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ORIGINAL ARTICLE

The generalized Newton's law of gravitation

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Abstract By introducing the gravitomagnetic field, we have shown that the generalized Newton's law of gravitation is the Lorentz force analogue of electromagnetism. The prediction of the generalized law for the precession of planets and binary pulsars coincides with the general theory of relativity. Moreover, the dilemma of the flat rotation curve is resolved in the framework of this law.

Keywords Generalized Newton's law · Gravitomagnetism · Modified Newton's law of gravitation · Rotation curve

1 Introduction

Newton's law of gravitation has been very successful in describing gravitational interaction related to the motion of celestial objects. However, this law is not invariant under Lorentz transformation of special relativity. Einstein introduces his theory of general relativity to remedy this defect. One of the conflicting observation of Newton's law was the precession of Mercury. Newton's prediction fell short to the value observed by a small amount of 43 arcsec/century. It is worth noting that the Einstein's theory of general relativity predicts a value consistent with observation. For this and other reasons the general theory of relativity is adopted as the correct theory of gravitation. However, we have recently (Arbab 2009a) shown that the existence of a gravitomagnetic field is responsible for the precession of planets (e.g., Mercury) and binary pulsars. This coincides with the Einstein's general theory if relativity prediction. Thus, the generalized (gravitational Lorentz-force) Newton's law of gravitation is consistent with the prediction of general relativity. Thus, Newton's law of gravitation can be rescued to suit the present observation. Moreover, the new generalized law can resolve several of the presently unsolicited problems with observation. One of these problems is the dark matter problem (Zwicky 1937). In the standard theory of gravity (general relativity) dark matter plays a vital role, explaining many observations that the standard theory can't explain by itself. But for many years, cosmologists have never observed dark matter, and the lack of direct observation has created skepticism about what is really out there. Perhaps a fundamental theory of gravity which differs from general relativity on large scales can explain the observations without recourse to the existence of dark matter. In the present paper we explore the need for the existence of dark matter. The inclusion of the gravitomagnetic force in the Newton's law of gravitation leads directly to the observational data pertaining to the rotation curve exhibited by galaxies. We remark here that static gravity is not the sole force that determines the dynamics of gravitating objects. Since, galactic objects move at very high speed the gravitomagnetic force will have significant contribution on the evolution of galactic bodies. The correct law of gravitation should be the one we provide here.

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2 The gravitomagnetic force and the modified Newton's law of gravitation

In electromagnetism, the force on a moving charge is given by Lorentz force. Analogously, the force on a moving mass is given by a Lorentz-like force. Such a force requires a priori the presence of the gravitomagnetic field produced by a central body. Hence, the gravitomagnetic force acting on any orbiting object of mass m about a central mass M, in the presence of a gravitomagnetic field, is given by Arbab (2009b)

$$\vec{F}_m = m\vec{v}\times\vec{B}_g, \quad F_m = \frac{mv^4}{c^2 r} \quad \Rightarrow \quad F_m = \frac{mv^2}{r}\left(\frac{v}{c}\right)^2, \tag{1}$$

upon using the relation (Arbab 2009c)

$$\vec{B}_g = \frac{\vec{v}}{c^2}\times\vec{E}_g \quad \Rightarrow \quad B_g = \frac{v^3}{c^2 r}, \qquad \vec{E}_g = \vec{a}, \tag{2}$$

for the circular motion. This is nothing but the generalized Newton's law of gravitation. Using (1) one can calculate the ratio between the centripetal force of an orbiting body to the gravitomagnetic force. This is given by

$$\left(\frac{F_m}{F_g}\right) = \left(\frac{v}{c}\right)^2. \tag{3}$$

This clearly shows that this gravitomagnetic force is a relativistic correction to the gravitational force. Equation (1) reveals that the gravitomagnetic force represents a relativistic correction of Newton's law of gravitation. Notice that the gravitomagnetic field is not a real magnetic field as we know it (arising from the motion of currents). It is an analogue of the ordinary magnetic field. This gravitomagnetic field has a unit of frequency. It is produced by the motion (neutral mass current) of a gravitating object. In 1916 Oppenheim and Laplace in 1805 considered a modification of Newton's equation by adding a velocity dependent term, but that couldn't give the correct Mercury precession (Gine 2008). The gravitational Lorentz force takes the general form (Arbab 2009b)

$$\vec{F}_{gm} = m(\vec{a} + \vec{v}\times\vec{B}_g) = \vec{F}_g + \vec{F}_m, \tag{4}$$

where $\vec{E}_g = \vec{a}$ is the gravitational field. Using the vector identity, $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$, (2) and the fact that $(\vec{a}\cdot\vec{v}) = 0$, (4) can be written as

$$F_{gm} = \frac{GmM}{r^2} - \frac{mv^4}{c^2 r}. \tag{5}$$

This shows that the force is not central, but depends on the object velocity as well. This is the modified Newton's law of gravitation. It must be applied when we study the motion of all gravitating objects. For circular motion one has

$$\frac{mv^2}{r} = \frac{GmM}{r^2} - \frac{mv^4}{c^2 r}, \tag{6}$$

which can be solved to find the velocity of the object in terms of its orbital distance. To this end, one has

$$v^2 = \frac{c^2}{2}\left(-1 + \sqrt{1 + \frac{4GM}{rc^2}}\right). \tag{7}$$

Substituting (7) in (5) yields the generalized gravitational force on the mass, viz.,

$$F_{gm} = -\frac{mc^2}{2}\left(\frac{1}{r} - \frac{1}{r}\sqrt{1 + \frac{4GM}{rc^2}}\right). \tag{8}$$

This is the generalized Newton's law of gravitation that should be used in studying any gravitational interaction of gravitating bodies. This force can be associated with a central potential energy (U_{gm}) of the form

$$\vec{F}_{gm} = -\vec{\nabla}U_{gm}, \tag{9}$$

which suggests that (see the Appendix)

$$U_{gm} = -\frac{mc^2}{2}\left(\ln r + 2\sqrt{1 + \frac{4GM}{rc^2}} + \ln\frac{\sqrt{1 + \frac{4GM}{rc^2}} - 1}{\sqrt{1 + \frac{4GM}{rc^2}} + 1}\right). \tag{10}$$

This is an effective potential describing the motion of the gravitating object. This support the assertion made by Wild (1996) that a central gravitational field can equally well be described by a modified Newtonian theory as by general relativity theory. Such a modification will satisfy the critical tests of general relativity. The curvature of space is a consequence of the force field and the Newton's equation determines this field. Hence, the two approaches are compliment to each other.

Employing (6) and (7) the gravitomagnetic force is given by

$$F_m = -\frac{mc^2}{2r}\left(1 + \frac{2GM}{rc^2} - \sqrt{1 + \frac{4GM}{rc^2}}\right). \tag{11}$$

The gravitomagnetic force vanishes for first and second order terms in $1/r$. We remark here that this potential energy is not a correction to the Newtonian potential energy but reduces to it in some particular case. It is evident from (6) that the gravitomagnetic force is opposite (repulsive force) to gravitational force. This equation is found to give the correct advance of perihelion of planets and binary pulsars (Arbab 2009a). The equation of the orbit of the gravitating object can be found by solving (8) with

$$m\ddot{r} - mr\dot{\varphi}^2 = F_{gm} \tag{12}$$

in the polar coordinates (r, φ). Such an orbit should give rise to an advance of the perihelion. It was shown by that the advance of perihelion of a planet can be obtained by considering a logarithmic correction term to the Newton potential energy (Mücket and Treder 1977). For the case $r > \frac{4GM}{c^2} = 2R_s$, (7) yields $v^2 = (\frac{GM}{r})$. This implies that far away from the central mass, the motion of orbiting object is governed by the ordinary Newton's law. For very large distance from the central mass the velocity v^2 remains approximately unchanged. This can resolve the problem of the dark matter occurring in Keplerian models. However, for short distance ($r \ll \frac{4GM}{c^2}$), (7) implies that the object velocity will be enormously large. Moreover, $v \to c$ when $r \to \frac{GM}{2c^2}$. As regards to dark matter, we see that (7) implies that the velocity far away from the central mass ($r \gg R_s$) is constant, and close distances to the central mass, $v^2 = c\sqrt{\frac{GM}{r}}$, i.e., $v \propto r^{-1/4}$. This latter relation does not hold for Keplerian orbit. This means that the matter at close distance from the gravitating central mass moves faster than the Keplerian law predicts. The above relation can be written as $v^2 = c^2\sqrt{\frac{GM}{rc^2}}$. We thus see that the absence of the gravitomagnetic force is responsible for the existence of dark matter. Therefore, we argue that the Generalized Newton's law of gravitation should replace the presently existing law, so that theoretical expectations from Newton's law agree with experimental facts.

One of potential explanation for the existence of dark matter emerges from the so-called Modified Newtonian Dynamics proposed by Milgrom (1983). It is considered in this theory that the possibility that there is not much hidden mass in galaxies. If a certain modified version of the Newtonian dynamics is used to describe the motion of bodies in a gravitational field the observational results are reproduced with no need to assume a hidden mass. The basis of the modification is the assumption that in the limit of small acceleration a very low characteristic acceleration, a_0, the acceleration of a particle at distance r from a mass M satisfies approximately the relation $\frac{a^2}{a_0} = \frac{GM}{r^2}$, where a_0 is some characteristic acceleration.

The second term in (4) corresponds to an acceleration of $a_m = (\frac{v}{c})^2\frac{v^2}{r}$. It is a second order relativistic correction to the centripetal acceleration. For the Earth-Sun system it amounts to $0.584 \times 10^{-10}\ \mathrm{ms}^{-1}$. The is approximately twice the value of the MOND acceleration. This can be compared with the Pioneer anomalous acceleration exhibited by Pioneer satellite having a value of $8.74 \times 10^{-10}\ \mathrm{ms}^{-1}$ directed toward the Sun (Anderson et al. 2002; Berman 2007). The existence of such a minute acceleration is a consequence of the gravitomagnetic effect of the planets in our solar system. Note that any gravitational radiation from the planets leads to an apparent acceleration that allows the orbital radius to decay with time. The orbital decay can also arise from the temporal increase in the gravitational constant (Arbab 2008). It is shown recently by Minguzzi that there is a possible relation between galactic flat rotational curves and the Pioneers anomalous acceleration (Minguzzi 2006). According to the generalized Newton's law, the gravitomagnetic acceleration for the planets in our solar system ranges from $10^{-9}\ \mathrm{ms}^{-1}$ to $10^{-15}\ \mathrm{ms}^{-1}$.

3 Concluding remarks

We have shown in this work that the generalized Newton's law of gravitation is of same form as Lorentz force in electromagnetism. The existence of the gravitomagnetic term accounts very well for the precession of planetary orbits and pulsars. The generalized law gives an acceptable interpretation of the flat rotation curve exhibited by galaxies. The new law, rules out the existence of dark matter to interpret the flat rotation curve. Therefore, static gravity is not the sole force that governs the dynamics of gravitating objects. The existence of a very minute acceleration could be liked to the gravitomagnetic force of the planets.

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Appendix

Let $X = \frac{4GM}{c^2}\frac{1}{r} + 1$ and $x = \frac{1}{r}$ the integral $\int \frac{1}{r}\sqrt{1 + \frac{4GM}{rc^2}}\,dr$ can be obtained using the formula

$$\int \frac{\sqrt{X}}{x}\,dx = 2\sqrt{X} + \int \frac{dx}{x\sqrt{X}} = 2\sqrt{X} + \ln\frac{\sqrt{X}-1}{\sqrt{X}+1},$$

where $X = \frac{4GM}{c^2}x + 1$.

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