value in any frame of reference, it does take different non-negative values in different frames of reference.

sured in an *inertial* frame. If vis viva is a real property (rather than a quantity relative to the arbitrary choice of a reference frame), then it must be that the real property is not simply mv^2 (which is relative), but rather *mv^2 as measured in some particular reference frame,* so that the empirical measurement of vis viva depends on an empirical determination of a privileged frame. But how can we empirically determine a privileged frame, prior to determining the absolute motions?

So Leibniz faces a dilemma: Vis viva is either reference frame-dependent, or it isn't. If it is, then it couldn't be a real metaphysical property of a substance. If it isn't, then its measure must be mv^2 relative to some particular coordinate frame, but since a privileged coordinate frame cannot be empirically determined, vis viva cannot be empirically measured. For this reason, it is difficult to see how it could play any role at all in the science of dynamics.

4. Force and mv^2. The first horn of the dilemma is untenable: Force could not simultaneously be a real property of a substance and have a value that depends on an arbitrary decision made on our part concerning which reference frame to use. It might be thought that this relativity is a harmless one, like the relativity of a quantity to a choice of unit, but this isn't so. Leibniz is quite clear that a substance has a vis viva of zero if and only if it has no power to cause motion, and whether or not a substance has this power is certainly not dependent on our choice of coordinates. If we want to find a reasonable view to attribute to Leibniz, then we must find a way to sit comfortably on the second horn. The questions we must face are these: How can mv^2 (as measured in some particular reference frame), which is manifestly a kinematical quantity and hence a denizen of the phenomenal realm, be a measure of something metaphysically real? How can a quantity that is not empirically measurable play a fundamental role in the science of dynamics? Finally, isn't Leibniz guilty of circularity when he defines absolute motion in terms of force, and specifies that this force is measured by mv^2 in some particular reference frame? For his procedure seems to require that before it can be settled which bodies are truly in motion, it must first be settled what the absolute mv^2 of each body is, but this would already require that the absolute state of motion of each body (its motion as measured in the preferred reference frame) is settled.

In order to address the first of these questions, we need to examine the argument Leibniz gives for using mv^2 as a measure of living force. This argument is presented, for example, in the "Brief Demonstration of a Notable Error of Descartes", (Loemker 1989, 296–301) and the "Specimen Dynamicum" of 1695 (Ariew and Garber 1989, 118–138). Leibniz's strategy is to measure the quantity of a force by means of the *violent effect* that it can produce—that is, by the magnitude of an effect it would produce in the course of expending itself. Leibniz examines a case in which violent effects can be easily decomposed into homogeneous parts, so that the magnitudes of these effects may be numerically compared with one another. The violent effect of raising a mass m through a vertical distance d seems to admit of exactly this kind of decomposition, since the effect of raising a mass of 2m through a distance d is evidently twice the effect of raising a mass of m through a distance d, as is the effect of raising a mass of m through a distance of 2d. At least, this is true if the weight of a body is independent of its height, which Leibniz admits is only true in an idealized case, though the error that results from this idealization is negligible for practical purposes. Hence, a force is to be measured by the vertical distance through which it could raise a given mass while expending itself. One way of determining this distance is by means of a simple pendulum. The force of the bob when at its lowest position is the force that will expend itself while producing the effect of raising the bob through a vertical distance until it reaches its highest point. Appealing to Galileo's treatment of free fall and pendula, Leibniz argues that for a given mass, this distance is proportional to mv^2, where m and v are the mass and velocity of the body possessing the force.[4]

Leibniz thus appeals to an empirical phenomenon, namely that for a body which, in a given inertial reference frame *F,* moves as simple pendulum (that is, is suspended by string from a fixed point and moves with a constant downward acceleration), the vertical distance through which the body moves is proportional to mv^2, where v is the body's velocity at its lowest point (all these quantities are to be measured using the frame *F*). Since that phenomenon holds true in *any* reference frame in which Galileo's law of free fall holds, it cannot by itself indicate the importance of mv^2 as measured in any *particular* reference frame (or any particular class of frames that all agree on the value of mv^2).

But Leibniz doesn't rest his case solely on this appeal to the observable phenomena; he makes appeal to metaphysical premises as well. This illustrates a view that Leibniz consistently maintains from 1686 on, namely that dynamics cannot be completed as a science without recourse to metaphysics. It is possible for the science of mechanics to state the laws of motion in purely empirical and mathematical terms, and thereby to arrive

4. Hence, Leibniz's argument for mv^2 as a measure of force is crucially dependent on the law of free fall. John Bernoulli notes this and criticizes Leibniz's argument on the grounds that the proof of such a fundamental result shouldn't rest on the "accidental" fact of Galileo's law. Westfall (1972, 301) gives a illuminating discussion of this issue, in which he argues that the dependence on Galileo's law reflects Leibniz's failure to formulate a general work-energy principle, which in turn is explained by Leibniz's resistance to integrating force (that is, Newtonian force, or Leibnizian "dead force") over space rather than over time.

at mechanical explanations of the phenomena of nature without recourse to metaphysical notions such as substantial forms; this is the point on which Leibniz agrees with the mechanical philosophy. Where Leibniz disagrees with the mechanical philosophy is in his view that in order to provide complete explanations of natural phenomena, we must have not only a mechanics encompassing the empirical laws of motion but also a dynamics which explains the laws of motion themselves, and this is possible only if we have recourse to metaphysical notions and metaphysical truths (see especially section 18 of the *Discourse on Metaphysics*). In the case at hand, Leibniz appeals to the metaphysical notion of living force, a (quasi-) causal property of substances that admits of degrees, and to the metaphysical premise that living force is always conserved in natural phenomena. The living force causing Leibniz's pendulum bob to move expends itself while producing an effect proportional to mv^2; in order to conclude from this that the living force possessed by the bob is proportional to mv^2, Leibniz needs to make one more crucial assumption: that the motion that we attribute to the bob is caused (or rather, quasi-caused) only by the bob itself, by virtue of the bob's own living force. This assumption requires that the reference frame we use to describe the motion of the bob only attributes motion to the bob that is caused by the bob itself. In other words, none of the motion we attribute to the bob can be an artifact of our choice of a moving reference frame. Leibniz doesn't state this assumption explicitly, but it is quite clear from his discussion that he is presupposing it. Further, Leibniz must assume that the frame of reference that he uses to describe the motion of the pendulum is one in which Galileo's law of free fall holds.

So, I propose the following reconstruction of Leibniz's argument. The following five statements are used as premises:

1. For any given natural phenomenon, there exists a reference frame such that when this frame is used to describe the motions involved in the phenomenon, each body is described as undergoing all and only those motions that are caused by itself, through its own living force.
2. In any reference frame satisfying 1, Galileo's law of free fall holds.
3. In any reference frame satisfying 1, the violent effect that a given living force can produce is a measure of the magnitude of that force.
4. The living force of bodies is conserved.
5. The laws of motion, when stated in terms of phenomenal quantities, are invariant under suitable coordinate transformations.

Premises 1 and 2 are non-trivial assumptions that Leibniz doesn't explicitly state. Nonetheless, it seems safe to attribute them both to Leibniz. His description of the pendulum in the "Specimen Dynamicum" would hardly make sense if 1 were false, and this description would be false if 2 were false. Premises 3 and 4 are metaphysical assumptions to which Leibniz is explicitly committed.

The situation with premise 5 is more complicated. In the "Specimen Dynamicum", Leibniz relies on a premise he calls the Principle of Continuity, which is stated as follows:

> *[I]f one case continually approaches another case among the givens, and finally vanishes into it, then it is necessary that the outcomes of the cases continually approach one another in that which is sought and finally merge with one another.* (Ariew and Garber 1989, 133)

As Leibniz interprets it, this principle implies that the mathematical form of the laws of motion should be the same no matter which of a group of uniformly moving bodies we choose to treat as being at rest. This is because we can continuously vary our hypothesis concerning the velocities of the bodies, keeping their relative velocities constant, until we have passed from a hypothesis in which one is at rest to one in which another, moving uniformly relative to the first, is at rest. "That which is sought" is just the mathematical form of the laws of motion, so these must be the same for the two cases. This implies what we now call Galilean invariance (or Galilean relativity).

However, it isn't obvious from the discussion in the "Critical Thoughts" just how clear Leibniz is about the importance of Galilean relativity. Given Leibniz's style of reasoning in this work, it might well be the case that he would hold that even if the bodies under consideration were moving non-uniformly, it would still make no difference to the form of the laws whether we treated one body or another as being at rest. After all, the fact that all of the bodies Leibniz considers are moving uniformly is a feature of the particular examples that he (following Descartes) considers; it is not built into the Principle of Continuity as Leibniz states it. This would imply an invariance principle much stronger than Galilean invariance, one which is inconsistent with Newtonian mechanics. Unfortunately, this makes Leibniz's views inconsistent, for the law of conservation of mv^2 cannot be preserved under arbitrary coordinate transformations. Suppose that in a two-particle system, mv^2 is conserved in a frame of reference *F;* now shift to a frame of reference *F*' accelerating uniformly with respect to *F;* relative to *F*', the velocities of both particles will soon begin increasing uniformly, violating the conservation law. For this reason, I have formulated premise 5 in terms of "suitable" coordinate transformations: If you read "suitable" as "Galilean", then you get a premise that Leibniz was certainly committed to—though one that is perhaps weaker than his preferred relativity principle—and the argument goes through; if you read "suitable" as "any", then you get something perhaps closer to what Leibniz had in mind, and the argument still goes through, but unfortunately the premises are inconsistent! I suggest that even if Leibniz was inconsistent in this way, it is a local inconsistency; we can correct it without doing much damage elsewhere, and it is worthwhile to examine Leibniz's system thus corrected.

Leibniz's demonstration shows that from 1, 2, and 3, it follows that there is a frame satisfying 1, and that in any such frame, living force is proportional to mv^2. This is what Leibniz states as the conclusion of his argument. But this conclusion is not applicable to any empirical phenomena, for we have no empirical method of determining whether a given frame of reference satisfies 1. Note that if the fixed end of the apparatus is attached to a vertical ruler, and this ruler and the fixed end are both moving inertially, then the height to which the bob rises as measured on this ruler is proportional to mu^2, where u is the velocity of the bob relative to the fixed end—but this could all be so even if the fixed end, and the bob, are moving with absolute speeds much greater than $|u|$, so that the true living force of the bob is much greater than mu^2. Hence, this kind of apparatus cannot be used to measure true living force. However, two more conclusions follow immediately. From the first conclusion and 4, it follows that in any frame satisfying 1, mv^2 is conserved. From this together with 5, it follows that in any frame in which Galileo's free fall law holds, the conservation of mv^2 holds as well. Since the law of inertia also holds in such frames, this amounts to a proof of the conservation of mv^2 in inertial (or "suitable") frames. This is an empirically useful conclusion.

The argument establishes that mv^2 is a conserved quantity in any inertial frame, but it leaves open the possibility that there is no way of empirically determining the true vis viva of any body or of establishing a reference frame that attributes to each body its own absolute speed. Importantly, it also allows vis viva to play a fundamental role in the science of dynamics, even though it is not Galilean invariant whereas the laws of motion are. Recall that for Leibniz, the laws of motion are to be stated as mathematical relations among phenomenal properties, whereas dynamics will be a complete science only to the extent that it explains these phenomenal laws by means of metaphysical principles. Given this, it couldn't fail that a quantity, like mv^2, which occurs in the mathematical expression of a law of dynamics, is a phenomenal, kinematic quantity whose value is relative to a choice of reference frame. The metaphysical notion of vis viva is crucial to the science of dynamics because of the role it plays in theoretically establishing and explaining the law of conservation of mv^2, even though the *real* value of vis viva cannot be empirically distinguished from mv^2-relative-to-some-frame. There is some frame of reference in which mv^2 is proportional to vis viva, and therefore it is conserved in that frame; we needn't ever be able to determine empirically which reference frame this is; but, given Galilean invariance, if mv^2 is conserved in that frame, then it is conserved in all inertial frames. In this way, the metaphysical notion of force is employed in the explanatory derivation of an empirical law of motion, even though force, strictly speaking, cannot be empirically measured. This way of putting the argument clears up a problem that Leibniz doesn't explicitly address; Leibniz typically speaks simply of "the law of conservation of *vis viva,*" whereas if the above reconstruction is right, then the law that actually gets applied in the study of particular phenomena is the law of conservation of mv^2, which is a *consequence* of the conservation of vis viva. Nonetheless, the reconstruction just offered is not inconsistent with what Leibniz says (modulo the above remarks about the Principle of Continuity), and it does reconcile many of Leibniz's commitments in a consistent manner, something that it is otherwise difficult to do. The moral is that Leibniz's stated position is defensible, but not necessarily that Leibniz was in possession of the argument in its defense.

Of the problems described at the beginning of this section, this leaves only that of Leibniz's apparent circularity. Again, Leibniz defines true motion in terms of force, which seems to be identified with mv^2—but mv^2 is not well-defined unless a privileged reference frame has been settled on, which would already fix the true motions of bodies. We are now in a position to see that this circularity is merely apparent. True motion is indeed defined in terms of living force. However, living force is not defined in terms of mv^2, but rather in terms of a body's ability to cause (or quasi-cause) changes in relative position. The connection between living force and mv^2 is a contingent empirical one, rather than a definitional one; the argument Leibniz uses to establish this connection depends on an empirical premise, Galileo's law of free fall.

Of course, this connection holds only if mv^2 is measured in one of a class of privileged reference frames (all members of which agree on the speeds of all bodies). How is it determined which reference frames are privileged? Well, the privileged frames are just the ones in which mv^2 is proportional to degree of living force. Why is Leibniz entitled to believe that there are such frames? Leibniz's argument, as reconstructed above, assumes that there exist reference frames in which the motions attributed to bodies are exactly the motions those bodies are quasi-causally responsible for; it demonstrates that in those reference frames, living force is proportional to mv^2. Again, these privileged reference frames cannot be identified by empirical means, but their existence nonetheless has empirically useful consequences.

What is the ontological status of these privileged reference frames? The frames themselves are ideal objects, mathematical abstractions we invent for the purpose of describing motions in the phenomenal world. But, given the phenomenal world and given the set of all reference frames, the question of which frames are the privileged ones is settled by the quasi-causal powers of bodies. For these determine the degrees of living force, so given the world of phenomenal motions and the set of reference frames that can be used for describing these motions, it is settled which frames distribute mv^2 in a way proportional to degree of living force. The quasi-causal powers of bodies, in turn, supervene on the real properties of the simple substances. So, though reference frames are ideal objects, there is a sense in which the privilege enjoyed by the privileged ones is grounded in what is metaphysically real.

The structure of Leibniz's account is thus not circular at all. At the foundation, we have the simple substances and their modes. Supervening upon these are phenomenal bodies, their quasi-causal powers, their spatial relations, and their relative motions. Living force is constituted by relative motions together with quasi-causal powers, and in turn determines what the true motions are. This is a hierarchical structure, in which each level determines what is at the level above it. The way in which the bottom level serves to determine all the things above it insures that there are reference frames in which mv^2 is proportional to living force. So mv^2 and living force do not circularly depend on one another; their correlation is due to the fact that both of them depend on something more fundamental.

5. Is Leibniz Committed to Denying Absolute Speed? This still leaves us with a final worry. I've argued that Leibniz's doctrine of force commits him to the doctrine of absolute speed, that absolute speed doesn't entail absolute space, and that Leibniz implicitly recognizes his commitment to the existence of absolute speed even while stating his relationalist theory of space. Still, Leibniz took himself to have very forceful reasons for denying absolute space, and these reasons for rejecting absolute space appear also to be powerful reasons for rejecting all absolute kinematical properties, including absolute speed. So, as far as I've shown thus far, there still seems to be a deep tension between Leibniz's views on force and motion on the one hand, and his views on the relational character of space on the other. My final task in this paper is to show that this tension is merely apparent.

Leibniz's principal argument against absolute position is that if position were absolute, then God would have had to make a choice between the actual arrangement of bodies and the arrangements that would result from switching east and west, moving everything to the left five meters, etc., and this would be an impossible choice for God to make since he would have no sufficient reason for choosing one such way of arranging the world rather than another. It might be thought that a similar argument can be given in the case of speed: 'If speed were absolute, then God would have had a choice between creating the universe as it is, or setting the whole universe in uniform motion in some direction, a choice which he would have found impossible to make.' In fact, in section 13 of his fourth letter to Clarke, Leibniz himself gives such an argument (Ariew and Garber 1989, 328–329; Alexander 1956, 38).

I reply that Leibniz should not have used this argument. It is worth noting that he only gives the argument as a direct response to remarks along similar lines made by Clarke in section 4 of his third letter (Alexander 1956, 32), and that the argument as originally presented by Leibniz, in section 5 of his third letter, is strictly and explicitly an argument against absolute position (Ariew and Garber 1989, 325; Alexander 1956, 26). Hence, it is possible to interpret the argument in question as an ad hominem polemical move against Clarke. And after all, we know that Leibniz allows some points of metaphysics to slide in his correspondence with Clarke, though he is strict about them elsewhere; for example, he is willing to speak of bodies absolutely at rest in this correspondence, though elsewhere he denies that there could be such bodies (Loemker 1989, 341).

But it might be wondered whether Leibniz is *entitled* to ignore this argument. For this argument seems to be only a minor variant of Leibniz's crucial argument against absolute space, and it seems to depend on nothing more than Leibniz's own principles of sufficient reason and identity of indiscernibles. Consistency seems to demand that Leibniz grant the argument against absolute speed, which contradicts the views I have attributed to Leibniz above.

Fortunately, it isn't so; given Leibniz's views, this argument against absolute speed does not go through. That argument depends on the premise that God could have created a world that differs from the actual world only in the addition of a constant vector to the velocity of every body, and that God had no reason not to do so. But on Leibniz's view, it is not clear that this is true. If you change the absolute speed of a given body, then you have to change its degree of living force, and to do that you have to tamper with the modes of simple substances. (For, living force is a matter of degree of activity, which is in turn a matter of the degree to which one substance's perceptions of another undergo an increase in distinctness while those of the substances it perceives undergo a decrease.) The phenomenal quantities, such as the relative quantities of motion, supervene on the modes of simple substances, so there is no guarantee that the kind of tampering needed to change the absolute speeds of all bodies would not result in a change in other phenomenal quantities such as relative positions. In short, given Leibniz's metaphysical views, there is no guarantee that there is a possible world that agrees with the actual world on all phenomenal quantities but differs in that each body has been given a constant velocity boost.

Moreover, there may be good reasons to think that Leibniz's views entail that there is no such possible world. The alleged possible world in question would be one in which no physical quantities have been changed except the absolute speeds. So, this world matches the actual world on the relative quantities of motion and the masses. But since this entire world has been given a uniform velocity boost (as compared with the actual world), the center of mass of this world has a different absolute speed. Hence, the total amount of living force in this world is not equal to the total amount in the actual world. For Leibniz, living force is a phenomenal manifestation of activity. Passivity and activity within the created world are reciprocal for Leibniz; wherever you have an instance of the one, you have a correlated instance of the other. (This is because, for Leibniz, activity and passivity correspond to the reciprocal relationship holding between two substances when the first comes to perceive the second more distinctly while the second comes to perceive the first less distinctly.) An increase in activity in one place would seem to require a decrease in activity someplace else. So it would seem to be impossible to increase the degree of activity of the entire world, while leaving all phenomena alone. Hence, the alleged possible world in which everything has been given a uniform boost, but otherwise all phenomena match the actual world, does not exist. This is only a sketch of an argument; whether such an argument can be worked out in detail depends on the nature of the reciprocity of passivity and activity, and I cannot attempt to work out all the details here. But doing so isn't necessary for the present point, which is that the velocity-boost argument of the Leibniz-Clarke correspondence is not sufficient to refute the view I am here attributing to Leibniz. That argument only goes through if the possible world it invokes really exists. By Leibniz's lights, it is far from obvious that it does, and we have seen the beginnings of a Leibnizian argument that it does not.

6. Conclusion. On the interpretation I have offered, the right translation of Leibniz's views into contemporary space-time vocabulary does not say that we live in a so-called Leibnizian space-time, but rather that we live in a richer space-time structure featuring absolute speeds. These absolute speeds are not defined as speeds relative to the eternal, fixed background space, but rather in terms of the active force possessed by the individual substances that dominate and unify those aggregates that appear to us as bodies. These quantities, and thus space-time structure of the world, are well-founded phenomena, supervening on the real properties of individual substances. However, space and time, considered as entities in their own right, are not well-founded phenomena. What is metaphysically real does not suffice to determine, for example, which part of absolute space a given body takes up (except insofar as it determines the location of the body relative to other bodies), or at which moment of absolute time a particular event occurs (except insofar as it locates that even relative to other events).

So the present interpretation is intermediate between the two-level and three-level interpretations mentioned in section 3: If space and time are to be thought of as entities in themselves then the only place for them is at a third, "ideal", level; but there is a rich spatiotemporal structure to the physical world that exists at the second level, the level of well-founded phenomena.

The link between the metaphysical world of individual substances and the phenomenal level studied by physics is mediated by force, which is a real property of individual substances and which has a quantitative measure that plays a central role in the laws of motion. This quantitative measure, mv^2, is an accurate measure of (derivative active) force only when it is measured using an appropriate reference frame. As far as Leibniz tells us, this appropriate reference frame cannot be empirically determined, though Leibniz's argument for mv^2 as the true measure of force presupposes that there exists such a reference frame. Our inability to determine the privileged reference frame is no obstacle to the empirical usefulness of the law of conservation of force, since mv^2 is also conserved in other reference frames, which we can make use of in empirical investigation. This interpretation underlines Leibniz's view, elaborated in sections 10–11 of the *Discourse on Metaphysics,* of the relation between metaphysics and mechanistic natural philosophy; the latter can proceed by giving explanations based on mathematical laws of motion, autonomously of the metaphysics of substances and substantial forms, but the true foundation of these laws can only be found in the metaphysics of simple substances.

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