AN UNIFIED QUANTA ENERGY FIELD THEORY

**Current „realities" of affected
physical and mathematical areas**

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Dedicated to my wife Vibhuta
on the occasion of her 61th birthday, August 25, 2022

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A. Preface

This paper is about the „reality" part of „aspiration and reality" of an unified field theory. It provides an overview of current „realities" on today's most relevant affected physical and mathematical areas. The humble author's opinion is, that an unified field theory with the aspiration to integrate all current particle-field interaction models requires a paradigma change in theoretical physics.

In classical and quantum mechanics there are two types of systems to deal with, particle systems and field systems. Their similarities are preliminary based on the fact that both types are governed by an action principle (Lagrange, Feynman-Schwinger-Dyson).

The quantum field theories, the plasma dynamics theories, as well as the two theories of relativity operate with different mathematical concepts. Those concepts were developed by a step by step approach, which started 1900, when Max Planck introduced the theory of „quanta with specific energies" to explain „radiation" effects. This process was and is governed by the following paradigm of physics, (DeP) p. 551:

physics is scale dependent and decoupling

- Physics is scale dependent and at each scale, there are different degrees of freedom and different dynamics
    Therefore, at each scale level to be studied, there is the need for a different theory (e.g. classical continuum mechanics, theory of granular structure, nucleus + electronic cloud, nuclear physics, QED, free-electron theory, modelling, e.g. the properties of metals, semiconductors, and insulators) to describe the behavior of the considered physical system depending on a scale (of energies, distances, momenta, etc.). For example, in quantum field theory, the dependence of the behavior on the scale is often expressed mathematically by the fact that in order to regularize (i.e. render finite) Feynman diagram integrals one must introduce auxiliary scales, cutoffs, etc. The effect of these choices on the physics is encoded into the renormalization group equation. This equation then becomes an important tool for the study of physical theories.

- Physics at large scale decouples from the physics at a smaller scale
    When passing from a smaller scale to a larger scale irrelevant degrees of freedom are averaged over. Mathematically this means that they become integration variables and thus disappear.
    - In classical mechanics one deals with three scales according to its 3 basic measurements: distance D, time T, mass M
    - In non-relativistic quantum theory and classical relativity it has two scales: D & T resp. D & M
      (mass M can be expressed through T & D using the Planck constant resp. T can be expressed via D using the speed of light)
    - In relativistic quantum theory there is only one scale: distance D

The consequences of the step-by-step development process governed by the above paradigm resulted into

- paradoxes (from a natural science perspective) with respect to contradicting predictions
- related „dualism" interpretation (from a mathematics perspective) of paradoxes and case specific dynamic particle definitions.

B. Narratives in physics

The Planck and the Fermi oscillators
(ScE) p. 20

The energy levels of the Planck oscillator are

$$E_n = \left(n + \frac{1}{2}\right) h\nu \, , \, n = 0,1,2, \ldots$$

The sum-over-states (Zustandsumme) is given by ($\mu = \frac{1}{kT}$, where k denotes the Boltzmann constant)

$$Z = \sum_{n=0}^{\infty} e^{-E_n/kT} = \sum_{n=0}^{\infty} e^{-\mu E_n} = \sum_{n=0}^{\infty} e^{-\mu h\nu \left(n+\frac{1}{2}\right)}.$$

Then

$$\Psi := k \log Z = -\frac{1}{T} \sum_{n=0}^{\infty} E_n$$

is the negative free energy divided by T. Putting $x := \mu h\nu = \frac{h\nu}{kT}$ one have

$$Z = e^{-\frac{x}{2}} \sum_{n=0}^{\infty} e^{-nx} = e^{-\frac{x}{2}} \frac{1}{1-e^{-x}} = \frac{1}{2} \frac{1}{\sinh\left(\frac{x}{2}\right)} \, .$$

Hence

$$\Psi = k \log Z = -k \log 2 \left(\sinh\left(\frac{x}{2}\right)\right),$$

$$U = \frac{E}{N} = T^2 \frac{\partial \Psi}{\partial T} = -kT^2 \frac{1}{2} \frac{\cosh\left(\frac{x}{2}\right)}{\sinh\left(\frac{x}{2}\right)} \left(-\frac{h\nu}{kT^2}\right)$$

$$= \frac{h\nu}{2} \frac{e^{x/2}+e^{-x/2}}{e^{x/2}-e^{-x/2}} = \frac{h\nu}{2} \frac{e^{x/2}+1}{e^{x/2}-1}$$

$$= \frac{h\nu}{2} + \frac{h\nu}{e^{\frac{h\nu}{kT}}-1} = \frac{h\nu}{2} + \frac{h\nu}{e^{\mu h\nu}-1} \, ,$$

which is the well-known expression, in which the „zero-point energy" is usually dropped.

The Fermi oscillator is a particularly simple system (invented, as we shall see later, to formulate „Fermi statistics"). It is a thing capable only of two levels, 0 and $\varepsilon = E_1 = E_2 = \cdots$). Hence

$$Z = 1 + e^{-\varepsilon/kT}$$

$$\Psi = k \log\left(1 + e^{-\frac{\varepsilon}{kT}}\right)$$

$$U = \frac{E}{N} = T^2 \frac{\partial \Psi}{\partial T} = -kT^2 \frac{e^{-\frac{\varepsilon}{kT}}}{1-e^{-\frac{\varepsilon}{kT}}} \frac{\varepsilon}{kT^2} = \frac{\varepsilon}{1+e^{\frac{\varepsilon}{kT}}} \, .$$

The corresponding term of the Planck oscillator (taking $\varepsilon = h\nu$) is given by

$$U = \frac{h\nu}{2} + \frac{h\nu}{e^{\mu h\nu} - 1}$$

There is just one remarkable difference in sign, ± 1 in the denominator. We shall see later that this consitutes the relevant difference between „Einstein-Bose statstics" and „Fermi-Dirac" statistics.

The thermodynamical functions of a system composed of L Planck oscillators or L Fermi oscillators would, of course, be obtained on multiplying by L.

The Coulomb potential model

Matter consists of atomic nuclei and electrons. The Coulomb force acts between atomic nuclei and electrons. It determines the several appearances of matter and most of the observable phenomena ((FlT) p. 37. From a mathematical perspective the Coulomb force corresponds to Newton's gravitational force. Both are equipped with the same functional 3D specific k^{-2} „distance" dependency. the 3D-Coulomb force is proportional to the charge of the considered electrons; the 3D-gravitation force is proportional to the masses of the considered mass particles.

The Coulomb potential of electromagnetism is an example of a Yukawa potential with factor $e^{-\alpha m r}$ equal to 1, everywhere. This can be interpreted as saying that the photon „mass" m (i.e. the force-carrier between interacting, charged particles) is equal to 0. The Yukawa potential concept was developed to explain the results of James Chadwick's atomic model, which consisted of positively charged protons and neutrons packed inside of a small nucleus, with a radius on the order of 10^{-14} meters. Finally, the Yukawa potential concept resulted into the concept of meson particles. The Coulomb potential resp. the Yukawa potential are accompanied by plane resp. spheric waves.

The Dirac „function" $\delta(\vec{r} - \vec{r}_0)$ is a linear continuous functional over the space of continuous functions, which maps a continuous function depending on $\vec{r}$, in this case the constant function q, to the specific function value $q(\vec{r}_0) = q$. The functional cannot be represented as an integral, however, in a distributional sense it holds $f(\vec{r}_0) = (\delta(\vec{r}_0, \cdot), f)$. The corresponding concept in Hilbert space theory are dual Hilbert spaces.

We note that the mathematical models of both physical concepts, the Dirac potential and the Coulomb potential, very much depend on the 3D-case.

The Heaviside function

$$\theta(x - x_0) = \begin{cases} 0 & for \ x < x_0 \\ 1 & for \ x \geq x_0 \end{cases}$$

is linked to the Dirac function by

$$\frac{d}{dx}\theta(x - x_0) = \delta(x - x_0).$$

From the identity

$$\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|^3} = -grad\frac{1}{|\vec{r}-\vec{r}_i|} \text{ and } \vec{E}(\vec{r}) := \int \rho(\vec{s})\frac{\vec{r}-\vec{s}}{|\vec{r}-\vec{s}|^3}d^3\vec{s} = -grad \int \rho(\vec{s})\frac{1}{|\vec{r}-\vec{r}_i|}d^3\vec{s} = -grad\Phi(\vec{r})$$

it follows that the 3D electric field can be represented by a (scalar) electrostatic potential $\Phi(\vec{r})$ in the form

$$\vec{E}(\vec{r}) = -grad\Phi(\vec{r})$$

where

$$\Phi(\vec{r}) := \int \frac{\rho(\vec{s})}{|\vec{r}-\vec{r}_i|}d^3\vec{s} \ .$$

The Coulomb potential model is a generalization of the Coulomb force law, where there are two Coulomb forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ acting in the direction of the connecting line between two charged particles, whereby $\boldsymbol{F}_1 = -\boldsymbol{F}_2$. The two Coulomb forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are proportional to the product of the two charges q_1 and q_2 and are inverse proportional to the square of the distance of both particles, i.e.

$$F_1 = -F_2 = kq_1q_2\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|^3}.$$

The constant k determines the interaction force between both particles. It depends from the chosen unit of those charges. If there is a repulsive interaction between those particles this implies $k > 0$.

The charge density of N point charges are expressed by Dirac functions in the form

$$\rho(\vec{r}) = \sum_{i=1}^{N} q_i\delta(\vec{r} - \vec{r}_i).$$

In case one identifies the electrons and atomic nuclei with those charge this corresponds to the charge density of an atom. A continuous charge density

$$\rho(\vec{r}) = \int \rho(\vec{s})\delta(\vec{r} - \vec{s})d^3\vec{s}$$

can be approximated by N point charges in related N space partition areas by

$$\rho(\vec{r}) = \int \rho(\vec{s})\delta(\vec{r} - \vec{s})d^3\vec{s} = \sum_{i=1}^{N} \int_{\Delta V_i} \rho(\vec{s})\delta(\vec{r} - \vec{s})d^3\vec{s} \approx \sum_{i=1}^{N} q_i\delta(\vec{r} - \vec{r}_i) \ ^{(*)}.$$

Then, the electric field (which is also called the electric field-force) $\vec{E}(\vec{r})$ defined by

$$\vec{E}(\vec{r}) := \int \rho(\vec{s})\frac{\vec{r}-\vec{s}}{|\vec{r}-\vec{s}|^3}d^3\vec{s}$$

describes the relationship between an arbitrary charge density ρ and the electric field $^{(**)}$.

A charged particle with charge q in an electric field is governed by N particles with charges q_i experiences a „field force" in the form

$$\vec{F}_N(\vec{r}) = \sum_{i=1}^{N} qq_i\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|^3} = q\vec{E}_N(\vec{r}) \qquad (\text{resp. } \vec{F}(\vec{r}) = q\vec{E}(\vec{r}))$$

The physical modelling assumption is that the considered charge q are that small, that its size is not influenced by the charges of the electric field and its underlying charges q_i over time.

The force density of a given distribution of charges are defined by

$$\vec{f}_N(\vec{r}) = \frac{\Delta \vec{F}_N(\vec{r})}{\Delta V} = \frac{\Delta q}{\Delta V}\vec{E}_N(\vec{r}) \qquad (\text{resp. } \vec{f}(\vec{r}) = \rho(\vec{r})\vec{E}(\vec{r}))$$

where ΔV denotes the volume element at $\vec{r}$ and Δq denotes the related charge of that volume element.

$^{(*)}$ this is because of

$$\sum_{i=1}^{N} \int_{\Delta V_i} \rho(\vec{s})\delta(\vec{r} - \vec{s})d^3\vec{s} \approx \sum_{i=1}^{N} \left[\int_{\Delta V_i} \rho(\vec{s})d^3\vec{s}\right]\delta(\vec{r} - \vec{s}) = \sum_{i=1}^{N} q_i\delta(\vec{r} - \vec{r}_i).$$

$^{(**)}$ this is because of

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N} q_i\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|^3} = \sum_{i=1}^{N} \left[\int_{\Delta V_i} \rho(\vec{s})d^3\vec{s}\right]\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|^3} = \sum_{i=1}^{N} \left[\int_{\Delta V_i} d^3\vec{s}\right]\Delta V_i\rho(\vec{r})\frac{\vec{r}-\vec{r}_i}{|\vec{r}-\vec{r}_i|^3} \approx \int \rho(\vec{s})\frac{\vec{r}-\vec{s}}{|\vec{r}-\vec{s}|^3}d^3\vec{s}.$$

The Dirac potential model

(FeE): „Dirac's theory of radiation is based on a very simple idea; instead of considering an atom and the radiation field with which it interacts as two distinct systems, he treats them as a single system whose energy is the sum of three terms: one representing the energy of the atom, a second representating the electromagnetic energy of the radiation field, and a small term representing the coupling energy of the atom and the radiation field".

The Dirac model consists of an „atom" particle equipped with „energy", a „field particle" surrounded by „field energy", and a small „coupling energy term" between both energies. The corresponding model components are particles, anti-particles, and „boson" particles. In the electromagnetism case those are the electrons, the electron neutrinos, and the photons.

Mathematically speaking, the Dirac model is about a mathematical point element $x \in R^k$ and a related (electromagnetical) point charge (distribution) function $c \cdot \delta \in H_{-k/2-\varepsilon}$, carrying a point charge defined the corresponding physical problem related coupling constant c. The simplest related corresponding potential equation is given by the Laplacian operator accompanied by the gravitational constant $c_{grav} \sim 6{,}6 \cdot 10^{-11} \frac{N \cdot m^2}{kg^2}$ in the form

$$-\Delta u = c_{grav} \delta.$$

The Dirac model provides the blueprint for the conceptual design of the SMEP, where each of the three considered „force" phenomena requires correspondingly grouped „affected particles", related „(force specific) energy field particles (anti-particles)", and a small „coupling energy term" between the pair of the two energy types per considered force phenomenon. This conceptual design results into the today's zoo of $17 = 3 + 14$ fermions and bosons of the SMEP „model" representated in the form $U(1) \times SU(2) \times SU(3)$. Each of those symmetry groups represents the physical (conservation of energies) „symmetry" rule, by which the corresponding „zoo member" group are affected by its related „group force".

The mathematical-physical laws, which govern the charges and the related energy factors are represented by variational potential operator equations based on appropriately defined spaces, (ChJ). The Hilbert space for thermo-statistical relevant particles is the standard Hilbert space $L_2 = H_0$ equipped with the Lebesgue integral based inner product $(\cdot,\cdot)_0$.

The standard H_0-based variational energy Hilbert space with respect to the Laplacian potential operator is H_1, equipped with the (Dirichlet integral) inner product $D(\cdot,\cdot) := (\cdot,\cdot)_1 := (\nabla \cdot, \nabla \cdot)_0$. The elements of the kinematical energy Hilbert space H_1 can be represented as superposition of Fourier waves $\in H_1$. Accordingly, the $H_{-1/2}$-variational energy Hilbert space with respect to the Laplacian potential operator is the Hilbert space $H_{1/2}$, accompanied by the concept of wavelets.

We note that the mathematical link between the Hilbert scales H_α and the Dirac radiation theory is given by the Sobolev embedding theorem in the form $H_{-k/2-\varepsilon} \subset C^0$.

The Dirac (distributional) function provides a density distribution of charged point particles. The mass density of a point mass m at $\vec{r}_0$ is given by $\rho_m(\vec{r}_0) = m\delta(\vec{r} - \vec{r}_0)$. Its relation to the Laplacian operator is given by

$$\Delta(\frac{1}{|\vec{r}-\vec{r}_0|}) = -4\pi m \delta(\vec{r} - \vec{r}_0).$$

It can be generalized in the form

$$(\Delta + k^2)(\frac{e^{\pm ik(\vec{r}-\vec{s})}}{|\vec{r}-\vec{s}|}) = -4\pi m \delta(\vec{r} - \vec{s}).$$

The potential energy of a continuous charge distribution in its own field
(FIT)

The amount of work to be performed to move a point charge in the field $\vec{E}(\vec{r})$ from $\vec{r}_1$ to $\vec{r}_2$ is given by the scalar potential difference

$$W_{1,2} = q(\Phi(\vec{r}_1) - \Phi(\vec{r}_2)).$$

This potential difference is called the „voltage". The „work" is the product of „charge" and „voltage. As the field is assumed to be rotation free the work does not depend from the path between $\vec{r}_1$ and $\vec{r}_2$. The work has the dimension of an energy. The term

$$W(\vec{r}) = q\Phi(\vec{r})$$

is called the potential energy of a charge q in an electric field, whereby the charge q does not contribute to the potential Φ governed by N particles with charges q_i.

As a generalization of this case a charge distribution $\rho(\vec{r})$ within an external field Φ_{ext} can be considered, where there is no contribution of $\rho(\vec{r})$ to that field. Then, every charge element $dq = \rho(\vec{r})d^3\vec{r}$ contributes according to formula (*). The summation over all charged elements then gives

$$W = \int \Phi_{ext}(\vec{r})\rho(\vec{r})d^3\vec{r}.$$

In order to determine the electrostatic energy of N particles with charges q_i one considers $i-1$ point charges q_j, which rest at $\vec{r}_i$ ($j = 1, \dots (i-1)$). Then, the potential energy of another point charge q_i in the field of the given charges q_j is given by

$$W_i(\vec{r}_i) = q_i \sum_{j=1}^{i-1} \frac{q_j}{|\vec{r}_i - \vec{r}_j|}.$$

Then, the potential energy of a system with N point charges is given by

$$(*) \quad W = \sum_{i=2}^{N} W_i(\vec{r}_i) = \sum_{i=2}^{N} q_i \sum_{j=1}^{i-1} \frac{q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_{\substack{i,j=1 \\ i \neq j}}^{N} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}.$$

In order to derive a continuous charge distribution $\rho(\vec{r})$ the distribution is replaced by N discrete charges $\Delta q_i = \rho(\vec{r}_i)\Delta V_i$. In case ρ is continuous, then for $N \to \infty$ and $\Delta V_i \to 0$ this lead to

$$W = \frac{1}{2}\int \frac{\rho(\vec{r})\rho(\vec{s})}{|\vec{r} - \vec{s}|} d^3\vec{s}\, d^3\vec{r}.$$

The limit process is only applicable in case of $\vec{r} \neq \vec{s}$; this corresponds to $i \neq j$

$$(*) \quad W = \sum_{i=2}^{N} W_i(\vec{r}_i) = \sum_{i=2}^{N} q_i \sum_{j=1}^{i-1} \frac{q_j}{|\vec{r}_i - \vec{r}_j|} = \frac{1}{2}\sum_{\substack{i,j=1 \\ i \neq j}}^{N} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}.$$

In case the charge distribution is continuous the case $\vec{r} = \vec{s}$ contributes only a negligible energy contribution.

Note: The „continuity" requirement for $\rho(\vec{r})$ cannot be omitted. A charge density of point charges in the form $\rho = \sum q_i \delta(\vec{r} - \vec{r}_i)$ would lead to an infinite energy of the point charges in their own field [*].

[*] The self-energy case (the case $\vec{r} = \vec{s}$) of a continuous charge distribution $\rho(\vec{r})$ can be approximately considered by the finite energy of the (approximation) model of a homogeneously charged ball with radius $r \to 0$:

$$W = \frac{1}{2}\int \Phi(\vec{r})\rho(\vec{r})\, d^3\vec{r}.$$

In this case, the potential $\Phi(\vec{r})$ is generated by $\rho(\vec{r}) = \Delta\Phi(\vec{r})$. It results into the „work" $W = -\frac{1}{2}\frac{1}{4\pi}\int \Phi(\vec{r})\Delta\Phi(\vec{r})\, d^3\vec{r} = \frac{1}{8\pi}\int |\vec{E}(\vec{r})|^2\, d^3\vec{r}$.

In other words, an approximating self-energy density of a continuous charge distribution in its own field is given by $w(\vec{r}) := |\vec{E}(\vec{r})|^2$.

We also note the (Helmholtz) formula $(\Delta + k^2)(\frac{e^{\pm ik(\vec{r}-\vec{s})}}{|\vec{r}-\vec{s}|}) = -4\pi m\delta(\vec{r} - \vec{s})$.

Heisenberg's lost key

Indefinite metrics in a Hilbert space
and the degeneracy of the ground state

In (HeW) a mathematical formalism for an unified field theory is proposed. Its cornerstones are an indefinite metric in a Hilbert space and the degeneracy of the ground state (accompanied with the action variable J), [*].

The corresponding classical and variational kinematical energy model world is governed by the (hyperbolic) wave equation accompanied by the physical (cosmic or objective) time variable [**]. It is connected to the ground state variable (which is defined as an differential) by, (HeW1),

$$t := \frac{1}{h\frac{\partial v}{\partial J}} \ .$$

The wave equation is accompanied by the Green function, which is connected to the quantum world by the Green function integral, underlying particle interaction terms, and related Weyl spinors, (HeW).

In (HeW) the deviation from iso-spin-symmetry in electrodynamics is taken as indication for an asymmetry of the ground state, (DüH):

(HeW) 7-1: „*The deviation from iso-spin-symmetry in electrodynamics shall be taken as indication for an asymmetry of the ground state, (DüH). In fact the number of protons in the world seems to be very different from the number of neutrons, the number of electrons is very different from the number of neutrinos. Even the matter and antimatter should be distributed in the universe with equal average density – many glaxies might be consist of matter, equally many of antimatter – and if total isospin should be small in this way, the big asymmetry would remain, since in matter the total isospin would point in one direction, in antimatter in the opposite direction. Hence there would be a macroscopic deviation from symmetry in isospace.*

An asymmetry of the ground state and therefore a degeneracy of this state is a well-known phenomenon in many systems discussed in conventional quantum mechanics. Ferromagnetism, superfluidity, superconductivity, crystal structure are obvious examples. In such cases two important new phenomena appear ….: The degeneracy of the ground state enforces the existence of bosons of rest mass zero, as has been pointed out in a mathematical form by Goldstone (the Goldstone theorem). Some property of the ground state can be attached to the particles thereby changing normal particles into strange particles."

(HeW) 8-1: „*The asymmetry of the ground state with respect to the isospin group has been used in chapter 7 as explanation for the strange particle poles in the Green's functions and as basis for the spurion formalism. … It has been emphasized already in earlier papers on this subject, that empirically the asymmetry of the ground state seems to be closely connected with the existence of long-range forces, i.e. of particles with rest mass zero, (DuH). The asymmetry with respect to the isospin group comes in through the long-range forces of electrodynamics, the asymmetry of with respect to the the space reflection parity appears in the weak interactions, and this is the first interaction which affects neutrinos. It can be well understood that short-range forces allow a clear separation of the particles from the rest of the world, while long-range forces may lead to a dependence of the properties of the particles on the state of the world in large dimensions. This connexion has been found a mathematical expression in the theorem of Goldstone. .. In the present theory the goldstone theorem is the basis for an understanding of quantum electrodymamics*".

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[*] Applying the concept and elementary properties of indefinite inner product spaces on quantum field theory first appeared in the papers of P. Dirac, (DiP), and W. Pauli, (PaW). Soon afterwards, L. Pontrjagin (PoL), gave the first mathematical treatment of an indefinite inner product space.

[**] Husserl differentiates between the objective time of appearing objects, the subjective or preempirical time of acts and experiences and the prephenominal absolute flow of the internal time consciousness, ((ZaD) chapter 3.

Renormalization groups
(DeP) p. 551

The behavior of a physical system depends on a scale (of energies, distances, momenta, etc.) at which the behavior is studied. The change of a behavior when the scale is changed, is described by the renormalization group equation. In quantum field theory, the dependence of the behavior on the scale is often expressed mathematically by the fact that in order to regularize (i.e. render finite) Feynman diagram integrals one must introduce auxiliary scales, cutoffs, etc. The effect of these choices on the physics is encoded into the renormalization group equation. This equation then becomes an important tool for the study of physical theories.

Very generally speaking, the method of renormalization group is a method designed how to describe how the dynamics of some system changes when we change the scale (distance, energies) at which we probe it.

Two basic facts about physics:

Scale dependence

Physics is scale dependent. For example, consider a fluid. At each scale of distances, we need a different theory to describe its behavior:

- classical continuum mechanics (NSE)
- theory of granular structure
- nucleus + electronic cloud
- nuclear physics
- quantum chromodynamics
- string theory.

At each scale, we have different degrees of freedom and different dynamics.

Decoupling

Physics at large scale (largely) decouples from the physics at a smaller level. For example, to describe the behavior of fluid at the scale ~ 1cm, we don't know about the granular structure, nor about the atoms and nucleons. The only things we need to know are th viscosity and the density of the fluid. ….

Similarly, if we want to describe atoms, we don't need to know anything about the nucleus except its mass and electric charge.

When we pass fom a smaller scale to a larger scale, we average over irrelevant degrees of freedom. Mathematically this means that they become integration variables and thus disappear in the answer. This decoupling is the reason why we are able to do physics. If there was no decoupling, it would be necessary for Newton to know string theory to describe the motion of a viscous fluid.

> In classical mechanics we deal with three scales according to its 3 basic measurements (distance D, time T, mass M)
>
> In non-relativistic quantum theory (M can be expressed through T & D using the Planck constant) and classical relativity (T can be expressed via D using the speed of light) we have 2 scales
>
> In relativistic quantum theory we only have 1 scale – the scale of distance D (or equivalently – the (inverse) scale of momenta).

Brownian motion

Brownian noise or random walk noise or red noise (with zero means, constant variance, serially correlated in time, and strong in longer wavelengths)) has lower frequencies than (uncorreleted) visible light (i.e., white noise with zero means, constant variance, and uncorrelated in time)) is the kind of signal noise produced by Brownian motion.

A Brownian motion (also called a Wiener process) is obtained as the integral of a white moise signa. It is a continuous stationary stochastic process having independent increments, i.e. $B(t) - B(0)$ is a normal random variable with mean μt and variance $\sigma^2 t$, μ, σ^2 constant real numbers. The density function of a Brownian motion is given by

$$f_{B(t)}(x) = \frac{1}{\sqrt{2\pi\sigma^2 t}} e^{-\frac{(x-\mu t)^2}{2\sigma^2 t}}$$

The sample paths of Brownian motion are not differentiable, a mathematical fact explaining the highly irregular motions of small particles. The total variation of Brownian motion over a finite interval $[0, T]$ is infinite. It holds

$$Var\left[\frac{B(t)}{t}\right] = \frac{\sigma^2}{t}.$$

If $W(t)$ is a Wiener process on the interval $[0, \infty)$, then, as well the process

$$W^*(t) := \begin{cases} tW(1/t) & t > 0 \\ 0 & t = 0 \end{cases}.$$

White noise can be defined as the derivative of a Brownian motion (i.e. a Wiener process) in the framework of infinite dimensional distribution theory, as the derivative $B'(t)$ of $B(t)$, does not exist in the ordinary sense. Not only $B'(t)$, but also all derivatives of Brownian motion are generalized functions on the same space. For each t, the white noise $B'(t)$ is defined as a generalized function (distribution) on an infinite dimensional space.

Let $F[B'](\omega)$ denote the Fourier transform of $B'(t)$. A Brownian motion is obtained as the integral of a white noise signal $dB(t)$, i.e.,

$$B(t) = \int_0^t dB(\tau) = \int_0^t \frac{dB(\tau)}{d\tau} d\tau$$

meaning that Brownian motion is the integral of the white noise $dB(t)$ whose power spectral density is flat

$$S_0 = \left|F[B'](\omega)\right|^2 = const.$$

The spectral density S_0 for white noise is flat , i.e. $S_0/\omega^0 = c$ i.e., it is inversely proportional to ω^0. As it holds $Fourier[B'](\omega) = i\omega F[B](\omega)$, the power spectrum of Brownian noise is given by

$$S(\omega) = |F[B](\omega)|^2 = \frac{S_0}{\omega^2}.$$

This means, that the spectral density of Brownian (red) noise is S_0/ω^2, i.e., it is inversely proportional to ω^2, meaning it has more energy at lower frequencies, even more so than pink noise.

**Harmonic music "noise" signals
between red (Brownian) and white noise**

Conjecture: the spectral density for harmonic music "noise" signals is given by

$$S^*(\omega) = |F[B^*](\omega)|^2 = \frac{S_0}{\omega}.$$

Friction or damping forces,
thermodynamics & Gibbs ensemble,
stochastic (Langevin) forces,
and the Fokker-Planck equation
(RiH)

(RiH) p. 1 ff.: The simplest form of the Brownian motion of particles is a deterministic differential equation, where a friction force act on a small particle of mass m with velocity v immersed in a fluid. The simplest expression for such a friction or damping force is given by Stokes's law $F_c = -\alpha v$. Therefore, the (deterministic) equation of motion for the particle in the absense of additional forces reads $\dot{v} + \gamma v = 0$ with $\gamma = \frac{\alpha}{m} = 1/\tau$, where τ denotes the relaxation time.

The Fokker-Planck equation is just an equation of motion for the distribution function of fluctuating macroscopic variable, (RiH) p. 5. A special Fokker-Planck equation is an equation for the distribution function $W(v, t)$ for a-one-dimensional Brownian motion, (RiH) p. 4.

The physics behind the friction is that the molecules of the fluid collide with the particle. The momentum of the particle is transferred to the molecules of the fluid and the velocity of the particle therefore decreases to zero. The velocity $v(t)$ at the time t is completely determined by its initial value. This deterministic equation is valid only if the mass of the particle is large so that ist velocity due to thermal fluctuations is negligible.

From the equipartition law, the mean energy of the particle is (in one dimension)

$$\tfrac{1}{2} m \langle v^2 \rangle = \tfrac{1}{2} kT$$

where k is Boltzmann's constant and T is the temparature.

For smaller mass m the thermal velocity may be observable and therefore the velocity of a „small" particle cannot be described exactly by $\dot{v} + \gamma v = 0$.

If the mass of the small particle is still large compared to the mass of the molecules, one expects $m\dot{v} + \alpha v = 0$ to be valid approximately. However, this equation must be modified so that it leads to the formal correct thermal energy $\frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} kT$. The modification consists in adding a fluctuation force $F_f(t)$ on the right-hand side of $m\dot{v} + \alpha v = 0$, i.e., the total force of the molecules acting on the small particle is decomposed into a contibuous damping force $F_c(t)$ and a fluctuating force $F_f(t)$ in the form

$$F(t) = F_c(t) + F_f(t) = -\alpha v(t) + F_f(t).$$

This force $F_f(t)$ is a stochastic or random force, the properties of which are given only in the average.

Why the stochastic (Langevin) forces occurs?

If we were to treat the problem exactly, we should have to solve coupled euqations of motion for all the molecules of the fluid and for the small particle, and no stochastic force would occur. Because of the large number of molecules in the fluid (the number is of order 10^{23}), however, we cannot generally solve these coupled equations. Furthermore, since we do not know the inital values of all the molecules of the fluid. If we were to use another system (particle and fluid) identical to the first except for the inital values of the fluid, a different motion of the small particle results. As usually done in thermodynamics, we consider an ensemble of such systems (Gibbs ensemble). The force $F_f(t)$ then varies from system to system and the only thing we can do is to consider averages of this force for the ensemble. Inserting $F(t) = F_c(t) + F_f(t) = -\alpha v(t) + F_f(t)$ into $m\dot{v} + \alpha v = 0$ and dividing by the mass we get the (stochastic differential) equation of motion $\dot{v} + \gamma v = \Gamma(t)$. Here we have introduced the fluctuation force per unit mass $\Gamma(t) = \frac{F_f(t)}{m}$, which is called the (stochastic) Langevin force.

First we assume that its average over the ensemble should be zero $\langle \Gamma(t) \rangle = 0$, because the equation of motion of the average velocity $\langle v(t) \rangle = 0$ should be given by $m\dot{v} + \alpha v = 0$. If we multiply two Langevin forces at

different times we assume that the average value is zero for time differences $t' - t$ which are larger than the duration time τ_0 of a collision, i.e., $\langle \Gamma(t)\Gamma(t')\rangle = 0$ for $|t' - t| \geq \tau_0$. This asumption seems to be reasonable, because the collisions of different molecules of the fluid with the small particle are approximately independent. Usually, the duration time τ_0 of a collision is much smalller than the relaxation time $\tau = 1/\gamma$ of the velocity of the small particle. We may therefore take the limit $\tau_0 \to 0$ as a reasonable approximation, giving the correlation

$$\langle \Gamma(t)\Gamma(t')\rangle = q\delta(t - t')$$

where $q = 2\gamma kT/m$ denotes the noise strength of the Langevin force. The δ function appears because otherwise the average energy of the small particle cannot be finite as it should be according to the equipartittion law $\frac{1}{2}m\langle v^2\rangle = \frac{1}{2}kT$. One usually assumes that the $\Gamma(t)$ have a Gaussian distribution with δ correlation. A noise strength with the δ correlation is called white noise, because the spectral distribution which is given by the Fourier transform of the correlation equation is then independent from the frequency ω. If the stochastics forces are not δ correlated, i.e., if the spectral density depends on the frequency, one uses the term colored noise.

Because in $\dot{v} + \gamma v = \Gamma(t)$ the force $\Gamma(t)$ varies from system to system in the ensemble, i.e., it is a stochastic quantity, the velocity will also vary from system to system, i.e., it will become a stochastic quantity, too. We therefore may ask for the probability to find the velocity in the interval $(v, v + dv)$, or in other words we may ask for the number of systems of the ensemble whose velocity are in the interval $(v, v + dv)$ divided by the total number of systems in the ensemble. Because v is a continuous variable we may ask for the probability density $W(v)$, also often called probability distribution. The probability density times the length dv is then the probability of finding the particle in the interval $(v, v + dv)$. The equation of motion for the distribution function $W(v, t)$ is given by (one of the simplest Fokker-Planck equations)

$$\frac{\partial W}{\partial t} = \gamma \frac{\partial(vW)}{\partial t} + \gamma \frac{kT}{m}\frac{\partial^2 W}{\partial v^2}.$$

Quantization of elastic waves in solid state physics
phonon $\cong$ photon

(KiC) p. 99: „*The energy of a lattice vibration is quantized. The quantum of energy is called a phonon in analogy with the photon of the electromagnetic wave. ... The energy of an elastic mode of angular frequency ω and the related zero point energy of the mode are equivalent to a quantum harmonic oscillator frequency, the energy eigenvalues in the form $\left(n + \frac{1}{2}\right)\frac{h}{2\pi}\omega$ and $\frac{1}{2}\frac{h}{2\pi}\omega$. .. A phonon of wavevector K will interact with particles such as photons, neutrons, and electrons as if it had a momentum $\frac{h}{2\pi}K$. However, a phonon does not carry physical momentum.*"

Quantum theory from Schrödinger to Sommerfeld and the Coulomb problem

The difference between the Schrödinger (bounded electron in a hydrogen atom related matter waves with smaller velocity than the light speed) and the earlier (relativistic) De Broglie „matter waves" (for free particles only) is, that Schrödinger indroduced an additional potential function $V(x)$ as model for the „exterior forces" affecting the bounded particles, i.e.,

$$E_{de\ Broglie} = c\sqrt{(mc)^2 + P^2} \approx mc^2 + \frac{P^2}{2m} \quad \rightarrow \quad E = c\sqrt{(mc)^2 + P^2} + V(x) \approx mc^2 + \frac{P^2}{2m} + V(x).$$

The Schrödinger equation

For the "Schrödinger" energy $E_{Schrö} = \frac{P^2}{2m} + V(x)$ the De Broglie relations between wave and particle theory $E = \hbar\omega$ and $P = \hbar k$ are only valid in a "variational sense" in the form $E\psi = -\frac{\hbar}{i}\frac{\partial}{\partial t}\psi$ and $P = \hbar k$ governed by the famous Schrödinger equation $-\frac{\hbar}{i}\frac{\partial}{\partial t}\psi = E\psi = \left[\frac{P^2}{2m} + V(x)\right]\psi$ and the related eigenvalues E_k of the corresponding Hamiltonian operator depending from quantum number k, (MaW1) S. 105.

The Stern-Gerlach experiment

Then the Stern-Gerlach experiments showed that there is a magnetic property of the observed "electron", which is not reflected by the "Schrödinger" energy term.

The Dirac equation

The conclusions out of the Stern-Gerlach experiment lead to the theoretical concept of the "hypothesis of an electron spin" accompanied by an additional assumed angular momentum (energy) of an "electron", resulting into Dirac's model of a bounded electron in a hydrogen atom. Later, the Dirac model turned out to be a relativistic model of an "electron" in contrast to the non-relativistic Schrödinger model.

The Lamb-shift phenomenon

Then the Lamb-shift phenomenon was observed, which falsified the Dirac model. Assuming that the bounded electron has two complementary kinetical energies, a spin-orbit momentum and a spin-angular momentum Sommerfeld derived an adapted energy (*approximation*) formula to the Dirac energy in the form

$$E = E_{k,\rho} = \frac{mc^2}{\sqrt{1 + \frac{\alpha^2}{\rho + \sqrt{k^2 - \alpha^2}}}} \ .$$

We use the term "energy approximation formula" as Sommerfeld derived his formula based on the following two physical and mathematical assumptions:

- the bounded electron is assumed to be a physical object with kinetical (orbit) movements and independent complementary kinetic (rotation) movements equipped with spin-orbit quantum numbers k resp. with the radial quantum numbers ρ
- the corresponding mathematical model assumed only classical convergent power series representations of wave function solutions of the Dirac equation for very *small* and very large distances r.

The mathematical assumptions lead to famous fine structure constant α. The subsequent physical interpretation of the *small* value of the fine structure constant was that it ensures that all bounding states of an electron in a hydroxen atom only differ very little from the restless energy mc^2. This ensures a good *physical* approximation model to the non-relativistic mathematical quantum mechanics model.

For an interpretation as a locally quantized wave function by the fine structure constant we refer to (XiK).

The Coulomb potential problem
(RoH) p. 147/ 166 ff. freely translated

Two-particle system play the role of paradigms in all areas of physics, from astronomy to sub-atomic physics. Therefore, the analysis of quantum properties of two-particle-systems became a fundamental part of quantum physics. … The approach is analogous to classical mechanics, where the two-particle system is treated as an effective one-particle system by spin-off of the center of gravity movement considering quantum mechanical states in imple potentials, like caste potentials with states in Coulomb or Oszillator potentials. The most simple assumptions are, that the occurring wave functions do not depend from angles, i.e. there is only an anguar momentum of $l = 0$. Giving up this premise leads to the concept of energy states for centrosymmetric one-particle systems. Mathematically speaking, this is about a required angular-symmetric Lagrange function resulting into qualitatively different consequences/conclusions for the cases $l = 0$ and $l > 0$. In the first case, there is only one state related to the energy $E_{n,0}$; in the second case there are at least $2l + 1$ linear independent states $\varphi_{l,m}$ with identical energy $E_{n,l}$. This is what is called a $2l + 1$ direction degeneration or a m-degeneration; the term "at least" is used, as it cannot excluded that for different l-values with same energy.

A more detailed analysis of the consequences of the angular invariance accompanied by the l-dependency of the wave equation and the related Hamiltonian operator shows the hidden symmetry of the Coulomb potential problem, where the energy levels E_n of the Coulomb spectrum with the main quantum number n contains all orbital angular momenta from $l = 0$ to $l = n - 1$.
(For the Coulomb problem there exists the symmetry group $SU(2) \times SU(2)$, which is related to an ellipse-orbit movement of a particle in a $1/r$ (Coulomb/Netwton) potential.)

The origin of the hidden symmetry of the Coulomb potential comes from the classical movement of a particle in a Coulomb field. Pauli's solution of the related movement equation goes back to W. Lenz 1924 (before the quantum mechanics) in the context of a kind of semi-classical treatment of an atomic structure. Thereby, Lenz's approach was based on a "new method" from C. Runge, to solve the movement equation of a central force field, where $f(r)$ is assumed to be an arbitrary rotationally symmetrical force function in combination with an assumed time-independency of the orbital angular momentum.

In simple words, Runge's mathematical method (assuming only classical convergent power series representations of wave function solutions of the Dirac equation for very *small* and very large distances r, (MaW) p. 65) is the origin of the physical concepts of spin-orbit quantum numbers k in combination with radial quantum numbers ρ from which Sommerfeld derived his famous energy formal introducing the famous fine structure constant α.

<h1 style="text-align:center">Schrödinger's concept of a heat-bath</h1>

Schrödinger's concept of a heat-bath
E. Schrödinger

Schrödinger's thermo-statistical dilemma of the Dirac model (particle – anti-particle – coupling energy) is the fact that the system I (particle – anti-particles) equipped with quantum numbers $n_1, n_2, n_3, \ldots n_s, \ldots$ determines (only) one quantum state, while the system II (coupling energy) is described by the Dirac point charge model equipped with a „quantum state" n_0. This means the overall set of quantum numbers of the Dirac model is $n_s = 0,1,2,3$. However, the zero-point energy" $\frac{1}{2}h\nu$ of a Planck oscillator is borne out directly observation in the cristal lattices inducing a set of quantum numbers of both systems in the form $n_s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$. We also note that the mathematical potential model to link both „systems" is given by the Coulomb-Newton potential. Its proper mathematical handling and usage require a restriction to the 3D case.

(ScE) p. 1: "*There is, essentially, only one problem in statistical thermodynamics: the distribution of a given amount of energy E over N identical systems. Or perhaps better: to determine the distribution of an assembly of N identical systems over the possible states in which this assembly can find itself, given that the energy of the assembly is a constant E. The idea is that there is weak interaction between them, so weak that the energy of interaction can be disregarded, that one can speak of the "private" energy of every one of them and that the sum of their "private" energies has to equal E. The distinguished role of the energy is, therefore, simply that it is a constant of the motion – the one that always exists, and, in general, the only one.*"

"To determine the distribution" .. mean in principle to make oneself familiar with any possible distribution-of-the-energy (or state-of-the-assembly) …. is (always the same) the mathematical problem; we shall (soon) present its general solution, from which in the case of every particular kind of system enery particular classification that may be desirable can be found as a special case:

But there are two different attitudes as regards the physical application of the mathematical result. …

The older and more naive application is to N actually existing physical systems in actual physical interaction with each other, e.g. gas molecules or electrons or Planck oscillators or degrees of freedom ("ether oscillators") of a "hohlraum". The N of them together represent the actual physical system under consideration. This original point of view is associated with the names of Maxwell, Boltzmann and others.

But it suffices only dealing with a very restricted class of physical systems – virtually only with gases. It is not applicable to a system which does not consist of a great number of identical constituents with "private" energies. …

Hence a second point of view … has been developed. It has a particular beauty of its own, is applicable quite generally to every physical system, and has some advantages to be mentioned forthwith. Here the N identical systems are mental copies of the one system under consideration – of the one macroscopic device that is actually erected on our laboratory table. Now what on earth could it mean, physically, to distribute a given amount of energy E over these N mental copies? The idea is, in my view, that you can, of course, imagine that you really had N copies of your system, that they really were in "weak interaction" with each other, but isolated from the rest of the world. Fixing your attention on one of them, you find it in a peculiar kind of "heat-bath" which consists of the $N-1$ others.

Now you have on the one hand, the experience that in thermodynamical equilibrium the behavior of a physical which you place in a heat-bath is always the same whatever be the nature of the heat-bath that keeps it at constant temperature, provided, of course, that the bath is chemically neutral towards your system, i.e., that there is nothing else but heat exchange between them. On the other hand, the statistical calculations do not refer to the mechanism of interaction: they only assume that it is "purely mechanical", that it does not affect the nature of the single systems (e.g., that it never blows them to pieces), but merely transfers energy from one to the other.

These considerations suggest that we may regard the behavior of any one of those N systems as describing the one actually existing system when placed in a heat-bath of given temperature. Moreover, since N systems are a likely and number similar conditions, we can then obviously, from their simultaneous statistics, judge of the probability of finding our system, when placed in a heat-bath of given temperature, in one or other of its private states. Hence all questions concerning the system in a heat-bath can be answered. …"

Statistical Thermodynamics
E. Schrödinger

(ScE) p. 44: *„The different cases in the evaluation of the sum over states „Z" arise thus: The values admitted for every n_s may be*

$n_s = 0,1,2,3,4, \dots$ *(Bose-Einstein gas);*
$n_s = 0,1$ *(Fermi-Dirac gas, Pauli´s exclusion principle).*

There may or may not be condition that the total number of particles is constant, $n = \sum_s n_s$.

(ScE) p. 50: *Not until the idea of photons had gained considerable ground did Bose (about 1924) point out that we could, alternatively to the „holhraum" oscillator statistics, speak of photon statistics, but then we ad to make it „bose statistics". Very soon after, Einstein applied the same to the particles of an ideal gas. And thereupon I pointed out that we could also in this case speak of ordinary statistics, applied to the wave-mechanical proper vibrations which correspond to the motion of the particles of the gas.*

The wave point of view in both cases, or at least in all Bose cases, raises another interesting question. Since in the Bose case we seem to be faced, mathematically, with simple oscillator of the Planck type, of which the n_s is the quantum number, we may ask whether we ought not to adopt for n_s half-odd integers

$$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \dots n + \frac{1}{2}, \dots$$

rather then integers. One must, I think, call that an open dilemma. From the point of analogy one would very much prefer to do so. For, the "zero point energy" of a Planck oscillator is not only borne out by direct observation in the case of crystal lattices, it is also so intimitely linked up with the Heisenberg uncertainty relation that one hates to dispense with it. On the other hand, if we adopt it straightaway, we get into serious trouble, especially on contemplating changes of the volume (e.g. adiabatic compression of a given volume of black-body radiation), because in this process the (infinite) zero-point energy seems to change by infinite amounts! So we do not adopt it, and we continue to take for the n_s the integers, beginning with 0."

The Lamb shift

The Lamb shift phenomenon says that the energy values of an electron in the hydrogen potential field shows slighly different values than the related discrete energy knots $\sqrt{\lambda_n} = n$. The fine structure constant of the hydrogen atom is a mathematical correction term in the Sommerfeld energy formula derived from the „hydrogen" Dirac equation governed by the Coulomb potential operator based on the „physical relevant" stationary solutions of this equation. It is derived from Dirac's spin-orbit momentum operator by a mathematical „trick", which may be called „two-component-separation of the angular momentum". Beside the restriction to only physical relevant solutions of the Dirac equation there is also a mathematical approximation error, which is caused by a finite power series approximation of this separation (phys., a shrinking of the set of quantum numbers).

The definition of plasma

(ChF) p. 3: *„Any ionized gas cannot be called a plasma, of cource; there is always some small degree of ionization in any gas. A useful definition is as follows:*

A plasma is a quasineutral gas of charged and
neutral particles which exhibits collective behavior.

Phase vs. group velocity of waves in a plasma
exceeding vs. not exceeding the velocity of light c

Plasma is an inoized gas consisting of approximately equal numbers of positively charged ions and negatively charged electrons. The nearly equal numbers of the plasma electron & positron elements is the most relevant physical differentiator between plasma matter states and „standard" matter states.

(DeR) p. 94: „*The Landau damping phenomenon is complementary to the properties of electro-magnetic forces, which weaken themselves spontaneously over time w/o increase of entropy or friction. Landau damping involves a flow of energy between single particles on the one hand side, and collective excitations of plasma on the other side*". Physically speaking, there is a wave damping without energy dissipation by elementary particle collisions, i.e. the Landau damping phenomena is about the possibility of resonance between the wave phase velocity and the velocity of individual electrons.

(ChF) p. 81: The phase velocity of a wave in a plasma often exceeds the velocity of light c. This does not violate the theory of relativity, because an infinitely long wave train of constant amplitude cannot carry information. The carrier of a radio wave, for instance, carries no information until it is modulated. The modulation information does not travel at the phase velocity but at the group velocity, which is always less than c. To illustrate this, we may consider a modulated wave formed by adding („beating") two waves of nearly equal frequencies. Let these waves be

$$E_1 = E_0 cos[(k + \Delta k)x - (\omega + \Delta\omega)t]$$

$$E_2 = E_0 cos[(k - \Delta k)x - (\omega - \Delta\omega)t]$$

E_1 and E_2 differ in frequency by $2\Delta\omega$. Since each wave must have the phase velocity ω/k appropriate to the medium in which they propagate, one must allow for the difference $2\Delta\omega$ in propagation constant. Using the abbreviation $a = kx - \omega t, b = (\Delta k)x - (\Delta\omega)t$ we have

$$E_1 + E_2 = E_0 \cos(a + b) + E_0 \cos(a - b)$$
$$= E_0[\cos a \cos b - \sin a \sin b + \cos a \cos b + \sin a \sin b]$$
$$= 2E_0 \cos a \cos b$$

i.e., $E_1 + E_2 = 2E_0(coskx - \omega t)cos(\Delta k)x - (\Delta\omega)t$. This is a sinusoidally modulated wave. The envelope of the wave, given by $cos(\Delta k)x - (\Delta\omega)t$, is what carries information; it travels at velocity $\Delta\omega/\Delta t$. Taking the limit $\Delta\omega \rightarrow 0$, we define the group velocity to be

$$v_g = d\omega/dk.$$

It is this quantity that cannot exceed c.

(ChF) p. 245: Landau damping is a characteristic of collisionsless plasmas, but it may also have application in other fields. For instance, in the kinetic treatment of galaxy formation, stars can be considered as atoms of a plasma interacting via graviational rather than electromagnetic forces. Instabilities of the gas of stars can cause spiral arms to form, but this process is limited by Landau damping [*].

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[*] (ShF) p. 402: The Landau damping phenomenon may be interpreted as
- a long term asymptotic approximation behavior to neutron elements
- the capability of stars to organize themselves in a stable arrangement in a vacuum
  "*In its purest form, Landau damping represents a phase-space behavior peculiar to collisionless systems. Analogs to Landau damping exist, for example, in the interactions of stars in a galaxy at the Lindblad resonances of a spiral downsity wave. Such resonances in an inhomogeneous medium can produce wave absorption (in space rather than in time), which does not usually happen in fluid systems in the absence of dissipative forces (an exception in the behavior of corotation resonances for density waves in a gaseous medium) … Landau amplification can take place only for charged-particle distribution functions that display some sort of anomalous behavior in phase space*".

**Linear vs. nonlinear Landau damping phenomena
arising from different physical effects**

(ChF) p. 248-249: There are actually two kinds of Landau damping: linear Landau damping, and nonlinear Landau damping. Both kinds are independent of dissipative collisional mechanisms. If a particle is caught in the potential well of a wave, the phenomenon is called „trapping". Particles can indeed gain or lose energy in trapping. However, trapping does not lie within the purview of the linear theory. …. Trapping is not in the linear theory. When a wave grows to a larger amplitude, collisonless damping with trapping occur. One then finds that the wave does not decay monotonically; rather the amplitutes fluctuates during the decay as the trapped particles bounce back and forth in the potential wells. This is nonlinear Landau damping. .. Since the linear Landau damping is derived from a linear theory, … the nonlinear Landau damping must arise from a different physical effect. The question is: Can untrapped electrons moving close to the phase velocity of the wave exchange energy with the wave?

(ChF) P. 254: Neither the untrapped particles nor particle trapping are responsible for linear Landau damping. … Indeed, there are particles in the original distribution which have velocities so close to v_φ that at the time t they have not yet gone half-wavelength relative to the wave. For these particles, one cannot take the average $\langle \Delta W_k \rangle$. These particles can absorb energy from the wave and are properly called the „resonant" particles. As time goes on, the number of resonant electrons increases, since an increasing number will have shifted more than $\frac{1}{2}\lambda$ from their original positions. The damping rate, however, can stay constant, since the amplitude is now smaller, and it takes fewer electrons to maintain a constant damping rate.

(ChF) p. 260: The resonant particles
We are now in a position to see precisely which are the resonant particles that contribute to linear damping. … These particles rapidly bcome spread out in phase, so that they contribute little to the average; the intial distribution is forgotten. … Those particles may include both trapped and untrapped particles. This phenomenon is unrelated to particle trapping.

(ChF) p. 260: Two paradoxes resolved
The function which describes the relative contribution of various velocity groups to Landau damping is an even function of $\omega - ku$ so that the particles going both faster than the wave and slower than the wave add to Landau damping. On the other hand, the slope of the curve curve of this function, … is an odd function of $\omega - ku$; and one would infer from this particles traveling faster than the wave give energy up to it, while those traveling slower than the wave takes energy from it. The two descriptions differ by an integration by parts. Both descriptions are correct; which one is the be chosen depends on whether one wishes to have $\hat{f}_0(v)$ or $\hat{f}_0'(v)$ in the integrand (of the formula of the rate of change of the wave energy density function).

A second paradox concerns the question of Galilean invariance. If we take the view that damping requires there be fewer particles traveling faster the wave than slower, there is no problem as long as one is in the frame in which the plasma is at rest. However, if one goes into another frame moving with velocity than V, there would appear to be more particles faster than the wave than slower, and one would expect the wave to grow instead of decay. This paradox is removed by reinserting the second term $\frac{2ku}{\omega - ku}$, which we neglected. This term can make $\langle \Delta W_k \rangle$ negative … and the wave appears to have negative energy (that is, there is more energy in the quiescent, drifting Maxwelllian distribution than in the presence of an oscillation). The wave „growth", but adding energy to negavtive energy wave makes ist amplitude decrease.

(ChF) p. 261: We have seen that Landau damping is directly connected to the requirement that $f_0(v)$ be initially uniform in space. On the other hand, one can generate undamped electron waves if $f_0(v, t = 0)$ is made to be constant along the particle trajectories initially. Those particled will neither gain nor lose energy, on the average, if the plasma is initially prepared to that the density is constant along each trajectory. Such a wave is called a BGK mode (I. B. Bernstein, J. M. Green, M. D. Krustal).

Microwave radiation pressure to plasma
Ponderomotive force

(ChF) p. 305, 307: Light waves exert radiation pressure which is usually very weak and hard to detect. … When high-powered microwaves or laser beams are used to heat or confine plasmas the radiation pressure can reach several hundred thousand atmospheres! When applied to plasma, this force is coupled to the particles in a somewhat subtle way and is called the ponderomotive force. Many nonlinear phenomena have a simple explanation in terms of the ponderomotive force.

Nonlinear Landau damping or growth
Potential due to ponderomotive force

(ChF) p. 328: When the amplitude of an electron or ion wave excited, say by a grid is followed in space, it is often found that the decay is not exponential, as predicted by linear theory, if the amplitude is large. Instead, one typically finds that the amplitute decays, grows again, and then oscillates before settling down to a steady value. … although other effects may also be operative, these oscillations in amplitudes are exactly what would be expected from the nonlinear effect of particle trapping discussed in section 7.5. Trapping of velocity occurs when its energy in the wave frame is smaller than the wave potential. Small waves will trap only these particles moving at high speeds near v_φ. … When the wave is large, its linear behavior can be exspected to be greatly modified. .. There is a bounce frequency ω_B of a sinusoidal potential well with corresponding potential and equation of motion, where the frequency is not constant unless x is small, and the potential is approximattely parabolic. … When the resonant particles are reflected by the potential, they give kinetic energy back to the wave, and the amplitude increases. When the particles bounce again from the other side, the energy goes back into the particles, and the wave is damped. Thus, one would expect oscillations in amplitutde at the frequence ω_B in the wave frame. … The condition $\omega_B \geq \omega$ turns out to define the breakdown of linear theory even when other processes besides particle trapping are responsible. Another typ of nonlinear Landau damping involves the beating of two waves. Suppose there are two high-frequency electron waves (ω_1, k_1) and (ω_2, k_2). These would beat to form an amplitutde envelope traveling at a velocity $\frac{\omega_2 - \omega_1}{k_2 - k_1} \approx \frac{d\omega}{dk} = v_g$. This velocity may be low enough to lie within the ion distribution function. There can then be an energy exchange with the resonant inos. The potential the ions see is the effective potential due to the ponderomotive force, and Landau damping or growth can occur. Damping provides an effective way to heat ions with high-frequency waves, which do not ordinary interact with ions. If the ion distribution is double-humped, it can excite the electron waves, Such an instability is called a modulational instability.

The Korteweg-de Vries equation and the nonlinear
Schrödinger equations of nonlinear plasma physics
Ponderomotive force forming isolated structures
called envelope solitary waves

(ChF) p. 330: There are two nonlinear equations that have been treated extensively in connection with nonlinear plasma waves: The Korteweg-de Vries equation and the nonlinear Schrödinger equation. Each concerns a different type of nonlinearity. …

When an electron plasma wave goes nonlinear, the dominant new effect is that the ponderomotive force of the plasma waves causes the background plasma to move away, causing a local depression in density called a caviton. Plasma waves trapped in this cavity then form an isolated structure called an envelope soliton or envelope solitary wave. Such solutions are described by the nonlinear Schrödinger equation. Considering the difference in both the physical model and the mathematical form of the governing equations, it is surprising that solitons and envelope solitons have almost the same shape.

α, β, γ-decay phenomena

The α, β, γ-decay phenomena may be also called α, β, γ- „energic particle transformations". In the proposed model those different energic particle transformations are governed by the table of affected sets of quantum numbers.

The β-decay is about an ionized radiation, which occurs by the β transformation of an energic β-particle of a nucleus (i.e., a „neutron") leaving the nucleus, while generating at the same time an anti-neutrino or a neutrino. On average this neutron decays into an electron and a proton within 15 minutes.

The only „standard models" of the SMEP, which do not annihilate each other are the electrons and the positrons. They emit photons in the form $e^+ + e^- \rightarrow \gamma\gamma$. In the proposed model the neutrino may be interpreted as a photon. It avoids the up/down attributes of the related „standard models" of the SMEP accompanied by the YME model.

The α-decay is about instable atomic nuclei. The related energic particle transformations are the change from one atomic nucleus to another, both governed by different kinetic potential operators. The mathematical tool governing this process with respect to the impact of such a change on the corresponding potential operator changes is given by the inequalities

$$\lambda^{-\alpha} \leq \delta^{2\alpha} + e^{\tau(\delta^{-1} - \sqrt{\lambda})} \quad \text{for any } \tau, \delta, \alpha > 0 \text{ and } \lambda \geq 1$$

$$\vartheta^{-2\alpha} \leq \theta^{-2\alpha} + e^{\tau(\theta - \vartheta)} \quad \text{for any } \tau, \theta, \alpha > 0 \text{ and } \vartheta \geq 1.$$

The lifespan of atomic states
(FlT)

In Bohr's atomic model an electron with a certain mass and charge $-e$ circles a proton with a certain mass and charge e, whereby the angular momentum is a multiple of the Planck constant. The balance of power and the angular momentum quantification (where the reduced mass has been approximated by m_e, because of $m_p \gg m_e$) is given by, ($\tilde{h} := \frac{h}{2\pi}$)

$$\frac{m_e v^2}{r} = \frac{e^2}{r^2} \quad , m_e v r = n, n \in N.$$

For $n = 1$ one gets the Bohr radius α_B and the fine structure constant α in the form

$$r := \alpha_B := \frac{\tilde{h}^2}{m_e e^2}, \quad v := v_{at} := \frac{e^2}{\tilde{h}} = \frac{e^2}{\tilde{h}c} c = \alpha c.$$

It follows that the rotation frequency of an atom is given by

$$\omega_{at} := \frac{v_{at}}{\alpha_B} = \frac{m_e e^4}{\tilde{h}^3} \sim 4 \cdot 10^{16} \frac{1}{sec}.$$

An excited atomic state ($n \geq 2$) goes into another lower state by radition of an photon. It there has a finite lifespan. This lifespan τ is determined by the time, to release the required energy during the circular motion

$$P = \frac{2}{3} \frac{e^2}{\alpha_B} \alpha^3 \omega_{at} \quad \text{i.e.,} \quad \tau \sim \frac{E_{at}}{P} \approx \frac{1}{\alpha^3} \frac{1}{\omega_{at}}.$$

Physical coupling constants

The Krein space framework provides the concept of „potential barriers" between $x^+_{\kappa.(\tau)}$ and $x^-_{\kappa.(\tau)}$ quantum elements. For given constants c the potential of the elements $x^+_{\kappa.(\tau)} + x^-_{\kappa.(\tau)}$ enables the definition of corresponding hyperboloids H_c, hyperbolic regions V_c, conical regions V_0, and the boundary of conical regions, which are asymptotic conical manifolds for the hyperboloids.

The most prominent candidates for those potential constants in the plasma case are the the permeability constants ε_0 resp. μ_0 of elementary particles of an electric resp. magnetic field in a vacuum. As there are two „wave types" affected the dynamics of both fields is governed by the telegraph equation.

The related „electric resp. magnetic mass" constants are the natural candidates for the two field in the Mie theory. Accordingly, their corresponding dynamics is governed by two wave equations in line with the Maxwell equations. Accordingly, the today's formula $\varepsilon_0 \cdot \mu_0 = \frac{1}{c^2}$ as a consequence of the two independent wave equations need to be revisited.
The Planck action constant requires the concept of „physical time" accompanied by kinematic (time-dependent) energy norms.

Superfluids, Superconductors, and Bose-Einstein-Condensates

The Ginzburg-Landau theory in a magnetic field accompanied with the concept of surfaces of superconductors. In (GiJ) Ginzburg has presented the notion of Krein spaces that is an extension of Hilbert spaces for studying in quantum mechanics.

The BCS theory of superconductors is accompanied by the concepts of Cooper pairs and a mean-field-Hamilton operator. In the proposed model the Cooper pairs may be interpreted as alternating (maximal) pairs. The counterpart of the mean-field-Hamilton operator might become the plasma potential operator.

F. Ehrenhaft's forgotten discovery

The phenomenon of „Photophoresis by electric and magnetic ions"

Key words: light negative and positive motion, light positive and light negative longitudinal photophoresis, trembling effect, difficulties of radiometer forces

(EhF) p. 238: „The speed of the magneto-photophoresis depends upon the intensity and the frequency of the light, and upon the intensity of the magnetic field as well as on the material itself"
(EhF) p. 238: „The Trembling-Effect: This rapidly changing movement of diamagnetic particles in low fields cannot be confused with Brownian movement in gases"
(EhF) p. 240: „This geomagnetic field alone without any artificial field caused nickel particles to move up"
(EhF) p. 242: „The prevailing opinion is that within an arbitrarily chosen geometric surface a real quantity of either kind of electricity can be inclosed, but no matter how the surface is chosen, it will always inclose the same amount of north and south magnetism. In other words there are true quantities of electricity of either sign, but not true magnetic quantities. Thus we have electric but not magnetic currents"
(EhF) p. 243: „In order to explain the phenomena of photophoresis one conclusion is drawn from the movement of illuminated particles in the homogeneous electric and magnetic fields. The light induces electric and magnetic charges (poles) upon the particles if they are illuminated by concentrated light preponderantly shorter wave lengths. …. For the magnetic charges this conclusion is new, but is justified because of the complete analogy of this phenomenon with the electric phenomenon.
(EhF) p. 245: „The trembling-effect is thus a very frequent chang of magnetic poles in the intense beam of light and the magnetic field."
(EhF) p. 250: „light thus magnetizes matter"
(EhF) p. 252: „6. The Ponderomotive Foreces upon Matter: If light induces electric and magnetic charges upon matter and if as under the conditions of the experiments, thre are also electric and magnetic fields in beams of light besides the well-known oscillating ones it follows that there are ponderomotive forces besides the light pressure which are producted by those stationary components and induced charges. These forces have attracting as well as repelling effects."

III. THE INTERPRETATION OF THE EXPERIMENTS
2. The Interpretation of the Photophoresis

(EhF): In order to explain the phenomena of photophoresis one conclusion is drawn from the movement of the illuminated particles in the homogeneous electric and magnetic fields.
The light induces electric and magnetic charges (poles) upon the particles if they are illuminated by concentrated light pre-ponderantly shorter wave lengths. …
For the magnetic charges this conclusion is new, but is justified because of the complete analogy of this phenomenon with the electric phenomenon.
It should be noted here that there are no empirical facts which contradict this conclusion as was shown above. The difficulties such an explanation encounters are not due to experimental facts but only to theoretical considerations which go back as far as Ampere who introduced the hypothesis of molecular currents and reduce the phenomena of magnetism to purely electrical phenomena.

V. Conclusion

(EhF): Particles of matter irradiated by light between electrodes behave as if they carry positive or negative electric charges. Therefore we can say that through the action of the light uncharged particles obtain unipolar charges, either negative or positive.

Particles of matter, sufficiently irradiated by light between magnetrodes behave as if they carry single south or north magnetic poles (charges). Light therefore produces unipolar magnets (magnetic ions, charges). Unipolar particles flow in homogeneous fields E or H in the direction of the field and reverse their movement with the field. Such a flow of particles simultaneously in both directions can be observed directly by means of a microscope (dark field). One can actually see the flow of an electric current in the above mentioned arrangement. It is very remarkable that the same picture appears if the magnetic field is applied as if the electric field is applied. From the visual appearance it is impossible to determine when an electric and when a

magnetic field is applied. The generality of this effect is not diminished by the necessity of using light to produce magnetic ions. It will be a question of further investigation to find out if magnetic ions exist also without light. It should be remembered that when electric currents were discovered, dissociation in the voltaic cell was considered all important, but nobody could explain it. No model to picture what happens in a voltaic cell was known to aid ones imagination and in the same way the author does not attempt to use a model to explain the mechanism of the production of the magnetic ions ....

It is evident that a great number of problems are suggested by the conclusions described above. Thus, for instance, one may think of the existence of conductors of magnetic ions, of the heat created by the flow of magnetic ions, etc.

In this paper, the attempt has been made to show that a beam of light causes or induces not only heat and electricity but also magnetism at the same time.

Although an attempt has been made to separate the thermal and mechanical forces from the electric and magnetic ones one cannot be certain, from a general point of view, whether this is entirely possible in the observation of physical phenomena.

V. Schauberger's forgotten discovery
(AlO)

Schauberger's concept of „living water" is about two types of „moving forces", the centrifugal force, which is in line with the concept of entropy, and a centripetal force (a kind of negative „entropy", which is in line with the concept of a „biological potential of a cell". In other words, physical and biological spiral movement phenomena (including spiral galaxies) can be interpreted as two related physical resp. biological force types, centrifugal (linear movements) and centripetal forces.

Newton (gravity) potential (field) equation
vs.
Einstein (gravity potential) field equations

The Einstein operator is given by

$$G = R_{ik} - R\frac{g_{ik}}{2}$$

with the corresponding gravity field equations

$$G = -\kappa T_{ik}$$

with the corresponding motion equations

$$\frac{d}{d\tau}\left(g_{\mu,\nu}\frac{dx^\mu}{d\tau}\right) = \frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial x^\nu}\frac{\partial x^\alpha}{\partial \tau}\frac{\partial x^\beta}{\partial \tau}$$

for the path $x^\mu = x^\mu(t)$ of a particle.

The change from the Newton model to the Einstein model is about a change from the potential equation to the Einstein equation

$$-\Delta\Phi = -4\pi k\rho \qquad \rightarrow \qquad G = -\kappa T_{ik}$$

and a change from the motion equations

$$\frac{d^2\vec{x}}{dt^2} = -grad\Phi \qquad \rightarrow \qquad \frac{d}{d\tau}\left(g_{\mu,\nu}\frac{dx^\mu}{d\tau}\right) = \frac{1}{2}\frac{\partial g_{\alpha\beta}}{\partial x^\nu}\frac{\partial x^\alpha}{\partial \tau}\frac{\partial x^\beta}{\partial \tau}.$$

Instead of one potential equation there are now 10 equations with 10 potentials Φ_{ik}; instead of the Newton potential, the Einstein gravity potential is no longer the sum of single gravitation potentials. Additionally there is a circle structure, i.e. the potentials are a functions of the T_{ik} ($\Phi_{ik} = f(T_{ik})$), while the space-time structure is a function of the potentials ($f(\Phi_{ik})$). The matter, as described by the energy-momentum tensor T_{ik}, reflecting the principles of energy and momentum conservation, generates a curvature of the space-time and particles move along of geodesics.

The idea of a physical electric-magnetic space-time decomposition
to prove the nonlinear gravitational stability of the Minkowski space-time
(ChD)

In a nutshell:
The ideas around the Bianchi equation of an E-V space-time (an electric-magnetic decomposition, a null decomposition of a Weyl field, a null-structure equation in space-time) are at the heart to prove the nonlinear gravitational stability of the Minkowski space-time applying the Bel-Robinson tensor

$$Q_{\alpha\beta\gamma\delta} = \frac{1}{2}(W_{\alpha\mu\beta\rho}W_{\gamma\delta}^{\mu\rho} +* W_{\alpha\mu\beta\rho} * W_{\gamma\delta}^{\mu\rho}).$$

„The problem of stability of the Minkowski space-time is closely related to that of characterizing the space-time solution of the E-V equations, which are globally asymptotically flat – as defined in physics literature, space-times that becomes flat as we approach infinity in any direction."

(KLS) 3.2: „The primary example of the solution of the Bianchi equations is the Riemann curvature tensor of an Einstein vacuum space-time „The Bianchi equations look complicated. This is obvious formally, but it becomes even apparent if we decompose the Weyl field W into its „electric" and „magnetic" parts. …. The two covariant symmetric traceless tensor fields $E = i_{(T,T)}W$ and $H = i_{(T,T)} * W$, tangent to the hypersurface determines completely the Weyl tensor field.

The corresponding Bianchi equations for this decomposition are given by the following Maxwell-type equations:

$$\Phi^{-1}\partial_t E + curlH = \rho(E,H), \ \Phi^{-1}\partial_t H - curlE = \sigma(E,H)$$
$$divE = k \therefore H , divH = -k \therefore E$$

The explicit expressions of $\rho(E,H)$ and $\sigma(E,H)$ can be found in (ChD), p. 146. The strong formal analogy with the Maxwell equations goes even further. In fact, the Bianchi equations possess a tensor analogous to the energy-momentum tensor of an electromagnetic field for the solutions of the Maxwell equations, the Bel-Robinson tensor

$$Q_{\alpha\beta\gamma\delta} = \frac{1}{2}(W_{\alpha\mu\beta\rho}W_{\gamma\delta}^{\mu\rho} +* W_{\alpha\mu\beta\rho} * W_{\gamma\delta}^{\mu\rho})"$$

„In the least precise version our main result asserts the following:

Theorem 1.0.1 (First version of Main Theorem, 2nd & 3rd versions pp. 17 & 298)
Any strongly asymptotically flat initial data set that satisfies, in addition, a global smallness assumption, leads to a unique, globally hyperbolic, smooth, and geodesically complete solution of the E-V equations. Moreover, this development is globally asymptotically flat, by which we mean that its Riemann curvature tensor approaches zero on any causial or spacelike geodesic, as the corresponding affine parameter tends to infinity."

The main difficulties one encounters in the proof of our result are

(1) The problem of coordinates, and
(2) The strongly nonlinear hyperbolic features of the Einstein equations.

(1) The problem of coordinates is the first major difficulty one has to overcome when trying to solve the Cauchy problem for the Einstein equations. In short, one is faced with the following dilemma: … coordinates seem to be necessary even to allow the formulation of well-posed Cauchy problems and a proof of a local in time existence result. Nevertheless, as the particular case of wave coordinates illustrates, the coordinates may lead, in the large, to problems of their own. ….

(2) The other major obstacle in the study of the Einstein equations consists in their hyperbolic and strongly nonlinear character. The only powerful analytic tool we have in the study of nonlinear hyperbolic equations in the physical space-time dimension are the energy estimates. Yet the classical energy estimates are limited to

proving results that are local in time. The difficulty has to do with the fact that, in order to control the higher energy norms of the solutions, one has to control the integral in time of their bounds in uniform norm. …new techniques were developed, based on modified energy estimates and the invariance property of corresponding linear equations, which were applied to prove global or long-term existence results for nonlinear wave equations… one uses the Killing and conformal Killing vectorfields generated by the conformal group of the Minkowski space-time to define a global energy norm that is invariant relative to the linear evolution. The precise asymptotic behavior, including the uniform bounds previously mentioned, are then an immediate consequence of a global version of the Sobolev inequalities (KlS2), (KlS3). ….The relevant linearized equations for the E-V equations are the Bianchi equations in Minkowski space-time. … Its complete asymptotic properties are analyzed by using only energy estimates and the conformal invariance properties of the equations. …To derive a global existence result, however, one also needs to investigate the structure of the nonlinear terms. It is well known that arbitrary quadratic nonlinear perturbances of the scalar wave, even when derivable from a Lagrangean, could lead to formation of singularities unless a certain structural condition, which we have called the null condition, is satisfied. It turns out that the appropriate, tensorial version of this structural condition is satisfied by the Einstein equations. Roughly speaking, one could say that the troublesome nonlinear terms, which could have led to formation of singularities, are in fact excluded due to the covariance and algebraic properties of the Einstein equations. … These basic algebraic properties of the Einstein equations, which allow us to prove the global existence result, are in sharp contrast with the nonlinear hyperbolic equations of classical continuum mechanics. Indeed, the equations of nonlinear elasticity (JoF) and compressible fluids (SiT), in four space-time dimensions, form singularities even for arbitrary small initial conditions."

<div align="center">The still existing conceptual challenge</div>

Despite the central importance that such space-times have in General Relativity as corresponding to isolated systems, it is not at all settled how to define them correctly, consistent with the field equations. ... The present state of understanding was set by R. Penrose (PeR2), (PeR3), who formulized the idea of asymptotics flatness by adding a boundary at infinity attached through a smooth conformal compactification. However, it remains questionable whether there exists any nontrivial solution of the field equations that satisfies the Penrose requirements. Indeed, his regularity assumptions translate into fall-off conditions of a curvature that may be too stringent and thus may fail to be satisfied by any solution that would allow gravitational waves. Moreover, the picture given by conformal compactification fails to address the crucial issue of the relationship between conditions in the past and behavior in the future. …. We believe that a real understanding of assymptotically flat spaces can only be accomplished by constructing them from initial data and studying their asymptotiv behavior. In addition, only such a construction can address the crucial issue of the relationship between conditions in the past and behavior in the future, an issue that the conformal compactification leaves entirely open. …"

The Cosmological Microwave Background Radiation (CMBR)
and Big Bang models

(LaM) p. 7: The CMBR provides us with the most important evidence supporting the big bang model. The rest of this book explains how and why this is so. … Big Bang models are on the basis of general relativity and follow from a number of assumptions. These are the following:

- homogeneity of space applies. Thus it is assumed that all points of space are equivalent and the properties assiciated with each point are the same. …
- isotropy of space applies. This means that there is no privileged direction in space. (Again this refers to large scales) …
- the matter in the universe can be described very simple in terms of what is called a perfect fluid. In this case its properties are completely given by ist density ρ and its pressure p
- the laws of physics are the same everywhere.

Models of the universe satisfying the above assumptions are called Friedmann-Lemaitre models and have a number of properties in common. They are completely fixed by three parameters, … the Hubble constant which measures the present rate of expansion of the universe, the density parameter, which measures the mean mass of the universe today, and the cosmological constant. … although observations hint at lower values, this (the Einstein-de Sitter) model (density parameter =1, cosmological constant =0) is often taken as a prototype because it renders calculations particularly simple. Cosmological calculations are often done for this very specific case.

C. Related mathematical tools

Primarily affected mathematical tools/equations

The Mie equation is acompanied with the concept of an „electric force", counterbalanced in the ether by an „electric pressure":

The problem of the field theory of matter as described by the Maxwell equations is, that they cannot explain (WeH) p. 170:, „why the field possesses a granular structure and why the knots of energy remain intact in spite of the back and forth flux of energy and momentum. The Maxwell equations will not do because they imply that negative charges compressed in an electron explode; to guarantee their coherence in spite of Coulomb's repulsive forces was the only service still required of the substance by H. A. Lorentz's theory of electrons. The preservation of the energy knots must result from the fact that the modified field laws admit only of one state of field equilibrium. ... The target of the Mie theory is to explain, (WeH) p. 170: „*why the field possesses a granular structure and why the knots of energy remain intact in spite of the back and forth flux of energy and momentum* [(*)].

In the proposed quantum potential model the „mass" of a quantum „particle" is modelled as an element of the „positive" sub-space of a Krein space accompanied by corresponding potential energy; its related „mass" is modelled by the discrete „energy knots" of the underlying kinetic potential operator. The Krein space based model supports the „preservation of the energy knots" requirement in spite of the back and forth flux of quantum energy governed by related angular operator (due to the compact embedding $H_{\kappa.\alpha}^+ \subset H_{\kappa.(\tau)}^+$ enabled by the Krein space intrinsic angular operator

Schrödinger's „only one problem" in statistical thermodynamics:
(ScE) p. 1: „*There is, essentially, only one problem in statistical thermodynamics: the distribution of a given amount of energy E over N identical systems. Or perhaps better: to determine the distribution of an assembly of N identical systems over the possible states in which this assembly can find itself, given that the energy of the assembly is a constant E*".
In the proposed quantum potential model every considered „elementary particle" object is an element of a Hilbert-Krein space governed by a corresponding potential operator. All considered Hilbert-Krein sub-spaces can be related to the statistical Hilbert space L_2. Additonally, the proposed quantum potential model provides some evidence for the famous Schrödinger momentum operator $\boldsymbol{p} = -ih\nabla = -ih\partial$, also supporting his concept of negotropy in the context of „living systems", (ScE1)

Dirac's theory of radiation of an electron:
E. Fermi: „Dirac's theory of radiation is based on a very simple idea; instead of considering an atom and the radiation field with which it interacts as two distinct systems, he treats them as a single system whose energy is the sum of three terms: one representing the energy of the atom, a second representating the electromagnetic energy of the radiation field, and a small term representing the coupling energy of the atom and the radiation field".
In the proposed quantum potential model the potential operator becomes an intrinsic part of the Krein space model and not a physical phenomenon specific appropriately chosen function, like the Coulomb potential function in Dirac's theory of radiation. The kinetical energy system is defined by the energy knots of an underlying kinetic energy operator. In other words, the need for a small term representing the coupling energy of the atom and the radiation field disappears

Pauli's spin(1/2) concept and Heisenberg's proposed mathematical tool of an indefinite metric
Pauli's spin(1/2) concept is about a „rotation" concept of an elementary particle, which looks the same only after the second „rotation". This „spin(1/2)-rotation" concept is the model for an angular (non-kinematical) momentum. However, in the current quantum theory translation and rotation operators are not interchangeable, which is a consequence of the quantification process of classical partial differential equations (PDE). Therefore, in order to characterize the angular momentum of a system about an axis by a quantum number it is neccessary that the perpendicular translation momentum vanishes or is unknown, (DüH), (HeW).

[(*)] (WeH) p. 170: „*since all physically important properties of an elementary material particle belong to the surrounding field rather than the substantial nucleus at the field center, the question becomes inevitable whether the existence of such a nucleus is not a presumption that may be completely dispensed with. ... This question is answered in the affirmative by the field theory of matter. According to the latter a material particle such as an electron is merely a small domain of the electric field within which the field strength assumes enormously high values, indicating that a comparatively huge field energy is concentrated in a very small space. Such an energy knot, which by no means is clearly delineated against the remaining field, propagates through empty space like water waves across the surface of a lake; there is no such thing as one and the same substance of which the electron exists at all times. .... It (the field) is of the essence of the continuum. Even the atomic nuclei and the electrons are not ultimate unchangeable elements that are pushed back and forth by natural forces acting upon them, but they are themselves spread out continuously and are subject to fine fluent changes*".

(WeH1) p. 208: „*the ponderomotive force occuring in the Mie equation is contrasted with the „electrical force" E by $E - \nabla\varphi = 0$, that is the electric force E is counterbalanced in the ether by an electric pressure φ. In other words, the potential appears as an electric pressure; this is the required cohesive pressure that keeps the electron together*".

The Lorentz transformation in special relativity is a simple type of rotation in hyperbolic space.

The Lorentz group L has four components, each of which is connected in the sense that any point can be connected to any other, but no Lorentz transformation in one component can be connected to another in another component. One of this components is the restricted Lorentz group, which is the group of $2x2$ complex matrices of determinant one, $SL(2,C)$. It is isomophic to the symmetry group $SU(2) \cong SL(2,C)$, containing as elements the complex-valued rotations, which can be written as a complex-valued matrix of type

$$\begin{pmatrix} a + ib & c + id \\ -c + id & a - ib \end{pmatrix} \text{ with determinant one.}$$

It is important in describing the transformation properties of spinors. In SMEP the group $SU(2) \cong SL(2,C)$ describes the weak force interaction with 3 bosons W^+, W^-, Z.

Another group associated with the Lorentz group L is the complex Lorentz group $L(C)$, [*]. It has just two connected components, $L_+(C)$ and $L_-(C)$. The transformations 1 and -1, which are disconnected in L are connected in $L(C)$. In other words, the complex Lorentz transformation connects

- the two components containing the 1-transformation and space-time inversion
- the two components containing the space inversion and the time inversion.

Just as the restricted Lorentz group is associated with $SL(2,C)$, the complex Lorentz group is associated with $SL(2,C) \otimes SL(2,C) \cong SU(2) \otimes SU(2)$. There is also a two-to-one homomorphism from $SL(2,C) \times SL(2,C)$ onto $L(+,C)$.

The spin of an elementary particle is its eigen-rotation with exactly two rotation axes, one parallel and one anti-parallel axis to a magnetic field. This is the 2×2 complex number scheme, where every „normal" rotation is contained twice. Consequently, an electron has a charge only half of the Planck's quantum of action.

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[*] The complex Lorentz group is composed of all complex matrices satisfying

$$\Lambda^\kappa{}_\mu \Lambda_{\kappa\nu} = g_{\mu\nu} \text{ or } \Lambda^T G \Lambda = G, \qquad (1\text{-}5).$$

It has just two connected components, $L_+(C)$ and $L_-(C)$ according to the sign of $\det(\Lambda)$. The transformations 1 and -1, which are disconnected in L are connected in $L(C)$. In other words, the complex Lorentz transformation connects

- the two components containing the 1-transformation and space-time inversion, i.e. the pair

$$\{\det(\Lambda) = +1, \det(\Lambda^0{}_0 = +1)\}, \{\det(\Lambda) = +1, \det(\Lambda^0{}_0 = -1)\},$$

- the two components containing the space inversion and the time inversion, i.e. the pair

$$\{\det(\Lambda) = -1, \det(\Lambda^0{}_0 = +1)\}, \{\det(\Lambda) = -1, \det(\Lambda^0{}_0 = -1)\}.$$

Summary:

While two (real) Lorentz transformations need to be connected to one another by an appropriately defined continuous curve of Lorentz transformations, there are two pairs of components of the complex Lorentz transform, which are both already connected by definition.

Just as the restricted Lorentz group is associated with $SL(2,C)$, the complex Lorentz group is associated with $SL(2,C) \otimes SL(2,C)$. The latter group is the set of all pairs of $2x2$ matrices of determinants one with the multiplication law

$$\{A_1, B_1\} \cdot \{A_2, B_2\} = \{A_1 A_2, B_1 B_2\}.$$

Is is easy to see that only matrix pairs which yield a given $\Lambda(A, B)$ are $(\pm A, \pm B)$. In particular,

$$\Lambda(-1,1) = \Lambda(1,-1) = -1.$$

The corresponding complex Poincare group admits complex translation but also the multiplication law

$$\{a_1, \Lambda_1\} \cdot \{a_2, \Lambda_2\} = \{a_1 + \Lambda_1 a_2, \Lambda_1 \Lambda_2\}.$$

It has two components $P_\pm(C)$, which are distinguished by $\det(\Lambda)$ and a corresponding inhomogeneous group to $SL(2,C)$.

Affected Millennium Problems of the Clay Mathematics Institute

- *Yang–Mills and Mass Gap*
  Experiment and computer simulations suggest the existence of a "mass gap" in the solution to the quantum versions of the Yang-Mills equations. But no proof of this property is known

  The physical „mass gap" problem of the three elementary particles $W^{\pm}$ and Z (whereby the charges of those particles have very similar properties with the positrons, electrons and photons) is due to the mathematical inconsistencies between all affected elementary particles.

  The proposed quantum potential model enabling the Mie equation model accompanied by the concept of discrete „energy knots" makes the YME obsolete

- *Navier–Stokes Equation*
  This is the equation which governs the flow of fluids such as water and air. However, there is no proof for the most basic questions one can ask: do solutions exist, and are they unique? Why ask for a proof? Because a proof gives not only certitude, but also understanding.

  All attempts to prove well-posed 3D NSE faled due to inappropriate (not bounded) energy norm estimates

  The variational representation of the 3D-NSE governed by the least energy principle with respect to the potential energy norm $\|x\|_{1/2}^2$ ensures convergent energy norm estimates (appendix). Additionally, Mie's electric pressure concept applied to the second unknown function in the NSE, the „pressure p" enables a reformulation of the Navier-Stokes Equations $\dot{u} - \Delta u + (u \cdot \nabla)u = \nabla p$ where $p = \sum_{i,j=1}^{3} R_i R_j (u_i u_j)$ and ∇p becomes the Calderón-Zygmund integrodifferential operator (EsG) p. 44.

- *Riemann Hypothesis*
  The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2.

  (DeJ) p. 292: The Montgomery-Odlyzko law: "The distribution of the spacing between successive non-trivial zeros of the Riemann zeta function (suitable normalized) is statistically identical with the distribution of eigenvalue spacing in a GUE (Gaussian Unitary Ensemble) operator.
  (DeJ) p. 295: "What on earth does the distribution of prime numbers have to do with the behavior of subatomic particles?"

  We claim, that the plasma potential operator provides an appropriate model for the existence of the conjectured Hermitian „Berry-Keating" operator Its mathematical counterpart is the Hilbert-Polya conjecture, which is equivalent to the Riemann Hypothesis, (BrK1). For supporting formulae we refer to appendix A.

  With respect to the Goldbach conjecture we note that the proposed sets of quantum number systems was motivated by the proposed two-semicircle method to replace the Hardy-Littlewood circle method enabling a proof of the Goldbach conjecture, (BrK2), (BrK3).

Potentialtheoretischen Untersuchungen
J. Plemelj, (PlJ)

§ 5 Der Begriff der Masse und der Strömung

Vom Integral $\oint \frac{\partial U}{\partial n} ds$ auf einer nichtgeschlossenen Kurve ergibt sich aus der Gleichung $\int \frac{\partial U}{\partial n} ds = 0$ eine Eigenschaft von großer Wichtigkeit. Das Integral hängt nämlich nur von den Endpunkten ab und nicht von der näheren Form der sie verbindenden Integrationskurve in der Weise, daß die Integrale alle gleich einander gleich sind, welche Integrationswege entsprechen, die durch stetige Deformation im Regularitätsgebiete auseinander hervorgehen. Sind also p und q zwei Punkte im Regularitätsgebiete und verbindet man sie durch irgendeine Kurve (die Tangenten hat), so ist $\int_p^q \frac{\partial U}{\partial n} ds$ wohl definiert und hat einen von der näheren Form der Kurve nicht abhängigen Wert. Dies erkennt man, wenn man durch p und q eine geschlossene Kurve legt und auf diese die Gleichung $\int \frac{\partial U}{\partial n} ds = 0$ anwendet.

Das Integral zwischen zwei Punkten p und q $\quad \bar{U}(q) = -\int_p^q \frac{\partial U}{\partial n} ds$ ist, weil von der Kurve unabhängig, eine wohl definierte Funktion der Grenzen p und q und soll in seiner Abhängigkeit von q mit $\bar{U}$ bezeichnet werden. Diese Funktion ist stetig bei einer stetigen Änderung der oberen Grenze q. Läßt man nun den Punkt q aus dem Innern des Regularitätsgebietes gegen einen *Randpunkt σ* rücken, so wird sich der Integralwert stetig ändern. Nun kann es vorkommen, daß *trotz der Nichtexistenz* eines bestimmten Wertes für die Ableitungen von U doch das Integral $\int_p^q \frac{\partial U}{\partial n} ds$ gegen einen ganz bestimmten Grenzwert strebt, der dann mit $\bar{U}(\sigma) = -\int_p^\sigma \frac{\partial U}{\partial n} d\sigma$ zu bezeichnen wäre. Ganz analoges gilt vom Einrücken des zweiten Punktes gegen einen Randpunkt σ_0. So erklärt sich die Bedeutung des Integrales $\quad \bar{U}(\sigma) = -\int_{\sigma_0}^\sigma \frac{dU}{dn} d\sigma \quad$ zwischen zwei Randpunkten selbst dann, wenn es längs der Berandung nicht zu bilden ist wegen Nichtexistenz von $\frac{dU}{dn}$. Wenn das Integral $\bar{U}(\sigma)$ bei stetiger Verlegung des σ eine stetige Änderung erfährt, so wird infinitesimalen Stücken $d\sigma$ der Berandung auch eine infinitesimale Änderung von $\bar{U}(\sigma)$ entsprechen, die wir mit $d\bar{U}(\sigma)$ bezeichnen können. Der Zusammenhang mit den normalen Ableitungen ist jetzt leicht herzustellen. Ist nämlich die Größe $\bar{U}(\sigma)$ längs des Randes differentiierbar, d.h. besteht $\frac{d\bar{U}(\sigma)}{d\sigma}$, so ist dieser Differentialquotient gleich $-\frac{dU}{dn}$. Wir haben also im neuen Begriff tatsächlich einen Ersatz für die normalen Ableitungen, aber von einer weitgrößeren Leistungsfähigkeit. Es kann nämlich $d\bar{U}(\sigma)$ einen guten Sinn haben, ohne daß ein solcher für $\frac{d\bar{U}(\sigma)}{d\sigma}$ besteht *Es handelt sich hier um eine Verallgemeinerung, wie es die Erweiterung differentiierbarer Funktionen auf die stetigen ist*. Die größte Tragweite des neuen Begriffes ist nicht schwer zu erkennen, denn jetzt ist man unabhängig wowohl von der Existenz der Normale als auch der Ableitungen am Rande. Man wird also z. B. eine Verallgemeinerung der Aufgabe, normale Ableitungen eines Potentials am Rande vorzugeben, darun haben, jedem element des Randes den ihm entsprechenden Betrag $d\bar{U}(\sigma)$ vorzuschreiben.

Dieser neue Begriff hat in der physikalischen Anwendung bei der Strömung der Elektrizität bereits den Namen „Stärke des Stromes" der durch dieses Stück der Berandung fließt. Ich will also in der Folge, mangels einer von der Anwendung freien Benennung die Bezeichnung „Strom" oder „Strömung" auf dem Randstücke gebrauchen.

§8 Das Potential der einfachen Schicht

Bisher war es üblich fuer das Potential $V(p)$ die Form $\quad V(p)(s) = \oint log \frac{1}{r_{ps}} \cdot \mu'(s) \cdot ds \quad$ vorauszusetzen, wobei dann $\mu'(s)$ die *Massendichtigkeit* der Belegung genannt wurde. *Eine solche Annahme erweist sich aber als eine derart folgenschwere Einschränkung, daß dadurch dem Potentials $V(p)$ der größte Teil seiner Leistungsfähigkeit hinweg genommen wird*. Für tiefergehende Untersuchungen erweist sich „*das Potential der einfachen Schicht*" oder Belegung nur in der Form

$$V(p)(s) = \oint log \frac{1}{r_{ps}} \cdot d\mu_s$$

verwendbar. Die infinitesimale Größe $d\mu_s$ kann man das *Massenelement* nennen, von Massendichtigkeit wird man aber nur dann sprechen können, wenn der Differentialquotient $\frac{d\mu_s}{ds} = \mu'(s)$ besteht.

The „pressure incompatibility problem of the classical NSE at the boundary walls or at the initial time"

The initial boundary value problem determines the initial pressure $p_0(x)$ by the Neumann problem

$$\Delta p_0 = (f_0 - u_0 \cdot \nabla u_0 \qquad \text{in } \Omega$$

$$\frac{\partial p_0}{\partial n} = [\Delta u_0 - u_0 \cdot \nabla)u_0 + f_0] \cdot n \quad \text{at } \partial\Omega$$

With $f_0 := \lim_{t \to 0} f(\cdot, t)$.

Applying formally the div-operator to the classical NSE the pressure field must satisfy the following Neumann problem, (GaG),

$$\Delta p = (u \cdot \nabla)u - f \qquad \text{in } \Omega$$

$$\frac{\partial p}{\partial n} = [\Delta u - (u \cdot \nabla)u + f] \cdot n \quad \text{at } \partial\Omega$$

where n denotes the outward unit normal to $\partial\Omega$.

As it holds that

$$[\Delta u - (u \cdot \nabla)u + f] \cdot n|_{\partial\Omega} \quad \rightarrow \quad [\Delta u_0 - (u._0 \nabla)u_0 + f_0] \cdot n|_{\partial\Omega} \quad \text{in } H_{-1/2}(\partial\Omega)$$

and

$$\nabla \cdot [f - u \cdot \nabla)u]|_{\partial\Omega} \quad \rightarrow \quad \nabla \cdot [f_0 - u_0 \cdot \nabla)u_0]|_{\partial\Omega} \qquad \text{in } H_{-1/2}(\partial\Omega)$$

the pressure p tends to p_0 in the sense that $\|\nabla(p(\cdot, t) - p_0\| \to 0$ as $t \to 0$.

As a consequence the prescription of the pressure at the boundary walls or at the initial time independently of u, could be incompatible with and, therefore, could retender the NSE problem ill-posed.

Electrodynamics in matter
Maxwell & telegraph equations

In electrodynamics the matter equations (Maxwell equations in discharged areas) are prescribed by the hyperbolic telegraph equations in the form

$$\Delta \vec{E} = \frac{\mu\varepsilon}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} + \sigma\mu_0\mu \frac{\partial \vec{E}}{\partial t}, \quad \Delta \vec{H} = \frac{\mu\varepsilon}{c^2} \frac{\partial^2 \vec{H}}{\partial t^2} + \sigma\mu_0\mu \frac{\partial \vec{H}}{\partial t}.$$

They are wave equations for a lossy dieletric, which correspond to standard wave equations in case of an isolator, i.e. $\sigma = 0$. The dielectric constant governs the resistence and the frequency depending damping phenomena.

The constant ε prescribes an assumed linear relation of the electric displacement field $\vec{D}$ (for homogeneous and isotropic materials) and the electric field by $\vec{D} = \varepsilon\vec{E}$. The constant ε_0 denotes the vacuum (electric) permitivity constant. The magnetic permeability is defined by the ratio μ of the magnetic flux density and the magnetic field strength. The vacuum (magnetic) permeability constant of free space is denoted by μ_0. The two vacuum constants are linked to the speed of light c by $c^2 = \frac{1}{\mu_0\varepsilon_0}$.

Singularities in the general relativity theory
Treder H.-J., (TrH)

Regular solutions of Einstein's equations mean very different things. In the case of the empty-space equations, $R_{ik} = 0$, such solutions must be metrics $g_{ik}(x^l)$ without additional singular „field sources" (Einstein's particle problem"). However the „phenomemological matter" is defined by the Einstein equations $R_{ik} - \frac{1}{2} g_{ik} R = -\mu T_{ik}$ itselves. Therefore if 10 regular functions $g_{ik}(x^l)$ are given (fulfilling the inequalities of Lorentz-signature) then these g_{ik} define 10 functions $T_{ik}(x^l)$ without singularities. But, the matter-tensor T_{ik} must fulfill only the two inequalitites $T \geq 0, T_0^0 \geq \frac{1}{2} T$, and therefore the Einstein equations with „phenomenological matter" must mean two inequalities $R \geq 0, R_0^0 \leq 0$, which are in general incompatible with permanently regular metrics with Lorentz-signature.

The Hilbert/Riesz transform operators
Translations, homothesis, rotation, and the Schrödinger momentum operator

The Riesz transform is the n-dimensional generalization of the Hilbert transform. The Riesz transform operators are related to the Caldéron- Zygmund operators $T(f) = S * F$ with a distribution S defined by a homogeneous function of degree zero, satisfying a kind of average mean zero condition on the unit sphere with its underlying rotation invariant probability measure (MeY). They are Pseudo Differential Operators with symbols $m(\omega) \in C^\infty(R^n - \{0\})$, where

i) $\quad m(\mu\omega) = m(\omega), \mu > 0$

ii) $\quad$ the mean of $m(\omega)$ on the unit sphere is zero

iii) $\quad m(\omega) = \frac{\omega_j}{|\omega|}$.

arising when study the Neumann problem in upper half-plane. The Riesz transform operators

$$R_k u = -ic_n p.v. \int_{-\infty}^{\infty} \frac{x_k - y_k}{|x-y|^{n+1}} u(y) dy \quad \text{with } c_n := \frac{\Gamma(\frac{n+1}{2})}{\pi^{(n+1)/2}}$$

commutes with translations and homothesis, having nice properties relative to rotation:

Let $SO(n)$ denote the rotation group, let $m := m(x) := (m_1(x), \dots m_n(x))$ be the vector of the Mikhlin multipliers of the Riesz operators and $\rho = \rho_{ik} \in SO(n)$, then it holds

$$m(\rho(x)) = \rho(m(x)) \text{ (i.e. } m_j(\rho(x)) = \sum \rho_{jk} m_k(x)).$$

Proof:

$$m(\rho(x)) = c_n \int_{S^{n-1}} (\frac{\pi i}{2} sign(x\rho^{-1}(y)) + log \left| \frac{1}{x\rho^{-1}(y)} \right|) \frac{y}{|y|} d\sigma(y)$$

$$= c_n \int_{S^{n-1}} (\frac{\pi i}{2} sign(xy) + log \left| \frac{1}{xy} \right|) \frac{y}{|y|} d\sigma(y).$$

The Riesz transform operators enables the following „Caldéron operator" representations

$$(\Lambda u)(x) = (\sum_{k=1}^{n} R_k D_k u)(x) = \frac{\Gamma(\frac{n+1}{2})}{\pi^{\frac{n+1}{2}}} \sum_{k=1}^{n} p.v. \int_{-\infty}^{\infty} \sum_{k=1}^{n} \frac{x_k - y_k}{|x-y|^{n+1}} \frac{\partial u(y)}{\partial y_k} dy$$

$$= -(\Delta \Lambda^{-1})u(x) = -\frac{\Gamma(\frac{n-1}{2})}{2\pi^{\frac{n+1}{2}}} p.v. \int_{-\infty}^{\infty} \frac{\Delta_y u(y)}{|x-y|^{n-1}} dy$$

with symbol $|v|$ and its inverse operator ((EsG) (3.15), (3.17), (3.35))

$$(\Lambda^{-1}u)(x) = \frac{\Gamma(\frac{n-1}{2})}{2\pi^{\frac{n+1}{2}}} p.v. \int_{-\infty}^{\infty} \frac{u(y)}{|x-y|^{n-1}} dy, n \geq 2.$$

Note: In dimension 1, this is about $\Lambda = DH$, where H denotes the Hilbert transform and D the Schrödinger momentum operator in the form $P := D = -i\frac{d}{dx}$, (MeY) p. 5.

The Schrödinger momentum operator in dimension n, and its related Hamiltonian operator is given by

$$P := -i\hbar\nabla = \frac{\hbar}{i}\nabla \text{ resp. } H := -\frac{\hbar^2}{2m}\Delta = \frac{1}{2m}\left(\frac{\hbar}{i}\nabla\right)^2.$$

**The completely open Courant-Hilbert conjecture of the specific role of
undistorted spherical travelling waves in 3+1 space+time dimension**

„Relatively undistorted spherical waves relate to the problem of transmitting with perfect fidelity signals in all directions. All we can do here is to formulate a conjecture which will be given some support in article 3

Courant-Hilbert conjecture:

„Families of sherical waves for arbitrary time-like lines exist only in the case of two or four variables, and then only if the differential equation is equivalent to the wave equation.

A proof of this conjecture would show that four-dimensional physical space-time world of classical physics enjoys an essential distinction, (CoR) p. 763

Radiation and Huygens' principle
(CoR) p. 765-766

„Huygens' principle stipulates that the solution at a point does not depend on the totality of initial data within the conoid of dependence but only on data on the characteristic rays …

For single differential equation of second order with constant coefficients we have seen: Only for the wave equation in 3,5,7,… space dimensions, and for equivalent equations, Huygens' principle is valid. For differential equations of second order with variable coefficients Hadamard's conjecture states that the same theorem holds even if the coefficients are not constant. Examples to the contrary shows that this conjecture cannot be completely true in this form, although it is highly plausible that somehow it is essentially correct.

Altogether, the question of Huygens's principle for second order equations should be considered in the light of the much more comprehensive problem of the exact domain of dependence and influence for any hyperbolic problem, a problem which is still completely open.

Concerning the transmission of signals which not only remain sharp but undistorted, the conjecture in article 2 stated that this phenomenon is possible only in three space dimensions. For an isotropic homogeneous medium, i.e., for constant coefficients (and second order equations), the proof of this conjecture is contained in the preceeding discussions. Thus our actual physical world, in which accoustic or electromagnetic signals are the basis of communication, seems to be singled out amoung other mathematically conceivable models by intrinsic simplicity and harmony.

… It would seem, therefore, that the generalized Huyghens' principle should be considered as the proper expression of physical reality".

Retarded potential: roots of the second law or due to reasons of probability?
W. Ritz contra A. Einstein

On the present status of the radiation problem
by W. Ritz and A. Einstein
(Physikalische Zeitschrift 10 (1909) 323-324)

To clarify the differences of opinion that came to light in our respective publications, we note the following. In the special cases in which an electromagnetic process remains restricted to a finte space, the process can be represented in the form

$$f = f_1 = \frac{1}{4\pi} \int \frac{\varphi(x', y', z', -\frac{r}{c})}{r} dx' dy' dz'$$

as well as in the form

$$f = f_2 = \frac{1}{4\pi} \int \frac{\varphi(x', y', z', +\frac{r}{c})}{r} dx' dy' dz'$$

and in other forms.

While Einstein believes that one could restrict oneself to this case without substancially limiting the generality of considerations, Ritz considers this restriction not to be permissible in principle. If one takes this standpoint, then experience compels one to consider the representation by means of retarded potentials as the only one possible, if one is inclined to the view that the fact of irreversibility of radiation processes must already find its expression in the fundamental equations. Ritz considers the restriction to the form of retarded potentials as one of the roots of second law, while Einstein believes that irreversibility is exclusively due to reasons of probability.

A new type of cosmological solutions of the gravity field equations
(GöK)

All cosmological solutions with non-vanishing density of matter known at present have the common property that, in a certain sense, they contain an „absolute" time coordinate, owing to the fact that there exists a one-parametric system of three-spaces everywhere orthogonal on the world lines of matter. It is easily seen that the non-existence of such a system of three-spaces is equivalent with a rotation of matter relatively to the compass of inertia. In this paper I am proposing a solution (with a cosmological term $\neq 0$) which exhibits such a rotation. This solution, or rather the four-dimensional space S which it defines, has the further properties

(1) S is homogeneous

(2) …. so that any two world lines of matter are equidistant

(3) S has rotational symmetry

(4) … That is, a positive direction of time can consistently be introduced in the whole solution

(5) It is not possible to assign a time coordinate to each space-time point in such a way that the coordinate always increases, if one moves in a positive time-like direction; …

(6) … it is theoretically possible in these worlds to travel into the past, or otherwise influence the past

(7) There exist no three-spaces which are everywhere space-like and intersect each world line of matter in one point

(8) … an absolute time does not exist, even if it is not required to agree in direction with the times of all possible observers (where absolute means: definable without reference to individual objects, such as e.g. a particular galactic system).

(9) Matter everywhere rotates relatively to the compass of inertia with the angular velocity $2\sqrt{\pi\mu\rho}$, where ρ is the mean density of matter and μ Newton's gravitational constant.

Geometric geometry structures of manifolds
(ThW)

„The universe is not made up of fields on space-time; it is made up of fields on fields", (RoC) p. 9.

Mathematically speaking, the Riemannian manifold is a pure metric space (where the metric depends from the coordinates, i.e., at every space-time point in the space-time universe there is a different metric influenced by the mathematical concept of a curvature at those points. In simple words, there is no geometric structue at all in today's „universe model".

A manifold geometry is defined as a pair (X, G), where X is a manifold and G acts transitively on X with compact point stabilisers (ScP).

In an universe model with appropriately connected geometric manifolds the corresponding symmetries breakdowns at those "connection dots" would govern corresponding different conservation laws in both of the two connected manifolds.

> *Thurston conjecture*:
> The interior of every compact 3-manifold has a canonical decomposition into pieces which
> have geometric structure (ThW).

(ScP): The Thurston conjecture asserts that any compact 3-manifold can be cut in a reasonably canonical way into a union of geometric pieces. In fact, the decomposition does exist. The point of the conjecture is that the pieces should all be geometric. There are precisely eight homogeneous spaces (X, G) which are needed for geometric structures on 3-manifolds. The symmetry group $SU(2)$ of quaternions of absolute value one (the model for the weak nuclear force interaction between an electron and a neutrino) is diffeomorph to S^3, the unit sphere in R^4. The latter one is one of the eight geometric manifolds above.

(ScP): Kulkarni (unpublished) has carried out a finer classification in which one considers pairs (G, H), where G is a Lie group, H is a compact subgroup and G/H is a simple connected 3-manifold and pairs (G_1, H_1) and (G_2, H_2) are equivalent if there is an isomorphism $G_1 \to G_2$ sending H_1 to a conjugate of H_2.

Thus for example, the geometry S^3 arises from three distinct such pairs, (S^3, e), $(U(2), SO(2))$, $(SO(4), SO(3))$. Another example is given by the Bianchi classification consisting of all simply connected 3-dimensional Lie groups up to an isomorphism.

(AnM): Looking from an Einstein field equation perspective the Ricci tensor is a second order tensor, which is very much linked to the Poincare conjecture, its solution by Perelman and to S^3.

(CiI): The geometrodynamics provides alternative (pseudo) tensor operators to the Weyl tensor related to H_3 .

In (CaJ) the concept of a Ricci potential is provided in the context of the Ricci curvature equation with rotational symmetry. The single scalar equation for the Ricci potential is equivalent to the original Ricci system in the rotationally symmetric case when the Ricci candidate is nonsingular. The single scalar equation for the Ricci potential might be interpreted as the counterpart of the CLM vorticity equation as a simple one-dimensional turbulent flow model in the context of the NSE.

For an overview of the Ricci flow regarding e.g. entropy formula, finite extinction time for solutions on certain 3-manifolds in the context of Prelman's proof of the Poincare conjecture we refer to (KlB), (MoJ).

Wavelets
(HoM), (LoA), (MeY)

The decomposition of the quantum state space $H_{-1/2} = H_0 \otimes H_0^\perp$ resp. the quantum energy space $H_{1/2} = H_1 \otimes H_1^\perp$ goes along with the Fourier wave resp. the Calderón wavelet tool [*]. While the Fourier waves enable an analysis of the test space H_0, wavelets enable an alternative analysis tool for a specific densely embedded subspace of H_0, as the (wavelet) admissibility condition for a $\psi \in H_0$ is a weak one, as for each $\psi, \widehat{\psi} \in H_0$: it holds $\|\psi_\varepsilon - \psi\|_{L_2}^2 \to 0$ for

$$\widehat{\psi}_\varepsilon := \begin{cases} \widehat{\psi}(\omega), & |\omega| \geq \varepsilon \\ 0, & else \end{cases}$$

There are at least two approaches to wavelet analysis, both are addressing the somehow contradiction by itself, that a function over the one-dimensional space R can be unfolded into a function over the two-dimensional half-plane. The Fourier transform of a wavelet transformed function f is given by, (LoA), (MeY),

$$\widehat{W_\vartheta[f]}(a, \omega) := (2\pi |a|)^{\frac{1}{2}} c_\vartheta^{-\frac{1}{2}} \widehat{\vartheta}(-a\omega) \widehat{f}(\omega) .$$

For $\varphi, \vartheta \in L_2(R)$, $f_1, f_2 \in L_2(R)$,

$$0 < |c_{\vartheta\varphi}| := 2\pi \left| \int_R \frac{\widehat{\vartheta}(\omega)\overline{\widehat{\varphi}}(\omega)}{|\omega|} d\omega \right| < \infty$$

and $|c_{\vartheta\varphi}| \leq c_\vartheta c_\varphi$ one gets the duality relationship, (LoA)

$$(W_\vartheta \, f_1, W_\varphi^* f_2)_{L_2(R^2, \frac{dadb}{a^2})} = c_{\vartheta\varphi}(f_1, f_2)_{L_2}$$

i.e.

$$W_\varphi^* W_\vartheta \, [f] = c_{\vartheta\varphi} f \quad \text{in a } L_2 - \text{sense.}$$

For $\varphi, \vartheta \in L_2(R)$, $f_1, f_2 \in L_2(R)$,

$$0 < |c_{\vartheta\varphi}| := 2\pi \left| \int_R \frac{\widehat{\vartheta}(\omega)\overline{\widehat{\varphi}}(\omega)}{|\omega|} d\omega \right| < \infty$$

and $|c_{\vartheta\varphi}| \leq c_\vartheta c_\varphi$ one gets the duality relationship (LoA)

$$(W_\vartheta \, f_1, W_\varphi^* f_2)_{L_2(R^2, \frac{dadb}{a^2})} = c_{\vartheta\varphi}(f_1, f_2)_{L_2}$$

i.e.

$$W_\varphi^* W_\vartheta \, [f] = c_{\vartheta\varphi} f \quad \text{in a } L_2 - \text{sense.}$$

This identity provides an additional degree of freedom to apply wavelet analysis with appropriately (problem specific) defined wavelets in a (distributional) Hilbert scale framework where the "microscope observations" of two wavelet (optics) functions ϑ, φ can be compared with each other by the above "reproducing" ("duality") formula.

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[*] (HoM) 1.2: „The idea of wavelet analysis is to look at the details are added if one goes from scale a to scale $a - da$ with $da > 0$ but infinitesimal small. ... Therefore, the wavelet transform allows us to unfold a function over the one-dimensional space R into a function over the two-dimensional half-plane $\boldsymbol{H}$ of positions and details (where is which details generated?). ... Therefore, the parameter space $\boldsymbol{H}$ of the wavelet analysis may also be called the position-scale half-plane since if g localized around zero with width Δ then $g_{b,a}$ is localized around the position b with width $a\Delta$. The wavelet transform itself may now be interpreted as a mathematical microscope where we identify

$$b \quad \leftrightarrow \text{ position; } \quad (a\Delta)^{-1} \quad \leftrightarrow \text{ enlargement; } \quad g \quad \leftrightarrow \text{optics. "}$$

An unknown relationship between the physical and mathematical world

The mathematical Hilbert-Polya conjecture
and the physical Montgomery-Odlyzko law

The Montgomery-Odlyzko law and its conjectured relation to the Hilbert-Polya conjecture may be the touch point, when "Number Theory meets Quantum Mechanics", answering the question

> *„What on earth does the distribution of prime numbers have to do with the behavior of subatomic particles?",* (DeJ) p. 295

Note: There is also an unknown behavior of the zeta function $\zeta(-s) = \sum_{n=1}^{\infty} n^{-s}$ at $-s = 1,3,5,\ldots$ with a potential common denominator to S. Joffily's proposal of a variant for the Hilbert-Polya spectral representation of the Riemann zeta function. The proposal is looking for the complex poles of the S matrix that mapped into the critical line in coincidence with nontrival Riemann zeros, where the associated quantum system …. can be interpreted as the quantum vacuum, (JoS).

The S^1, S^2, S^3, S^7 unit spheres

The 1-dimensional unit sphere in R^2 corresponds to the Lie group $U(1)$. The related number grid is built by the Eisenstein numbers. The compactification of the field of complex numbers C, the Riemann sphere, is homeomorphic to S^2. It plays a key role in the Teichmueller theory. We note the relationship of the Teichmüller space with the fractional Hilbert space $H_{1/2}$, (NaS). The S^1 and S^3 are the only spheres with a "continuous" group structure, (EbH) 7.2. The groups S^1 and S^3 have parameter representations, (EbH) 3.5.4 (2'), 7.3.2 (3). The spheres S^0, S^1, S^3, S^7 are the only parallelizable spheres.

The 3-dimensional S^3 unit sphere is isomorphic to $SU(2)$. The symmetry group $SU(2)$ is connected with the strong interaction holding together the nucleons in atomic nuclei. It describes the „how" of the β-decay process. This is the (about 15 minutes) decay of a neutron into a proton, an electron, and an antineutrino. Nobody is known until today, why this process occurs and takes only 15 minutes. The „how-process" described by the symmetry group $SU(2)$ is based on the idea of the nucleon is the underlying physical substance with two states, called „neutron" and „proton" (the two „spin-states" of a nucleon), and where the root cause of their „folding over/flipping" is called „weak interaction" (which is not a „force" in a true sense of this word), (UnA3) p. 189.

> *„In the standard model the weak and the electromagnetic interactions are unified in what is called electroweak theory, where there is a special symmetry related to W^+, W^-, Z^0, and the photon γ, according to the groups $SU(2) \times U(1)$ or, more correctly, $U(2)$. The group might be expressed as $SU(2) \times U(1)/Z_2$, where the '$/Z_2$' means „factor out by a Z_2 subgroup". However, there is more than one such subgroup, so this notation is not fully explicit. The notation '$U(2)$' automatically picks out the correct one. (I am grateful to Florence Tsou for this observation.) It seems that the reason that the electroweak symmetry group is not conventionally referred to as '$U(2)$' is that this does not easily extend to the symmetry of the full standard model, which also incorporates the strong symmetry group $SU(3)$, the full group being a version $SU(3) \times SU(2) \times U(1)/Z_6$,"* (PeR4) p. 641, 654.

There are epimorphisms between S^3 and $SO(3)$, resp. between $S^3 \times S^3$ and $SO(4)$. The group $SO(4)$ contains isomorphic normal subgroups to the group S^3, i.e. it is a not „simple" Lie group. The groups $SO(n)$, $n > 4$, are all „simple", i.e. they have not trivial coherent normal subgroups. The groups $SO(2n+1)$ have no normal subgroup unequal (e). The groups $SO(4)$ have exactly the not trivial normal subgroup $\{e, -e\}$, (EbH) 7.3.4.

From the fundamental theorem of algebra for quaternions it follows that there are exactly n roots of any quaternion with not vanishing imaginary part, (EbH) 7.1.8. For each quaternion of S^3 there is a quaternion representation as a sum of two product terms in the form $e \cdot cos\left(\frac{\omega}{2}\right) + q \cdot sin\left(\frac{\omega}{2}\right)$, where e denotes the „real" quaternion unit, q denotes a purely imaginary quaternion with norm equal one, and ω denotes an angle between zero and 2π, (EbH) 7.3.

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