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TAUBERIAN THEOREMS FOR INTEGRAL TRANSFORMS OF DISTRIBUTIONS

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1. Introduction

Integral transforms of Schwartz's distributions and other generalized functions have been used in the last thirty years as a powerful tool especially in mathematical physics. The monograph of Zemanian [18] is the first one which gives systematic and general approach to different integral transforms of generalized functions. Tauberian type results, with the use of generalized asymptotic behaviours, have been elaborated only for some special integral transforms of distributions. Vladimirov, Drozhinov and Zav'yalov have given in [15] Abelian and Tauberian theorems for the Laplace transform. Various generalized asymptotics as well as Abelian and Tauberian type results for the Stiejtjes transform are investigated in [7]. Wiener type theorems (cf. [17], [10]) were studied in [6] and [8] via integral transforms with appropriate kernels.

We will give in this paper Tauberian theorems for integral transforms which are of Mellin convolution type and whose kernels belong to suitable test function spaces. The results are based on the Wiener–Tauberian theorems for distributions which are proved in [8]. The integral transforms under consideration will be the Laplace, Stieltjes, Weierstrass and Poisson transforms. We will give Tauberian type results for all these transforms.

2. Notation and definitions

We denote by L a slowly varying function ([1]). Then L is measurable and positive on $(0,\infty)$ and

$$\frac{L(xh)}{L(x)} \to 1, \quad h \to \infty, \quad \text{for every} \quad x \in \mathbf{R}_+ = (0,\infty).$$

As usual (cf. [11]), $\mathcal{D}(\mathbf{R})$ and $\mathcal{D}'(\mathbf{R})$ denote spaces of test functions and of Schwartz's distributions, respectively, and $\mathcal{D}_{L^1}(\mathbf{R})$ is the space of smooth

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