3.5 Hyperbolic Functions

3.51 Hyperbolic functions

3.511

1. $\int_0^\infty \frac{dx}{\operatorname{ch} ax} = \frac{\pi}{2a} \quad [a > 0].$

2. $\int_0^\infty \frac{\operatorname{sh} ax}{\operatorname{sh} bx}\, dx = \frac{\pi}{2b} \operatorname{tg} \frac{a\pi}{2b} \quad [b > |a|].$ BI ((27))(10)a

3. $\int_0^\infty \frac{\operatorname{sh} ax}{\operatorname{ch} bx}\, dx = \frac{\pi}{2b} \sec \frac{a\pi}{2b} - \frac{1}{b} \beta\left(\frac{a+b}{2b}\right) \quad [b > |a|].$ GW ((351))(3b)

4. $\int_0^\infty \frac{\operatorname{ch} ax}{\operatorname{ch} bx}\, dx = \frac{\pi}{2b} \sec \frac{a\pi}{2b} \quad [b > |a|].$ BI ((4))(14)a

5. $\int_0^\infty \frac{\operatorname{sh} ax \operatorname{ch} bx}{\operatorname{sh} cx}\, dx = \frac{\pi}{2c} \frac{\sin \frac{a\pi}{c}}{\cos \frac{a\pi}{c} + \cos \frac{b\pi}{c}}$

$[c > |a| + |b|].$ BI ((27))(11)

6. $\int_0^\infty \frac{\operatorname{ch} ax \operatorname{ch} bx}{\operatorname{ch} cx}\, dx = \frac{\pi}{c} \frac{\cos \frac{a\pi}{2c} \cos \frac{b\pi}{2c}}{\cos \frac{a\pi}{c} + \cos \frac{b\pi}{c}}$

$[c > |a| + |b|].$ BI ((27))(5)a

7. $\int_0^\infty \frac{\operatorname{sh} ax \operatorname{sh} bx}{\operatorname{ch} cx}\, dx = \frac{\pi}{c} \frac{\sin \frac{a\pi}{2c} \sin \frac{b\pi}{2c}}{\cos \frac{a\pi}{c} + \cos \frac{b\pi}{c}}$

$[c > |a| + |b|].$ BI ((27))(6)a

8. $\int_0^\infty \frac{dx}{\operatorname{ch} x^2} = \sqrt{\pi} \sum_{k=0}^\infty \frac{(-1)^k}{\sqrt{2k+1}}$ BI ((98))(25)

9. $\int_{-\infty}^\infty \frac{\operatorname{sh}^2 ax}{\operatorname{sh}^2 x}\, dx = 1 - a\pi \operatorname{ctg} a\pi \quad [a^2 < 1].$ BI ((16))(3)a

10. $\int_0^\infty \frac{\operatorname{sh} ax \operatorname{sh} bx}{\operatorname{ch}^2 bx}\, dx = \frac{a\pi}{2b^2} \sec \frac{a\pi}{2b} \quad [b > |a|].$ BI ((27))(16)a

3.512

1. $\int_0^\infty \frac{\operatorname{ch} 2\beta x}{\operatorname{ch}^{2\nu} ax}\, dx = \frac{4^{\nu-1}}{a} \mathrm{B}\left(\nu + \frac{\beta}{a}, \nu - \frac{\beta}{a}\right)$

$[\operatorname{Re}(\nu \pm \beta) > 0,\ a > 0,\ \beta > 0].$ LI((27))(17)a, EH I 11(26)

2. $\int_0^\infty \frac{\operatorname{sh}^\mu x}{\operatorname{ch}^\nu x}\, dx = \frac{1}{2} \mathrm{B}\left(\frac{\mu+1}{2}, \frac{\nu-\mu}{2}\right) \quad [\operatorname{Re} \mu > -1,\ \operatorname{Re}(\mu - \nu) < 0].$ EH I 11(23)

3.513

1. $\int_0^\infty \frac{dx}{a + b \operatorname{sh} x} = \frac{1}{\sqrt{a^2+b^2}} \ln \frac{a+b+\sqrt{a^2+b^2}}{a+b-\sqrt{a^2+b^2}} \quad [ab \neq 0].$ GW ((351))(8)

2. $\int_0^\infty \frac{dx}{a + b \operatorname{ch} x} = \frac{2}{\sqrt{b^2-a^2}} \operatorname{arctg} \frac{\sqrt{b^2-a^2}}{a+b} \quad [b^2 > a^2];$

$= \frac{1}{\sqrt{a^2-b^2}} \ln \frac{a+b+\sqrt{a^2-b^2}}{a+b-\sqrt{a^2-b^2}} \quad [b^2 < a^2].$ GW ((351))(7)

3. $\int_0^\infty \frac{dx}{a \operatorname{sh} x + b \operatorname{ch} x} = \frac{2}{\sqrt{b^2-a^2}} \operatorname{arctg} \frac{\sqrt{b^2-a^2}}{a+b} \quad [b^2 > a^2];$

$= \frac{1}{\sqrt{a^2-b^2}} \ln \frac{a+b+\sqrt{a^2-b^2}}{a+b-\sqrt{a^2-b^2}} \quad [a^2 > b^2].$ GW ((351))(9)

4. $\int_0^\infty \frac{dx}{a + b \operatorname{ch} x + c \operatorname{sh} x} = \frac{2}{\sqrt{b^2-a^2-c^2}} \left[\operatorname{arctg} \frac{\sqrt{b^2-a^2-c^2}}{a+b+c} + \varepsilon\pi\right]$

$[b^2 > a^2 + c^2;\ \varepsilon = 0$ for $(b-a)(a+b+c) > 0$,
$|\varepsilon| = 1$ for $(b-a)(a+b+c) < 0$, also $\varepsilon = 1$ for $a < b + c$
and $\varepsilon = -1$ for $a > b + c]$;

$= \frac{1}{\sqrt{a^2-b^2+c^2}} \ln \frac{a+b+c+\sqrt{a^2-b^2+c^2}}{a+b+c-\sqrt{a^2-b^2+c^2}}$
$[b^2 < a^2 + c^2,\ a^2 \neq b^2];$

$= \frac{1}{c} \ln \frac{a+c}{a} \quad [a = b \neq 0,\ c \neq 0];$

$= \frac{2(a-b)}{c(a-b-c)} \quad [b^2 = a^2 + c^2,\ c(a-b-c) < 0].$

GW ((351))(6)

3.514

1. $\int_0^\infty \frac{dx}{\operatorname{ch} ax + \cos t} = \frac{t}{a} \operatorname{cosec} t \quad [0 < t < \pi].$ BI ((27))(22)a

2. $\int_0^\infty \frac{\operatorname{ch} ax - \cos t_1}{\operatorname{ch} bx - \cos t_2}\, dx = \frac{\pi}{b} \frac{\sin \frac{a(\pi - t_2)}{b}}{\sin t_2 \sin \frac{a}{b}\pi} - \frac{\pi - t_2}{b \sin t_2} \cos t_1 \quad [b > |a|,\ 0 < t < \pi].$

BI ((6))(20)a

3. $\int_0^\infty \frac{\operatorname{ch} ax\, dx}{(\operatorname{ch} x + \cos t)^2} = \frac{\pi(-\cos t \sin at + a \sin t \cos at)}{\sin^3 t \sin a\pi}$

$[a^2 < 1,\ 0 < t < \pi].$ BI ((6))(18)a

4. $\int_0^\infty \frac{\operatorname{sh} ax \operatorname{sh} bx}{(\operatorname{ch} ax + \cos t)^2}\, dx = \frac{b\pi}{a^2} \operatorname{cosec} t \operatorname{cosec} \frac{b\pi}{a} \sin \frac{bt}{a}$

$[a > |b|,\ 0 < t < \pi].$ BI ((27))(27)a

3. $$\int_0^\infty H_\mu^{(1)}\left(\frac{a^2}{x}\right) H_\mu^{(2)}\left(\frac{a^2}{x}\right) J_0(bx)\,dx = 16\pi^{-2}b^{-1}\cos\mu\pi K_{2\mu}\left(2e^{\frac{1}{4}\pi i}a\sqrt{b}\right) K_{2\mu}\left(2e^{-\frac{1}{4}\pi i}a\sqrt{b}\right)$$
$\left[|\arg a| < \frac{\pi}{4},\ b > 0,\ |\operatorname{Re}\mu| < \frac{1}{4}\right].$ ET II 17(36)

6.516

1. $$\int_0^\infty J_{2\nu}(a\sqrt{x})\,J_\nu(bx)\,dx = b^{-1}J_\nu\left(\frac{a^2}{4b}\right)$$
$\left[a > 0,\ b > 0,\ \operatorname{Re}\nu > -\frac{1}{2}\right].$ ET II 58(16)

2. $$\int_0^\infty J_{2\nu}(a\sqrt{x})\,N_\nu(bx)\,dx = -b^{-1}\mathbf{H}_\nu\left(\frac{a^2}{4b}\right)$$
$\left[a > 0,\ b > 0,\ \operatorname{Re}\nu > -\frac{1}{2}\right].$ ET II 111(18)

3. $$\int_0^\infty J_{2\nu}(a\sqrt{x})\,K_\nu(bx)\,dx = \frac{\pi}{2}b^{-1}\left[I_\nu\left(\frac{a^2}{4b}\right) - \mathbf{L}_\nu\left(\frac{a^2}{4b}\right)\right]$$
$\left[\operatorname{Re}b > 0,\ \operatorname{Re}\nu > -\frac{1}{2}\right].$ ET II 144(45)

4. $$\int_0^\infty N_{2\nu}(a\sqrt{x})\,J_\nu(bx)\,dx = 2\sec(\nu\pi)\,b^{-1} \times \left[\frac{1}{2}\cos(\nu\pi)N_\nu\left(\frac{a^2}{4b}\right) - N_{-\nu}\left(\frac{a^2}{4b}\right) + \mathbf{H}_{-\nu}\left(\frac{a^2}{4b}\right)\right]$$
$\left[a > 0,\ b > 0,\ \operatorname{Re}\nu > -\frac{1}{2}\right].$ ET II 62(39)

5. $$\int_0^\infty N_{2\nu}(a\sqrt{x})\,N_\nu(bx)\,dx = \frac{b^{-1}}{2}\left[\sec(\nu\pi)J_{-\nu}\left(\frac{a^2}{4b}\right) + \operatorname{cosec}(\nu\pi)\mathbf{H}_{-\nu}\left(\frac{a^2}{4b}\right) - 2\operatorname{ctg}(2\nu\pi)\,\mathbf{H}_\nu\left(\frac{a^2}{4b}\right)\right]$$
$\left[a > 0,\ b > 0,\ |\operatorname{Re}\nu| < \frac{1}{2}\right].$ ET II 111(19)

6. $$\int_0^\infty N_{2\nu}(a\sqrt{x})\,K_\nu(bx)\,dx = \frac{\pi b^{-1}}{2}\left[\operatorname{cosec}(2\nu\pi)\,\mathbf{L}_{-\nu}\left(\frac{a^2}{4b}\right) - \operatorname{ctg}(2\nu\pi)\,\mathbf{L}_\nu\left(\frac{a^2}{4b}\right) - \operatorname{tg}(\nu\pi)\,I_\nu\left(\frac{a^2}{4b}\right) - \frac{\sec(\nu\pi)}{\pi}K_\nu\left(\frac{a^2}{4b}\right)\right]$$
$\left[\operatorname{Re}b > 0,\ |\operatorname{Re}\nu| < \frac{1}{2}\right].$ ET II 144(46)

7. $$\int_0^\infty K_{2\nu}(a\sqrt{x})\,J_\nu(bx)\,dx = \frac{1}{4}\pi b^{-1}\sec(\nu\pi)\left[\mathbf{H}_{-\nu}\left(\frac{a^2}{4b}\right) - N_{-\nu}\left(\frac{a^2}{4b}\right)\right]$$
$\left[\operatorname{Re}a > 0,\ b > 0,\ \operatorname{Re}\nu > -\frac{1}{2}\right].$ ET II 70(22)

8. $$\int_0^\infty K_{2\nu}(a\sqrt{x})\,N_\nu(bx)\,dx = -\frac{1}{4}\pi b^{-1}\left[\sec(\nu\pi)J_{-\nu}\left(\frac{a^2}{4b}\right) - \operatorname{cosec}(\nu\pi)\,\mathbf{H}_{-\nu}\left(\frac{a^2}{4b}\right) + 2\operatorname{cosec}(2\nu\pi)\,\mathbf{H}_\nu\left(\frac{a^2}{4b}\right)\right]$$
$\left[\operatorname{Re}a > 0,\ b > 0,\ |\operatorname{Re}\nu| < \frac{1}{2}\right].$ ET II 114(34)

9. $$\int_0^\infty K_{2\nu}(a\sqrt{x})\,K_\nu(bx)\,dx = \frac{\pi b^{-1}}{4\cos(\nu\pi)}\left\{K_\nu\left(\frac{a^2}{4b}\right) + \frac{\pi}{2\sin(\nu\pi)}\left[\mathbf{L}_{-\nu}\left(\frac{a^2}{4b}\right) - \mathbf{L}_\nu\left(\frac{a^2}{4b}\right)\right]\right\}$$
$\left[\operatorname{Re}b > 0,\ |\operatorname{Re}\nu| < \frac{1}{2}\right].$ ET II 147(63)

10. $$\int_0^\infty I_{2\nu}(a\sqrt{x})\,K_\nu(bx)\,dx = \frac{\pi b^{-1}}{2}\left[I_\nu\left(\frac{a^2}{4b}\right) + \mathbf{L}_\nu\left(\frac{a^2}{4b}\right)\right]$$
$\left[\operatorname{Re}b > 0,\ \operatorname{Re}\nu > -\frac{1}{2}\right].$ ET II 147(60)

6.517 $$\int_0^z J_0\left(\sqrt{z^2 - x^2}\right)dx = \sin z.$$ MO 48

6.518 $$\int_0^\infty K_{2\nu}(2z\operatorname{sh}x)\,dx = \frac{\pi^2}{8\cos\nu\pi}\left(J_\nu^2(z) + N_\nu^2(z)\right)$$
$\left[\operatorname{Re}z > 0,\quad -\frac{1}{2} < \operatorname{Re}\nu < \frac{1}{2}\right].$ MO 45

6.519

1. $$\int_0^{\frac{\pi}{2}} J_{2\nu}(2z\cos x)\,dx = \frac{\pi}{2}J_\nu^2(z) \qquad \left[\operatorname{Re}\nu > -\frac{1}{2}\right].$$ WH

2. $$\int_0^{\frac{\pi}{2}} J_{2\nu}(2z\sin x)\,dx = \frac{\pi}{2}J_\nu^2(z) \qquad \left[\operatorname{Re}\nu > -\frac{1}{2}\right].$$ WA 42(1)a