RH, NSE, YME solutions

homepage overview

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http://www.fuchs-braun.com/721801/645601.html

Albert Einstein, "*we can't solve problems by using the same kind of thinking we used when we created them*",

Wolfgang E. Pauli, "*all things reach the one who knows how to wait*".

This homepage addresses the following three Millennium problems (resp. links to corresponding homepages):

A. The Riemann Hypothesis (RH)
B. The 3D-Navier-Stokes equations (NSE)
C. The Yang-Mills equations (YME)

A helicopter view

From a helicopter point of view there is a common denominator of all solution concepts of this homepage: it is about a common mathematical frame to govern the "infinitesimal small" with respect to truly infinitesimal small "elements" and related "functions" and truly geometrical (i.e. equipped with an inner product) "function spaces", enabling a truly geometrical mathematical modelling framework with corresponding operators (including well defined domains and ranges). The two applied central "objects" are

- the well-established "differentials", with its number theoretical counterpart, Leibniz' ideal numbers/monads field, and its link to the Stieltjes integral (Plemelj)

- the well-established distributional "Hilbert scale" concept with its corresponding strong and weak variational theory for PDE (approximation) solutions, but also for Pseudo-Differential and Fourier multiplier (weak and strong singular integral) equations (Calderón).

The "infinitesimal small" is and will be all the time out of scope for any human observations. Mathematics is a purely descriptive science with well-established concepts to deal with any kind and "size" of "infinity" (e.g. Cauchy, Dedekind, Bolzano, Weierstrass, Kronecker, Cantor, Gödel, Brouwer). The mathematical tool managing physical "observations" are Partial Differential Equations (PDE), mathematical statistics and approximation theory. Those concepts are also applied in quantum mechanics and quantum field theory. The essential mathematical "objects" are the real numbers (while "nearly all" of those objects are far away from being "real") and the Lebesgue integral building the Lebesgue (Hilbert) space L(2), where all rational numbers build a null set measured by its corresponding norm.

The "helicopter view" statement would be now the following:

- the real numbers are replaced by differentials with same cardinality as the real number field, but allowing infinitesimal small number objects in the neighborhood of each real number (see the Riesz theorem below for the analogue properties with respect to the "Hilbert scale" objects

- the "measurement/observation/statistical" function space L(2) is replaced by the "much larger" distributional Hilbert space H(-1/2).

The advantages are the following

- both conceptual changes do not increase the already existing "infinity" characters of "real" numbers (measured by Cantor's cardinality concept) and the L(2) space, which is a separable Hilbert space, in the same way, as all of the considered Hilbert scale "objects"

- the "measurement of real numbers is already an approximation by rational numbers, i.e. truly "observations" of irrational number "objects" are not possible; each irrational number is already a full universe, i.e. an approximation of an infinite numbers of rational numbers; extending those number field to ideal numbers is just the same mystery with same cardinality; the key differentiator is related to a measurement of length axiom by given "unit of measure" length

- the H(-1/2) provides an alternative model to the (space dimension depending Dirac function regularity)

- the L(2) is a closed subspace of H(-1/2), i.e. state of the art statistical analysis is guaranteed

- the L(2) is a closed subspace of H(-1/2), i.e. compactness arguments (e.g. based on Garding type inequalities) and the theorem of Riesz ensure "quasi-optimal" approximation properties of each "object" of the H(-1/2) space by an object in the L(2) space:

Theorem (Riesz): *let (((x,x))) denote the norm of H(-1/2). Then for each e with $0<e<1$ there exists a y with (((y,y)))=1 and inf(((x-y,x-y)))>=e ,for all x of L(2)).*

Remark*: as 0 is an element of L(2) this means that the inf-term above is at* most *equal 1; therefore the theorem states that this value can be arbitrarily close approximated.*

A separable Hilbert scale can be built from the solutions of the eigenvalue equation K(x) = l*x, where K denotes a symmetric and compact operator:

Lemma*: for no more than countable values l(i) the equation K(x)=l*x possesses non-trivial solutions x(i), where lim(l(i)) is equal zero.*

In case the domain of such a compact operator is the L(2) Hilbert space the corresponding eigenfunctions build the basis of this Hilbert space. The concept of "wave package" enables also continuous spectra. Therefore, such "wave packages" require a domain extension (e.g. L(2) --> H(-1/2) in order to ensure convergent inner products and related norms. "Wave packages" are also called "eigen-differentials" (H. Weyl), playing a key role in quantum mechanics in the context of the spectral representation of Hermitian operators (D. Hilbert, J. von Neumann, P. A. M. Dirac).

Schrödinger's "purely quantum wave" vision

This is about half-odd integers, rather than integers to be applied to wave-mechanical vibrations which correspond to the motion of particles of a gas resp. the eigenvalues and eigenfunctions of the harmonic quantum oscillator still governed by the Heisenberg uncertainty inequality. The alternatively proposed H(1/2) energy space enables Schrödinger's vision ((ScE) (7.23) ff):

let w denotes the angular frequency, h the (h-bar) Planck constant and e := w*h/2): then Schrödinger's "half-odd integer vision" is about the following replacement:

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 n=0:    E(0) = e                    -->        E(1/2) =   1 *  e
 n>0:    E(1) = 1* w*h               -->        E(3/2) =   2 *  e
         E(2) = 2* w*h               -->        E(5/2) =   3 *  e
         ...
         E(n) = n* w*h  , n>0    -->  E ((n+1)/2) = (n+1) * e  , n=0,1.2....
         ...
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As a consequence the corresponding eigenvalue and eigenfunction solutions of the number operator (i.e. the product of generation and annihilation operators) start with index n=1, not already with n=0.

The generalized Hermite polynomials satisfy different differential equations for even and odd polynomials. In (KrA) the spectral analysis for those generalized (even and odd) Hermite polynomials is provided. For the special case of the Schrödinger differential equation the spectrum of its related Schrödinger equation operator L is discrete, consisting of the odd integers. The corresponding eigenfunctions form a complete orthogonal set in the weighted-L(2) space (DaD).

Our alternatively proposed Schrödinger (Calderón) equation operator differs from the standard operator L by its combination with the Hilbert-transform operator H (which is an unitary operator with corresponding spectral theorem and a representation H = cos(A)+i*sin(A) with A being a Hermitian operator and a corresponding spectrum on the unit circle) and an extension of the L(2) space. This enables a spectral representation of the alternatively proposed Schrödinger equation operator with *vanishing* (!) constant Fourier term being replaced by a continuous spectrum summand (modelling the "ground state zero" eigenfunction/ eigendifferential) "governing" the corresponding complementary space of L(2).

The Berry conjecture is about the Riemann Zeta function as a model for the quantum chaos (BeM).

A "space lab" view comment ...:)

We can think (hear and watch) the Yoda quote "may the FORCE be with us" and mathematics can model this FORCE/POWER/ENERGY in a way that all corresponding physical (law) models are consistent; ... the bad (or good?) news is, that's it and that's all!

From a philosophical perspective we are back to

- **Leibniz's** "*Fünf Schriften zur Logik und Metaphysik*" and "*Monadologie*"
"*the primitive active and passive forces, the form and matter are in the monadological view understood as features of the perceptions of the monads … in this way the notion of force, … loses its foundational status: primitive force gets folded into the perceptual life of non-extended perceiving things*", Garber's monograph: Leibniz: Body, Substance, Monad, 2009
- **Kant's** conception of physical matter, based on the existence of an ether which fills the whole space and time with its moving forces (WoW)
- **Schrödinger's** "*view of the world*" with respect to "*reasons for abandoning the dualism of thought and existence, or mind and matter*".

A. The Riemann Hypothesis

All nontrivial zeros of the analytical continuation of the Riemann zeta function have a real part of 1/2. The *Hilbert-Polya conjecture* states that the imaginary parts of the zeros of the Zeta function corresponds to eigenvalues of an unbounded self adjoint operator.

We provide a solution for the RH building on a new *Kummer function based Zeta function theory*, alternatively to the current Gauss-Weierstrass function based Zeta function theory. This primarily enables a proof of the *Hilbert-Polya conjecture* (but also of other RH criteria like the *Bagchi formulation of the Nyman-Beurling criterion* or Polya criteria), whereby the imaginary parts of the zeros of the corresponding alternative Zeta function definition corresponds to eigenvalues of a *bounded,* self adjoint operator with (newly) distributional Hilbert space domain.

The proposed framework also provides an answer to Derbyshine's question, ("Prime Obsession")

... "*The non-trivial zeros of Riemann's zeta function arise from inquiries into the distribution of prime numbers. The eigenvalues of a random Hermitian matrix arise from inquiries into the behavior of systems of subatomic particles under the laws of quantum mechanics. What on earth does the distribution of prime numbers have to do with the behavior of subatomic particles?*"

The answer, in a nutshell:

"identifying "fluids" or "sub-atomic particles" not with real numbers (scalar field, I. Newton), but with hyper-real numbers (G. W. Leibniz) enables a truly infinitesimal (geometric) distributional Hilbert space framework (H. Weyl) which corresponds to the Teichmüller theory, the Bounded Mean Oscillation (BMO) and the Harmonic Analysis theory. The distributional Hilbert scale framework enables the full power of spectral theory, while still keeping the standard L(2)=H(0)-Hilbert space as test space to "measure" particles' locations. At the same time, the Ritz-Galerkin (energy or operator norm minimization) method and its counterpart, the methods of Trefftz/Noble to solve PDE by complementary variational principles (A. M. Arthurs, K. Friedrichs, L. B. Rall, P. D. Robinson, W. Velte) w/o anticipating boundary values) enables an alternative "quantization" method of PDE models (P. Ehrenfest), e.g. being applied to the Wheeler-de-Witt operator.

Regarding the proposed alternative quantization approach we also refer to the Berry-Keating conjecture. This is about an unknown quantization ***H*** of the classical Hamiltonian H=xp, that the Riemann zeros coincide with the spectrum of the operator 1/2+i***H***. This is in contrast to canonical quantization, which leads to the Heisenberg uncertainty principle and the natural numbers as spectrum of the harmonic quantum oscillator. The Hamiltonian needs to be self-adjoint so that the quantization can be a realization of the Hilbert-Polya conjecture.

B. The Navier-Stokes Equations

The Navier-Stokes equations describe the motion of fluids. The *Navier–Stokes existence and smoothness* problem for the three-dimensional NSE, given some initial conditions, is to prove that smooth solutions always exist, or that if they do exist, they have bounded energy per unit mass.

We provide a global unique (weak, generalized Hopf) H(-1/2)-solution of the generalized 3D Navier-Stokes initial value problem. The global boundedness of a generalized energy inequality with respect to the energy Hilbert space H(1/2) is a consequence of the Sobolevskii estimate of the non-linear term (1959):

http://www.navier-stokes-equations.com

The "standard" weak Hopf solution is not well posed due to not appropriately defined domains of the underlying velocity and pressure operators. Therefore, this is also the case for the corresponding classical solution(s).

The proposed solution also overcomes the "Serin gap" issue, as a consequence of the bounded non-linear term with respect to the appropriate energy norm.

C. The Yang-Mills Equations

The YME are concerned with quantum field theory. Its related Millennium problem is about an appropriate mathematical model to govern the current "mass gap" of the YME, which is the difference in energy between the vacuum and the next lowest energy field.

We propose to apply the same solution concept to solve the "mass gap" issue of the YME. This provides a truly infinitesimal geometry (H. Weyl), enabling the concept of Riemann that force is a pseudo force only, which results from distortions of the geometrical structure. The baseline is a common Hilbert space framework (for all (nearby action) differential equations)

- providing the mathematical concept of a geometrical structure (while Riemann's manifold concept provides only a metric space and related affine connections)

- replacing "force type" specific gauge fields and its combination model(s) for the electromagnetic, the strong and the weak nuclear power "forces"

- building an integrated (no longer "force" dependent dynamical matter-field interaction laws) universal field model (including the gravity "force")

As a consequence there is no "mass" and therefore no (YME-) "mass gap" anymore, but there is an appropriate vacuum (Hilbert) energy space, which is governed by the Heisenberg uncertainty principle.

With respect to the proposed alternative mathematical framework (distributional Hilbert scale and corresponding appropriate domains for Pseudo-differential and Fourier multiplier operators) we note the following:

- The Maxwell equations are represented by differential equations or integral equations. Both representations are considered as equivalent.

- The Lagrange (“force”) and the Hamiltonian (“energy”) formalisms are considered as equivalent. The rational / mathematical proof is based on the Legendre transform.

In both cases there are underlying regularity assumptions enabling those propositions. Restricting the regularity of the domains of the corresponding operators according to the proposals of this page leads to no longer well-defined classical differential equations resp. to no longer valid Lagrange formalism. In other words, if there is a consistent model in the distributional framework, the corresponding classical solutions of the several differential equations are only approximations to it.

http://www.quantum-gravitation.de/

D. The Boltzmann Entropy

The Boltzmann equation is a (non-linear) integro-differential equation which forms the basis for the kinetic theory of gases. This not only covers classical gases, but also electron /neutron /photon transport in solids & plasmas / in nuclear reactors / in super-fluids and radiative transfer in planetary and stellar atmospheres. The Boltzmann equation is derived from the Liouville equation for a gas of rigid spheres, without the assumption of "molecular chaos"; the basic properties of the Boltzmann equation are then expounded and the idea of model equations introduced. Related equations are e.g. the Boltzmann equations for polyatomic gases, mixtures, neutrons, radiative transfer as well as the Fokker-Planck and Vlasov equations. The treatment of corresponding boundary conditions leads to the discussion of the phenomena of gas-surface interactions and the related role played by proof of the Boltzmann H-theorem.

We provide the relationships of the Boltzmann and Landau equations to the proposed alternative quantum state Hilbert space frame H(-1/2) resp. its corresponding quantum energy space H(1/2) and the related NSE global solution, as well as to the proposed alternative Schrödinger (Calderón) momentum operator. At the same point in time we suggest to apply the same frame for a proof of the non-linear Landau damping for the Vlasov equation with only physical relevant mathematical assumptions.

E. Wavelets

Wavelets are proposed as appropriate analysis tool for the proposed NMEP, additionally to Fourier analysis technique. There are at least two approaches to wavelet analysis, both are addressing the somehow contradiction by itself, that a function over the one-dimensional space R can be unfolded into a function over the two-dimensional half-plane.

A wavelet transform W(g)(v) of a function v with respect to a wavelet function g is an isometric mapping, whereby the corresponding adjoint operator is given by the inverse wavelet transform on its range. Let u,v denote two elements of a Hilbert space with inner product (u,v), let ((*,*)) denote the inner product of the Hilbert space H(-1/2). Let further f,g denote two wavelets with bounded inner product ((f,g)) and let (((*,*))) denote the inner product of the corresponding wavelet transforms W(f)(u), W(g)(v) with respect to the underlying Haar measure. Then (up to a constant) it holds

(*) (((W(f)(u),W(g)(v)))) = ((f,g)) * (u,v) .

This identity enables a combined Fourier wave & Calderon wavelet tool set for analyzing the H(-1/2) = H + G, whereby H denotes the L(2) Hilbert space and G its corresponding complement space with respect to the H(-1/2) norm.

The mathematical microscope approach enables a purely (distributional) Hilbert scale framework where the "microscope observations" of two wavelet (optics) functions f, g can be compared with each other by the corresponding "reproducing" ("duality") formula (*), whereby

- the "bra(c)"-wavelet transform W(f) is inverted by the adjoint operator of the "(c)ket"-wavelet transform W(g) (given corresponding admissibility conditions are valid)

- the identity (*) provides also some additional degree of freedom in the way that in order to analyze a signal s(t) the wavelet f can be chosen properly according to the special situation of the underlying mathematical model. The prize to be paid is only later, when the "re-building" wavelet g needs to be built accordingly to enable the corresponding "synthesis"

- the Hilbert transform operator (which is valid for every Hilbert scale) is a "natural" partner of the wavelet-transform operator, as it is skew-symmetric, rotation invariant and each Hilbert transformed "function" has vanishing constant Fourier term. The example in the context above is the Hilbert transform of the Gaussian distribution function, the (odd) Dawson function, with the "polynomial degree" point of zero at +/-infinite.

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