On the Null Energy Condition and Cosmology

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Abstract

Field theories which violate the null energy condition (NEC) are of interest for the solution of the cosmological singularity problem and for models of cosmological dark energy with the equation of state parameter $w < -1$. We discuss the consistency of two recently proposed models that violate the NEC. The ghost condensate model requires higher-order derivative terms in the action. It leads to a heavy ghost field and unbounded energy. We estimate the rates of particles decay and discuss possible mass limitations to protect stability of matter in the ghost condensate model. The nonlocal stringy model that arises from a cubic string field theory and exhibits a phantom behavior also leads to unbounded energy. In this case the spectrum of energy is continuous and there are no particle like excitations. This model admits a natural UV completion since it comes from superstring theory.

1 Introduction

There are general restrictions to the energy-momentum tensor $T_{\mu\nu}$ of a physical system. Such restrictions are referred to as energy conditions. They play an important role in general relativity, in particular in considerations of the black holes and cosmological singularities [1, 2] (see also [3]-[7] and refs. therein). A weak form of the energy condition states that $T_{\mu\nu}n^{\mu}n^{\nu} \geq 0$ for any null vector n^{μ}; it is called the null energy condition (NEC).

The NEC is directly related with a restriction on the dark energy equation of state parameter $w = p/\rho$. The energy-momentum tensor of dark energy is $T_{\mu\nu} = diag(\rho, p, p, p)$ with positive energy density ρ and negative pressure p [8, 9]. The condition $w < -1$ implies violation of the NEC. Since experimental data allows [1] a possibility of $w < -1$ the study of such models attracts a lot of attention. Models of bouncing from cosmological singularity also require violation of the NEC [2, 11].

There are general results that coupled scalar-gravity models which violate the NEC are unstable [3, 4, 5, 7, 12, 15] but recently there have been proposed the ghost condensate model [13, 14, 15, 16] and the nonlocal stringy model [17, 19, 18, 20] which apparently lead to consistent effective theories. Both these models include higher-order derivative terms. In this note we analyze these two models using the Ostrogradski method.

The ghost condensate model requires higher-order derivative terms in the action hence it leads to a massive ghost field and unbounded energy and as a result it could violate the stability of matter. We estimate the rates of particles decay and discuss possible mass limitations to protect stability of matter in the ghost condensate model.

The nonlocal stringy model that exhibits a phantom behavior also leads to unbounded energy but in this case the spectrum of energy is continuous and there are no particle like excitations. This model admits a UV completion since it comes from the superstring theory.

In more details, the model is based on SFT formulation of a fermionic NSR string with the GSO$-$ sector [21]. In this model a scalar field is the open string tachyon [22], which describes according to the Sen conjecture [23] a dynamical transition of a non-BPS D-brane to a stable vacuum (see [24] for review). This stable string theory is supposed to be described by a VSFT (Vacuum String Field Theory). The model that we are going to present is an approximation to VSFT which is stable and has no particle excitations corresponding to the open string.

A vector-scalar model violating NEC with excitations modes stable in some region has been proposed in [25]. There is a tensor-scalar dark energy model admitting phantom behavior at small redshifts[26]. String-inspired and braneworld dark energy models are also the subject of intensive study in the last years (see for example [27, 28, 29] and refs. therein).

The paper is organized as follows. In Section 2 we discuss the ghost condensate model. We show that the model actually includes two non-relativistic scalar fields. One field is a massive ghost while the other field has a positive energy. We evaluate the

[1] A direct search strategy to test the inequality $w < -1$ has been proposed [10].

decay rate of an ordinary particle in the ghost condensate model and obtain a rather stringent limit to the mass of the particle if the lifetime is greater then the Hubble time. In Section 3 we consider the nonlocal stringy model. We used the Weierstrass product representation and the Ostrogradski method to display the spectrum of the model. We obtain that also for this model the energy is unbounded from below but in this case there is no particle like excitations.

2 Ghost Condensate Model

2.1 Setup

The ghost condensate model in Minkowski space (signature is $(-,+,+,+)$) deals with the Lagrangian [13, 14, 15, 16]

$$\mathcal{L} = M^4 P(X) + M^2 S_1(X)(\Box\phi)^2 + M^2 S_2(X)\partial^\mu\partial^\nu\phi\partial_\mu\partial_\nu\phi \tag{1}$$

Here ϕ is a scalar field,

$$X = -\partial_\mu\phi\partial^\mu\phi = \dot{\phi}^2 - (\nabla\phi)^2 \tag{2}$$

and M an arbitrary mass scale. Ghosts (phantoms) could come from the term $P(X)$. There are higher-derivative terms in the Lagrangian proportional to S_1 and S_2 which also lead to ghosts as it will be discussed below.

If

$$P(X) = \frac{1}{8}(X-1)^2 = -\frac{1}{4}\dot{\phi}^2 + ... \tag{3}$$

then the kinetic term has the "wrong" sign and one has ghosts. This corresponds to the "wrong" vacuum state solution of the field equations

$$\phi_0 = 0 \tag{4}$$

It was proposed in [13] that to cure the theory, by analogy with the Higgs mechanism, one can consider the configurations with $P'(X) = 0$ as a candidate ground state. There is a solution of the field equations of the form

$$\phi_0 = t \tag{5}$$

which satisfies $P'(X) = 0$.

If we consider small fluctuations $\pi(t,x)$ about this solution

$$\phi = t + \pi(t,x) \tag{6}$$

then the Lagrangian (3) for quadratic fluctuations leads to the kinetic term with the "proper" sign

$$\mathcal{L} = \frac{1}{2}M^4\dot{\pi}^2 \tag{7}$$

There is no the spatial kinetic term $(\nabla\pi)^2$ in the quadratic Lagrangian for π. The higher order terms proportional to S_1 and S_2 in (1) are added to P to get a spatial kinetic term. The following quadratic Lagrangian is considered in [13, 15, 14, 16]

$$\mathcal{L} = \frac{1}{2}M^4\dot{\pi}^2 - \frac{1}{2}\gamma^2(\nabla^2\pi)^2 \tag{8}$$

where γ^2 is a constant,

$$\gamma^2 = -2M^2(S_1(1) + S_2(1)) \tag{9}$$

We note, however, that the original Lagrangian (1) leads not to (8) but to the following quadratic Lagrangian

$$\mathcal{L} = \frac{1}{2}M^4\dot{\pi}^2 - \frac{1}{2}\gamma^2(\Box\pi)^2 \tag{10}$$

where

$$\Box\pi = -\ddot{\pi} + \nabla^2\pi.$$

The Lagrangian (10) includes the higher derivative term which leads to ghosts for all momenta. In fact it describes not a single but two scalar fields. To demonstrate it let us consider theories with higher derivatives.

2.2 Ostrogradski Method for Equations of Higher Order

Let us consider the Lagrangian

$$\mathcal{L} = \varphi\Box(\alpha + \beta\Box)\varphi \tag{11}$$

where α and β are real parameters. By using the Ostrogradski method [30, 31] we introduce the fields

$$\psi = (\alpha + \beta\Box)\varphi, \quad \phi = \Box\varphi \tag{12}$$

Then, modulo the surface terms, the Lagrangian (11) can be written as

$$\mathcal{L} = \frac{1}{\alpha}\psi\Box\psi - \frac{\beta}{\alpha}\phi(\alpha + \beta\Box)\phi \tag{13}$$

We see that independently of the signs of α and β one of fields ϕ and ψ is a ghost. For instance if $\alpha > 0$ and $\beta < 0$ then we have the massless ordinary field ψ and the massive ghost ϕ.

The Lagrangian

$$\mathcal{L} = \varphi\Box^2\varphi \tag{14}$$

is more complicated for studies. The limit $\alpha \to 0$ is singular and to compute it one has first to make a canonical transformation [32, 33].

2.3 Ghosts in the Ghost Condensate Model

Now let us discuss the Lagrangian (10)

$$\mathcal{L} = \frac{1}{2}M^4\dot{\pi}^2 - \frac{1}{2}\gamma^2(\Box\pi)^2$$

Equation of motion is

$$(\Box^2 + \mu^2\partial_t^2)\pi = 0 \tag{15}$$

where $\mu^2 = M^4/\gamma^2$ or

$$(\partial_t^4 - 2\partial_t^2\nabla^2 + \nabla^4 + \mu^2\partial_t^2)\pi = 0 \tag{16}$$

For the spatial Fourier transform of the field $\pi(t,k)$ the equation (16) reads

$$(\partial_t^4 + (2k^2 + \mu^2)\partial_t^2 + k^4)\pi = 0 \tag{17}$$

The characteristic equation

$$\omega^4 - (2k^2 + \mu^2)\omega^2 + k^4 = 0 \tag{18}$$

has only real roots

$$\omega_{1,2}^2(k) = \frac{1}{2}\mu^2[1 + 2\frac{k^2}{\mu^2} \pm \sqrt{1 + 4\frac{k^2}{\mu^2}}] \tag{19}$$

and we obtain

$$\partial_t^4 + (2k^2 + \mu^2)\partial_t^2 + k^4 = (\partial_t^2 + \omega_1^2)(\partial_t^2 + \omega_2^2) \tag{20}$$

Denote

$$\sigma^2 = \omega_1^2 - \omega_2^2 = \mu^2\sqrt{1 + 4\frac{k^2}{\mu^2}} > 0 \tag{21}$$

By using the Ostrogradski method we introduce the fields

$$\psi = (\partial_t^2 + \omega_1^2)\frac{1}{\sigma}\pi, \quad \phi = (\partial_t^2 + \omega_2^2)\frac{1}{\sigma}\pi \tag{22}$$

and obtain that the Lagrangian (10)

$$\mathcal{L} = -\frac{1}{2}\gamma^2\pi[\partial_t^4 + (2k^2 + \mu^2)\partial_t^2 + k^4]\pi \tag{23}$$

modulo the surface terms can be represented as

$$\mathcal{L} = \frac{1}{4}\gamma^2\phi(\partial_t^2 + \omega_1^2)\phi - \frac{1}{4}\gamma^2\psi(\partial_t^2 + \omega_2^2)\psi \tag{24}$$

We obtain two non-relativistic fields ϕ and ψ. If $\gamma^2 > 0$ then the field ϕ leads to ghosts for any momenta k. Note that there is a mass gap for ghosts since $\omega_1(k)^2 \geq \mu^2$. For large k one gets a relativistic dispersion law $\omega_1(k) \sim \sqrt{k^2 + \mu^2}$. The field ψ does not have a mass gap.

2.4 Decay of Particles

Ghosts (phantoms) could have negative energy. Therefore ordinary particles can decay into heavier particles plus phantoms. This is discussed in [3, 4].

We want to estimate the lifetime of the ordinary particle under this decay. We do not have a direct interaction describing this decay. What we have for sure is an interaction of the phantom with gravity and interaction of gravity with the usual particles. A simple diagram describing a decay of an ordinary particle ψ into usual particles ψ and χ and two phantoms is presented in Fig.1. There h is a graviton.

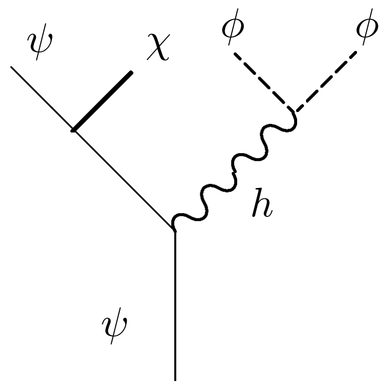

Figure 1: Decay of ψ particle into two phantoms and two other particles ψ and χ

We suppose that the interactions "gravity-phantom" and "gravity-ordinary fields" come from the mass terms in the action

$$S = M_p^2 \int d^4x\sqrt{-g}R + \frac{1}{2}\int d^4x\sqrt{-g}[(\partial\phi)^2 + M^2\phi^2 \tag{25}$$

$$-(\partial\psi)^2 - m_\psi^2\psi^2 - (\partial\psi)^2 - M_\chi^2\chi^2 + \lambda_{\psi^2\chi}\psi^2\chi]$$

Here ϕ is a phantom and ψ and χ are ordinary fields. There is a cubic interaction of the ordinary fields but they couple with the phantom only through gravity. We assume that the mass M_χ of the field χ is of the order of the large phantom mass M and the mass m_ψ of the field ψ is much smaller. From this action we get the following interaction of gravity with matter fields

$$\sqrt{-g}\frac{M^2}{2}\phi^2 \Rightarrow \frac{M^2}{M_p}\phi^2 h \tag{26}$$

$$\sqrt{-g}\frac{m_\psi^2}{2}\psi^2 \Rightarrow \frac{m_\psi^2}{M_p}\psi^2 h, \tag{27}$$

where h is a scalar field symbolically describing graviton. Therefore, the coupling constants of gravity with the ϕ and ψ fields are

$$\lambda_{h\phi^2} = \frac{M^2}{M_p}, \quad \lambda_{h\psi^2} = \frac{m_\psi^2}{M_p} \tag{28}$$

The decay rate of a particle to phantoms through the channel (see Fig.1)

$$\psi \to \psi + \chi + \phi_1 + \phi_2 \tag{29}$$

is

$$\Gamma = \frac{1}{m_\psi} \int^\Lambda \frac{d^3 p_{\phi_1}}{(2\pi)^3 2E_{\phi_1}} \frac{d^3 p_{\phi_2}}{(2\pi)^3 2E_{\phi_2}} \frac{d^3 p_\psi}{(2\pi)^3 2E_\psi} \frac{d^3 p_\chi}{(2\pi)^3 2E_\chi} |\mathcal{M}|^2 \tag{30}$$

$$(2\pi)^4 \delta^{(4)}(p_{\psi_{in}} - p_{\phi_1} - p_{\phi_2} - p_\psi - p_\chi)\ ,$$

where the matrix element $\mathcal{M}$ is

$$\mathcal{M} = \lambda_{h\phi^2} \lambda_{h\psi^2} \lambda_{\chi\psi^2} \frac{1}{p^2} \frac{1}{q^2 + m_\psi^2} \tag{31}$$

Here Λ is a momentum cutoff, p is the momenta of the internal graviton line and q of the internal ψ line. We assume that the dimensional coupling constant $\lambda_{\chi\psi^2}$ is of the order

$$\lambda_{\chi\psi^2} \sim m_\psi.$$

Using (26) and (27) we get

$$\mathcal{M} = \frac{m_\psi^3 M^2}{M_p^2} \frac{1}{p^2} \frac{1}{q^2 + m_\psi^2}. \tag{32}$$

The integral is convergent even without cutoff. To estimate it we can take

$$M \sim M_p \sim \Lambda$$

We estimate the decay rate as

$$\Gamma \sim \frac{m_\psi^5}{M_p^5} M_p \tag{33}$$

The timescale for decay is $\tau_\psi = 1/\Gamma$. A model will be phenomenologically viable if the lifetime is greater than the Hubble time H_0^{-1}, i.e.

$$\Gamma \sim \frac{m_\psi^5}{M_p^5} M_p < H_0 \sim 10^{-60} M_p \tag{34}$$

Using $M_p \sim 10^{19}$GeV we get

$$m_\psi < 10^7 \mathrm{GeV} \tag{35}$$

A stringent limit to the mass one can obtain from the investigation of the quartic interaction of the ordinary particles of the form $\lambda\psi^3\chi$ with the coupling constant λ of the order one. Then instead of (33) and (35) we get

$$\Gamma \sim \frac{m_\psi^3}{M_p^3} M_p \tag{36}$$

and an uncomfortable result

$$m_\psi < 10^{-1} \mathrm{GeV}. \tag{37}$$

3 Nonlocal Stringy Model

3.1 Example of Field Equations of the Exponential Type

Consider first the following action

$$\mathcal{L} = \phi(e^{\Box} - 1)\phi \tag{38}$$

The field equations are nonlocal since one can write them as integral equations. It serves as an example for the nonlocal stringy Lagrangian which will be considered below. We don't consider here the rigorous mathematical approach to such equations, see recent papers [34]. By using the Weierstrass product

$$e^z - 1 = e^{z/2} z \prod_{j=1}^{\infty} (1 + \frac{z^2}{\omega_j^2}) \tag{39}$$

where

$$\omega_j^2 = 4\pi^2 j^2$$

we write

$$e^{\Box} - 1 = e^{\Box/2} \Box \prod_{j=1}^{\infty} (1 + \frac{\Box^2}{\omega_j^2}) \tag{40}$$

Therefore [32] the Lagrangian (38) has the same spectrum as

$$\mathcal{L} = \psi_0 \Box \psi_0 + \sum_{j=1}^{\infty} \epsilon_j \psi_j (\Box^2 + \omega_j^2) \psi_j \tag{41}$$

Here

$$\epsilon_j = 1/\omega_j^4 F'(-\omega_j^2) \tag{42}$$

where

$$F(z^2) = (e^z - 1)\frac{e^{-z/2}}{z} = 2\frac{\sinh z/2}{z} \tag{43}$$

More general Lagrangian

$$\mathcal{L} = \phi(e^{\alpha\Box} - \lambda)\phi \tag{44}$$

appears in p-adic string theory (see [35, 36] and refs therein). Here α and λ are constants. The spectrum of the theory can be read out of the Weierstrass product

$$e^{\alpha\Box} - \lambda = \lambda^{1/2} e^{\alpha\Box/2} (\alpha\Box - \log\lambda) \prod_{j=1}^{\infty} (1 + \frac{(\alpha\Box - \log\lambda)^2}{\omega_j^2}) \tag{45}$$

3.2 Nonlocal Tachyon Field

An effective Lagrangian for the tachyon field coming from GSO− sector of the fermionic NSR string which is described by cubic string field theory [21] has the following form [22]

$$\mathcal{L} = \frac{1}{2}\phi e^{-\frac{1}{4}\Box}(\Box + m^2)\phi - \frac{\lambda}{4}\phi^4 \tag{46}$$

There is the "wrong" vacuum state solution of the field equations

$$\phi_0 = 0 \tag{47}$$

which leads to the tachyon equation for fluctuations about this vacuum:

$$(\Box + m^2)\phi = 0 \tag{48}$$

There exists also another vacuum solution

$$\phi_0 = m/\sqrt{\lambda} \tag{49}$$

One obtains the following Lagrangian for the small fluctuations about this solution:

$$\mathcal{L} = \frac{m^2}{2}\phi F(-\Box)\phi \tag{50}$$

where

$$F(z) = \left(-\xi^2 z + 1\right) e^{\frac{1}{4}z} - 3, \tag{51}$$

and we set $\xi^2 = 1/m^2$.

The characteristic equation

$$F(z) = 0 \tag{52}$$

has an infinite number of complex conjugated pairs of complex roots $z = \kappa_j^2$ (see [20] for the consideration of these roots in the case corresponding to the action (50) with (51) and [37] for the case of arbitrary constants). The roots κ_j^2 are expressed by means of the Lambert function $W(z)$ satisfying the equation $W(z)e^{W(z)} = z$.

We represent (51) in terms of the Weierstrass product

$$F(z) = -2e^{\frac{1}{8}(4\xi^2-1)z}\prod_j(1-\frac{z}{\kappa_j^2})(1-\frac{z}{\kappa_j^{*2}})\exp[z(\frac{1}{\kappa_j^2}+\frac{1}{\kappa_j^{*2}})] \tag{53}$$

Now we can write an appropriate quadratic Lagrangian (up to total derivatives) as

$$\mathcal{L} = \sum_j \epsilon_j \psi_j \left(\Box - \kappa_j^2\right)\left(\Box - \kappa_j^{*2}\right)\psi_j \tag{54}$$

Equations of motion with complex distinct frequencies were studied in [32]. Consider the following equation:

$$\left(\Box - \kappa^2\right)\left(\Box - \kappa^{*2}\right)\phi = 0 \tag{55}$$

where $\kappa = \nu + i\alpha$. The corresponding Hamiltonian is a linear superposition of pairs of complex conjugated oscillators of the form:

$$H = \frac{1}{2}\sum_k [P_1^2(k) + \omega(k)^2 Q_1^2(k) + P_2^2(k) + \omega(k)^{*2} Q_2^2(k)] \tag{56}$$

where

$$P_1^* = P_2, \quad Q_1^* = Q_2$$

and

$$\omega(k)^2 = \kappa^2 + k^2$$

The spectrum of the Hamiltonian has the form

$$n\nu(k) + \rho\alpha(k) \tag{57}$$

where we put $\omega(k) = \nu(k) + i\alpha(k)$. Here $n = 0, \pm 1, ...$ while ρ is continuous and ranges from $-\infty$ to $+\infty$. The energy spectrum is indefinite and continuous. Therefore no particle attributes can be ascribed to these modes [32].

4 Conclusions

We have considered two recently proposed models which violate the NEC. In both models there are higher derivative terms and we have used the Ostrogradski method to study them. A straightforward application of this method to linear theories indicates that energies are unbounded. Note that the problems with unbounded energy considered in this paper are mathematically similar to the eigenvalue problem for the non positively defined hyperbolic Klein-Gordon equation on Lorentzian manifolds which is solved in [38].

One can expect that an incorporation of nonlinear terms could drastically change the situation. In particular, in nonlinear theories could exist islands of stability [3, 39, 40, 41].

In the ghost condensate model with higher-order derivative terms it would be interesting to find a UV completion of the theory and study islands of stability.

The nonlocal stringy model has been proposed to describe a decay of D-brane and though it also leads to unbounded energy in this case the spectrum of the energy is continuous and there are no particle like excitations. This model admits a UV completion since it comes form superstring theory but it would be very interesting to find a direct mechanism of compensation of unbounded continuous spectrum.

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