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A simple model for the prediction of CO_2 concentrations in the atmosphere, depending on global CO_2 emissions

M. Vollmer[1], W. Eberhardt[2]
1: UAS Brandenburg / Germany, vollmer@th-brandenburg.de
2: TU Berlin / Germany

Abstract

We present a very simple model for estimating time dependent atmospheric CO_2 concentrations *c(t)* from global carbon emission scenarios, serving as single input data. We derive a single linear differential equation of 1st order, based on parameters which are estimated from quantitative data of the global carbon project and Mauna Loa data for CO_2 concentrations. The model is tested first by comparing it to the 1960 to 2021 period with reasonably good quantitative agreement and, second to two of the typical current IPCC scenarios with good qualitative agreement. Finally, some new emission scenarios are modelled. Despite several drawbacks concerning absolute quantitative predictions, there are two important advantages of the model. First, it can be easily executed by students already with simple programmable spreadsheet programs such as Excel. Second input emission scenarios can be changed easily and expected changes are immediately seen for discussion during undergraduate and graduate courses on the carbon cycle and climate change.

1. Introduction

Complexity governs our world. One of the most important and relevant phenomena concerning impact on living conditions on earth is climate change. It requires quite sophisticated physical models to quantitatively relate carbon dioxide concentrations in the atmosphere to global anthropogenic greenhouse gas emissions. The atmospheric concentrations of the greenhouse gases in turn serve as input to other complex models predicting respective changes of climate parameters such as temperature, precipitation patterns, shifting climate zones on earth, extreme weather events, sea level rise and many more. The more complex the models, the more difficult it is for non-experts to quantitatively understand all aspects, even for other scientists in the fields of physics, chemistry, biology, mathematics etc.

The greenhouse effect and climate change with all related physics processes have of course been covered in physics education journals (e.g. [Ono11, Wil12]) and respective resource materials have been reviewed [Schw18]). The global carbon cycle on earth with the modeling of carbon uptake by the ocean and the biosphere as input to all climate models is one of the more complex issues. Besides a large number of research papers on the topic (see e.g. [Roe80, Joo96, Ben19] and many others) and also IPCC reviews (e.g. [IPCC01], Chapter 5 in [IPCC21,] and refs. therein), simplified models for the carbon uptake of ocean and biosphere have also been introduced to university level education (e.g. [Hop92, Hol99, Tom09, Fan10]). They usually treat the carbon cycle with box models. For such models, the various sinks for CO_2 are treated as boxes and differential equations describe the exchange at the interfaces between the box boundaries. More sophisticated models add vertical diffusion and convection processes in the ocean. In the educational models, up to 7 boxes were introduced: atmosphere, biosphere, soil, ocean surface, intermediate ocean, deep ocean and ocean sediments. Obviously, the more boxes there are, the more differential equations are needed and the more complex the models get.

Furthermore it is known, that the exchange between various carbon reservoirs can only be described by linear models if the carbon is well mixed in each reservoir. In this respect, the ocean CO_2 exchanges need to be described by nonlinear equations due to aquatic carbon chemistry [Joo96].

Obviously, a large number of differential equations, some of them being nonlinear, is kind of a hindrance for teaching the topic at an introductory level. Furthermore, there are so many parameters in these equations that these hardly can be regarded as unique. Accordingly these equations do not really provide an in-depth understanding of the underlying physical processes. Therefore we here want to introduce a most simple linear two-box model, one box being the atmosphere, the other describes the combined CO_2 sink of ocean and biosphere. This is partly justified by the fact that for example the exchange between the top ocean layer and deep ocean is occurring only on very long timescales of several hundred years [IPCC05].

This basic model can help to promote an intuitive understanding of the first step of climate change, i.e. the prediction of carbon dioxide concentrations in the atmosphere from extremely simple arguments of the global carbon cycle and a reasonable assumption, which is supported by data of the global carbon project [GCP22], of how global sinks for CO_2 depend on atmospheric CO_2 concentrations.

As starting point at a specific time, the model uses the empirical emission and sink data for 2021 (see Fig. 4 of [GCP22]). In 2021, the combined terrestrial and ocean sinks have taken up 6.4 Gt of carbon or 23.5 Gt of CO_2. The CO_2 from global emissions, remaining in the atmosphere after deposition, is around 19 Gt of CO_2, corresponding to α_{Atm} = 45%. In the past, this fraction α_{Atm} has varied and it will also vary considerably in the future, depending on emissions. Surprisingly it was about constant during the past 60 years (see Fig. 9 in [GCP22]).

The main argument of the model is based on empirical data from the period 1960 to 2020 which show that the annual fraction of global CO_2 emissions, being deposited in the ocean and biosphere combined, was roughly proportional to the atmospheric CO_2 concentration.

The latter assumption leads to a description by one simple differential equation, wherein the change of CO_2 concentrations in the atmosphere depends on time dependent global anthropogenic CO_2 emissions. The model easily provides a natural time scale for changes of atmospheric concentrations. It also allows to estimate typical expected maximum CO_2 concentrations and changes as a function of time, for a number of emission scenarios.

The input parameters of the model are extracted from CO_2 emission and CO_2 sink data of the period 1960 to 2020. In order to get confidence in these model results despite its extreme simplicity, we first compare model predictions depending on emission scenarios for the model input period 1960 to 2020. Results, using only the starting conditions at 1960 and the model parameters, are in quite good agreement with literature data [NOAA]. Hence, we are convinced that the model may serve as reasonable starting point for analyzing atmospheric CO_2 concentrations for a variety of future emission scenarios.

Therefore we briefly compare model results to those of two current IPCC scenarios and also other emission scenarios and discuss limitations and potential problems, but also advantages of the model. Concerning teaching of the topic, the model is particularly helpful in providing an intuitive insight in changes of atmospheric CO_2 concentrations depending on various emission scenarios. Due to its mathematical simplicity, it may also be especially suitable for introductory teaching of the fundamentals of climate change. Students can solve the equation already with a simple spreadsheet program such as Excel.

2. The model

On average there is a global equilibrium between sources and sinks of CO_2 on earth (see Fig. 4e, [GCP22]). The most relevant sources are the emissions from the use of fossil fuels and the emission from land-use change, in particular the burning of forests. The sinks are the net growth of vegetation on land and the uptake by the oceans due to direct absorption, carbonate formation and growth of algae. Recent studies, denoted as global carbon project, present results for the global carbon budget from 1850 to 2021 with a projection for 2022 [GCP22]. Respective data summarize schematically how global anthropogenic CO_2 emissions Q_{tot} within the past six decades (natural emissions are much smaller and neglected in this period) and from 1850 to 2021 were counterbalanced within the planetary system by deposition in the oceans, on land as well as in the atmosphere. From these data of the global carbon cycle (see Fig. 9 in [GCP22]) one can deduce, that the deposition of CO_2 in the atmosphere after around 1960 amounted to a surprisingly constant fraction of $\alpha_{1960-2020}\approx 0.45$ of the total emissions (for simplicity, we neglect the 1% of unaccounted carbon budget). In addition, the ocean and land sinks have grown roughly linearly with increasing atmospheric CO_2 and at present take up slightly more than half (the fraction $(1-\alpha)$, i.e. 55%) of all emissions. This statement is directly supported by the data of the 'Global Carbon Project' [GCP22]. For the future and in our model in general we do not assume that this fraction α is a constant, but it may vary with time. A schematic of the model is depicted in Fig. 1.

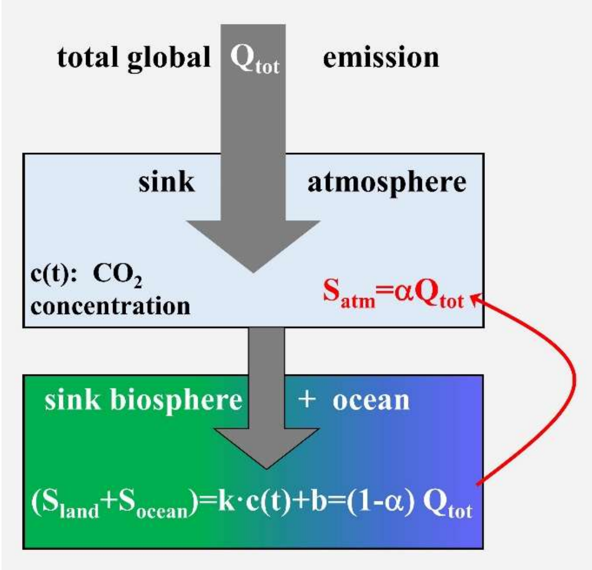

Fig. 1: Schematic representation of the model. The total global emission $Q_{tot.}$ into the atmosphere enters the atmosphere. A fraction (1-α) therefrom ends up in the biosphere and ocean combined. The remaining part S_{atm} stays in the atmosphere. Details, see text.

a) Model input data and assumptions

In order to estimate, how the deposition Q_{tot} into the atmosphere leads to a change of the CO_2 concentration, we use the following approach:

The total global emission is $Q_{tot}=Q_{land}+Q_{fossil}$ with the fossil contribution Q_{fossil} being presently much larger than Q_{land} from land use changes. For the year 2021, Q_{fossil} amounted to 37.0 Gt CO_2/y and the additional land use change of around 4 GT CO_2/y, lead to a total of 41 Gt CO_2/y [GCP22].

The total deposition (sinks) is $S_{tot}=S_{ocean}+S_{land}+S_{atm}$ and in equilibrium, $Q_{tot}= S_{tot}$, i.e.

$$S_{atm} = (Q_{tot} - (S_{ocean} + S_{land})) \qquad (1)$$

For the change in atmospheric CO_2 concentration, we assume

$$\frac{dc}{dt}(t) = \beta S_{atm} = \beta(Q_{tot} - (S_{ocean} + S_{land}) \qquad (2)$$

where the constant β (in ppm/Gt) describes the conversion from deposition in Gt/y to change of atmospheric concentration in (ppm/y) .

In the period 1960 to 2021, the atmosphere, ocean and vegetation sinks could be approximated as

$$S_{atm} = \alpha_{1960-2020} \cdot Q_{tot} \approx 0.45 \cdot Q_{tot} \quad \text{and} \quad S_{ocean} + S_{land} = (1 - \alpha_{1960-2020}) \cdot Q_{tot} \approx 0.55 \cdot Q_{tot} \qquad (3).$$

This means that we treat the atmosphere as one box and ocean and land combined as a second box in the notation of usual models. Due to the North-South asymmetry of the land masses, vegetation uptake shows a periodic annual time dependence, clearly shown in the saw-tooth profile of the Mauna Loa global CO_2 data [NOAA]. The effect around the averaged mean value increased from about ± 2.5 ppm around 1960 to about ± 3.5 ppm around 2020 and clearly indicates the important effect of varying land sinks. As we want to model long term changes only, we discuss the averaged trends and hence only treat yearly averages, i.e. we do not discuss seasonal variations.

CO_2 deposition in the ocean and biosphere occur via the intermediate step of initial deposition into the atmosphere. The transport from CO_2 in air to the water of the ocean or vegetation happen via the respective interface. It is obvious, that the deposition depends on the number of CO_2 molecules striking this interface, i.e. the deposition will depend on the CO_2 concentration c_{atm} of the atmosphere. In the most simple model, we assume that the deposition will just linearly depend on c_{atm}. Obviously, each component (ocean or biosphere) may itself have a different proportionality constant. For the sake of simplicity and to show the benefit of this basic model assumption, we just use a single proportionality constant for the combined uptake by ocean and land, i.e. they serve as one box, only. Quantitatively, we therefore assume that all planetary time dependent CO_2 deposition in ocean and biosphere is described by the linear relationship

$$(S_{ocean} + S_{land}) = k \cdot c(t) + b \qquad (4)$$

We will show below in Fig. 2 that this model assumption is well justified by the available global carbon data over the past 60 years. Using Eq. 4, Eq. 2 can be rewritten as

$$\frac{dc}{dt}(t) = \beta(Q_{tot} - k \cdot c(t) - b) \quad (5a).$$

We end up with a differential equation of first order

$$\frac{dc}{dt}(t) + \beta \cdot k\, c(t) = \beta(Q_{tot}(t) - b) \quad (5b)$$

The whole subsequent discussion will be based on this model approach with the three parameters *β, k* and *b* and the input data Q_{tot} of various emission scenarios.

b) Model parameters derived from actual data

Before solving the simple differential equation (Eq.5b) for various emission scenarios $Q_{tot}(t)$, we discuss the relevant constant parameters *β*, *k* and *b* within the model. They can be derived from known (or estimated) quantities of the earth system which are:

- global carbon or CO_2 emissions per year: estimates are around $Q_{tot,2021} \approx$ 41 Gt/y in 2021, [GCP22] (the IPCC model projection, to which we will later compare our model, start around $Q_{tot} \approx$40Gt/y which is closeby [IPCC21]).
- global atmospheric CO_2 concentration at Mauna Loa with the annual mean value $c_{atm,2021} \approx$ 416.4 ppm for 2021 as well as the plot of atmospheric CO_2 concentrations from 1960 to 2022 [NOAA].
- current growth rate of year averaged CO_2 concentration in the atmosphere: currently at 2021 around $\frac{dc}{dt}(t) \approx 2.37 ppm/y$ [NOAA].

Similar to the emissions Q, sinks S are given in Gt/y.

From Eqs. 2,3 for the year 2021 we therefore find 2.37 ppm/y = $\beta \cdot \alpha_{2021} \cdot 41$ Gt/y, which, with α_{2021}=0.45, gives *β*=0,128 ppm/Gt. In the following we assume this constant β–value for our model in the emission range above around 20 Gt/y. The inverse value 1/β ≈ 7.8 Gt/ppm represents the global CO_2 deposition in Gt into the atmosphere, leading to an increase in atmospheric CO_2 concentration by 1 ppm.

The other constants *k* and *b* are derived from empirical sink data and fit procedures using Eq. 4. In particular we used the data for decadal averaged annual uptake of CO_2 by vegetation and the oceans as a function of time from table 6 of [GCP22]. The time scale was transferred into an atmospheric CO_2 concentration scale by using the famous Keeling curve from Mauna Loa data [NOAA]. The result is shown in Fig. 2, which depicts the annual ocean and land CO_2 uptake as a function of atmospheric CO_2 concentration between 1960 and 2020 averaged over a decade each.

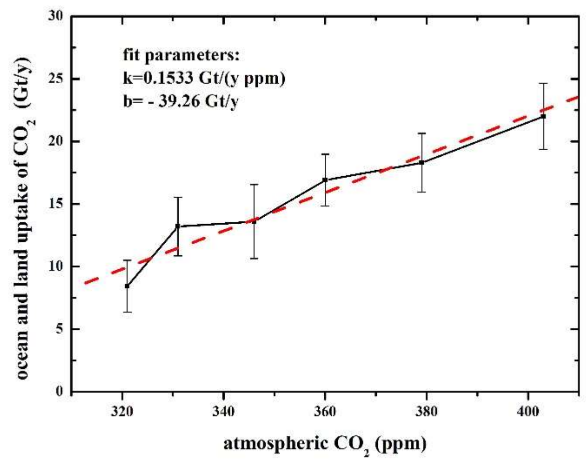

Fig. 2: Combined and 10 year averaged annual CO_2 uptake of oceans and land as a function of atmospheric CO_2 concentration in the period from 1960 to 2022. The sink data are from [GCP22]. The dashed line denotes a linear fit to the data including uncertainty bars.

We performed a linear fit procedure to the data (with R^2=0.916, resembling a rather good linear dependence) to find the parameters of the linear relationship of Eq. 3. The respective fit parameters were used in all of the following simulations. The product βk=0.0196/y is independent of the assumed Q_{tot}. The inverse value defines a time constant of the model

$$\tau = \frac{1}{\beta\, k} = 51\, a \qquad (6).$$

Incidentally, this time constant describes the system response time to any changes. It does not reflect the 'residence time' of CO_2 in the atmosphere, which is substantially shorter, because of the large fluxes between the various reservoirs.

We summarize all model parameters in Table 1.

$Q_{tot,\,2021} \approx 41$ Gt/y	$c_{Atm,2021} \approx 416.4$ ppm	$\frac{dc}{dt_{2021}} \approx 2.37\mathrm{ppm/y}$
$\alpha_{2021} = 0.45$	$k = 0.1533$ Gt/(y·ppm) ± 0.0205	
β= 0,128 ppm/Gt	b = - 39.26 Gt/y ± 7.29	
Tab. 1: Input data and model parameters, used for simulations		

c) Range of validity of model and time dependence of α

First, we note that probably due to the quite large error bars of the available global carbon data, the fit curve for global CO_2 in ocean and land sink gives theoretical CO_2 uptake estimates of ocean and land in Gt/y which – being interpreted as 55% of total emissions - are a bit above, i.e. not in complete agreement, with the known global CO_2 emissions. For c_{2021}=416.4 ppm, Eq. 5 would give an ocean and land uptake of around 24,6 Gt/y which according to 41 Gt/y emission would mean a 60 % uptake rather than the expected 55%. So we do indeed expect deviations of the order of 5% for land and ocean uptake when comparing the model to emission scenarios. Nevertheless, the trends should provide some valuable insights.

Second the model parameters k and b are assumed as constant. Obviously, this empirically supported assumption only holds reasonably well starting around the 1960-ies after the big acceleration started, i.e. for concentrations above 315 ppm and during times with substantial emissions due to the burning of fossil fuels where the linear relationship of Eq. 4 for sinks versus concentration applies. This is already obvious from the fact, that the total ocean and land sinks would become negative for concentrations below around 255 ppm.

From 1960 to 2020, the atmospheric uptake portion α was found to be approximately constant. Before around 1950, however, emission and sink data are much smaller, fluctuating and more uncertain (see Fig. 3 in [GCP22]). This is not described in our simple model. Knowing that in preindustrial times before 1850, CO_2 concentration was stable around 280 ppm, and assuming that natural emissions were nonzero, the atmospheric uptake portion α must have been zero. Assuming constant, though low natural emissions then also means constant uptake by ocean and biosphere, i.e. the fit parameters will also vary with time and k must approach zero as well. Therefore not only α, but k and b as well must change as function of time between 1750 and sometime before 1960. In our simple model we want to avoid any such complications for CO_2 concentrations below those of around 1960. We therefore restrict all simulations to emission amounts which apply to data after 1960, i.e. total CO_2 emissions of above around 16 Gt/y. As a consequence all our simulation scenarios with emission decreases will usually end with a decrease at around 50% of present emissions, i.e. 0.5 x 41 Gt/y ≈ 20 Gt/y.

Third, α must in general be time dependent in the future. Let´s come back to the empirical data of Fig. 2 of the fraction of CO_2 uptake by ocean and vegetation being proportional to atmospheric CO_2 concentration. If this holds, the value of α will automatically depend on the chosen emission scenario.

For example, let´s assume global emissions would be constant. The respective increase of atmospheric CO_2 concentration would lead to an increase of the uptake by ocean and land (Eq.4). For constant emission, the fraction of the atmospheric uptake α must decrease accordingly. If in contrast, global emissions increase roughly proportional to concentration similar as the ocean and land uptake does, α may be constant as was roughly the case for the past 60 years. For a steeper increase, α may increase. We conclude, that the fraction α of atmospheric CO_2 uptake from global emissions will indeed depend on the emission scenario as a function of time. For any scenario with constant emission or a decrease, α_{atm} must decrease.

Such a behavior of changing atmospheric uptake is also expected for much more sophisticated earth system models which include more than 2 boxes and also assume nonlinear uptake by the ocean as summarized in the most recent IPCC report.

d) Modelling time dependent global emissions

Usually, the global emissions $Q_{tot}(t)$ in Eq. 5b depend on time. Whenever we have a scenario for $Q_{tot}(t)$, we can then use Eq. 5b to numerically solve for c(t). In order to get rid of seasonal effects we used time steps of Δt=1a. Starting point is computation of $\frac{dc}{dt}(t)$. This gives

$$c(t + \Delta t) = c(t) + \frac{dc}{dt}(t) \cdot \Delta t \quad . \tag{7}$$

Using Eq. 5b and Eq. 7, we can model any chosen emission scenario.

Due to the simplicity of the model, it may even be solved by undergraduate students using e.g. a programmable spreadsheet program such as Excel. Table 2 depicts a potential Excel file.

column 1: time [year]	column 2: total global emission = input [Gt CO_2 per year]	column 3: right hand side of Eq. 5b [ppm per year]	column 4: 2nd term of left side of Eq. 5b [ppm per year]	column 5: column 3-column 4, dc/dt from Eq. 5b [ppm per year]	column 6 : solving Eq.7, CO_2 concentration [ppm]
t	Q_{tot}	$\beta(Q_{tot}-b)$	β k c(t)	dc/dt(t)= $\beta(Q_{tot}-b-kc)$	c(t+Δt)=c(t)+dc/dt
					Start 1.1.60: 316
1960	17	7.20…	6.19…	1.007…	317.0…
1961	17.39…	7.25…	6.21…	1.038…	318.0…
1962	17.78…	7.30…	6.23…	1.068…	319.1…
…	…	…	…	…	…

Table 2. Potential schematic Excel table: for given model parameters (of Tab. 1), the input emission in column 2 determines the change dc/dt in column 5 and therefrom the new concentration at the end of the year (column6)

The first column gives the year, the second is the input in terms of total global emissions $Q_{tot}(t)$, the third computes $\beta(Q_{tot}(t) - b)$ from the second column. The fourth computes $\beta \cdot k\, c(t)$, while the fifth gives the difference between fourth and third, i.e. $\frac{dc}{dt}(t)$. Finally the sixth column gives concentration with the first line being the input starting parameter of the relevant starting year. For example 316 ppm is the start at 1st of January and 317 ppm is the concentration at the end of the year.

3) Results

a) The case of constant emissions

Eq. 5b has the most simple solution for constant emission Q_{tot}. In this case, the inhomogeneous equation has a solution which is the sum of the solution of the homogeneous equation plus one special one of the inhomogeneous equation. Let us discuss this case first by assuming, that the world – quite unrealistically - would have a constant global emission for the next few hundred years which is equal to the emission of 2021, i.e. $Q_{tot}(t)$=41 Gt/y.

The special solution is found for infinite time, when dc/dt = 0. In this case

$$c_\infty = \frac{(Q_{tot} - b)}{k} \tag{8a}$$

Using k=0,1533 Gt/(ppm y), Q_{tot}=41 Gt/y and b=-39.26Gt, we find the equilibrium value $c(t \rightarrow \infty) \approx 524\ ppm$. At that concentration, atmospheric uptake α will be zero, i.e. we expect a strong decrease of α with time.

Similarly, Eq. 8a gives the result for equilibrium CO_2 concentrations at t → ∞ for any situation where $Q_{tot}(t)$ finally approaches a constant value. For example, with Q_{tot}=20 Gt/y, i.e. ≈ 50% of present emissions, earth would finally end up with around 387 ppm, i.e. concentrations, we had around the 1980-ies.

The solution of the homogeneous equation is simply

$$c(t) = c_0\, e^{-\beta k t} = c_0\, e^{-\frac{t}{\tau}} \tag{8b}$$

with time constant τ=1/(βk) ≈ 51 a, i.e. the change of atmospheric concentrations is governed by a time constant of around 50 years. This time scale will also have an impact on other emission scenarios. Using the boundary condition, that $c(t=0)=c_{2021}$ we find for the solution of Eq. 5b and constant Q_{tot}

$$c(t) = (c_{2021} - c_\infty)\, e^{-\beta k t} + c_\infty \quad (8c)$$

Fig. 3a depicts the plot of this analytical solution as function of time. About 95% of the concentration difference $(c_{2021} - c_{\infty)})$ is reached after three time constants, i.e., around 2170. Fig. 3b plots expected equilibrium values as a function of constant emission rate Q_{tot}. Obviously, if the world would double its current emissions and kept them constant, CO_2 concentration would ultimately reach values around 790 ppm.

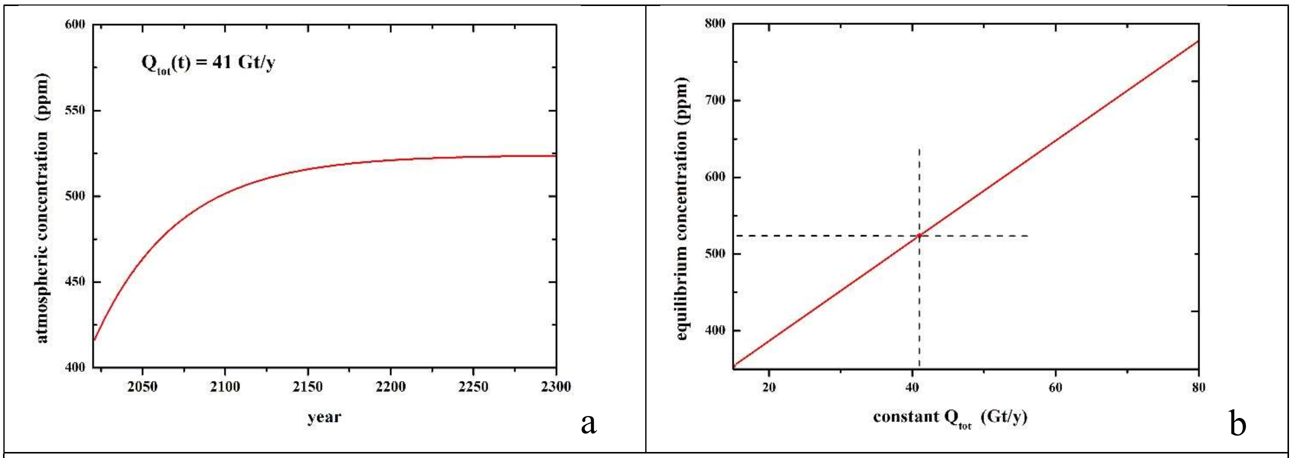

Fig.3: a) Model result for constant emission rate of 41 Gt/y. The CO_2 concentration in the atmosphere will ultimately reach an equilibrium value of 524 ppm (horizontal line). b) Change of equilibrium value as function of constant emission rate. Present emission data are indicated by the vertical dashed line and the respective equilibrium value by the horizontal dashed line.

Of course, assuming constant emission for several hundred years at present levels is completely unrealistic. Fig. 3 should only show the most simple mathematical solution of the model. If at all, it may serve as a prediction of what may happen until 2100 if current emissions persist within this century, and ocean and land uptake still increase linearly with atmospheric CO_2 concentration.

However, this is nevertheless a very important result. **According to our present knowledge of the carbon cycle, which has accumulated data over more than 60 years until now [GCP22] any constant emission scenario will result in a stable constant concentration of CO_2 in the atmosphere. Even, if we keep emitting substantial amounts of CO_2, the atmospheric concentration of CO_2 will saturate and will not rise indefinitely.** According to Fig. 3b, our simple model predicts that constant emissions of around 20 Gt per year would only result in equilibrium concentrations slightly below 390 ppm.

b) Comparison of model results to the period 1960 to 2022

In order to see how well our model performs, we first compare the model prediction with those of the recorded Mauna Loa measurement period of atmospheric CO_2 concentrations (Fig. 4).

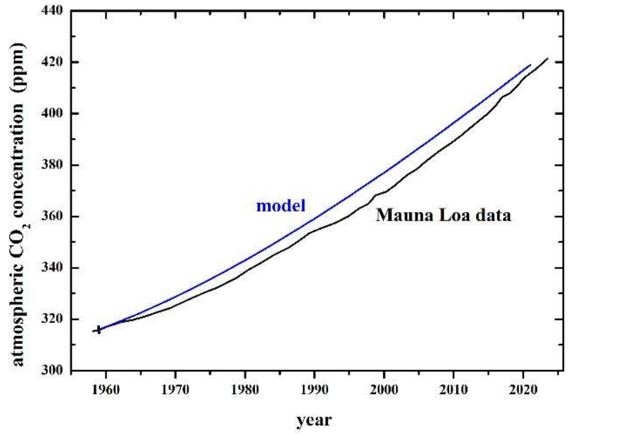

Fig. 4: Comparison of Mauna Loa data for atmospheric CO_2 concentration (lower line) with model results (upper line). The model starts with 316 ppm in 1960. Total emissions Q_{tot} increase linearly from 17 Gt/y in 1960 to 41 Gt/y in 2021.

The model used Q_{tot} values ranging linearly from 17 to 41 Gt/y (blue). Start was at the year 1959 with 316 ppm. The period up to 2021 was then computed from the model.

Considering the simplicity of the model, the quantitative agreement with the measured data is reasonably good. The trend is obvious and deviations amount to only a few ppm. This encouraged us to also apply the model to future emission scenarios of the 21st century.

c) Comparison to IPCC scenarios

The latest IPCC report from 2021 discusses a number of global carbon dioxide emission scenarios starting from 40 Gt/y in 2015. As publication of such a report is time consuming, we therefore assume that the presented IPCC models may date back to 2015. We compare our model results to two of these, the SSP1-2.6 and the SSP2-4.5 scenario (briefly denoted as 126 and 245) Comparison to the other scenarios are similar (see Figs. 5.25a,b,c in [IPCC21]).

From 2015 to 2085 we used the global emissions from the IPCC scenarios (see also below). As our model is only reasonable for emissions larger than those around 1960, we had, however, to use slightly modified $Q_{tot}(t)$ input values in our model. We do not treat emission values below 50%, i.e. here below 20 Gt/y. Therefore we used constant values of 20 Gt/y after around 2085. As the IPCC scenario started in 2015, we also used a respective atmospheric start concentration of around 402 ppm.

Our model shows the trends of the IPCC predictions but gives consistently lower values for the concentrations (see Fig. 5). The maximum concentrations for our model occur in 2040 (for 126) and in 2073 (for 245), i.e. roughly 25 years earlier as the IPCC results, and they are lower by about 40 ppm (126) and more than 100 ppm (245) compared to the IPCC prediction.

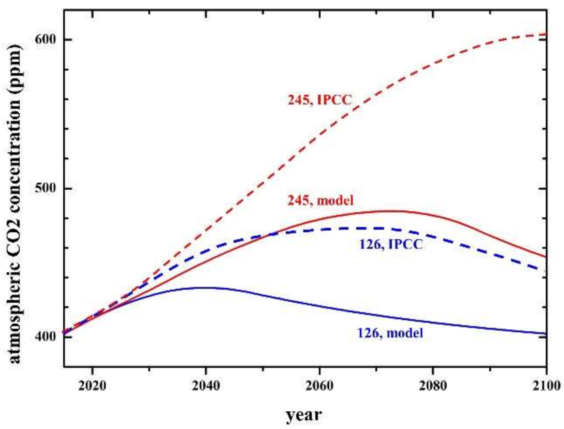

Fig. 5: Comparison of IPCC and model CO_2 concentrations for IPCC scenarios 126 and 245. The last two digits of the scenario characterize the additional radiative forcing, here 2.6 W/m^2 and 4.5 W/m^2. For emission input data, see Fig. 6and text.

These rather large differences are due to the fact that our model deviates appreciably from the IPCC assumptions concerning the combined uptake from ocean and land. Let´s have a closer look.

In our model, any time dependent emission starting now at the level of 40 Gt/y and subsequently slowly decreasing emissions, initially always leads to increasing atmospheric CO_2 concentration c_{atm}, as the combined ocean and land uptake is right now below around 60% and the remaining part (above 40%) stays in the atmosphere. An increased c_{atm}, however, leads to an increase in ocean and land uptake according to Eq. 4, assuming that the linear trend continues. The fraction of ocean and land uptake will therefore increase, slowing down the increase in the atmosphere. For decreasing emission Q_{tot}, there must therefore be a maximum concentration c_{atm}, where the combined ocean and land uptake is equal to the decreasing emission. In this case, land and ocean uptake amounts to a fraction of 100%, i.e. $\alpha_{atm}=0$. For further decreasing emissions, ocean and land can uptake more than is emitted. As a consequence CO_2 from the atmosphere can be transferred to the sinks and c_{atm} will decrease. This behavior is of course well-known and has also been stated e.g. in [NAS19]: atmospheric CO_2 will decline once net anthropogenic emissions (emissions minus sinks from negative emission technologies) become smaller than the annual uptake by the natural sinks.

This explains the general trend of our model results as well as those of the IPCC scenarios. The differences lie in the inherent assumptions of the model in comparison to the IPCC scenario, which lead to different time dependences of $\alpha(t)$ and respectively of the ocean and land fraction $(1-\alpha(t))$.

The assumptions of the IPCC model are not explicitly mentioned in the latest IPCC report, but can be derived from Fig. 5.25 of [IPCC21], which gives predictions for sink data of ocean and land - though with quite large uncertainty bars, in particular for land data. Fig. 6a,b shows a respective comparison of the combined ocean and land sink in our model and the IPCC models. The IPCC models assume much lower sinks in the ocean and biosphere, i.e. a much larger increase of atmospheric CO_2. As a result, the maxima of c_{atm}, which should occur for $Q_{sinks}(t)=Q_{tot}(t)$ differ.

Presumably, this behavior can be attributed to the expected nonlinear response of the sinks which, according to the IPCC (Fig. 5,25) are assumed to decrease already now and in the near future. The IPCC assumes reduced uptake from land due to land use changes (e.g deforestation) as well as reduced uptake by the oceans, e.g. if uptake depends also nonlinearly on upper ocean CO_2 concentration. This is expected due to a reduced buffer capacity of the carbonic system in the ocean as well as different time scales of mixing surface water with the deeper ocean water. These nonlinearities [IPCC21] come from the used Earth system models (ESMs) of the carbon cycle. These assume different emission driven respective changes of ocean and land uptake, which lead to a decrease of the respective sink fraction. As the sink gap difference between our model and the IPCC scenario increases, so does the concentration differences in Fig. 5.

We note that although used for the IPCC predictions, the report explicitly states that "*there is currently no direct evidence that the natural sinks are slowing down*"... (IPCC-AR6-1, The Physical Science Case on page 772). One recent publication has indeed shown a very small trend of decreasing sink capacity, though yet not very pronounced with around 0,5% per year [Ben19], whereas another one [Wan23] reports that the ocean sink is larger than presently assumed.

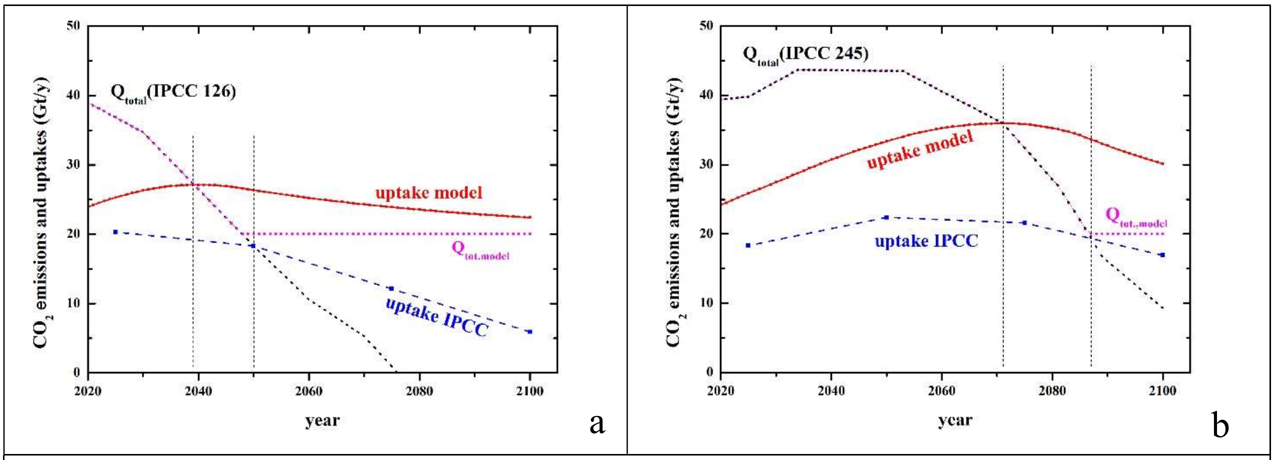

Fig. 6: Total emissions Q_{tot} as well as CO_2 uptake by ocean and biosphere due to our model and the IPCC scenarios 126 (a) and 245 (b) (dotted lines). Deviations between our model and the IPCC scenarios are due to a much lower assumed uptake of CO_2 from ocean and biosphere.

We summarize the comparison to the 1960 to 2020 period and the IPCC models. The period of the past six decades is reasonably well modelled. This is not surprising, as during this period, the fraction of CO_2 deposited in the atmosphere α_{atm} was about constant. In comparison to the IPCC models 126 and 245, the major features of atmospheric CO_2 concentrations can still be roughly approximated with our model. If the assumption that the combined sink of ocean and land will still increase linearly with atmospheric CO_2 concentration also applies for future emissions, the increase in concentrations will, however, always be appreciably lower than in IPCC scenarios. The reasons for the obvious deviations (differences in time dependent α between our model and IPCC earth system models) could be elaborated further on in a teaching sequence on emission scenarios. This does however, require a much deeper discussion of the IPCC earth system models and their assumptions. This is way beyond the scope of the present paper.

Although there are quantitative differences to present state of the art IPCC models, one may learn a lot about how changing parameters in scenarios will lead to changes of the respective atmospheric concentrations. Compared to IPCC scenarios, our model results point to a more optimistic prediction for any given future emission scenario. We want to point out again here that our model is based on presently available global carbon data up to the end of 2021 whereas the IPCC models started with available data around 2015 and assume a weakening of the sinks.

c) Results for other selected time dependent global emission scenarios

Once students have written the needed spreadsheet program or solved the differential equation 5b in another way, the model may be used in teaching to discuss selected new emission pathways, defined by their $Q_{tot}(t)$ values within the 21st century. Our starting point is always end of 2021with an initial CO_2 concentration of 416.4 ppm.

First, we want to illustrate how delaying the timescale for effective emission reduction will change the atmospheric concentrations. We start with $Q_{tot}(2021)$ = 41 Gt/y of global emissions and analyze five different scenarios.

First we assume just linear decreases of global CO_2 emissions $Q_{tot}(t)$ characterized by the time to reach around 50% of current emissions (precisely 20 Gt/y) either within $t_{50\%}$ =50 years (green) or $t_{50\%}$ =100 years (pink) (see Fig. 7a). After reaching this level, emissions are assumed to stay constant. Second we assume at first still constant emissions for 20 years before a $t_{50\%}$ =50 year decrease will proceed (blue). Two final examples assume first a 10 year increase up to 50 Gt/y (black) or 60 Gt (red), then a short 10 year plateau period before a $t_{50\%}$ =50 year decrease to 50% of current levels. The resulting atmospheric CO_2 concentrations are depicted in Fig. 7b.

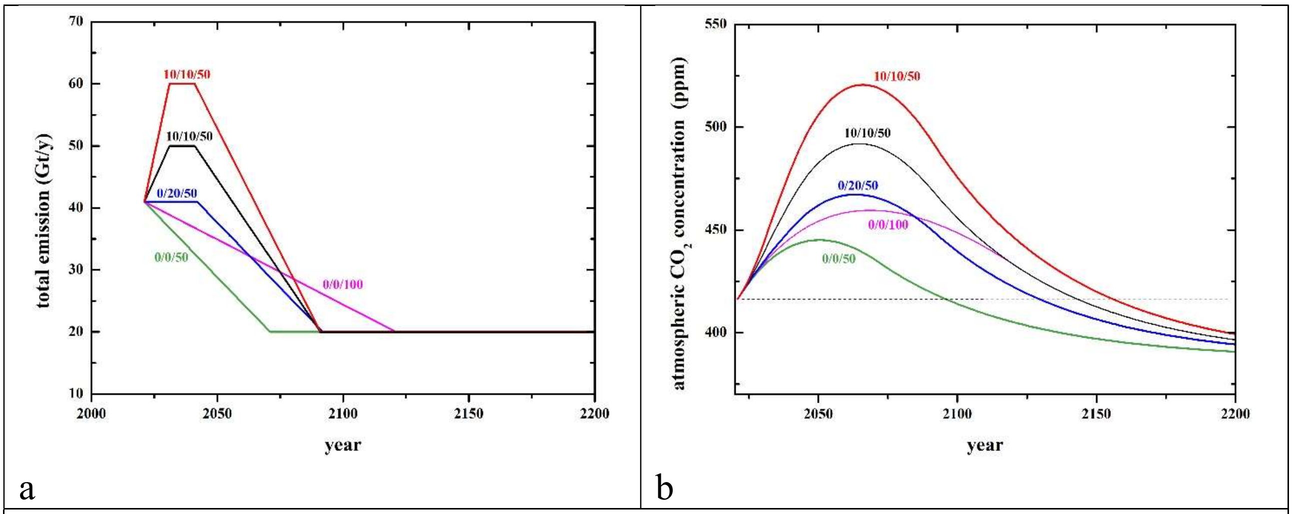

Fig. 7) Several emission scenarios (a) of Q_{tot} from 2021 levels of 41 Gt/y to ultimately 20 Gt/y and respective atmospheric concentrations (b). Numbers denoting the curves refer to period for increase/period for constant emission/period for decrease.

The most obvious change for just delaying the drop to 20 Gt from 50a (green) to 100a (pink) is a higher maximum concentration in the atmosphere, rising from 445 ppm to ≈ 460 ppm. In addition it also takes much longer times to reach again current CO_2 levels. For the most optimistic scenario of reduction 0/0/50, it takes already more than 65 years before atmospheric concentrations reach again current levels of, say, 420 ppm. For 0/0/100, this period has increased already to around 115 years.

The more realistic emission scenarios, though still optimistic, assume an initial 20 year plateau or even a 10 year increase followed by a 10 year plateau before a 50 year drop follows. All of these scenarios have larger maximum concentrations of 467 ppm (0/20/50), 492 ppm (10/10/50), and even nearly 521 ppm (10/10/50). All scenarios also need quite long times beyond 100 years to again reach current levels, i.e. appreciably longer than the most optimistic immediate $t_{50\%}$ =50 year decrease.

As maximum concentration relates to temperature increases, it is obvious that if at all, only the $t_{50\%}$ =50a scenario has a chance to meet the Paris goal of 2°C. The decrease period of global emissions is crucial, any appreciable extension beyond 30 to 50 years must be avoided.

Our model results indicate – always assuming that the underlying assumptions also hold for larger emissions - that already a reduction of emission to 50% of current values will have an impact and will finally reduce atmospheric concentrations again below our current levels. Unfortunately, the time period to reach these concentrations is quite long. During this time, we have to tolerate the effects of climate change. Nevertheless, as we discussed for the equilibrium concentrations in Fig. 3b, if

- we first act now to decrease global emission,
- second the model assumption that CO_2 sinks in ocean and biosphere remain about linearly dependent on atmospheric concentrations,
- third there will be no additional emissions due to tipping point events, and
- fourth, we are willing to tolerate a level of 400 ppm in the atmosphere, which is less than todays value

we may - after a transition period of around 100 years - still afford emissions of 20 Gt of carbon annually. This may make the transformation of the energy system much easier, even though still a dramatically huge effort is needed now.

Summary and Conclusions

We have described a simple linear two-box model for CO_2 uptake in atmosphere and ocean/biosphere. It allows to estimate atmospheric CO_2 concentrations as a function of global time dependent CO_2 emissions. Its parameters are based on known data of the global carbon budget as well as CO_2 atmospheric concentrations from Mauna Loa for the period 1960 to 2021. Its applicability is limited to global emissions above around 50% of current emissions.

The basic model assumption is that CO_2 uptake by ocean and land depends linearly on atmospheric CO_2 concentration. Regarding the simplicity of the model, results are in reasonable quantitative agreement with measured data when applied to the 1960 to 2021 period and in still good qualitative agreement with predictions of current IPCC model scenarios. Respective quantitative deviations are due to the differences in the fraction of global sink CO_2 uptake between our model and the assumed behavior of sinks in earth system models of the IPCC scenarios.

In this respect, our model behaves similar as many other simple climate models (SCMs). A comparison of many SCMs (mostly models with many boxes) with full-fledged earth system models (ESMs) shows [Mel23], that ESMs systematically estimate lower ocean carbon uptake than SCMs, depending on the mixing from surface to deep ocean in the models. Similarly, differences arise for land uptake. As a consequence, differences between SCMs and ESMs are to be expected. However, SCMs and hence also our model, have one particular advantage compared to ESMs.

Our model easily allows very fast predictions of atmospheric CO_2 concentration as a function of any given time dependent emission scenario, as has been demonstrated for a number of cases. We find, that after a transition period of around 100 to 150 years, depending on the used emissions scenario, new equilibrium atmospheric CO_2 concentrations below current levels of 420 ppm seem possible for reduced but constant emissions of 50% of todays values.

Besides, our model has quite a few merits. Its simplicity offers an easy applicability to introductory teaching of the topic, it also allows a lot of student modeling e.g. in lab courses or even as home work. Finally, this model offers an easy and fast method to study the impact of new reasonable emission scenarios on atmospheric concentrations as a first order approximation.

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