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Hermann Weyl's "Purely Infinitesimal Geometry"

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Weyl's view of the continuum

The years 1916 and 1921/22 delimit a phase of Weyl's work during which he made his most radical contributions to the foundations of mathematics as well as of highly innovative contributions to differential geometry and classical field theory. All of this work had a strongly speculative background. Although Weyl lived and worked in Zürich from 1913 onward, he continued to be a German citizen and had been drafted into the German army in May 1915. He was discharged from the service after an intervention by the Swiss government in August 1916.¹ The deep crisis of European culture in the years surrounding World War I was deeply felt in a very personal way by Weyl, although his reaction had much in common with the way this crisis was felt by other German academic intellectuals, particularly in the cultural sciences.²

Weyl's conception of the continuum during that time shifted from the semi-constructivist position in [W3] to a clear adherence to Brouwer's intuitionism and became more cautious with some openness to Hilbert's position from the middle of the 1920s. A common feature during the shifts was that he wanted to link the mathematical concept of continuum to a "directly experienced continuity" [W5, p. 527] so that the latter could never be identified with an extensionally closed realm or even object of thought. Weyl prized Brouwer's theory as it offered a means of representing the continuum as a "medium of free becoming", an expression carrying rather clear Fichtean connotations.³ And, in fact, he chose deliberately to link the continuum with a concept structure elaborated in the dialectical philosophy of the early nineteenth century, a notion whose logical features may appear surprising for a twentieth-century mathematician.

A general concept, *the whole* (here the continuum), has to be presupposed in order to give meaning to an individual determination, the particular, or the part (here the point).⁴ On the other hand, *the whole*, the general concept (the continuum) is constituted in a process of common generation by the particulars, (the parts).

¹[S, 61ff.]; [FS, 20ff.]

²A broad cultural picture of the self-understanding of cultural scientists in Germany has been given by Ringer [R].

³Weyl had already admitted the Fichtean inspiration of his thoughts in the foundations of

This conceptual figure came close to those procedures that had been called "impredicative" by Russell and Poincaré and that had been blamed as being responsible for contradictions of the type of Russell's antinomy. One example in real analysis, debated by Weyl himself, was the definition of a real number a as $a := \sup(A)$ for a bounded set $A \in \mathbb{R}$, as in this definition reference to the totality of the reals is made. So, for Weyl, it was clear that an attempt to translate too directly what he regarded to be the "essence of the continuum" into mathematical form might easily lead to logical difficulties.⁵ Weyl saw and discussed two conceptual strategies to avoid the dilemma:

Strategy I: Design of an "atomistic" theory of the continuum (either contructive or axiomatic).

This approach might be logically consistent but would, in Weyl's view, never lead to sufficiently rich expressions of the intuitive idea of the continuum, as Weyl immediately remarked in a discussion of his own "atomistic" theory in [W3, p. 83]; [W5, p. 527]. A set-theoretic approach with a topological characterization of continuity appeared even less convincing to him. Even in 1925, when his most radical phase in the foundational debate had already ended, Weyl made quite clear that such an approach was not convincing to him because a set-theoretic approach:

... contradicts the essence of the continuum, which by its very nature cannot be battered into a set of separated elements. Not the relationship of an element to a set, but that of a part to the whole should serve as the basis for an analysis of the continuum. [W6, p. 5]

But what, then, might be understood as the "relationship of a part to the whole" in the case of the continuum, independent from set theory? Weyl never gave a final answer to that obviously very difficult and perhaps unanswerable question, but he did outline how he wanted to approach it.

Strategy II: Development of mathematical theories that symbolically explore the "relationship between the part and the whole" for the case of the continuum.

Fundamental for this strategy was an idea expressed by Weyl in his 1925 article for the Lobachevsky prize as follows:

... a manifold is continuous if the points are joined together in such a way that it is impossible to single out a point just for itself, but always only together with a vaguely delimited surrounding halo (Hof), with a neighbourhood. [W6, p. 2]

As examples, Weyl referred to the characterization of a function element in complex analysis and the characterization of neighborhood systems of a point as a limit structure in topology. His "infinitesimal geometry" of 1918 was designed as another, and he hoped for a time, far-reaching, contribution to this strategy. This work should therefore be read with this context in mind.

The design of a "purely infinitesimal geometry"

From the perspective of Weyl's view of the continuum around 1918, differential geometry may be considered as one line of access to the question of how to link the infinitesimal "halos" of the point to the structure of the *whole*, the manifold. From this point of view, the differential geometric structures should be defined such that only relations in each infinitesimal neighborhood are immediately meaningful. Relations between quantities in different neighborhoods (of finite distance) ought to be considered meaningful only by mediation of the *whole*, more technically expressed by an integration process over paths joining the two points at the centers of the neighborhoods. From the point of view of building the continuum from its *smallest parts*, so Weyl claimed over and over again in the years following 1918, Riemann's differential geometry did not appear completely convincing. In Riemannian geometry the relationship between lengths of vectors ξ and η is well defined, independent of the points p and q of the manifold to which they are attached ($\xi \in T_p M, \eta \in T_q M, p \neq q$). From Weyl's continuum-based view of differential geometry such a comparison appeared unmotivated, and he stipulated instead that a

... truly infinitesimal geometry (wahrhafte Nahegeometrie) ... should know a transfer principle for length measurements between infinitely close points only. [W2, p. 30]

In this formulation Weyl alluded to Levi-Civita's transfer principle of direction in a Riemannian manifold embedded in a sufficiently high-dimensional Euclidean space, locally given by

$$\xi^i = \xi^i - \Gamma^i_{jk} \xi^j dx^k$$

with the dx^i to be interpreted as the coordinate representation of a displacement vector between two infinitesimally close points so that the direction vector ξ^i has been transferred to ξ^i .

Weyl immediately recognized that Levi-Civita's concept of parallel displacement wonderfully suited his nascent ideas about how to build differential geometry strictly on the basis of infinitesimal neighborhoods. He discussed it in this light during his lecture on general relativity in the summer of 1917 at Zürich, not yet knowing how to proceed similarly for the measurement and comparison of lengths.⁶

This was the motivation behind Weyl's effort to separate logically the concept of parallel displacement from metrics and to introduce what he called an *affine connection* Γ on a (differentiable) manifold as, speaking in later terminology, a linear torsion-free connection. Guided by the example of affine connections Weyl also proceeded to build up the metric in a manifold from a "purely infinitesimal view". The result was his introduction of a generalized Riemannian metric, a *Weylian metric* on a differentiable manifold M , which is given by:

1. a *conformal structure* on M , i.e. a class of (semi-) Riemannian metrics $[g]$ in local coordinates given by $g_{ij}(x)$ or $\tilde{g}_{ij}(x) = \lambda(x)g_{ij}(x)$, with multiplication by $\lambda(x) > 0$ (real valued) representing what Weyl considered to be a *gauge transformation* of the representative of $[g]$, and
2. a *length connection* on M , i.e. a class of differential 1-forms φ in local coordinates represented by $\varphi_i dx_i$, $\tilde{\varphi}_i dx_i = \varphi_i dx_i - d \log \lambda$ (representing the *gauge transformation* of the representative of φ).

In modern terms, Weyl introduced a connection ω in the line bundle $\pi: L \rightarrow M$ associated to the $GL(n, \mathbf{R})$ principal bundle of frames in the tangent bundle TM or — assuming orientability — in the principal bundle of positive 1-frames in L with group $G = (\mathbf{R}_+, \cdot)$. Here ω assumes values in the Lie algebra $\mathcal{G} = \mathbf{R}$ with trivial multiplication ($[x, y] = 0$) and can, after local trivialization, be identified with a 1-form ω . Change of trivialization is here given by a pointwise multiplication by $\lambda(x)$ with values in $\mathbf{R}_+$. That leads to a gauge transformation

$$\tilde{\omega} = \lambda^{-1} d\lambda + \lambda^{-1} \omega \lambda = \lambda^{-1} d\lambda + \omega = d \log \lambda + \omega,$$

which coincides up to sign with Weyl's formula. Weyl, of course, derived the gauge transformation as a compatibility condition for length transfer expressed in different representatives of the conformal metric.

The length transfer by a connection form φ on M was introduced by Weyl in close analogy to the direction transfer in M by an affine connection. Specification of length calibration in some point $p \in M$ may be represented as the determination of a positive real value $l(p)$. Without further specifications this calibration is meaningful only as a measuring device at the point p in M . In order to compare lengths of vectors attached to different points such a measure has to be transported from one point to another. For two infinitesimally close points p, p' with displacement vector α (for a modern reader to be understood as $\alpha \in T_p M$) transport of length measure was introduced by Weyl as fulfilling the condition

$$l(p') = l(p) - \varphi(\alpha)l(p) = l(p)(1 - \varphi(\alpha));$$

i.e. length has to be recalibrated in an infinitesimal transfer along the displacement vector α by the factor $(1 - \varphi(\alpha))$. For two finitely distant points $p, q \in M$, recalibration is given by a factor of the form

$$1 - \int_I \varphi(\dot{c}(t)) dt$$

integrated along a path parameterized by I . So length comparison between finitely distant points becomes in general path dependent, just like comparison of direction for affine connections.

In consequence, Weyl introduced *length curvature* as giving the length difference $l - l'$ for transfer along the periphery of an infinitely small parallelogram with sides α, β . This turned out to be a 2-form⁷

Weyl explored his new geometry and derived, among others, the following basic properties.

THEOREM 1 *A Weylian metric is (semi-)Riemannian⁸ if and only if $f \equiv 0$.*

THEOREM 2 *In a Weylian manifold there exists exactly one compatible affine connection Γ .⁹*

So there exists a curvature tensor $F^i{}_{jkl}$ of Γ that is moreover, as Weyl remarked, gauge invariant. This appeared quite remarkable for Weyl as he considered gauge invariance an important criterion for physical relevance of quantities in his new geometrical structure, in addition to general covariance.

The derivation of the length curvature from the length connection is structurally identical with the relationship between the relativistic electromagnetic field f_{ij} and its 4-potential φ_i . Thus, obviously, the first set of *Maxwell equations* hold

$$df = dd\varphi = 0, \quad \frac{\partial f_{jk}}{\partial x^i} + \frac{\partial f_{ki}}{\partial x^j} + \frac{\partial f_{ij}}{\partial x^k} = 0.$$

Guided by Mie's relativistic formulation of Maxwell's theory Weyl considered the vector densities $f^{ij} := \sqrt{-\det(g_{ij})} f^{ij}$ and the the source equation for them as an expression of the second set of *Maxwell's equations* in his theoretical framework

$$\frac{\partial \tilde{f}^{ij}}{\partial x^j} = \rho^i.$$

For readers of Weyl's papers of about 1918 such an approach may have appeared as a nice but still rather formal analogy. But for Weyl it appeared to be much more; in fact he appeared to be carried away by this analogy and took it as an indicator of a more or less obvious and forceful semantical bridge between mathematics and physics. So, for example, in his first paper on the subject in *Mathematische Zeitschrift*, addressed to a mathematical audience, Weyl barely made any distinction between mathematical concepts and physical interpretation [W1]. He seemed to be completely sure that his analysis had led him to a

... world metric from which not only gravitation but also the electromagnetic effects result, which therefore gives account of all physical processes, as one may assume with good reason. [W1, p. 2]

For the reader the "good reasons" for such a realistic semantical turn in the argument could not be at all so clear as they seemed to be for Weyl. Taken at face value they were not much more than analogies between the curvature equations and Mie's theory of the electromagnetic field. That was, of course, sufficient reason to investigate whether it was possible to extend this connection to a more elaborate theoretical link. For Weyl, however, there appeared to be not so much a question

⁸A Weylian metric reduces to the semi-Riemannian case, if $\varphi \equiv 0$ after proper choice of gauge, so that the calibration can be integrated independently of the path.

but rather already an answer. Surprising as such a semantical jump may appear for later readers, we want to take a glance at Weyl's "good reasons" for accepting the connection between his new geometry and physics on the spot. For that we have to take Weyl's philosophical background, in particular his Fichtean studies, into account.

Weyl's Fichtean background

In the period between 1916 and 1922 Weyl was strongly fascinated by Fichte's work because in it he found "metaphysical idealism in its most unreserved and forceful expression". With that position Weyl took up a motif that was shared by a group of intellectuals in Germany at the beginning of the twentieth century who felt the rise of positivism as an indicator of a cultural crisis and looked for strong idealist counterforces in classical German philosophy. For Weyl, Husserl's phenomenology, which he had started to study even earlier, shifted into the background during these years. It continued to be represented in his personal universe of discourse, however, by his wife Helene, who had been a student of Husserl's and was now an expert in phenomenology. Her views thus served Weyl as a sort of critical counterbalance to Fichte's highly speculative version of idealism. In our context Fichte's derivation of the concepts of space, time, and matter, in addition to those features of his philosophy that made Fichte "a constructivist of the purest water", as Weyl later said [W8, p. 637], are of particular concern.

The constructivist aspect in Fichte's philosophy of knowledge is most clearly expressed in the latter's essay on "transcendental logic" [F2]. Here Fichte discussed the relationship between classical logic and *transcendental logic*, which Fichte used synonymously with the terms *philosophical* or *dialectical* logic. Reading Fichte today and from the point of view of the history of mathematics, we find striking similarities between Fichte's opposition between *classical* and *transcendental logic* and the differences between Hilbert and Weyl with respect to formalist versus intuitionist mathematics.¹⁰ Fichte criticized classical logic because it took concepts as something given and analyzed their relationships by formal means only. Philosophical (*transcendental*) logic would, on the contrary, generate the concepts in a sort of reconstruction ("Nachconstruction") always trying to go back to some presupposed final origin ("das Ursprüngliche") [F2, p. 122]. In this context Fichte gave a very general explication of what he thought *construction* should achieve:

Construction is the instruction to invent the concept by the power of imagination (Einbildungskraft) such that evidence is gained. [F2, p. 188]

So, for Fichte, *construction* is not reducible to technical means of concept generation that are specified once and for all, although such technical means are obviously not excluded. Transcendental logic or dialectics should even give a "lawful method"

¹⁰Even the rather well-known chess metaphor used by Weyl to describe the nature of Hilbert's formalist proof theory (W8, p. 595ff.) had been used in a somewhat similar manner by Fichte.

for such a construction, i.e. characterize the conditions of reproducibility for a generation process of the mind. For a reader like Weyl, the correlation with Brouwers' intuitive view of mathematical concepts could not be closer. Obviously it might be quite instructive to reread Weyl's "constructivist" approach to mathematics in light of this Fichtean background, but that is not the goal of this paper.

Fichte exemplified his general views by considering the construction of the concepts of space, time, force, and matter; notions that appear over and over again in his *Wissenschaftslehre*. The basic figure of Fichte's generative dialectics of knowledge was the opposition of a so-called *self* (Ich) to a *non-self* (Nicht-Ich), which lies at the bottom of the Fichtean generative myth of all the concepts of the world. At the beginning and at the end of this generation process, the self is considered as "absolute", i.e. not opposed to any non-self, and was declared by Fichte as "absolutely free". During the transitional steps the self becomes limited and its freedom restricted by the opposition to the non-self. All the derivations in Fichte's *Wissenschaftslehre* deal with different steps toward an assimilation of certain features of the non-self, limiting the activities of the self. Part of these derivations is the generation of the concept of space, which Fichte described as follows:

— Take separate "products of the non-self". They are posed by the imagination (by the "self") into separate points X, Y^{11} and endowed with separate "spheres of action (Sphären der Wirksamkeit)" zx, zy , mutually excluding each other.

— Yet, on the other hand, between the mutually excluding "spheres of activity" there is, according to Fichte, "necessarily continuity". In this sense a unity between any such "spheres of activity" is posed by the imagination.

— The unity is produced in a medium of "continuous free activity" of the mind. This medium of free activity is *space*, symbolized as O by Fichte. [F1, p. 194–196]

Thus, in Fichte's derivation, Kant's transcendental space concept was taken up (*space* as unifying principle of the mind for determinations of localized activities) and radicalized (medium of "continuous free activity"). Thus Fichte's notion of space was devoid of any specific a priori structure like, for example, Euclidean geometry. With respect to mathematical structure his approach could be considered as generalizing Kant's, and Fichte went on from here to the construction of the concept of matter, considering the "spheres of action" as *intensities* irreducible to, but united in *extension* O . He considered the "intensities" of the "spheres of action" as being "forces" that fill the space with their product [F1, p. 201]. Matter, finally, was to him nothing but the external manifestation of the activities of forces.

The last motif, matter as nothing but the manifestation of the activities of forces. [F2, p. 356f.], was rather widespread in German natural philosophy around the turn of the eighteenth to the nineteenth century. It had been expressed by Kant, radicalized by Fichte, turned into a more subtle, qualitatively differentiated version of productivity by Schelling, and finally assimilated by Hegel into his system. For Fichte, as for the other post-Kantians, there was no empty space as an a priori form in which the senses could locate their empirical information. Space was the connecting activity between different "spheres of action" reconstructed by the self. The latter flourished for Fichte as a sort of *semantical meeting point for quite different*

features. They constituted the "infinitely small parts of space" and the building forces of matter. They represented the "interior forces of the non-self acting in absolute freedom"¹² but had to be reconstructed by the self as "spheres of action" of the productive imagination [F1, p. 201f.].¹³

When Weyl tried, beginning around 1918, to conceive the mathematical continuum as the uniting medium of the individual determinations that cannot be "battered" into its single elements without loss of meaning, he was taking up an old topic in the philosophy of mathematics, which can be traced back to Aristotle. But in Weyl's rhetoric (the continuum as "medium of free becoming") the ties to Fichte's conception of continuity as the medium of free activity of the self were apparent to those who had read the latter. Fichte's "spheres of activity" bound inseparably to the points stood in close affinity to Weyl's goal of constituting the concept of mathematical continuum by some structural link binding the points inseparably to the general concept. But of course such a reference was not enough in itself to become mathematically productive. That the conceptual figures elaborated in nuce in an adaptation of Fichtean topics could take hold for several years so strongly on Weyl's mind and actually bear mathematical fruit depended on another development that had taken place in mathematical physics and that fit nicely into his philosophical scheme.

Mie – Hilbert – Weyl

In 1912 the Greifswald physicist Gustav Mie had formulated a research strategy to explain the basic phenomena of matter on a purely electromagnetic basis, in particular the existence, mass, and stability of the electron [M1]. This program continued pre-relativistic attempts by Lorentz, Kaufmann, and others to develop a purely electromagnetic theory of the electron and its mass. Mie started in 1912 from the established concept of special relativity and in particular its energy-mass equivalence. He considered Lorentz-invariant scalar density functions L on Minkowski space-time depending on the values of a 4-potential φ and its first derivatives $D\varphi$ (Mie, of course, expressed these in coordinates $\varphi_i, \frac{\partial \varphi_i}{\partial x^k}, i, k = 1, \dots, 4$). This led him to a variational criterion for potentials

$$\delta \int L(\varphi, D\varphi) dx = 0$$

with respect to infinitesimal variations of φ . Starting from the observation that the Maxwell vacuum equations result from the particular case

$$L(\varphi, D\varphi) = \frac{1}{2} F_{ik} F^{ik}, \quad F_{ik} = \frac{\partial \varphi_i}{\partial x^k}$$

¹² "Absolute" freedom of the non-self to be understood as unlimited by the activities of the self.

¹³ The concept of space was thus generated by Fichte in a figure very close to the one used by him (and the other dialectical philosophers of the early nineteenth century) to characterize the relationship between the individual and society (the state, in Fichte's language). Such a social

he proposed to look for modifications of L leading to nonlinear terms in the field equation and solutions with space-like local energy concentrations ("energy knots"), which, as Mie hoped, might turn out to be stable, i.e. nondispersive in time.

Mie thus tried to bring the old dream of a dynamistic matter explanation, which had been philosophically expressed by Kant, to physico-mathematical maturity, although the technical framework in which Mie located his approach reduced the general concept of force to the electromagnetic setting.¹⁴ This reduction of the dynamistic matter explanation to purely electromagnetic actions had also enjoyed a long tradition among natural scientists going back deep into the nineteenth century. As a conjectural approach it had already been expressed by Weber, Riemann's teacher in physics. It was actively pursued by Zöllner, and to a certain degree by Riemann. Around the turn of the century, Kaufmann, Wiechert, and again to a certain degree Lorentz, had taken this approach up, placing it in the setting of Maxwell's theory of electrodynamics.¹⁵

The daring mathematical design of Mie's program, combining special relativity with variational viewpoints, and with the Kant-Weber-Riemann connection looming in the natural philosophical background, may have contributed to its enthusiastic reception by Hilbert. As is well known, Hilbert's research for a generally covariant formulation of relativity in 1915 was built on and modified Mie's program [EG], [V, p. 54ff.]. Hilbert extended Mie's variational principle to a Lagrangian $L(g, Dg, D^2g, \varphi, D\varphi)$ depending on a varying Lorentz metric g (expressed, of course, in coordinates as $g_{ij}(x)$), as well as its derivatives up to the second order, and a 1-form φ with its first derivatives, where the Lagrangian had to fulfill the condition of invariance under general coordinate change. So Hilbert's Lagrangian L lived on a Lorentz manifold, and for Hilbert the underlying invariance postulate had a rather direct semantical justification. Combining Riemann's and Minkowski's views, physically meaningful functions are defined on space-time events and thus on a physical manifold, which ought to be infinitesimally Minkowskian rather than Euclidean. Read today, that may seem a rather apparent step. That it was not so at the time, however, is shown by the long journey Einstein had to take between 1912 to 1915 before he finally convinced himself that (semi-)Riemannian geometry was more than a formal game and could be endowed with strong physical meaning.¹⁶ Hilbert's goal, moreover, in that respect also differed from Einstein's, as it was not primarily directed towards gravitation theory.¹⁷ Hilbert was much more ambitious; following Mie he wanted to outline the basis for a universal theory of matter. He was quite frank in expressing this ambitious goal, and, even more, he actually believed for some time that he had come close to it. He thus stated his conviction that by means of his generalized version of Mie's field theory of matter

¹⁴Mie was apparently completely aware of the neo-Kantian connotations of his research program. An explicit formulation is to be found in [M2]; and there is no reason to doubt that he was already aware of this connection in 1912.

¹⁵See [McC] for more details.

¹⁶Compare [Sta] and [Rea].

... the most intimate hidden processes in the interior of the atom can (...) be clarified.

Taking such strong hopes into account, expressed by Hilbert, it may appear less surprising that the much more "emphatic" thinker¹⁸ Weyl was nearly overwhelmed by the prospect of a close link between his Fichtean-inspired researches on the conceptual structures of the continuum and the Mie-Hilbert theory of matter. Surely he must have felt strong and surprising semantical resonances between the two different worlds of discourse in which he lived in 1918, the mathematical/physical one centered at Göttingen and the philosophical/cultural circles he moved in at that time in his local Zürich environment.¹⁹ These resonances must have appeared even more seductive for Weyl, as with regard to foundational questions proper the two realms of discourse brought him into strong opposition with his former teacher Hilbert, stronger than he would have liked personally. Weyl could now hope for a basic agreement between him and Hilbert, at least in questions of mathematical philosophy of nature as well as a place for himself in a renewed Göttingen universe of mathematics.

Instead, however, he had to live through disillusionment in several respects before he could continue to explore his basic ideas in a direction that found acceptance at Göttingen. With respect to the theory of matter the reaction of physicists was split. For a short time he found some strong support from Sommerfeld, Pauli and Eddington, whereas Einstein was more than skeptical from the outset. But even the early supporters turned away from Weyl's theory after the first possibilities of directly tapping its explicative power for physical phenomena had to be weakened.²⁰ Even more disillusioning for Weyl may have been Hilbert's rather strong rejection of Weyl's theory in the first years after its publication. Thus, in a lecture course given in the winter semester 1919/1920 at Göttingen, Hilbert criticized it as a kind

... of Hegelian physics trying to predetermine the whole world process which would not transcend the limited content of a finite thought, if one takes that view seriously. [H3, p. 100]

This flat rejection cannot be understood from a purely mathematical point of view,²¹ and in fact it was explicitly formulated by Hilbert as a difference in the mathematical philosophy of nature. Of course, it was easy for Hilbert to identify a strong idealist reductionism in Weyl's approach,²² as Weyl quite frankly expressed his Fichte-inspired drive to give direct physical meaning to the concepts of "infinitesimal geometry". But, in effect and read more carefully, Hilbert's strong rejection of Weyl's theory argued (perhaps unknowingly) against temptations he, Hilbert, in fact shared with Weyl. Hilbert himself was a strong proponent

¹⁸The characterization of Weyl's style of thought as "emphatic" goes back to E. Brieskorn in a comparison with the thought style of Hausdorff.

¹⁹To give names: F. Medicus and Helene Weyl.

²⁰[Str], [S], [V].

²¹Compare Rowe's remark in [H3, p. xif].

²²As a result of this, Weyl's philosophical motivations ("Hegelian physics")

of mathematical reductionism in the study of nature (and probably a more rigid one than Weyl), and he was himself much more prone to shortcut solutions when it came to the dialectics of finiteness and infiniteness in mathematical theories, as was shown by his involvement in the debates and research on the foundations of mathematics. Hilbert's brilliant former student, Weyl, now (around 1918) deviated from central tenets governing the Göttingen mathematical discourse. He was thus ideally suited as an external target for a projection of basic problems inherent in these rules, problems which were only vaguely intuited by Hilbert and never accepted by him as inherent to his own approach.

Only after Weyl had partially withdrawn his radical opposition to Hilbert's views with regard to foundational questions and outlined how a compatibility between both approaches could be established on a pragmatic level [W5] did Hilbert offer a positive reevaluation of Weyl's contributions to differential geometry and the Mic-Hilbert theory of matter [H2]. On the other hand, Weyl's own revision of his matter concept starting in 1922/23 and his later contribution to Dirac's electron theory at the end of the 1920s transformed his early gauge-theoretic ideas in a way that made them much more accessible and capable of a piecemeal assimilation by physicists. This complicated development of change, transfer, and transformation from Weyl's early gauge geometry to the gauge theories of the 1960s is a different story, although it is linked to the young Weyl's reformulation of Riemann's geometry following his dreams of a philosophically more acceptable concept of the continuum.²³

References

- [EG] Earman, John, and Glymour, Clark, *Einstein and Hilbert: Two months in the history of general relativity*, Arch. Hist. Exact Sci. **1** (1978), 291–308.
- [F1] Fichte, Johann Gottlieb, Grundriss des Eigenthümlichen der Wissenschaftslehre in Rücksicht auf das theoretische Vermögen. Jena – Leipzig 1802. Gesamtausgabe I.3. Hrsg. Reinhard Lauth, Hans Jacob. Stuttgart – Bad Cannstadt, 142–208.
- [F2] Fichte, Johann Gottlieb, *Ueber das Verhältnis der Logik zur Philosophie oder transcendentalen Logik*. Nachgelassene Werke I. Hrsg. I. H. Fichte. Berlin 1834 (Nachdruck Berlin: de Gruyter 1971), 103–400.
- [FS] Frei, Günther, and Stammbach, Urs, Hermann Weyl und die Mathematik an der ETH Zürich, 1913–1930. Basel usw.: Birkhäuser 1992.
- [H1] Hilbert, David, *Die Grundlagen der Physik. Erste Mitteilung*, Nachrichten Gesellschaft der Wissenschaften Göttingen (1915), 395–407.
- [H2] Hilbert, David, *Referat über die geometrischen Schriften und Abhandlungen Hermann Weyls, erstattet der Physiko-Mathematischen Gesellschaft an der Universität Kasan*, Bulletin de la Société Phys. Math. de Kazan (3) **2** (1927), 66–70.
- [H3] Hilbert, David, *Natur und mathematisches Erkennen*, Vorlesung 1919–1920 Göttingen, nach der Ausarbeitung von P. Bernays. Hrsg. D. Rowe. Basel etc.: Birkhäuser 1992.
- [McC] McCornack, Russell, H. A. Lorentz and the electromagnetic view of nature, Isis **61** (1970), 459–497.

- [M1] Mie, Gustav, *Grundlagen einer Theorie der Materie. 3 Teile*, Ann. Physik **37** (1912), 511–534, **39** (1912), 1–40, **40** (1913), 1–66.
- [M2] Mie, Gustav, *Aus meinem Leben*, Zeitwende **19** (1948), 733–743.
- [Rea] Renn, Jürgen, e.a. *Das Züricher Notizbuch*, in G. Castagnetti, P. Damerow, W. Heinrich, J. Renn, T. Sauer. Wissenschaft zwischen Grundlagenkrise und Politik: Einstein in Berlin. Arbeitsbericht der Arbeitsstelle Albert Einstein. MPI für Bildungsforschung Berlin 1994. 24–57.
- [R] Ringer, Fritz, Die Gelehrten. Der Niedergang der deutschen Mandarine, 1890–1933, München: DTV 1987.
- [S] Sigurdsson, Skuli, Hermann Weyl, *Mathematics and physics, 1900–1927*, dissertation, Harvard University, Cambridge, MA, 1991.
- [Sta] Stachel, John, *Einstein's search for general covariance, 1912–1915*, in Howard, Don; Stachel, John, Einstein and the History of General Relativity, Basel etc.: Birkhäuser 1989, 63–100.
- [Str] Straumann, Norbert, *Zum Ursprung der Eichtheorien bei Hermann Weyl*, Physikalische Blätter **43** (11) (1987), 414–421.
- [V] Vizgin, Vladimir P., Unified Field Theories in the First Third of the 20th Century, Translated from the Russian by J. B. Barbour, Basel etc.: Birkhäuser 1994.
- [W1] Weyl, Hermann, *Reine Infinitesimalgeometrie*, Math. Z. **2** (1918), 384–411. [W9] **2**, 1–28 [30].
- [W2] Weyl, Hermann, *Gravitation und Elektrizität*, Sitzungsberichte Akademie der Wissenschaften Berlin, 465–480. [W9] **2** (1918), 29–42 [31].
- [W3] Weyl, Hermann, Das Kontinuum. Kritische Untersuchungen über die Grundlagen der Analysis, Leipzig 1918. Neudruck New York 1960.
- [W4] Weyl, Hermann, *Der circulus vitiosus in der heutigen Begründung der Analysis*. Jber. DMV **28** (1919), 85–92. [W9] **2**, 43–50 [32].
- [W5] Weyl, Hermann, *Die heutige Erkenntnislage in der Mathematik*, Symposion **1** (1925), 1–32. [W9] **2**, 511–542 [67].
- [W6] Weyl, Hermann, Riemanns geometrische Ideen, ihre Auswirkungen und ihre Verknüpfung mit der Gruppentheorie, Ed. K. Chandrasekharan, Berlin etc. Springer 1988.
- [W7] Weyl, Hermann, Mathematische Analyse des Raumproblems, Vorlesungen gehalten in Barcelona und Madrid. Berlin etc.: Springer 1923, Neudruck Darmstadt (Wissenschaftliche Buchgesellschaft) 1963.
- [W8] Weyl, Hermann, *Erkenntnis und Besinnung*, Studia Philosophica 1954. [W9] **4** 631–649. [166]
- [W9] Weyl, Hermann, *Gesammelte Abhandlungen*, 4 Bände, Hrsg. K. Chandrasekharan, Berlin etc.: Springer 1968.