About a H(1/2) –(quantum field energy theory) Hilbert space

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A quantum gravity theory needs to address the two challenges of gravity theory and quantum theory: general relativity cannot account for quanta and quantum mechanics cannot deal with the curvature of space-time ((RoC2) p.126).

.. Space-time are manifestations of a physical field, the gravitational field. At the same time physical fields have quantum character: granular, probabilistic, manifesting through interactions. The solution concept of LQT is basically about *"A spin network represents a quantum state of a gravitational field: a quantum state of space; it is characterized by a volume "v" for every node and a half-integer "j" for every line. ... The crucial difference between photons (the quanta of the electromagnetic field) and the nodes of the graph (the quanta of gravity) is, that photons exists in space, whereas the quanta of gravity constitute space themselves ((RoC2) p. 148)*.

"*The nature of the elementary particles of the SMEP, and the way they move, is desribed by quantum mechanics ((RoC3) fourth lesson). They are elementary excitations of moving substratum similar to the Maxwell field: minuscule moving wavelets*".

The overall concept is about a mass generating non-zero "vacuum" energy with respect to an extended Hilbert (energy) space H(1/2), where the concepts of "time", "space", "light" and "action/causality" occur after the first "symmetry break down" of a purely "dark energy" model "situation".

The current quantum state Hilbert space L(2)=H(0) is extended to the Hilbert space H(-1/2) including „fluid, plasma, fermion, photon, boson“ states. Its dual space H(1/2) provides the corresponding quantum energy space, whereby the „mass-less EPs“ (hot plasma) are (meta-physical, ground state (dark) energy) „elements“ of the closed orthogonal subspace of H(1/2) with respect to the H(1/2) inner product. The standard (variational) energy space H(1) is defined by the selfadjoint Friedrichs extension of the Laplacian operator in the standard H(0)=L(2) variational (statistics) framework. It keeps being valid for the quantum energy of the EPs *with* mass, including cold plasma. The corresponding mass/energy Hilbert space is given by the decomposition H(1/2) = H(1)+H(1, ortho) into the „fermions“ space and the orthogonal „bosons“ space. The latter one includes the Higgs boson. Analogue to the quantum mechanics in the standard H(0) Hilbert space framework Heisenberg's uncertainly relation is also valid in the H(-1/2) framework, but the current ("state of the art") H(0) Hilbert space is now newly compactly (!!) embedded into H(-1/2).

Earliest examples of complementary variational principles are provided by the energy principle of Dirichlet in the theory of electrostatics, together with the Thomson principles of complementary energy. As a short cut reference in the context of the considered Maxwell equations we refer to (ShM1).

The Friedrichs extension of the Laplacian operator is a selfadjoint, bounded operator B with domain H(1). Thus, the operator B induces a decomposition of H(1) into the direct sum of two subspaces, enabling the definition of a potential and a corresponding „grad“ potential operator. Then a potential criterion defines a manifold, which represents a hyperboloid in the Hilbert space H(1) with corresponding hyperbolic and conical regions ((VaM) 11.2). The direct sum of the corresponding two subspaces of H(1) are proposed as a model to define a decomposition of the „fermions“ space H(1) into H(1)=H(attractive)+H(repulsive), whereby the potential criterion defines repulsive resp. attractive elementary mass particles.

To conquer the potential barrier provides an alternative (bi-directional) quantum tunnel model. In this context we note that the current "tunnel effect model" is neccessary for the sun to be a star, which is the neccessary condition for all life on our planet. The sun produces heat and light by "fusing" atoms and, beforehand protons. Without this tunnel effect the required energy to produce (or better to "manifest", see below) a helium atom out of (attractive) protons would require about 1 million electric volt, which corresponds to a temperature of about 8 billion kelvin. But near the csun's enter its temperature is about 15 million kelvin only.

The Hilbert space framework enables a (weak H(-1/2) variational) Cauchy problem representation of the Einstein-Vacuum field equation with an initial „inflation-field“ with regularity g(inflaton) an element of H(1,ortho) without singularities for t --> t(Planck) (as t=0 is not defined in the conical region of the hyperboloid (conical and hyperbolic regions), avoiding current early universe state model singularities. The Hamiltonian formalism is only equivalent to the Lagrange formalism in case the Legendre transform is valid. This is not the case in the H(-1/2) framework. At the same point in time the H(-1/2) Hilbert space is more regular than the Delta function Hilbert space for all space dimensions n. Therefore, the reduced regularity requirements of weak solution(s) of a H(-1/2)-based variational representation allows only the Hamiltonian formalism.

Dirac introduced the Delta "function" (distribution) to model the charge of a "point". The string theory is basically about a "wave model", modelling energy transportation/interaction on quantum "level". The H(-1/2) "quantum elements" with corresponding quantum potentials in H(1/2) allows Hamiltonian based quantum (wave) mechanics independently from space (-time) dimensions. Therefore, the Huygens/Minkowski/SRT 4-dimensional space-time continuum "space" keeps to be the relevant Hilbert space framework, while the (hyperbolic/causality/time-space depending) "wave" behaviour is only part of the relevant hyperbolic region of the appropriate hyperboloid (see below).

There is a relationship between the Hamiltonian mechanics and the symplectic geometry with its underling antisymmetric symplectic (differential) form. The geometry of a closed skew-symmetric form (the symplectic geometry) and the positivity of pseudo-differential operators is provided in

Fefferman Phong Symplectic geometry and positivity of pseudo-diferential operators

An antisymmetric form is also related to Hamilton's quaternions.

"*In physics, quaternions are correlated to the nature of the universe at the level of quantum mechanics. They lead to elegant expressions of the Lorentz transformations, which form the basis of the modern theory of relativity. A quaternion is a 4-tuple, which is a more concise representation than a rotation matrix. Its geometric meaning is also more obvious as the rotation axis and angle can be trivially recovered. The quaternion algebra to be introduced will also allow us to easily compose rotations. This is because quaternion composition takes merely sixteen multiplications and twelve additions*" (Yan-Bin Jia, "*Quaternions and Rotations*").

The algebra of quaternions can be equipped with a scalar product to build a Hilbert space. With respect to the relationship of quaternions to modular forms we refer to Stankewicz J., "*Quarternion algebras and modular forms*".

J.Lewis C. Parenti Pseudodifferential operators and Hardy kernels on L(p).pdf

One considered Kummer function of the proposed alternative entire Zeta funtion theory is the Hilbert transform of the Gaussian function. The Hilbert transform of a Fourier series has always a vanishing constant Fourier term. Consequently, the corresponding Kummer function based alternative "Theta" function has a vanishing constant Fourier term.

The link to the proposed quantum field model is about the wavelets. Any L(2) function with vanishing constant Fourier term defines a wavelet function.

In the context of a modular form (a modular function, where its Fourier series is a Taylor series) we note that this modular form is a cuspidal modular form (cusp form with the related Petersson inner product, (CoG) pp. 76, 81), if the constant Fourier term is vanishing. Therefore, the counterpart of Hilbert transform property is about the restriction of the Hilbert space of entire module functions (Hecke-Petersson theory with its underlying Petersson inner product based on the hyperbolic volume element) to its ("wavelet H(1/2)-sub-space" corresponding) cusp-sub-space of cusp forms. For an introduction to spectral theory on hyperbolic surfaces we refer to (**).

(*) Cornell G., Silverman J. H., Stevens G., Modular Forms and Fermat's Last Theorem, Springer, 1997

(**) Borthwick D., Introduction to Spectral Theory on Hyperbolic Surfaces.

We further note that the H(1/2) space as first cohomology is fundamental to explain the properties of period mapping on the universal Teichmüller space. We also note that a vector space and any linear subspace are convex cones, i.e. the tool „convex analysis and general vector spaces“ can be applied.

Nag S., Sullivan D., Teichmüller theory and the universal period mapping via quantum calculus and the H(1 2) space on the circle

Biswas I., Nag S., Jacobians of Riemann Surfaces and the Sobolev H(1-2) on the Circle

We mention the relationship of the H(1/2) Hilbert space to the winding number around zero of a continuous cycle f in C-(0):

"*The solution of many problems in hydrodynamics requires a thorough understanding of the structure of the solutions of the divergence problem with homogeneous Dirichlet data*", where the Bogovskii operator in Sobolev spaces of negative order plays a key role (GeM).

The proposed quantum gravity model overcomes current handicaps to unite the quantum field theory and the (classical, differentiable manifolds based) Einstein field equations, e.g.

(1) a missing quantum theory with a non-zero zero point energy of the quantum vacuum containing the full information about all kinds of dynamic energies

(2) current quantum theory with a zero point energy radiation, while the Casimir effect shows a non-zero radiation

(3) the mass gap problem of the classical Yang-Mills field theory with solutions which travel at the speed of light so that its quantum version should describe massless particles/gluons; the problem is to establish rigorously the existence of the quantum Yang-Mills theory and a mass gap

(4) the rough shortcoming of the Higgs mechanism of particle mass generation that the origin of the Higgs mechanism itself is not elaborated leading to a vicious circle (see below)

(5) no current model of the (extended) definition of "dark energy", which is called "dark", because it does not appear to interact with observable "electro"-waves. This dark energy then covers >4.6%+23%+72%=99.6% of the total universe energy, being modelled by the H(1, ortho) Hilbert space; at the same point in time hydrogen and helium represent 99.9% of all atoms in the universe

(6) a quantum gravity theory must be a time-asymmetric (differential form based (BrK), (DrT)) quantum theory and the classical Weyl tensor (vector valued 1-form) should be zero at "Big Bang" and infinite for later dark wholes (R. Penrose).

(DrT) Dray T., Differential Forms and the Geometry of General Relativity, CRC Press, Taylor & Francis Group, 2015

The new quantum gravity model also addresses the dilemma as pointed out by E. Schrödinger:

"Since in the Bose case we seem to be faced, mathematically, with simple oscillator of Planck type, we may ask whether we ought not to adopt for half-odd integers quantum numbers *rather than integers. Once must, I think, call that an open dilemma. From the point of view of analogy one would very much prefer to do so. For, the „zero-point energy“ of a Planck oscillator is not only borne out by direct observation in the case of crystal lattices, it is also so intimately linked up with the Heisenberg uncertainty relation that one hates to dispense with it“.*

The proposed distributional quantum state H(-1/2) with corresponding inner product admits and requires *infinite* linear combinations of LQT "loop states" (which we "promoted" becoming "quantum fluid / quantum element / fermion & boson / rotating differential / ideal point / monad" states). The physical LQT space (which is a quantum superposition of the QLT "spin networks") corresponds to an orthogonal projection of H(-1/2) onto H(0). This othogonal projection can be interpreted as a general model for a "spontaneous symmetry break down". In this sense the orthogonal projection is the "manifestation" operator of the statement:

"mass is essentially the manifestation of the vacuum energy".

The orthogonal (mass manifestation) projector provides also a (H(1/2) wave (-let) compatible, see below) model of the Einstein photoelectric effect.

From the original famous paper of Higgs we recall the following sentences:

...."*the idea, that the apparently approximate nature of the internal symmetries of elementary-particle physics is the result of asymmetries in the stable solution of exactly symmetric dynamical equations .... is an attractive one. .... Within the framework of quantum field theory such a "spontaneous" breakdown of symmetry occurs if a Lagrangian, fully invariant under the internal symmetry group, has a structure that the physical vacuum is a member of a set of (physically equivalent) states which transform according the a nontrivial representation of the group. .... That vacuum expectation values of scalar fields, ... might play such a role in the breaking of symmetries.... in a theory of this type the breakdown of symmetry occurs already at the level of classical field theory...."*

The Higgs mechnism is about an ether like field pervading the whole universe. Its Lagrange density is given by Klein-Gordon equation determined by the electro-weak interaction and the Higgs potential. Its motivation is the Lagrange density of the union of all electro-weak interacting particles with its group theoretical model SU(2)xU(1). The mass term in the Lagrange density prevents the neccessary invariance under local phase tranformations of SU(2). In other words, not only fermions but also all interaction "particles" need to be massless. The Higgs mechnism overcomes this contradiction. The expectation value of its ground state energy is greater than zero (~246,2 GeV/(c*c)).

"A *rough shortcoming of the Higgs mechanism of particle mass generation is that the origin of the Higgs mechanism itself is not elaborated and this leads to a vicious circle*." The approach in "*Physics of Transcendental Numbers*" (Müller Hartmut, progress in Physics, Vol. 15, 2019) "*allows deriving the mass ratios of the fundamental elementary particles electron, proton, W(+/-), Z(0) and H(0)-boson as well as the temperature of the cosmic microwave background from Euler's number and its rational powers.*"

The Higgs boson combines the existence of mass together with the action of the weak force. But why it provides especially to the quarks that much mass, is still a mystery.

The SMEP is about the group theoretical model SU(3)xSU(2)xU(1) of the one electro-strong and the two electro-weak interaction particles. The abelian group U(1) is about the interaction of photon particles only. It is isomorph to the set of complex numbers with length 1. The non-abelian group SU(2) is isomorph to SO(3) and *S*(3), whereby *S*(3) denotes the set of quaternions with length 1. We note that *S*(3)x*S*(3) is isomorph to SO(4). The generalization of SO(n) is about the orthogonal sub-group of all orthogonal projections in an Euclidean vector space framework (Koecher M., Remmert R., "*Hamilton's Quaternions*" in Ebbinghaus H.-D. et. al., "*Numbers*", Springer-Verlag New York, 1991).

Our proposed model does not require the "adding mechanism" of the SMEP (i.e. the "sum" of the Lagrange "force" concepts SU(3)xSU(2)xU(1), plus a still missing fourth one for the "*graviton*"), but use the same concept of an ether like field, pervading the whole universe while generating two types of particles, attractive and repulsive particles with mass. Each generation "event" (modelled as a orthogonal projection operator) "generates" the forms of the observed universe like "time", "space", "continuity", "velocity", "forces", "causality", "entropy".... The corresponding observed "forces" are modelled/defined/represented by corresponding phenomenon specific PDE, which are (even in its weak (Langrange formalism based variational representations) only approximations to a common underlying Hilbert space based Hamiltonian "energy/action" minimization
/variational representation.

Bourgain J., Kozma G., One cannot hear the winding number

With respect to the two parameters characterizing a spin network we refer to a corresponding wavelet properties (BrK6): The (Calderón) wavelet reproducing ("duality") formula provides an additional (second) degree of freedom (compared to a Fourier (transform) wave and its related Fourier transform inverse; see also (BrK7) Notes 9/10) to apply wavelet analysis with appropriately (problem specific) defined wavelets in a (distributional) Hilbert scale framework, where the "*microscope observations*" of *two* wavelet (optics) functions can be compared with each other. We note that for a convenient choice of the two wavelet functions the Gibbs phenomenon disappears (see also (RoC) 5.5, "Complements").

Here we are

Braun K., 3D-NSE, YME, GUT solutions