

The Maxwell equations including magnetic monopoles

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Abstract:

The derivation of the Maxwell equations is reproduced whereby magnetic charges are included. This ansatz yields following results:

- 1) Longitudinal Ampère forces in a differential magnetostatic force law are improbable. Otherwise a electric current would generate magnetic charges.
- 2) Simple magnetic and electric induced polarization phenomena are completely analogous and are described by a Laplace equation.
- 3) Permanent magnetic fields can be understood to be caused by magnetic charges. Consequently, a moving permanent magnet represents a magnetic current which generates a electric field.
- 4) The electromagnetic tensors of energy and momentum have some additional terms which are written down generally.
- 5) If the electric material parameters are influenced by non-electric variables (for instance temperature or pressure), the formalism of electrodynamics is not sufficient to describe the system and has to be completed by further differential equations from the other areas of physics.
- 6) Nonlinear electro-thermodynamic systems may violate the second law of thermodynamics. This is illustrated by a electric cycle with a data storing FET invented by Yusa & Sakaki.

1) Introduction

The Maxwell equations are about 150 Jahre old. They are the mathematical compilation of the experiments and considerations based on the original work of Cavendish, Coulomb, Poisson, Ampère, Faraday and others [1]. Mathematically they are partial differential equations. Different notations exist for them: most popular is the vector notation (O. Heaviside), which replaced the original notations in quaternions (J.C. Maxwell). More modern is the tensor notation (H. Minkowski, A. Einstein), which is able to describe situations which are discussed in the theory of relativity [2]. All notations are equivalent in the non-relativistic limit.

The Maxwell equations were and still are very successful. Until today their range of applicability grows permanently.

Here a short derivation is given which especially takes account for the newer developments of material descriptions. Furthermore, monopoles are included because Ehrenhaft proved their existence already 50 years ago [3-6]. It will be shown that the theory needs also their existence for a full description of all problems. This explains perhaps effects which are regarded generally as dubious because they cannot be understood in a conventional approach.

2) The equations of the electromagnetic field

a) The laws of Coulomb and the equation of Poisson

The so called Coulomb law describes the force between electric charges. It was discovered by Priestley in 1767 [1, 7]. Cavendish rediscovered it again and measured as well the dielectricity constant. However, due to many contributions to the knowledge about electricity it has the name of the third discoverer Coulomb [1].

The Coulomb law in the notation of today is [8]

$$F = \frac{1}{2} \sum_{i \neq j} q_i q_j \frac{(\mathbf{x}_i - \mathbf{x}_j)}{|\mathbf{x}_i - \mathbf{x}_j|^3} = \iint \rho(\mathbf{x}) \rho(\mathbf{x}') \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d\mathbf{x}'^3 d\mathbf{x}^3 \quad (1)$$

with the definitions $F :=$ force, $q :=$ single charge, \mathbf{x}' space coordinate and i, j are indices. It can also be written as

$$F = \int \rho(\mathbf{x}') \mathbf{E}(\mathbf{x}') d\mathbf{x}'^3 \quad (2)$$

by using the definition of the electric field \mathbf{E}

$$\mathbf{E} = \int \rho(\mathbf{x}') \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d\mathbf{x}'^3 \quad (3)$$

The electric field \mathbf{E} can be derived from a potential $\Phi_{\mathbf{E}}$ by using the definition

$$\mathbf{E} = -\nabla \Phi_{\mathbf{E}} \quad (4)$$

Then, the \mathbf{E} -field is defined by

$$\Phi_{\mathbf{E}} = \int \frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d\mathbf{x}'^3 \quad (5)$$

$\Phi_{\mathbf{E}}$ has an empirical meaning. The \mathbf{E} -field can be measured experimentally by the difference of voltage $\Phi_{\mathbf{E}}(r) - \Phi_{\mathbf{E}}(r_{ref})$ between a point in space at r and a reference point at r_{ref} which is often set to infinity where no field exists. Using the Poisson equation the charges the field can be calculated from the potential

$$\Delta \Phi_{\mathbf{E}} = -\nabla \cdot \mathbf{E} = -4\pi\rho \quad (6)$$

If matter is in the field the empirical potential $\Phi_{\mathbf{E}}$ consists of the induced charges $\rho_{matter}(\mathbf{x}')$ and the contributions from the charged surface of the conductors $\rho_{conductor}(\mathbf{x}')$

$$\Phi_{\mathbf{E}} = \Phi_{conductor} + \Phi_{\mathbf{P}} = \int (\rho_{conductor}(\mathbf{x}') + \rho_{matter}(\mathbf{x}')) \frac{1}{|\mathbf{x}-\mathbf{x}'|} d\mathbf{x}'^3 \quad (7)$$

where $\Phi_{\mathbf{P}}$ is the "mean field" of the material charges. Thus, per definition only the charges on the conductor are detected in the experiment. In order to obtain an expression with empirical variables similar to (6) the equation (7) is rewritten

$$\Phi_{\mathbf{D}} := \Phi_{\mathbf{E}} - \Phi_{\mathbf{P}} = \Phi_{conductor} = \int \frac{\rho_{conductor}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d\mathbf{x}'^3 \quad (8)$$

Contrary to the empirical meaning of $\Phi_{\mathbf{E}}$, $\Phi_{\mathbf{D}}$ has only a formal character. Using $\Phi_{\mathbf{D}}$ in the Poisson equation the charges to be measured in or on the conductors can be calculated. One defines

$$\mathbf{D} := \varepsilon_{ik} \mathbf{E} := \mathbf{E} + 4\pi \mathbf{P} := -\nabla \Phi_{\mathbf{D}} = \int \rho_{conductor}(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} d\mathbf{x}'^3 \quad (9)$$

where ε_{ik} is the dielectric tensor of the material.

Using the mathematical relations $\nabla |\mathbf{x} - \mathbf{x}'|^{-1} = -\nabla' |\mathbf{x} - \mathbf{x}'|^{-1}$ and $\nabla'^2 |\mathbf{x} - \mathbf{x}'|^{-1} = -4\pi \delta(\mathbf{x} - \mathbf{x}')$ and the redefinition $\rho := \rho_{conductor}$ the Poisson equation is

$$\Delta \Phi_{\mathbf{D}}(\mathbf{x}) = \nabla \cdot [\varepsilon_{ik}(\mathbf{x}) \nabla \Phi_{\mathbf{E}}(\mathbf{x})] = -4\pi \rho(\mathbf{x}) \quad (10)$$

Using (9) and (10) follows

$$\nabla \cdot \mathbf{D}(\mathbf{x}) = 4\pi \rho(\mathbf{x}) \quad (11)$$

Important special cases:

surface charges

An electric potential can exist due to a surface density σ

$$\Phi_{\mathbf{D}} := \Phi_{conductor} = \int \frac{\sigma_{conductor}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}'^2 \quad (12)$$

Then, the electric field

$$\mathbf{D} := -\nabla \Phi_{\mathbf{D}} = \int \sigma(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} da' \quad (13)$$

constraints for the material properties

In the most cases it is possible to make simplifying constraints for the material properties. In order to explain this it is necessary to write down the potential $\Phi_{\mathbf{p}}(\mathbf{x})$ of the multipole

expansion of the charges in the material [9]. A multipole expansion of the potential calculates the distribution of charge in space about a origin 0 as a serie of moments

$$\Phi_{\mathbf{P}}(\mathbf{x}) = \frac{\rho}{r} + \frac{\mathbf{P}_{\text{dipol}} \cdot \mathbf{x}}{r^3} + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{x_i x_j}{r^5} + \dots \quad (14)$$

cf. appendix 1. Here the definitions of the dipol moment $\mathbf{p}_{\text{dipol}}$ and the quadrupol moment Q_{ij} are

$$\mathbf{p}_{\text{dipol}} := \int \mathbf{x}' \rho(\mathbf{x}') dx'^3 \quad Q_{ij} := \int (3x'_i x'_j - r'^2 \delta_{ij}) \rho(\mathbf{x}') dx'^3 \quad (15)$$

This consideration is done for all points in space. Using $\mathbf{P}_{\text{dipol}}$ as density of polarisation then follows

$$\Delta \Phi_{\mathbf{P}}(\mathbf{x}', \mathbf{x}) = \left[\frac{\rho(\mathbf{x}')}{|\mathbf{x}_i - \mathbf{x}'_i|} + \frac{\mathbf{P}_{\text{dipol}}(\mathbf{x}') \cdot (\mathbf{x} - \mathbf{x}')}{|\mathbf{x}_i - \mathbf{x}'_i|^3} + \frac{1}{2} \sum_{i,j} Q_{ij}(\mathbf{x}') \frac{(x_i - x'_i)(x_j - x'_j)}{|\mathbf{x}_i - \mathbf{x}'_i|^5} + \dots \right] dV \quad (16)$$

The first term represents induced charges for instance if recombination processes in semiconductors have to be accounted for. However, for the most problems electric neutrality can be assumed and the first term becomes zero. Furthermore oftenly higher terms are neglected because they are quantitatively irrelevant. Then, after integration over the whole space holds

$$\Phi_{\mathbf{P}}(\mathbf{x}) = \int \mathbf{P}_{\text{dipol}}(\mathbf{x}') \cdot \frac{(\mathbf{x} - \mathbf{x}')}{|\mathbf{x}_i - \mathbf{x}'_i|^3} dx'^3 = - \int \frac{\nabla' \mathbf{P}_{\text{dipol}}(\mathbf{x}')}{|\mathbf{x}_i - \mathbf{x}'_i|} dx'^3 \quad (17)$$

If (17) is inserted in (8) one can identify: $\mathbf{P} = \mathbf{P}_{\text{dipol}}$

b) Ampère's law

The discovery of electromagnetism by Oersted [10, 11] in 1820 inspired some researchers in France to find the quantitative laws of these effects. Especially, Biot&Savart and Ampère tackled the task to solve this problem by intelligent experiments [1, 11]. In order to fit their experiment by the theory they made additional assumptions which filled up some lacking observations. This led to different laws for the forces between differential current elements of a circuit. For closed circuits, however, the different laws coincided in one. The discussion of this problem is running until today.

Biot and Savart [12-14] found out that "the total force which is exerted by a file of infinite length under current on a element of austral or boreal magnetism in the distance FA or FB, is perpendicular on the shortest distance between the molecule and

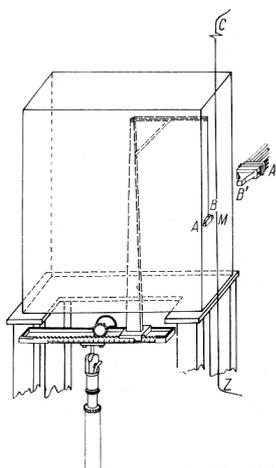


fig.1a: the Biot-Savart - setup

A magnetic needle is under the influence of the field of current CZ . A cover protects against the movement of air. The magnet A'B' compensates the magnetism of earth where the needle is located.

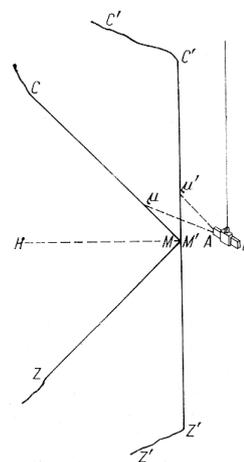


Fig.1b: the Biot-Savart - setup

measuring the time constant of the torsion pendulum it is concluded on the force of the field on the needle, if the current flows. Distance and angle of the file are varied in the experiments.

the file (see figs.1)". This law is written today in a form which goes back to Grassmann [11, 15]. It holds [8]

$$d\mathbf{F} \sim i_1 i_2 \frac{d\mathbf{s}_1 \times (d\mathbf{s}_2 \times \mathbf{r})}{|\mathbf{r}|^3} = i_1 i_2 \left[-(d\mathbf{s}_1 \cdot d\mathbf{s}_2) \frac{\mathbf{r}}{|\mathbf{r}|^3} + \left(\frac{d\mathbf{s}_1 \cdot \mathbf{r}}{|\mathbf{r}|^3} \right) d\mathbf{s}_2 \right] \quad (18)$$

with $i_{1/2} :=$ current, $d\mathbf{s}_{1/2} :=$ length of file element, $\mathbf{r} :=$ distance between file elements.

Ampère idolized Newton a little bit. So he overtook, 1) that Newton's 3. axiom (actio-reactio) of mechanics also holds for electromagnetism, and 2) that the force between single elements of current is a central force and lies on the distance line between the elements.

Based on his own experiments on closed circuits Ampère included the following observations [16, 17] in his theory, see fig.2a-d:

1) The force of a file under current reverses if the current reverses, see fig.2a.

2) the forces of a current, which flows in a smooth circular circuit, is the same, if the "circle" of the current is not smooth but sinoidal, see fig.2b.

3) the force of a closed current on a single current element is perpendicular to it, see fig.2c .

4) the force between two current elements does not change if all spatial dimensions of the setup are enlarged by a constant factor, see fig. 2d.

Applying these observations Ampère constructed his force law.

Based on Ampere's assumption it holds for the force $\mathbf{F} \sim \mathbf{r}$. The observations 1)+ 2) suggest for first order

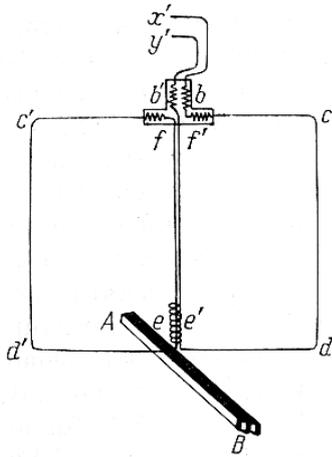


fig.2a: Ampère's first experiment
 AB is a fixed conductor under current. The circuits d'c'fe and cde'f' are stiffly connected, are symmetrical over AB and can rotate about the axis x'y'. Their orientation of the current is opposite in these circuits; experimental result: no rotation due to complete balance of opposite forces

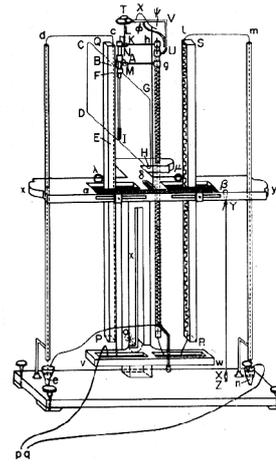


fig.2b: Ampère's second experiment
 in a trench PQ flows a current straight on in a conductor, in the trench SR in a sinoidal conductor. The circuits BCDE and FGHI mounted stiffly together, but can rotate around the Axis AK. The same current flows through them, however in opposite direction. experimental result: only if the circuit is exactly in the middle between the conductors all forces compensate and no movement is observable.

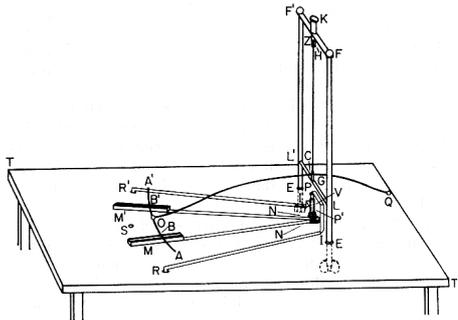


fig.2c: Ampère's third Experiment
 M and M' are trenches filled with mercury, arm OC can be turned. The current flows over the troughs M back to the arm OC. The arm turns into the middle, where a equilibrium of torque exists and where all forces on OC apply perpendiculary.

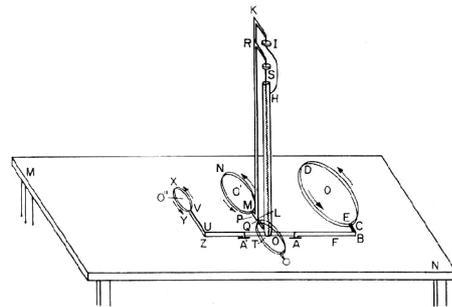


fig.2d: Ampère's fourth Experiment
 the outer circuits are fixed, the circuit in the middle can move. Only, if the diameters fullfil the relation $d_{left} \cdot d_{middle} = d_{middle} \cdot d_{right}$, all forces compensate and the circuit in the middle NOM does not move.

$\mathbf{F} \sim i_1 i_2 [\varphi(r) \cdot (d\mathbf{s}_1 \cdot d\mathbf{s}_2) + \psi(r) (d\mathbf{s}_1 \cdot \mathbf{r}) \cdot (d\mathbf{s}_2 \cdot \mathbf{r})]$, the combination of both proportionalities result in $\mathbf{F} \sim i_1 i_2 \mathbf{r} [\varphi(r) \cdot (d\mathbf{s}_1 \cdot d\mathbf{s}_2) + \psi(r) (d\mathbf{s}_1 \cdot \mathbf{r}) \cdot (d\mathbf{s}_2 \cdot \mathbf{r})]$. Observation 4) implies $\varphi(r) = A/r^3$ and $\psi(r) = B/r^5$ with A and B as constants to be determined. These can be calculated applying observation 3) as shown in the proof below. So follows $B = -3A/2$.

Proof[1]:

Imagine two circuits located with an angle of 90° between. Due to observation 3) holds for a closed circuit

$$\mathbf{F} \sim \int i_1 i_2 \mathbf{r} \left[\frac{A}{r^3} (d\mathbf{s}_1 \cdot d\mathbf{s}_2) + \frac{B}{r^5} (d\mathbf{s}_1 \cdot \mathbf{r}) \cdot (d\mathbf{s}_2 \cdot \mathbf{r}) \right] d\mathbf{s}_2 = 0$$

This equation is rewritten as

$$\frac{A (d\mathbf{s}_1 \cdot d\mathbf{s}_2) \cdot (d\mathbf{s}_2 \cdot \mathbf{r})}{r^3} + \frac{B (d\mathbf{s}_1 \cdot \mathbf{r}) \cdot (d\mathbf{s}_2 \cdot \mathbf{r})^2}{r^5}$$

Because the integral over the circuit is zero, a potential exist and consequently also a total differential. If the circuit is chosen to be a round circuit one can replace by $d\mathbf{s}_1 = -d\mathbf{r}$ and write

$$-\frac{A d(\mathbf{r} \cdot d\mathbf{s}_2) \cdot (d\mathbf{s}_2 \cdot \mathbf{r})}{r^3} + \frac{B (d\mathbf{s}_1 \cdot \mathbf{r}) \cdot (d\mathbf{s}_2 \cdot \mathbf{r})^2}{r^5}$$

Due to the potential property follows $\varphi_{xy} = \varphi_{yx}$ and then

$$d \frac{A}{2r^3} = -\frac{B}{r^5} (d\mathbf{s}_1 \cdot \mathbf{r})$$

With $d\mathbf{s}_1 = -d\mathbf{r}$ this becomes

$$-\frac{3A}{2r^4} dr = \frac{B}{r^4} dr$$

and $B = -3A/2$ follows.

q.e.d □

So Ampère's law is written :

$$\mathbf{F} = \frac{i_1 i_2}{c^2} \mathbf{r} \left[\frac{2}{r^3} \cdot (d\mathbf{s}_1 \cdot d\mathbf{s}_2) - \frac{3}{r^5} (d\mathbf{s}_1 \cdot \mathbf{r}) \cdot (d\mathbf{s}_2 \cdot \mathbf{r}) \right] \quad (19)$$

Riemann [18] and Whittaker [1] checked this derivation and realized, that Ampère's workout is only one possible ansatz to explain the observations. They doubted in Ampère's assumption, that the force between current elements is a central force, because the forces could be as well angular moments [19]. They found other possible formulas, which could explain all observations. Whittaker enlarged Ampère's formula, by adding terms, which were in accordance with the observations on closed current loops, because they were zero after integration over a closed loop. So he made the general ansatz:

$$\begin{aligned} \mathbf{F} = & -\frac{i i'}{c^2} \mathbf{r} \left[\frac{2}{r^3} \cdot (d\mathbf{s} \cdot d\mathbf{s}') - \frac{3}{r^5} (d\mathbf{s} \cdot \mathbf{r}) \cdot (d\mathbf{s}' \cdot \mathbf{r}) \right] \\ & + \chi(r) (d\mathbf{s}' \cdot \mathbf{r}) \cdot d\mathbf{s} + \chi(r) (d\mathbf{s} \cdot \mathbf{r}) ds' + \chi(r) \cdot (d\mathbf{s} \cdot d\mathbf{s}') \cdot \mathbf{r} \\ & + \frac{1}{r} \chi'(r) \cdot (d\mathbf{s} \cdot \mathbf{r}) \cdot (d\mathbf{s}' \cdot \mathbf{r}) \end{aligned} \quad (20)$$

Whittaker dropped Ampère's assumption, that the force should be a central force and he applied only Newton's law actio-reactio. He made the most simple possible choices $\chi(r) = i i' / (c^2 r^3)$, $\chi'(r) = -3 i i' / (c^2 r^3)$ and obtained the force law

$$\mathbf{F} = \frac{i i'}{c^2 r^3} [(d\mathbf{s} \cdot \mathbf{r}) ds' + (d\mathbf{s}' \cdot \mathbf{r}) ds - \mathbf{r} (d\mathbf{s} \cdot d\mathbf{s}')] \quad (21)$$

Tabel 1: different versions of magnetostatic force law between current elements ([20] and [1])
 general form of the magnetostatic force law:

$$F = k \frac{i i'}{r^3} [\mathbf{r} \cdot (A. (d\mathbf{s} \cdot d\mathbf{s}') + B. (\mathbf{r} \cdot d\mathbf{s})(\mathbf{r} \cdot d\mathbf{s}')/r^2) + C. (\mathbf{r} \cdot d\mathbf{s}') d\mathbf{s} + D. (\mathbf{r} \cdot d\mathbf{s}) d\mathbf{s}']$$

name	year	ref.	A	B	C	D	comment	
Ampere	1823	[17]	-2	3	0	0	central force	
Grassmann	1845	[11, 15]	-1	0	0	1	no monopoles	
Riemann	1875	[18]	-1	0	1	1	moment conserved	
Whittaker	1912	[1, 15]	see under Riemann					
Brown	1955	????	1	-6	6	6	?????	
Aspden	1987	[21]	-1	0	1	-1	cons. angular moment	
Marinov	1993	[22]	-1	0	0.5	0.5	experiment	
Cavallieri	1998	[23]	see under Grassmann					experiment

Of course this force law was not convincing as well.

For the basic idea of Riemann and Whittaker was used by many others who built their "own" force laws using other assumptions. The discussion is running until today, see [23] and tab.1. All different forms yielded the same result for the magnetic field \mathbf{H} , if they were integrated over a closed circuit.

$$\mathbf{H} = \frac{I}{c} \oint \frac{\mathbf{x} \times d\mathbf{s}'}{|\mathbf{x}|^3} \quad \text{or generally} \quad \mathbf{H} = \frac{1}{c} \oint \mathbf{j}(\mathbf{x}') \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} d^3x' \quad (22)$$

general scheme of proof for every magnetostatic force law:

According to a general theorem of vector analysis, see appendix 2, every vector field can be decomposed into a vortex field and a potential field. The vortex field is caused by currents, the potential field by charges. If this is compared with theorem 2 in appendix 2, then the Biot-Savart law generates a vortex field. All other fields deviating from Biot-Savart, have to be written as

$$\text{field law} = \text{Biot-Savart-law} + \text{additional terms}$$

These additional terms must be identified as a potential field. If the current is integrated over a closed circle the potential terms cancel to zero¹ . □

The force of a closed circuit on a differential current element is according to Biot-Savart, see (18) and (22),

$$d\mathbf{F} = \frac{i}{c} \mathbf{H} \times d\mathbf{s} \quad (23)$$

If one integrates over two interacting closed circuits the von Neumann force law is obtained [1, 8, 24, 25]

$$\mathbf{F} = \frac{I_1 I_2}{c^2} \oint \oint \frac{\mathbf{x}_{12}}{|\mathbf{x}_{12}|^3} d\mathbf{s}_1 d\mathbf{s}_2 \quad (24)$$

From (22) also follows, that the magnetic field can be calculated

¹

The Biot-Savart law is probably the correct version for physical currents. It does not generate "magnetic charges" and coincides with the \mathbf{B} -field of a moving charge according to Lienard-Wiechert (in the special case of zero acceleration). Experimentally the Biot-Savart - law is supported by the measurements of Cavallieri [23].

from a vector potential $\mathbf{H} := \nabla \times \mathbf{A}$ with

$$\mathbf{A}(\mathbf{x}) = \frac{1}{c} \oint \frac{\mathbf{j}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x' \quad (25)$$

Then follows

$$\operatorname{div} \operatorname{rot} \mathbf{A} = \operatorname{div} \mathbf{H} = 0 \quad (26)$$

So Ampère concluded: The cause of the magnetic field are not magnetic charges but only currents.

Ampère's theory includes as well para-, dia- oder ferromagnetic "excited" materials. The total magnetic field \mathbf{B} includes the field from the measurable currents \mathbf{j} and the field \mathbf{M} of the magnetism of the material, where the field \mathbf{M} (according to Ampère) is generated exclusively by currents in the material.

Then follows

$$\operatorname{rot} \mathbf{B} = \frac{4\pi}{c} (\mathbf{j}_{\text{conductor}} + \mathbf{j}_{\text{material}}) \quad (27)$$

with

$$\mathbf{B} := \mu \mathbf{H} \quad \text{oder} \quad \mathbf{B} := \mathbf{H} + 4\pi \mathbf{M} \quad (28)$$

Analogously like for charges a relation is sought between the empirical variables. So the unknown current $\mathbf{j}_{\text{material}}$ is eliminated.

If compared with electrostatics, see eq. (14) to (17), it can be derived for currents, that for magnetostatics holds

$\int \mathbf{j} d^3\mathbf{x} = \int \nabla \cdot \mathbf{j} d^2\mathbf{x} = -\int \dot{\rho} d^2\mathbf{x} = 0$. Here is applied $\nabla \cdot \mathbf{j} + \dot{\rho} = 0$ and $\mathbf{j}(\infty) = 0$, i.e.

no currents exist at the boundary in the infinite. Thus no charges can be built up there and only dipol terms and terms of higher order can contribute to the result. So a definition (29) analogous to (17) is used for the magnetization \mathbf{M} of the material

$$\nabla \times \mathbf{M} := \mathbf{j}_{\text{material}}/c \quad (29)$$

Then, using (27), (28) and (29) Ampère's laws are derived

$$\text{rot} \mathbf{H} = \frac{4\pi}{c} \mathbf{j}_{\text{conductor}} \quad \text{div} \mathbf{B} = 0 \quad (30)$$

In order to derive the present version Ampère's law is rewritten as [8]:

$$\begin{aligned} \nabla \times \mathbf{H} &= \text{rot rot} \mathbf{A} = \text{grad div} \mathbf{A} - \nabla^2 \mathbf{A} \\ &= \nabla \int \frac{\mathbf{j}(\mathbf{x}')}{c} \cdot \nabla \left(\frac{1}{|\mathbf{x} - \mathbf{x}'|} \right) d^3x' - \int \frac{\mathbf{j}(\mathbf{x}')}{c} \nabla^2 \left(\frac{1}{|\mathbf{x} - \mathbf{x}'|} \right) d^3x' \end{aligned} \quad (31)$$

With the mathematical relations $\nabla |\mathbf{x} - \mathbf{x}'|^{-1} = -\nabla |\mathbf{x} - \mathbf{x}'|^{-1}$ and $\nabla^2 |\mathbf{x} - \mathbf{x}'|^{-1} = -4\pi \delta(\mathbf{x} - \mathbf{x}')$ this becomes

$$\nabla \times \mathbf{H} = -\nabla \int \frac{\mathbf{j}(\mathbf{x}')}{c} \cdot \nabla \left(\frac{1}{|\mathbf{x} - \mathbf{x}'|} \right) d^3x' + \frac{4\pi}{c} \mathbf{j} \quad (32)$$

because \mathbf{A} also fulfills the Poisson equation $\nabla^2 \mathbf{A} = -4\pi \mathbf{j}/c$.

If the integral in (32) is integrated partially using that \mathbf{j} vanishes at boundary in the infinite, then follows

$$\nabla \times \mathbf{H} = \frac{4\pi \mathbf{j}}{c} - \nabla \int \frac{\nabla' \cdot \mathbf{j}(\mathbf{x}')}{c |\mathbf{x} - \mathbf{x}'|} d^3x' \quad (33)$$

Now, the observation is used that no charges build up during magnetostatic experiments. Using the continuity equation this fact can be translated into mathematics by $\nabla \cdot \mathbf{j} + \dot{\rho} = 0$.

This yields Ampère's law of magnetostatics:

$$\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} \quad \text{oder} \quad \oint \mathbf{H} \cdot d\mathbf{s} = \frac{4\pi}{c} \int_S \mathbf{j} \cdot d\mathbf{A} \quad (34)$$

Comparing the coefficients of (31) and (33) follows $\text{grad div } \mathbf{A} = 0$. Often, it is assumed $\text{div } \mathbf{A} = 0$. This expression is known as Coulomb-gauge. The vector potential \mathbf{A} is not a unique function, because \mathbf{A} can be replaced by $\mathbf{A}^* = \mathbf{A} + \nabla f(\mathbf{x})$. The important point for the choice of vector potential \mathbf{A} is that $\text{grad div } \mathbf{A} = ?$ has to be chosen such, that a physically motivated constraint is fulfilled - the continuity equation [26].

At the time of Biot&Savart and Ampère this was not known fully and only the closed circuits could be tested out. So the result (22) for the \mathbf{H} -Field was ok. However later, after the discovery of the electron by J.J. Thomson [27], discussions came up due to the basic problem behind the approaches of Biot&Savart and Ampère: Not every magnetic problem could be discussed by a closed electric circuit. Freely moving charges (as differential current elements) could exist and the question for their field had to be solved. So observations were published that longitudinal forces existed in railguns [28, 29] and in plasma tubes [30, 31] (See also the review article [32]). These forces seemed to be explained by Ampère's differential force law, but not by Biot-

Savart's version. Although these problems seem to be solved today not in favour for longitudinal forces² the problem will be left open here for further considerations. So all mathematically possible field configurations will be included in the discussion by adding a magnetic potential to the magnetic vector field. So any vector field \mathbf{F} can be decomposed into two terms \mathbf{F}_c and \mathbf{F}_v , derived from a potential (for \mathbf{F}_c) and a vector potential of vortex field (for \mathbf{F}_v), see the proof in appendix 2 [26] and [35, 36]. Thus any \mathbf{H} -field is described by

$$\mathbf{H} = \mathbf{H}_v + \mathbf{H}_c = \frac{1}{c} \int \mathbf{j}(\mathbf{x}') \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} d^3x' - \int \varrho_H(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} d^3x' \quad (35)$$

Here ϱ_H is the magnetic charge distribution due to the deviation from Biot-Savart's differential law, see eq. (18).

If a concrete system is solved with a boundary problem, a Laplace field \mathbf{H}_L has to be added which satisfies $\text{rot } \mathbf{H}_L = 0$ and $\text{div } \mathbf{H}_L = 0$.

$$\mathbf{H} = \mathbf{H}_v + \mathbf{H}_c + \mathbf{H}_L = \frac{1}{c} \int \mathbf{j}(\mathbf{x}') \times \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} d^3x' - \int \varrho_H(\mathbf{x}') \frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} d^3x' - \nabla\varphi(\mathbf{x}) \quad (36)$$

Here is $\mathbf{H}_L := -\nabla\varphi(\mathbf{x})$ the Laplace field. This potential describes a field, which is generated outside of the defined area of the problem. The field \mathbf{H}_L helps to adapt the solution to the given

2

Both observations were explained later by Rambaut & Vigier [33], see as well [34]. They pointed out, that these observations do not answer the question, because a closed moving circuit shows a "longitudinal" mechanical expansion due to a "expansion" pressure of a loop due to the Lorenz force.

boundary condition of the problem. Then (36) changes to

$$\mathbf{H} = \mathbf{H}_V + \mathbf{H}_C + \mathbf{H}_L = \nabla \times \mathbf{A} - \nabla \Xi - \nabla \varphi(\mathbf{x}) = \nabla \times \frac{1}{c} \int \frac{\mathbf{j}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x' - \nabla \int \frac{\varrho_H(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x' - \nabla \varphi(\mathbf{x}) \quad (37)$$

with $\mathbf{A} := c^{-1} \int \mathbf{j}(\mathbf{x}')/|\mathbf{x} - \mathbf{x}'| d^3x'$ as magnetic vector potential and $\Xi := \int \varrho_H(\mathbf{x}')/|\mathbf{x} - \mathbf{x}'| d^3x'$ as potential function of the magnetic charges. If magnetic charges are included the magnetic field becomes a general field and loses all symmetry properties with respect of parity. So every field configuration can be described generally. It will be shown here that this is useful for problems with induced and permanent magnetization. Only a reinterpretation of the conventional point of view leads to a Poisson equation for magnetic charges.

Proof:

The conventional theory for problems with permanent magnetization [8] (without exciting field from outside) assumes, that

$$\nabla \cdot \mathbf{B}_0 = \nabla \cdot (\mathbf{H}_0 + 4\pi \mathbf{M}) = 0 \quad (38)$$

Here is \mathbf{M} the magnetization of the material and \mathbf{H}_0 the inner magnetic field which generates the magnetization. If a field is applied additionally from outside, this equation is enlarged

$$\nabla \cdot \mathbf{B}_0 = \nabla \cdot (\mathbf{H}_0 + \mathbf{H} + 4\pi \mathbf{M}) = 0 \quad (39)$$

with \mathbf{H} a the exciting \mathbf{H} -field from outside, which is added. Because no currents are obvious in matter as cause for the inner \mathbf{H}_0 -field it holds $\nabla \times \mathbf{H}_0 = 0$. This means that \mathbf{H}_0 can be derived from a potential $\Xi_{\mathbf{H}_0}$ according to $\mathbf{H}_0 = -\nabla \Xi_{\mathbf{H}_0}$ and the magnetostatic Poisson-equation follows [8]

$$\Delta \Xi_{\mathbf{H}_0} = -4\pi\rho_M \quad (40)$$

with $\nabla \cdot \mathbf{H}_0 := 4\pi\rho_M$ defined as "effective magnetic charge density" in [8].

From (39) and (40) follows

$$\nabla \cdot \mathbf{B} := \nabla \cdot (\mathbf{H} + 4\pi\mathbf{M}) = -4\pi\rho_M := 4\pi\rho_H \quad (41)$$

With these redefinitions the conventional equation (39) is written down with magnetic charges in a form, which is completely analog to electrostatics. Then, analogously to electrostatics, the empirical field is the \mathbf{H} -field contrary to the conventional interpretation taking the \mathbf{B} -field. q.e.d. \square

Thus, magnetostatic boundary problems can be worked out analogously to electrostatics with changed boundary conditions. Textbooks show [8], that the solution of magnetic boundary problems are sometimes completely analogous to electrostatics. Similarly, permanent magnets (like an analog to ferroelectrets)

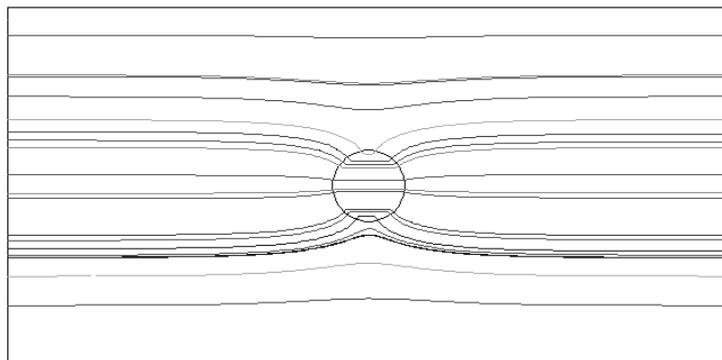


fig.3: polarized bowl in a potential field

boundary condition between inner of bowl and outside: no charges and no currents, i.e.

$\mathbf{B}_{\text{innen}} = \mathbf{B}_{\text{außen}}$, similar like in electrostatics the field is given by the charge distribution at the outer boundary: $B_y = 0$, $\nabla \mathbf{B}_{\text{links}} = -\nabla \mathbf{B}_{\text{rechts}}$. It holds the Laplace equation $\Delta \varphi = \nabla \mathbf{B} = 0$. For the equations of a metal bowl in the electric field the magnetic variables have to be replaced by electric ones.

can be regarded to consist of magnetic charges.

For the simple phenomena of induced polarization, however, see fig.3, the fields are derived by the assumptions, that \mathbf{B} is the solution of the equation $\nabla \cdot \mathbf{B} = 0$. The boundary condition represent either given current distributions exciting the material, either they are the existing field in a distance far from the object under consideration. This means, that the magnetic field \mathbf{H} in the neighborhood of a magnetic material fullfils locally always $\Delta \varphi = \nabla \cdot \mathbf{B} = 0$ and $\nabla \times \mathbf{H} = 0$. Or the Laplace equation holds for the induced magnetism.

Only, if the experiment deviates from the theory, magnetic charges are probable. This is the case for the ferromagnetic hysteresis of iron. If compared with the conventional parity tabel, see tab.2, the \mathbf{B} -field has (-1) parity under time inversion, i.e. if the current is inversed, the field has to be inversed as well. If a hysteresis exists, this is not the case, because the hysteresis line $\mathbf{B}(\mathbf{H})$ is not unique. For a change of parity with fields of the strenght of the coercitivity, the change in parity can easily be disproved. In this case inhomogenities or gradients of magnetic permeability $\mu(\mathbf{x})$ can induce magnetic charges. Then it holds

$$\nabla \cdot \mathbf{B}(\mathbf{x}) = \nabla \cdot (\mu(\mathbf{x}) \cdot \mathbf{H}(\mathbf{x})) = \mu(\mathbf{x}) \nabla \cdot \mathbf{H}(\mathbf{x}) + \mathbf{H}(\mathbf{x}) \cdot \nabla \mu(\mathbf{x}) = 4\pi q_M \neq 0 \quad (42)$$

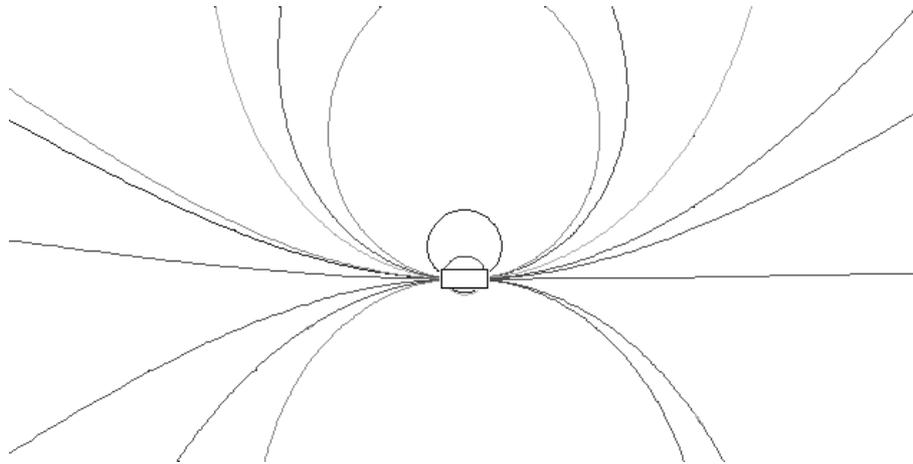


fig.4: field lines of magnetic H-field of a cylindric permanent magnet

the magnet is modeled here as capacity of magnetic charges. The magnetic charges are distributed on the surfaces of north and south pole. The iron has a permeability of $\mu=10000$

Tab.2: symmetry properties of conventional electrodynamics

It holds generally: $F(u) = P \cdot F(-u)$

variable	u	field	F	parity P	sort of field
x	--> -x	E		-1	potential
		D		-1	potential
		H		1	vortex
		B		1	vortex
t	--> -t	E		1	charge
		D		1	charge
		H		-1	current
		B		-1	current

Consequently, a magnet can be modeled as well using magnetic charges, see fig.4. Any permanent magnetism destroys any parity of a **B**-field similar like it is proved in the original experiments of violation of parity in beta-decay.

Of course a general **B**-field with no parity cannot be explained solely by a vector potential **A**,

1) because $\mathbf{B} = \text{rot } \mathbf{A}$ has a defined parity;

2) because $\mathbf{B} := \text{rot } \mathbf{A}$ implies $\text{div rot } \mathbf{A} = \text{div } \mathbf{B} = 0$ follows, which is contradicting to the physical result $\text{div } \mathbf{B} \neq 0$.

So, for magnetic charges the magnetic potential \mathbf{E} has to be introduced. Similar like the magnetic vector potential **A** it has a more formal character, because it is not known very much about magnetic charges except of Ehrenhaft's [5, 6] and Mikhailov's experiments [37-48]. Important questions about concentrating, storing and conducting of magnetic charges are open.

The potentials of the magnetic field are

$$\mathbf{A}_{\mathbf{B},\mathbf{H},\mathbf{M}} = \frac{1}{c} \int \frac{\mathbf{j}_{\mathbf{B},\mathbf{H},\mathbf{M}}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x' \quad \mathbf{E}_{\mathbf{B},\mathbf{H},\mathbf{M}} = \int \frac{q_{\mathbf{B},\mathbf{H},\mathbf{M}}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x' \quad (43)$$

Then, the magnetic fields can be derived

$$\begin{aligned} \mathbf{B}_{\mathbf{V}} &= \text{rot } \mathbf{A}_{\mathbf{B}} , & \mathbf{B}_{\mathbf{C}} &= -\nabla \mathbf{E}_{\mathbf{B}} \\ \mathbf{H}_{\mathbf{V}} &= \text{rot } \mathbf{A}_{\mathbf{H}} , & \mathbf{H}_{\mathbf{C}} &= -\nabla \mathbf{E}_{\mathbf{H}} \\ \mathbf{M}_{\mathbf{V}} &= \text{rot } \mathbf{A}_{\mathbf{M}} / 4\pi , & \mathbf{M}_{\mathbf{C}} &= -\nabla \mathbf{E}_{\mathbf{M}} / 4\pi \end{aligned} \quad (44)$$

using the definitions $\mathbf{B}_{\mathbf{C}} := \mathbf{H}_{\mathbf{C}} + 4\pi \mathbf{M}_{\mathbf{C}}$, $\mathbf{B}_{\mathbf{V}} := \mathbf{H}_{\mathbf{V}} + 4\pi \mathbf{M}_{\mathbf{V}}$, $\mathbf{A}_{\mathbf{B}} := \mathbf{A}_{\mathbf{H}} + \mathbf{A}_{\mathbf{M}}$ and $\mathbf{E}_{\mathbf{B}} := \mathbf{E}_{\mathbf{H}} + \mathbf{E}_{\mathbf{M}}$. The empirical magnetic field is

$$\mathbf{H}=\mathbf{H}_C+\mathbf{H}_V+\mathbf{H}_L \quad (45)$$

For the \mathbf{B} -field holds:

$$\mathbf{B}:=\mathbf{B}_V+\mathbf{B}_C+\mathbf{B}_L \quad (46)$$

Ampère's law are written (using $\text{rot } \mathbf{H}_{C/L}=0$ and $\text{div } \mathbf{B}_{V/L}=0$):

$$\nabla \times \mathbf{H}=\nabla \times \mathbf{H}_V=\frac{4 \pi}{c} \mathbf{j} \quad \nabla \cdot \mathbf{B}=\nabla \cdot \mathbf{B}_C=4 \pi \rho_B \quad (47)$$

Then the general force law of magnetism is:

$$\mathbf{F}=\frac{1}{c} \int \mathbf{j} \times \mathbf{B}_V d\mathbf{x}'^3+\int \rho_H \cdot(\mathbf{H}_C+\mathbf{H}_L) d\mathbf{x}'^3 \quad (48)$$

Later Ampère's law $\nabla \times \mathbf{H}_V=4 \pi \mathbf{j} / c$ was extended by Maxwell. Maxwell realized [49], that it could not describe cases, where electric charge appeared, which were stored in capacitances. Maxwell solved the problem by a hypothesis, which turned out to be very useful, especially with respect to the theory of electromagnetic waves. He changed Ampère's equation to

$$\nabla \times \mathbf{H}_V=\frac{4 \pi}{c} \mathbf{j}+\frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} \quad (49)$$

Introducing the dielectric displacement $d\mathbf{D} / dt$ Maxwell removed a contradiction between physics and mathematics, because the continuity equation as a constraint could always be fulfilled

$$\text{div rot } \mathbf{H}_V=\text{div}\left(\frac{4 \pi}{c} \mathbf{j}+\frac{1}{c} \frac{d \mathbf{D}}{d t}\right)=\frac{4 \pi}{c}\left(\text{div } \mathbf{j}+\frac{d \rho_E}{d t}\right)=0 \quad (50)$$

This form of Ampère's law holds until today. It can describe as well the cases where charges are generated, for instance electron-positron pairs in high energy physics, electron-hole pairs in semiconductors, or dissociations into ions in chemistry. Maxwell's improvement does not change as well the gauge relation, because using (31) it can be calculated

$$\text{grad div}\mathbf{A} = -\nabla \int \frac{\nabla' \cdot \mathbf{j}(\mathbf{x}')}{c|\mathbf{x}-\mathbf{x}'|} d^3x' = -\nabla \int \frac{\dot{\rho}_E(\mathbf{x}')}{c|\mathbf{x}-\mathbf{x}'|} d^3x' = \frac{1}{c} \frac{d\mathbf{D}}{dt} \quad (51)$$

So the vector potential for Ampère's law (34) can be retained.

c) Faraday's law

The induction law has been found by Faraday. Using his formulation it is written

$$U = -\frac{d\Psi}{dt} \quad (52)$$

For Faraday the flux $\Psi = \int \mathbf{B} \cdot d\mathbf{A}$ were the number of field lines, which go through a closed circuit. For an expanding or contracting circuit this is written today [9]

$$-\oint_C \mathbf{E} \cdot d\mathbf{s} = \frac{1}{c} \frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{A} = \frac{1}{c} \int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{A} + \frac{1}{c} \int_S \nabla \times (\mathbf{B} \times \mathbf{v}) \cdot d\mathbf{A} + \frac{1}{c} \int_S \mathbf{v} \cdot (\nabla \cdot \mathbf{B}) \cdot d\mathbf{A} \quad (53)$$

A simple derivation can be done using the formalism of special relativity, see section e). This law can be formulated alternatively using (53), $4\pi\rho_H = \nabla \cdot \mathbf{B}$ and $\mathbf{j}_H = \rho_H \mathbf{v}$

$$-\nabla \times \mathbf{E} = \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} - \frac{1}{c} \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{4\pi}{c} \mathbf{j}_H \quad (54)$$

It will be shown in the next section, that this equation is consistent with a gauge by a continuity equation for magnetic monopoles.

d) the complete Maxwell equations

The Maxwell equation describe the coupling of fields with moving charges in space. They can be generalized that they hold for solids and for gases and liquids.

The notations for the indices here are C:=charge, V:=vortex, E:=electric field and H:=magnetic field.

If magnetic charges are included the Maxwell equations are (using the definitions \mathbf{v} =velocity and $\mathbf{j}:=\mathbf{v}\rho$)

$$\begin{aligned} -\oint \mathbf{E}_V ds &= \frac{d\Psi}{dt} := \frac{1}{c} \frac{d}{dt} \int \mathbf{B} d\mathbf{A} & \oint \mathbf{H}_V ds &= \frac{d\Theta}{dt} := \frac{1}{c} \frac{d}{dt} \int \mathbf{D} d\mathbf{A} \quad \text{oder} \\ -\nabla \times \mathbf{E}_V &= \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} - \nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{B} \right) + \frac{4\pi}{c} \rho_H \mathbf{v} & \nabla \times \mathbf{H}_V &= \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} - \nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{D} \right) + \frac{4\pi}{c} \rho_E \mathbf{v} \\ \operatorname{div} \mathbf{D}_C &= 4\pi \rho_E & \operatorname{div} \mathbf{B}_C &= 4\pi \rho_H & (55) \\ \dot{\rho}_E + \nabla \cdot \mathbf{j}_E &= \operatorname{div} \nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{B} \right) = 0 & \dot{\rho}_H + \nabla \cdot \mathbf{j}_H &= \operatorname{div} \nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{D} \right) = 0 \\ \Delta \phi_D &= \operatorname{div} \mathbf{D}_L = 0 & \Delta \phi_B &= \operatorname{div} \mathbf{B}_L = 0 \\ \mathbf{D} &= \mathbf{D}_V + \mathbf{D}_C + \mathbf{D}_L & \mathbf{B} &= \mathbf{B}_V + \mathbf{B}_C + \mathbf{B}_L \end{aligned}$$

For a mixed system of charged particles the individual equations of each sort of particle have to be added together.

In the version above Ampere's law is extended by the so called

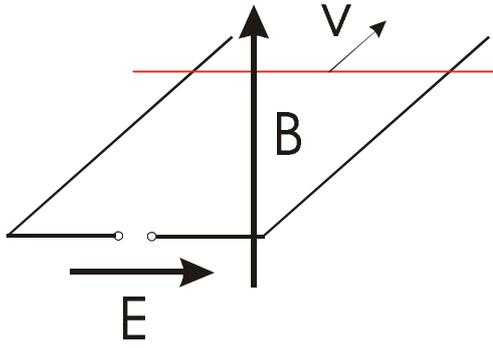


fig.5a: E-field due to the Lorenz force
at the expansion (or contraction) of a circuit in a magnetic field

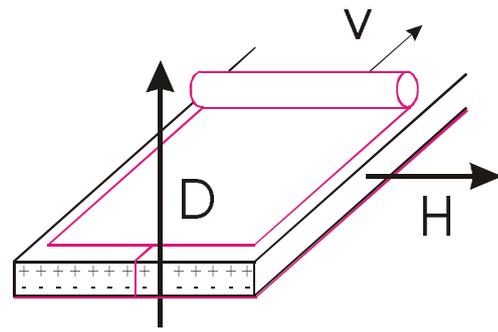


fig.5b: H-field due to the Rowlands force
at the roll out of a conducting foil over a polarized electret material

Rowlands term which is electric analog to the Lorenz force. This term takes account for a **H**-field, which is generated, if a capacitance grows in an electric field, see fig.5b.

Similarly the Laplace field is accounted for in (55).

To complete the theory an electric vector potential must also be introduced. It is generated by magnetic currents.

All generating potentials are listed in (56)

$$\begin{aligned}
 \Phi_{D,E,P} &= \int \frac{Q_{D,E,P}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x' & \Gamma_{D,E,P} &= \frac{1}{c} \int \frac{\mathbf{j}_{B,H,M}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x' \\
 \mathbf{A}_{B,H,M} &= \frac{1}{c} \int \frac{\mathbf{j}_{D,E,P}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x' & \Xi_{B,H,M} &= \int \frac{Q_{B,H,M}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x'
 \end{aligned}
 \tag{56}$$

They are interconnected with the fields by

$$\begin{aligned}
\mathbf{B}_V &= \text{rot} \mathbf{A}_B, & \mathbf{B}_C &= -\nabla \Xi_B \\
\mathbf{H}_V &= \text{rot} \mathbf{A}_H, & \mathbf{H}_C &= -\nabla \Xi_H \\
\mathbf{M}_V &= \text{rot} \mathbf{A}_M / 4\pi, & \mathbf{M}_C &= -\nabla \Xi_M / 4\pi \\
\mathbf{D}_V &= \text{rot} \Gamma_D, & \mathbf{D}_C &= -\nabla \Phi_D \\
\mathbf{E}_V &= \text{rot} \Gamma_E, & \mathbf{E}_C &= -\nabla \Phi_E \\
\mathbf{P}_V &= \text{rot} \Gamma_P / 4\pi, & \mathbf{P}_C &= -\nabla \Phi_P / 4\pi
\end{aligned}
\tag{57}$$

Summarizing it can be said about the Maxwell equations:

Electric and magnetic fields can be described mathematically as general fields. Their causes are charges and currents of electric and magnetic particles, which fulfill the continuity equation as a constraint. Due to the mathematics the electric and magnetic fields can be decomposed into a vortex, a potential field and a Laplace field. The charges build up the potential fields, the currents the vortex field and the Laplace field adapts to the boundary conditions.

e) The Maxwell equations and the theory of relativity

In the theory of relativity the Maxwell equations are formulated in the terminology of tensor calculus.

The theory of relativity relates the variables measured in a reference system to the variables of another system which moves relative to the first system. The transformation applies for a movement in z-direction (using the definitions $\beta := v/c$, $\gamma := 1/\sqrt{1-\beta^2}$)

$$a_{ij} = \frac{\partial x'_i}{\partial x_j} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \quad (58)$$

Similarly vectors are transformed (using the Einstein convention)

$$A'_i = a_{ij} \cdot A_j \quad (59)$$

Tensors T'_{ij} are transformed by

$$T'_{ij} = a_{ik} \cdot a_{jl} \cdot T_{kl} \quad (60)$$

The 4-vectors of the theory of relativity are, cf. appendix 3,

$$\begin{aligned} \text{space coordinates : } & \mathbf{x} = (x, y, z, ict) \\ \text{momentum : } & \mathbf{p} = (p_x, p_y, p_z, imc) \\ \text{wave number : } & \mathbf{k} = (k_x, k_y, k_z, \frac{i}{c}\omega) \\ \text{electric 4-current : } & \mathbf{j}_E = (j_x^E, j_y^E, j_z^E, ic\rho^E) \\ \text{magnetic 4-current : } & \mathbf{j}_H = (j_x^H, j_y^H, j_z^H, ic\rho^H) \\ \text{electric Lorenz vector : } & \mathbf{L}_E = (A_x^E, A_y^E, A_z^E, ic\Phi^D) \\ \text{magnetic Lorenz vector : } & \mathbf{L}_H = (\Gamma_x^H, \Gamma_y^H, \Gamma_z^H, ic\Xi^B) \end{aligned} \quad (61)$$

The 4-vectors are invariant, i.e. the length of a vector is independent from the state of movement of the reference system.

From this property and from (61) follows the continuity equation

$$\frac{d}{dx_i} j^i_E = \text{div } \mathbf{j}_E + \frac{dq_E}{dt} = 0 \quad \frac{d}{dx_i} j^i_H = \text{div } \mathbf{j}_H + \frac{dq_H}{dt} = 0 \quad (62)$$

An analogous equation - the Lorenz gauge - holds as well for Lorenz vectors, see appendix 3. The definitions for the

electromagnetic tensor field at no current ($\mathbf{v}=0$) are

$$F^{ij} := \begin{pmatrix} 0 & -E_3 & E_2 & -iB_1 \\ E_3 & 0 & -E_1 & -iB_2 \\ -E_2 & E_1 & 0 & -iB_3 \\ iB_1 & iB_2 & iB_3 & 0 \end{pmatrix} \quad G^{ij} := \begin{pmatrix} 0 & H_3 & -H_2 & -iD_1 \\ -H_3 & 0 & H_1 & -iD_2 \\ H_2 & -H_1 & 0 & -iD_3 \\ iD_1 & iD_2 & iD_3 & 0 \end{pmatrix} \quad (63)$$

If there no current is flowing, ($\mathbf{j}=\rho\mathbf{v}=0$) the 4-currents are

$$\mathbf{j}_E = (0, 0, 0, ic\rho^E) \quad \mathbf{j}_H = (0, 0, 0, ic\rho^H) \quad (64)$$

Then the Maxwell equations can be written

$$\frac{d}{dx_j} F^{ij} = 4\pi j_H^i \quad \frac{d}{dx_j} G^{ij} = 4\pi j_E^i \quad (65)$$

The complete system (55) of Maxwell equations follows if the charges move. This is described by the following coordinate transformation

$$\begin{aligned} \frac{d}{dx'_n} F'^{kn} &= \frac{d}{dx'_n} \frac{\partial x'_n}{\partial x_j} \frac{\partial x'_k}{\partial x_i} F^{ij} = 4\pi \frac{\partial x'_k}{\partial x_i} j_H^i = 4\pi j'^k_H \\ \frac{d}{dx'_n} G'^{kn} &= \frac{d}{dx'_n} \frac{\partial x'_n}{\partial x_j} \frac{\partial x'_k}{\partial x_i} G^{ij} = 4\pi \frac{\partial x'_k}{\partial x_i} j_E^i = 4\pi j'^k_E \end{aligned} \quad (66)$$

(66) represents the complete Maxwell equations in tensor notation, cf. (55). One consequence should be emphasized: if currents exist the complete Maxwell equations have to be applied including the terms of Lorenz and Rowlands force.

f) the electromagnetic tensors of momentum and energy

The electromagnetic conservation of energy

The power of a electrically and magnetically charged particle is

(using $\mathbf{F}_H = q_H \left(\mathbf{H} - \frac{\mathbf{v}}{c} \times \mathbf{D} \right)$, $\mathbf{F}_E = q_E \left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right)$ and $\mathbf{F} = \mathbf{F}_E + \mathbf{F}_H$)

$$\frac{dE_{\text{mech}}}{dt} = \mathbf{F} \cdot \mathbf{v} := q_E \cdot \mathbf{E} \cdot \mathbf{v} + q_H \cdot \mathbf{H} \cdot \mathbf{v} \quad (67)$$

This equation integrated over the whole space yields with $\mathbf{j} := \mathbf{v}\rho$

$$\frac{dE_{\text{mech}}}{dt} = \mathbf{F} \cdot \mathbf{v} := \int (\mathbf{j}_E \cdot \mathbf{E} + \mathbf{j}_H \cdot \mathbf{H}) dx^3 \quad (68)$$

If the Maxwell equations are solved for the currents, (i.e.

$\mathbf{j}_H = \frac{c}{4\pi} \left[-\nabla \times \mathbf{E} - \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} + \nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{B} \right) \right]$ and $\mathbf{j}_E = \frac{c}{4\pi} \left[\nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} + \nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{D} \right) \right]$) and inserted in (67), and using $\nabla \cdot (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \cdot (\nabla \times \mathbf{a}) - \mathbf{a} \cdot (\nabla \times \mathbf{b})$, it follows a modified

Poynting energy conservation equation for the energy density:

$$\frac{de_{\text{mech}}}{dt} = -\nabla \cdot \mathbf{S} - \frac{\partial U}{\partial t} + \frac{c}{4\pi} \left(\nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{B} \right) \right) \cdot \mathbf{H} + \frac{c}{4\pi} \left(\nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{D} \right) \right) \cdot \mathbf{E} \quad (69)$$

Here the following definitions have been used

$$\begin{aligned} \mathbf{S} &:= \frac{c}{4\pi} (\mathbf{E} \times \mathbf{H}) \\ \frac{dU}{dt} &:= \frac{1}{4\pi} \left(\mathbf{E} \frac{d\mathbf{D}}{dt} + \mathbf{H} \frac{d\mathbf{B}}{dt} \right) \end{aligned} \quad (70)$$

The last two terms in (69) are non-standard, because the energy conservation is derived always without Rowlands and Lorenz terms.

The electromagnetic conservation of momentum

The force on a charge distribution of electromagnetic charge is

$$F_{mech} = \int \rho_E (\mathbf{E} + \frac{\mathbf{j}_E}{c} \times \mathbf{B}) + \rho_H (\mathbf{H} + \frac{\mathbf{j}_H}{c} \times \mathbf{D}) dx'^3 \quad (71)$$

Using again Maxwell's equations solved for \mathbf{j} this can be written

$$F_{mech} = \int [\mathbf{E} \nabla \cdot \mathbf{D} + \mathbf{H} \nabla \cdot \mathbf{B} + (\nabla \times \mathbf{H}^*) \times \mathbf{B} + (\nabla \times \mathbf{E}^*) \times \mathbf{D} - \frac{1}{c} \left(\frac{\partial \mathbf{D}}{\partial t} \times \mathbf{B} \right) + \frac{1}{c} \left(\frac{\partial \mathbf{B}}{\partial t} \times \mathbf{D} \right)] dx'^3 \quad (72)$$

Using the definitions $\mathbf{E}^* := \mathbf{E} + \mathbf{v}/c \times \mathbf{B}$ and $\mathbf{H}^* := \mathbf{H} - \mathbf{v}/c \times \mathbf{D}$ and the calculation in footnote³ follows

$$F_{mech} = \int \left[\frac{d}{dx_k} \mathbf{T}_{ik}^* - \mathbf{D} \frac{d\mathbf{E}}{dx_k} - \mathbf{B} \frac{d\mathbf{H}}{dx_k} - \frac{d}{dt} \frac{\mathbf{D} \times \mathbf{B}}{4\pi c} + \left((\nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{B} \right)) \times \mathbf{B} \right) + \left((\nabla \times \left(\frac{\mathbf{v}}{c} \times \mathbf{D} \right)) \times \mathbf{D} \right) \right] dx'^3 \quad (73)$$

Here \mathbf{T}_{ik}^* is defined as $\mathbf{T}_{ik}^* := E_i D_j + H_i B_j$. The fourth term of the first line of (73) is $\mathbf{p}_{Feld} := (\mathbf{D} \times \mathbf{B}) / (4\pi c)$ which is defined as the

³ The first three vector terms can be written in the terminology of the tensor calculus:

$$\mathbf{E} \cdot \nabla \cdot \mathbf{D} + \mathbf{H} \cdot \nabla \cdot \mathbf{B} + (\nabla \times \mathbf{H}) \times \mathbf{B} + (\nabla \times \mathbf{E}) \times \mathbf{D} = \varepsilon_{ijk} \varepsilon_{jls} \frac{\partial E_s}{\partial x_l} D_k + \varepsilon_{ijk} \varepsilon_{jls} \frac{\partial H_s}{\partial x_l} B_k.$$

Using $\varepsilon_{ijk} \varepsilon_{jls} = \delta_{kl} \delta_{is} - \delta_{ks} \delta_{il}$ the first term is transformed to

$$\varepsilon_{ijk} \varepsilon_{jls} \frac{\partial E_s}{\partial x_l} D_k + \varepsilon_{ijk} \varepsilon_{jls} \frac{\partial H_s}{\partial x_l} B_k = E_i \frac{\partial D_j}{\partial x_j} + D_i \frac{\partial E_k}{\partial x_k} - D_k \frac{\partial E_k}{\partial x_i} + H_i \frac{\partial B_j}{\partial x_j} + B_i \frac{\partial H_k}{\partial x_k} - B_k \frac{\partial H_k}{\partial x_i}$$

In the 2nd and 5th term k can be exchanged with j without changing the result. Then follow the first three terms of (73).

electromagnetic momentum $\mathbf{P}_{\text{Field}}$ of the field.

If the generality of (73) is restricted (i.e. if only materials are used with purely linear constitutive relations like $\mathbf{B}=\mu\mathbf{H}$ and $\mathbf{D}=\varepsilon\mathbf{E}$) then the first three terms of (73) represent the Maxwell energy tensor:

$$\frac{d\mathbf{T}_{ik}}{dx_k} := \frac{d}{dx_k} (E_i D_k + H_i B_k - \frac{\delta_{ik}}{2} (\varepsilon \mathbf{E}^2 + \mu \mathbf{H}^2)) \quad (74)$$

This equation is found in the textbooks normally. The last two terms of (73) are omitted always, because "shorted" Maxwell equation are used which is wrong in the general case according to the author's opinion.

The equations of conservation of energy and momentum describe the behaviour of a generalized capacitive-inductive- electronic element. Special cases for the energy equation are the pure capacitance (if $\mathbf{H}=0$ and $\mathbf{B}=0$) and the pure coil (if $\mathbf{E}=0$ and $\mathbf{D}=0$), see (73). For these cases the equation says, that the energy flowing into the electronic element can be identified with the electric or magnetic field energy.

The definition of electromagnetic work can be done if (69) is applied

$$W_{el} = \int \frac{dE_{\text{mech}}}{dt} dt \quad (75)$$

It should be said that the discussion about the "correct" equations (69) and (73) is alive until today, cf. [50].

It is remarkable that the derivation with monopoles yields the same result as without. The cause of this may be, that many Maxwell equations are the solutions from the the theory of general relativity, because one degree of freedom remains undetermined during the derivation [8] [51, 52]. These considerations were done for the shorted Maxwell equations (55), i.e.

$$\begin{aligned}
\nabla \cdot \mathbf{D} &= 4\pi\rho_E & \nabla \times \mathbf{H} &= \frac{1}{c} \frac{d\mathbf{B}}{dt} + \frac{4\pi}{c} \mathbf{j}_H \\
\nabla \cdot \mathbf{B} &= 4\pi\rho_H & -\nabla \times \mathbf{E} &= \frac{1}{c} \frac{d\mathbf{D}}{dt} + \frac{4\pi}{c} \mathbf{j}_E
\end{aligned}
\tag{76}$$

It can be shown, that all these equation can be transformed by

$$\begin{aligned}
\mathbf{E} &= \mathbf{E}' \cos\zeta + \mathbf{H}' \sin\zeta & \mathbf{D} &= \mathbf{D}' \cos\zeta + \mathbf{B}' \sin\zeta \\
\mathbf{H} &= -\mathbf{E}' \sin\zeta + \mathbf{H}' \cos\zeta & \mathbf{B} &= -\mathbf{D}' \sin\zeta + \mathbf{B}' \cos\zeta \\
\rho_E &= \rho'_E \cos\zeta + \rho'_H \sin\zeta & \mathbf{j}_E &= \mathbf{j}'_E \cos\zeta + \mathbf{j}'_H \sin\zeta \\
\rho_H &= -\rho'_E \sin\zeta + \rho'_H \cos\zeta & \mathbf{j}_H &= -\mathbf{j}'_E \sin\zeta + \mathbf{j}'_H \cos\zeta
\end{aligned}
\tag{77}$$

If the parameter ζ in (77) is chosen appropriately, the conventional Maxwell equations without magnetic charges are the result. It is shown that relativistic pressure tensor (shorted calculation without Lorenz and Rowlands terms !) is invariant under these transformations.

If it is believed, that every electric charge is in a constant proportion with a magnetic charge, -so the argumentation and the calculation of Harrison[53] and Katz [52]- the combined charge is regarded as a new "elementary charge", and built up a transformed (shorted) system of Maxwell equations with $\text{div } \mathbf{B}=0$

[8, 52, 53]. So it is understandable, that Mikhailov [38, 48, 54] tried to determine the proportion between electric and magnetic charge, especially because the first workout of his measurements [38] spoke against the generally accepted theoretical value of Dirac [55, 56]. Anyway, in the light of these opinions of Harrison[53] and Katz [52], one can ask why Mikhailov sees any effects at all. For author the discussion is not at the end here. Perhaps, parity checks can solve this question.

g) boundary conditions

stationary discontinuous boundary conditions by charges

In order to derive boundary condition equation (13) is applied on a fictive "pillbox" at the boundary between two materials of a potential field [8], see fig.6a .

So one obtains the relation (with σ :=surface charge density)

$$\int \nabla \cdot \mathbf{F}_C \, dV = \int_S \mathbf{F}_C \cdot \mathbf{n} \, da = (\mathbf{F}_C(1) - \mathbf{F}_C(2)) \cdot \mathbf{n} \, \Delta a = 4\pi\sigma\Delta a \quad (78)$$

Equation (78) shows a relation between vector components of the field \mathbf{F}_1 in region 1 and \mathbf{F}_2 in region 2 which both are normal to the surface. This yields for the vertical components of dielectric displacement \mathbf{D}_C :

$$(\mathbf{D}_C(1) - \mathbf{D}_C(2)) \cdot \mathbf{\bar{n}} = 4\pi\sigma_E \quad (79)$$

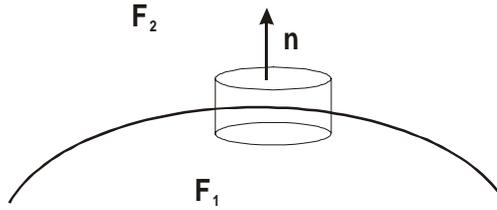


fig.6a the pillbox - construction
for the determination of boundary conditions
due to charges

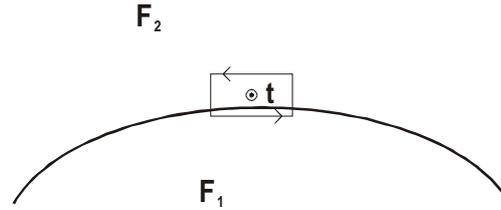


fig.6b the circuit - construction
for the determination of boundary conditions
due to currents

i.e. at the boundary there is a discontinuity which is determined by the surface charge density. An analog holds for the vertical component of the magnetic field \mathbf{B}_c :

$$(\mathbf{B}_c(1) - \mathbf{B}_c(2)) \cdot \mathbf{\bar{n}} = 4\pi\sigma_H \quad (80)$$

For a electric or magnetic conducting surface holds

$$\Phi = constant \quad \mathbf{E} = constant \quad (81)$$

stationary boundary conditions by currents

Equation (29) can be applied to derive a boundary condition if a surface current \mathbf{k} flows at the boundary between regions of different materials, see fig.6b . So one obtains [8]

$$\int \nabla \times \mathbf{F}_v \, dA = \int_S \mathbf{F}_v \, ds = (\mathbf{n} \times \mathbf{t}) \cdot (\mathbf{F}_v(1) - \mathbf{F}_v(2)) \Delta l = \frac{4\pi}{c} \mathbf{k} \cdot \mathbf{t} \Delta l \quad (82)$$

Equation (82) is a relation between the vector components \mathbf{F}_1 and \mathbf{F}_2 which flow tangentially on the surface of the boundary between two regions 1 and 2 of different materials.

discontinuities of the magnetic vortex field
for tangents to the surface

$$\bar{\mathbf{n}} \times (\mathbf{H}_V(1) - \mathbf{H}_V(2)) = \frac{4\pi}{c} \mathbf{K}_E \quad (83)$$

discontinuity of the electric vortex field
for tangents to the surface

$$\bar{\mathbf{n}} \times (\mathbf{E}_V(1) - \mathbf{E}_V(2)) = \frac{4\pi}{c} \mathbf{K}_H \quad (84)$$

For more general, nonstationary boundary conditions at moving surfaces, see [8].

h) the constitutive equations of the material

The system of Maxwell equations can be solved after the constitutive equation are known which describe the material properties. They couple the electric variables (\mathbf{E}, \mathbf{D}) and the magnetic variables (\mathbf{B}, \mathbf{H}) which can be represented generally by

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = \text{coupling} \circ \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \quad (85)$$

In the most cases these couplings are simple, i.e.

$$\text{as resistor: } \mathbf{j} = \sigma \mathbf{E}$$

$$\text{or capacitively: } \mathbf{D} = \varepsilon \mathbf{E} \quad (86)$$

$$\text{or inductively: } \mathbf{B} = \mu \mathbf{H}$$

Initially the material constant were constants which described the simple cases of material properties. Later more complicated nonlinear functions were found which could generate phase transitions, i.e.

$$\sigma = \sigma(\mathbf{E}), \quad \varepsilon = \varepsilon(\mathbf{E}), \quad \mu = \mu(\mathbf{H}) \quad (87)$$

After the fundamental crystal structures were known, the material properties could be correlated to the symmetry of the crystals. Then, the constitutive equation were described by tensors

$$\sigma = \sigma_{ik}(\mathbf{E}), \quad \varepsilon = \varepsilon_{ik}(\mathbf{E}), \quad \mu = \mu_{ik}(\mathbf{H}) \quad (88)$$

which were first linear, then non-linear.

Then, materials were discovered whose properties were magnetic and electric, and where an electric field influenced the magnetic properties and vice versa [57] [58].

The theory of relativity found out that dielectric or magnetic polarized material behaved different if it was set in motion. The following equations are from [59]

$$\begin{aligned} \mathbf{E}' &= \mathbf{E} - \frac{\mathbf{v}}{c} \times \mathbf{M} \\ \mathbf{H}' &= \mathbf{H} - \frac{\mathbf{v}}{c} \times \mathbf{P} \end{aligned} \quad (89)$$

A further complication of the constitutive relations are space-dependence of the material properties which are realized for instance as electronic elements.

Furthermore all materials had their own dynamics in time in the

form of relaxation time.

If all material properties are accounted for then the general constitutive equations can be abstracted as additional differential equations which help to solve the complete system of partial differential equations. This system can be written as

$$\begin{pmatrix} \dot{\mathbf{E}} \\ \dot{\mathbf{H}} \end{pmatrix} = \begin{pmatrix} f_1(\mathbf{E}, \mathbf{D}, \mathbf{H}, \mathbf{B}; T, \rho, \dot{\mathbf{x}}, \omega, \dots)(\mathbf{x}, t) \\ f_2(\mathbf{E}, \mathbf{D}, \mathbf{H}, \mathbf{B}; T, \rho, \dot{\mathbf{x}}, \omega, \dots)(\mathbf{x}, t) \end{pmatrix} \quad (90)$$

or

$$\begin{pmatrix} \dot{\mathbf{D}} \\ \dot{\mathbf{B}} \end{pmatrix} = \begin{pmatrix} f_1(\mathbf{E}, \mathbf{D}, \mathbf{H}, \mathbf{B}; T, \rho, \dot{\mathbf{x}}, \omega, \dots)(\mathbf{x}, t) \\ f_2(\mathbf{E}, \mathbf{D}, \mathbf{H}, \mathbf{B}; T, \rho, \dot{\mathbf{x}}, \omega, \dots)(\mathbf{x}, t) \end{pmatrix} \quad (91)$$

The variables after the semicolon show that the constitutive equations may not depend only from electromagnetic parameters, but can depend as well from mechanic or thermodynamic material properties. This means that the electrodynamics cannot be separated from the other areas of physics. If these the material properties drift under the influence of electromagnetic fields then a purely electrodynamic description is not sufficient and further differential equations from other areas of physics have to be added to a complete partial differential equation system to be solved.

Examples:

1) Known examples are electric motors and generators. Here the mechanic equations of motion of the motors are added. They describe the motion by the angular coordinate of the rotor.

2) Other systems are magnetic materials, for which the Landau-Lifshitz-Gilbert - equation [57] [60] hold

$$\dot{\mathbf{M}} = -\gamma \cdot \mathbf{M} \times \mathbf{H}_{\text{eff}} + \alpha \cdot \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}) \quad (92)$$

It generates a system of partial differential equation (:=PDE) if it is combined with the equation of the magnetostatic potential (41) [61]. It allows to calculate magnetic domains in ferromagnetic materials.

3) A homogeneous thermostatic system like a polymer solution is described by a free energy density f . The system plus field is described by the free energy density $f^* = f + \rho_E \cdot \Phi_E$. Then using the definitions of the global chemical potential $\mu_i^* := df^*/dx_i$ and $x_i := \text{volume ratio}$ the PDE-system hold

$$\begin{aligned} \Delta \Phi_{\mathbf{D}}(x_i(\mathbf{r}), \mathbf{r}) &= -4\pi\rho_E \\ \frac{\partial \mu_i^*}{\partial \mathbf{r}}(x_i(\mathbf{r}), \Phi_{\mathbf{E}}(\mathbf{r})) &= 0 \end{aligned} \quad (93)$$

For a magnetic system (for instance a ferrofluid solution) the electric variables (\mathbf{E}, \mathbf{D}) are replaced by magnetic ones (\mathbf{H}, \mathbf{B}). The magnetic charge density ρ is set to zero, because no magnetic charges can be detected during the magnetization, see [62].

4) If the problem depends from time additionally, it is necessary to replace the second equation of (93) by the thermodynamic functions for non-equilibrium. Then one can write

$$\Delta \Phi_D(x_i(\mathbf{r}), \mathbf{r}) = -4\pi\rho_E$$

$$j_i = -D_i \frac{\partial n_i(r)}{\partial r} + \lambda_i \frac{Z_i e n_i(r) \mathbf{E}(r)}{RT} \quad (94)$$

Here hold the definitions n :=concentration, r :=space coordinate
 Z :=number of charges per ion, e :=elementary charge, \mathbf{E} := electric
field, \mathbf{R} :=Avogadro-constant, T :=temperature, D :=diffusion
constant, λ :=mobility. The second equation of (94) is the Nernst-
Planck equation, which should coincide with the second equation
(93) for $\mathbf{j}=0$. So electrochemical problems are discussed, cf.[63].

5) In semiconductors the charge densities depend from chemical
potential or quasi-Fermi level, which can be influenced by the
electric potential. A good example for such a system is a InAs-
quantum dot-dotted FET invented by Yusa&Sakaki [64]. Its structure
is shown in fig.7. The FET can be used for storing data by
charging the gate capacitance.

The theoretical model of this FET stems from Rack et al.[65].
The PDE's of the system is:

$$\begin{aligned} \text{Poisson-equation: } & \varepsilon_0 \partial_z [\varepsilon(z) \cdot \partial_z \Phi(z)] = -\rho(z) \text{ with } \rho(z) = e[N_D^+(z) - n^{3d}(z) - n_{QD}(z)] \\ \text{current:} & \partial_t n(z) = \frac{1}{e} \partial_z j(z) - f(n_{QD}(z,t), n(z)) = 0 \\ \text{recombinations:} & \partial_t n_{QD}(z,t) = f(n_{QD}(z,t), n(z)) \end{aligned} \quad (95)$$

Here are ε_0 := dielectric constant of vacuum, ε :=dielectric
constant of the material, ρ :=charge density, N_D :=density of
donators, n_{3d} :=charge density of electrons, n_{QD} :=charge density of

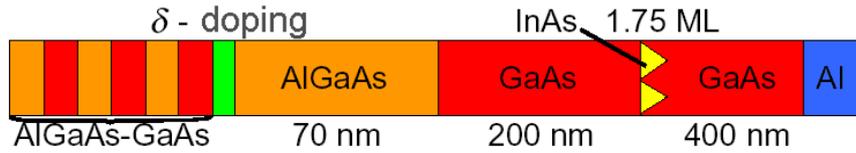


fig.7: structure of a InAs-quantum dot-dotted GaAs-FET

a two-dimensional electron gas (2DEG) is located in the boundary between AlGaAs and GaAs. It represents the zero potential of the system. The electric potential is applied to the Al layer, cf. figs.9

electron trapped in quantum dots, $n(z)$:=free electron density function specified in the article, j :=current in the FET, and $f(n_{\text{QD}}, n)$ is a specific function, which characterizes the recombination process, see [65]. Figs.8 show the electron density in the 2DEG versus voltage. Remarkable is the orientation of the electric cycle which is opposite to the ferroelectric loss hysteresis. This suggests a "gain hysteresis".

It is known that electric work can be changed to mechanic work

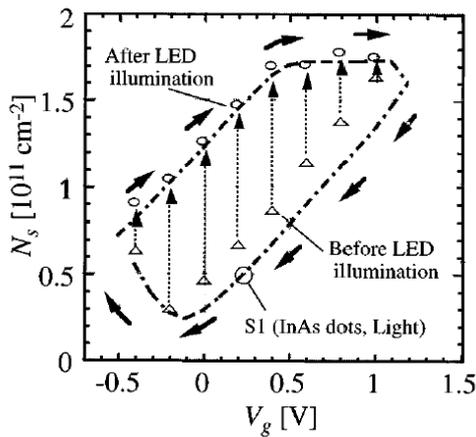


fig.8a the experiment of Yusa-Sakaki- cf. [64] hysteresis of a InAs-quantum dot-dotted FET electron charge density of the two-dimensional electron gas (2DEG) vs. gate voltage

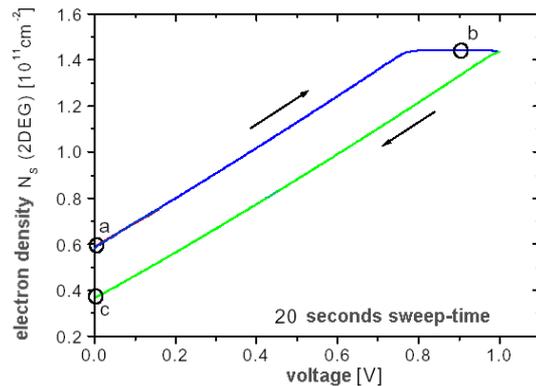


fig.8b the theoretical calculation of the Yusa-Sakaki-FET by Rack et al. electron charge density of the two-dimensional electron gas (2DEG) vs. gate voltage

with efficiencies until 100% in the best electromotors. So electric work should be equivalent to mechanical work in a thermodynamical sense. An "isothermically" proceeded electric cycle with an orientation like in fig.8a can fulfill the energy balance only if heat flows in from outside. Thus, the FET is a candidate for second law violation because only heat and electricity can be exchanged. According to own recent work [62] such cycles could be possible and further evidence can be found: Cooling effects in semiconductors have been predicted by [66]. These considerations support the considerations for the FET discussed above. According to [66] the FET is cooled down if it is set under voltage. So the electrons are enforced into the quantum dots below the quasi-Fermi niveau, where they stick due to their binding energy. After the electric discharge of the FET-capacitance the FET goes back to the equilibrium either if the voltage is slightly inverted, cf.fig.8a, either if the wavelength of the thermal radiation is sufficiently high to overcome the binding energy of 0.25eV, which holds the electrons in the quantum dot potentials. So, the system can be regarded also as a concretisation of Maxwell's demon. The electric energy is lended probably from the quantum dots to be paid back after some time from the thermic influx of environment. Further evidence for this idea can be found from the results of fig. 9a-c, which show the conduction band edge (which is here equivalent to the potential) in the FET at the beginning of the cycle, after charging it with voltage, and after discharging the capacitance.

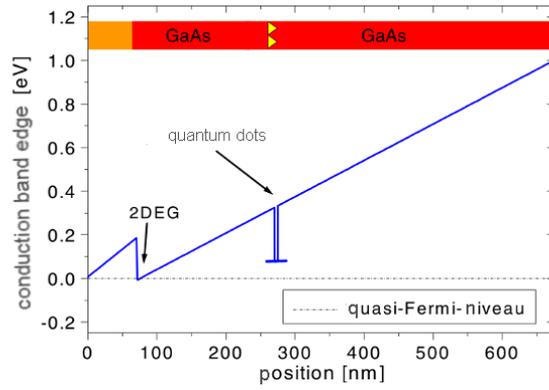


fig.9a the conduction band edge vs. position in the FET of Yusa&Sakaki before the cycle: voltage $U=0$ V

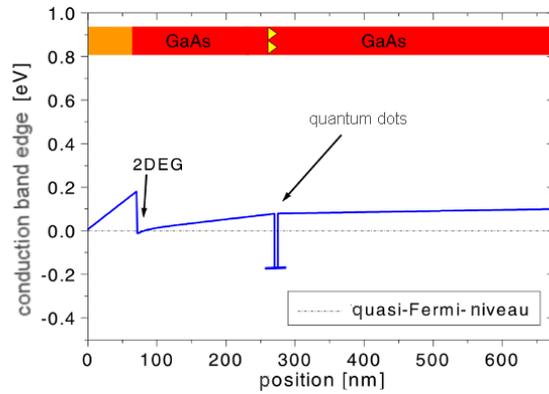


fig.9b the conduction band edge vs. position in the FET of Yusa&Sakaki in the cycle: voltage $U=0.9$ V

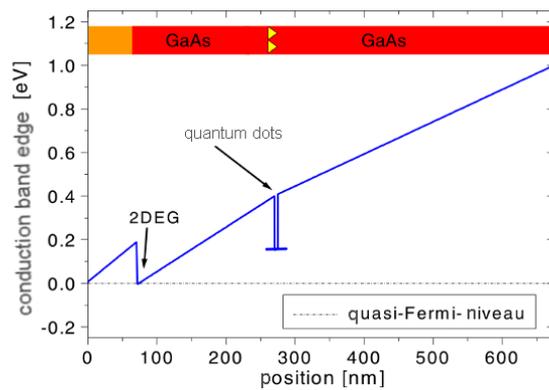


fig.9c the conduction band edge vs. position in the FET of Yusa&Sakaki after the cycle: voltage $U=0$ V
the band edge is changed due to the storage of charges in the quantum dots, cf. fig.9a

From the slope in the diagrams one calculates the electric fields in the FET. If one regards the FET as a capacitance and applies (69) one can estimate the energy exchanged after a cycle. From (69) follows for a pure capacitance

$$\Delta W = \int_0^T U \cdot I dt = -\frac{1}{4\pi} \iint \mathbf{E} d\mathbf{D} dV \quad (96)$$

If one reads off electric field values from the slopes in fig. 9a to fig. 9c one obtains the electric field energies in the FET: before charging the gate capacitance

$$W_1 \sim \mathbf{E}^2 \cdot V \sim (1V/600nm)^2 \cdot 600nm = 0.00166666$$

after discharging the gate capacitance

$$W_2 \sim \sum \mathbf{E}_i^2 \cdot V_i \sim (.38V/200nm)^2 \cdot 200nm + (.62V/400nm)^2 \cdot 0.400nm = 0.001683$$

energy balance

$$\Delta W \sim -(W_2 - W_1) \sim -0.00001633$$

The energy difference of ~1% is negative meaning that electric energy is released by the FET after the electric cycle is closed. The Second Law is violated by the hysteresis of the equilibrium state. The effect is due to the nonlinear behaviour of the FET. Of course, all evidence of the experiment with the Yusa-Sakaki FET is indirectly concluded here. More decisive would be a full balance of all electrons in the calculation or the experiment. Herewith, the constitutive equations are characterized from the simple case to the most complicated systems. Generally, the description of a system may be very sophisticated. However, normally the description is made as simple as possible.

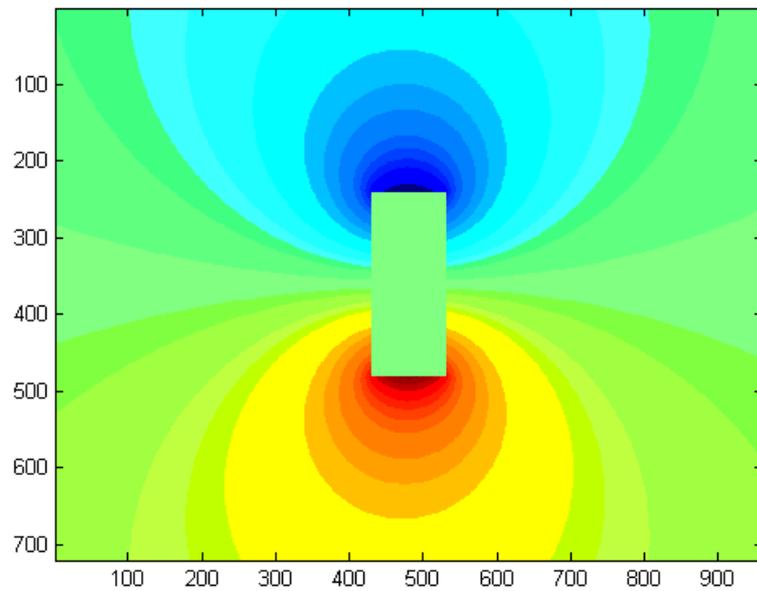


fig.10: strength of the Φ -component of the electric vortex field Γ of a rotating magnetic ring
 cross section view: ring radius 1m, ring width 5cm, ring height 12cm, center of rotation is to the left, (not to be seen in picture). rainbow scale: blue is minus min., red is plus max., see appendix 4.

3. Conclusions

It has been shown that the existence of magnetic charges is justified at least theoretically especially if fields of permanent magnetism are described.

This result suggests the following consequences to be proved:

If magnetic charges can be separated in space - for instance in the form of charges of polarisation in a permanent magnet - and if this magnet moves in a circle, two opposite magnetic currents are generated which itself should generate a electric field according to Faraday's law extended for magnetic charge currents. Measurement of the electric field from moving permanent magnets

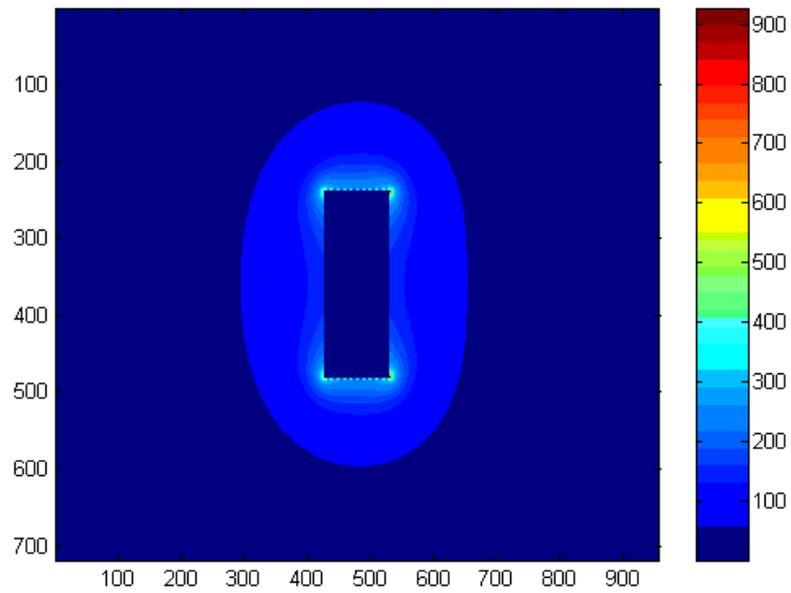


fig.11a: E-field strength around a rotating magnetic ring (cross section)
 ring radius 1m, ring width 5cm, ring height 12cm, center of rotation is to the left, (not shown in the picture). Arbitrary units. Picture is calculated from the data of fig.10, see appendix 4.

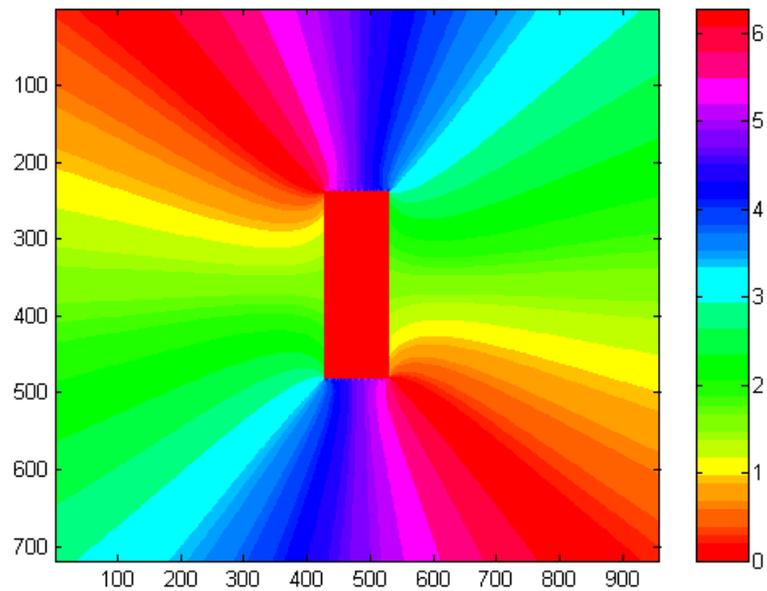


fig.11b position angle of the E-field around a rotating magnetic ring (cross section)
 radius 1m,ring width 5cm,ring height 12cm,center of rotation is to the left,(not shown in the picture)

can answer the question whether the field stems from currents of moving magnetic charges or from the dB/dt - term. According to the theory above both possibilities can be calculated.

The setup of such experiments would be similar to constructions which are known in the unofficial subscene of physics. J. Searl[67-70], D. Hamel [71] and Godin&Roschin [72, 73] claim, that they have observed strong electrostatic effects around fast moving permanent magnets.

Fig.10 shows the calculated electrical vortex field Γ due to a moving permanent magnet ring representing two currents of opposite magnetic surface charge which are placed on top and bottom of the ring, cf. fig.4. The electric field is calculated from the electric vortex field by $\mathbf{E} = \text{rot } \Gamma$, see fig.11a and fig. 11b: the pictures show the electric field strength and the position angle of the field around the cross section of the right half of the ring. In appendix 4 the method of the calculation is shown. If one compares the order of magnitude of the calculation with the data of Godin&Roschin [72, 73] then this suggests for the electric field values an agreement between theory and experiment.

Appendix 1: the derivation of the multipole expansion

First the term $1/|\mathbf{x}-\mathbf{x}'|$ is written as:

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{\sqrt{\mathbf{x}^2 + \mathbf{x}'^2 - 2\mathbf{x}\cdot\mathbf{x}'}} = \frac{1}{|\mathbf{x}|} \frac{1}{\sqrt{1 + \frac{\mathbf{x}'^2 - 2\mathbf{x}\cdot\mathbf{x}'}{|\mathbf{x}|^2}}}$$

with the abbreviation $\alpha := (\mathbf{x}'^2 - 2\mathbf{x}\cdot\mathbf{x}')/|\mathbf{x}|^2 \ll 1$.

This expression is expanded in a series

$$\frac{1}{\sqrt{1+\alpha}} = 1 - \frac{\alpha}{2} + \frac{3}{8}\alpha^2 \pm \dots = 1 - \frac{1}{2} \frac{\mathbf{x}'^2}{|\mathbf{x}|^2} + \frac{2}{2} \frac{\mathbf{x}\cdot\mathbf{x}'}{|\mathbf{x}|^2} + \frac{3}{8} \left[\frac{\mathbf{x}'^2 - 2\mathbf{x}\cdot\mathbf{x}'}{|\mathbf{x}|^2} \right]^2 \pm \dots$$

Using the definitions $\mathbf{x}_0 := \mathbf{x}/|\mathbf{x}|$ and $|\mathbf{x}| := r$ one obtains

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \frac{1}{r} + \frac{1}{r^2}(\mathbf{x}'\cdot\mathbf{x}_0) + \frac{1}{r^3} \left[\frac{3}{2}(\mathbf{x}'\cdot\mathbf{x}_0)^2 - \frac{1}{2}\mathbf{x}'^2 \right] + O\left(\frac{1}{r^4}\right)$$

If this result is applied to the potential definition one gets

$$\Phi(\mathbf{x}) = \frac{1}{r} \int \rho d^3\mathbf{x}' + \frac{1}{r^2} \mathbf{x}_0 \int \rho \mathbf{x}' d^3\mathbf{x}' + \frac{x_{0i}x_{0j}}{2r^3} \int \rho [3x'_i x'_j - x'_n x'_n \delta_{ij}] d^3\mathbf{x}' + O\left(\frac{1}{r^4}\right)$$

This can be written as well

$$\Phi(\mathbf{x}) = \frac{q}{r} + \frac{\mathbf{p}\cdot\mathbf{x}_0}{r^2} + \frac{Q_{ij}x_{0i}x_{0j}}{2r^3} + O\left(\frac{1}{r^4}\right)$$

using the definitions

$$q := \int \rho(\mathbf{x}') d^3\mathbf{x}' \quad \mathbf{p} := \int \mathbf{x}' \rho(\mathbf{x}') d^3\mathbf{x}' \quad Q_{ij} := \int (3x'_i x'_j - x'_n x'_n \delta_{ij}) \rho(\mathbf{x}') d^3\mathbf{x}'$$

Appendix 2: decomposition of a general vector field into a potential field and a vortex field

Theorem 1:

The derivative of a vector field \mathbf{F} can be decomposed in a symmetric (index=C) and a antisymmetric part (index=V), i.e.

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_i^V}{\partial x_j} + \frac{\partial F_i^C}{\partial x_j}$$

\mathbf{F}_C is the symmetric part and is a gradient of a potential field

$$\frac{\partial F_i^C}{\partial x_j} = \frac{\partial F_j^C}{\partial x_i} \quad \text{or} \quad \text{rot } \mathbf{F}_C = 0 \quad \text{with} \quad F_i^C = \frac{\partial U(x_l)}{\partial x_l}$$

(with $U(x_l) :=$ potential function)

\mathbf{F}_V is a antisymmetric vortex field

$$\frac{\partial F_i^V}{\partial x_j} = -\frac{\partial F_j^V}{\partial x_i}$$

Proof:

The derivatives of the field \mathbf{F} can be decomposed according to

$$\frac{\partial F_i}{\partial x_j} = \frac{1}{2} \left(\frac{\partial F_i}{\partial x_j} + \frac{\partial F_j}{\partial x_i} \right) + \frac{1}{2} \left(\frac{\partial F_i}{\partial x_j} - \frac{\partial F_j}{\partial x_i} \right)$$

for the symmetric part holds:

$$\frac{1}{2} \left(\frac{\partial F_i}{\partial x_j} + \frac{\partial F_j}{\partial x_i} \right) = \frac{1}{2} \left(\frac{\partial F_i^C}{\partial x_j} + \frac{\partial F_i^V}{\partial x_j} + \frac{\partial F_j^C}{\partial x_i} + \frac{\partial F_j^V}{\partial x_i} \right) = \frac{\partial F_i^C}{\partial x_j} = \frac{\partial^2 U}{\partial x_i \partial x_j}$$

for the antisymmetric part holds:

$$\frac{1}{2} \left(\frac{\partial F_i}{\partial x_j} - \frac{\partial F_j}{\partial x_i} \right) = \frac{1}{2} \text{rot} \mathbf{F} = \frac{1}{2} \left(\frac{\partial F_i^C}{\partial x_j} + \frac{\partial F_i^V}{\partial x_j} - \frac{\partial F_j^C}{\partial x_i} - \frac{\partial F_j^V}{\partial x_i} \right) = \frac{\partial F_i^V}{\partial x_j}$$

It can be checked, that

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_i^V}{\partial x_j} + \frac{\partial F_i^C}{\partial x_j}$$

q.e.d.

Theorem 2:

\mathbf{F} is a field with a defined boundary condition ∂F around the space which is interesting for the problem. Divergence and rotation are defined according to

$$\nabla \times \mathbf{F} = \mathbf{j}(\mathbf{x}) \quad \nabla \cdot \mathbf{F} = \rho(\mathbf{x})$$

and the boundary condition ∂F

$$\partial F: \mathbf{F} \cdot \mathbf{n} = f(\mathbf{r})$$

Then it holds:

\mathbf{F} can be calculated as sum of a gradient \mathbf{F}_C of a potential, plus rotation of a vector potential \mathbf{F}_V , plus a Laplace field \mathbf{F}_L according to

$$\begin{aligned}\mathbf{F} &:= \mathbf{F}_C + \mathbf{F}_V + \mathbf{F}_L \\ \mathbf{F}_C &= \frac{1}{4\pi} \int \frac{\rho(\mathbf{x}')(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d^3\mathbf{x}' = -\frac{1}{4\pi} \nabla \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3\mathbf{x}' = -\frac{1}{4\pi} \nabla \Phi \\ \mathbf{F}_V &= \frac{1}{4\pi} \int \frac{\mathbf{j}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d^3\mathbf{x}' = \frac{1}{4\pi} \nabla \times \int \frac{\mathbf{j}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3\mathbf{x}' = \frac{1}{4\pi} \nabla \times \mathbf{A} \\ \mathbf{F}_L &= \nabla \varphi\end{aligned}$$

It holds:

$$\begin{aligned}\nabla \times \mathbf{F}_C &= 0 & \nabla \cdot \mathbf{F}_C &= \rho(\mathbf{x}) \\ \nabla \cdot \mathbf{F}_V &= 0 & \nabla \times \mathbf{F}_V &= \mathbf{j}(\mathbf{x}) \\ \nabla \cdot \mathbf{F}_L &= \Delta \varphi = 0\end{aligned}$$

scheme of the proof [26]:

1) It is looked for the solution of

$$\nabla \times \mathbf{F}_C = 0 \quad \nabla \cdot \mathbf{F}_C = \rho(\mathbf{x})$$

This is the potential field

$$\mathbf{F}_C = \frac{1}{4\pi} \int \frac{\rho(\mathbf{x}')(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d^3\mathbf{x}' = -\frac{1}{4\pi} \nabla \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3\mathbf{x}'$$

2) It is looked for the solution of

$$\nabla \cdot \mathbf{F}_V = 0 \quad \nabla \times \mathbf{F}_V = \mathbf{j}(\mathbf{x})$$

This is the vortex field

$$\mathbf{F}_V = \frac{1}{4\pi} \int \frac{\mathbf{j}(\mathbf{x}') \times (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d^3\mathbf{x}' = \frac{1}{4\pi} \nabla \times \int \frac{\mathbf{j}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3\mathbf{x}'$$

3) It is looked for

$$\nabla \cdot \mathbf{F}_L = 0 \quad \nabla \times \mathbf{F}_L = 0$$

using the boundary condition

$$\mathbf{F}_L \cdot \mathbf{n} = \mathbf{F} \cdot \mathbf{n} - \mathbf{F}_C \cdot \mathbf{n} - \mathbf{F}_V \cdot \mathbf{n}$$

The solution is the Laplace field

$$\nabla \cdot \mathbf{F}_L = \Delta \phi = 0$$

4) The general solution is the sum of 1) - 3). This can be checked using the vector relations $\text{div rot } \mathbf{A} = 0$ and $\text{rot grad } \phi = 0$. So one obtains

$$\mathbf{F} = \mathbf{F}_C + \mathbf{F}_V + \mathbf{F}_L$$

q.e.d

The Laplace field is a "generalized constant of integration". It allows to adapt to the boundary conditions. It is needed, if boundary conditions for \mathbf{F} exist which are non-zero in the infinite, see fig.3.

Appendix 3: Derivation of the Lorenz gauge

The continuity equation is

$$\operatorname{div} \mathbf{j}(\mathbf{x}') + \frac{d\rho_E(\mathbf{x}')}{dt} = 0$$

It can be written as

$$\int \frac{\operatorname{div} \mathbf{j}(\mathbf{x}') + \dot{\rho}_E(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}'^3 = 0$$

The divergence term is changed using partial integration. One term can be canceled during partial integration, because $\mathbf{j}(\mathbf{x}') = 0$ holds for $\mathbf{x}' = \infty$. So it is obtained

$$\int -\mathbf{j}(\mathbf{x}') \nabla' \frac{1}{|\mathbf{x} - \mathbf{x}'|} + \frac{\dot{\rho}_E(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}'^3 = 0$$

With $\nabla |\mathbf{x} - \mathbf{x}'|^{-1} = -\nabla' |\mathbf{x} - \mathbf{x}'|^{-1}$ one yields

$$\int \nabla' \frac{\mathbf{j}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} + \frac{\dot{\rho}_E(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}'^3 = 0$$

This is the Lorenz gauge

$$\nabla \cdot \mathbf{A}_H + \frac{1}{c} \frac{\partial \Phi_E}{\partial t} = 0$$

Appendix 4:

Model calculation 1:

In order to estimate the field, which could be generated by a magnetic current, we calculate here the non-real case of a coil of one turn which is driven by a current of magnetic charge. The coil is modelled as a rotating tube which is charged with magnetic surface charges.

We use the formulas of magnetostatics applied for magnetic currents (in SI-units) by exchanging the magnetic variables by the analogous electric variables.

Datas of the setup:

1 magnetic tube charged with magnetic charges

diameter: $d = 2\text{m}$

height: $h = 10\text{cm}$

number of turns: $n = 1$

magnetic field strength at the surface: $\mathbf{B}_0 = 1\text{T} = 1 \text{ Vs/m}^2$

magnetic permeability: $\mu = 10001$

speed of rotation: $f = 10\text{Hz}$.

Using this data the magnetic current I_H can be calculated to

$$I_H = \text{surface charge} * \text{speed of rotation} = (\mu - 1) * \mathbf{B}_0 * d * \pi * h * f$$

Then, the electrical field of a magnetic current, cf. (80)

$$\mathbf{E} = I_H * n / h = (\mu - 1) * \mathbf{B}_0 * d * \pi * f = 2 * \pi * 10^5 \text{ V/m}$$

This means: electrical fields generated by magnetic currents

should be sufficiently strong to be detected easily. It should be possible to reach the breakdown voltage of air (30 kV/cm at 1 bar) if the parameters are chosen accordingly high.

Model calculation 2:

We estimate here the field of a magnetic cylinder ring which turns around its central axis. The upper surface of the ring is the north pole, the lower the south pole.

Data of the setup:

- 1 magnetic ring magnet
- upper rim: north-, lower rim: south pole
- diameter d= 2m
- height h= 12cm
- width b= 5cm
- number of turns n= 1
- magnetic field strength at the pole surfaces: $B_0= 1T$
- magnetic permeability $\mu= 10000$
- speed of rotation: f= 10Hz.

The origin of the coordinate system is the centre of symmetry on the middle of the central axis. The distribution of the electric field lines of the setup can be calculated by using a known example and adapting it for the present setup. For a simple ring current, see fig. 12, Jackson[8]

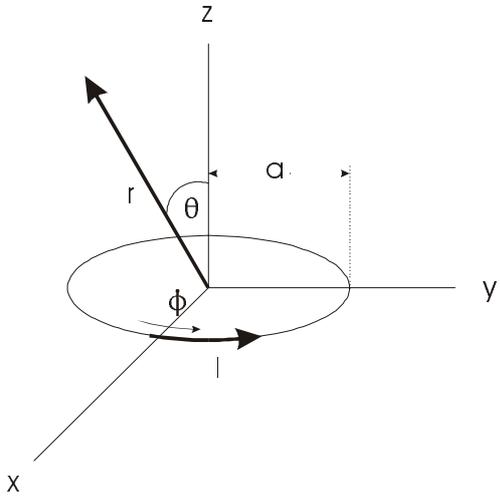
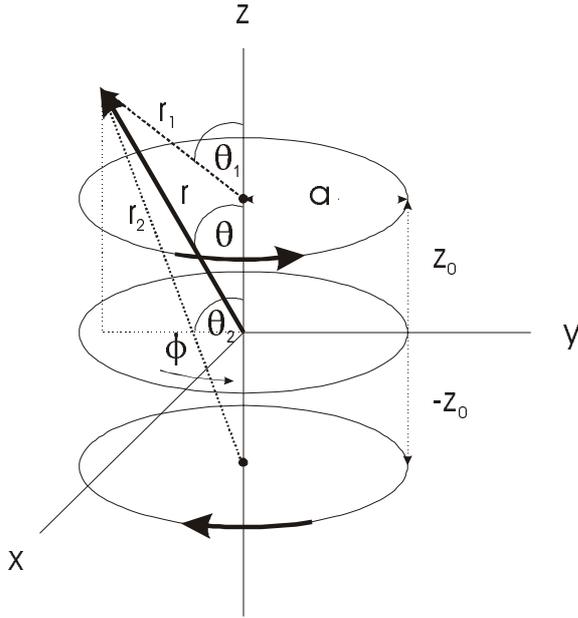


Fig.12: the coordinate system

calculates a vortex vector-field in chapter 5.5, equation 5.37. The formula transferred to magnetic currents yields



Due to geometry it holds:

$$\begin{aligned} z\text{-coordinates: } r_1 \cdot \cos\theta_1 + z_0 &= r \cdot \cos\theta \\ r_2 \cdot \cos\theta_2 - z_0 &= r \cdot \cos\theta \end{aligned}$$

$$\begin{aligned} \text{radius projection} \quad r_1 \cdot \sin\theta_1 &= r \cdot \sin\theta \\ \text{into } x\text{-}y\text{-plane: } r_2 \cdot \sin\theta_2 &= r \cdot \sin\theta \end{aligned}$$

fig.13 the geometric situation of a field point due to a circulating magnetic dipol
the field is composed from two opposite circulating magnetic currents

$$\Gamma_{\Phi}(r,\theta) = \frac{4I_H a}{\sqrt{m}} \left[\frac{(2-m)K(m) - 2E(m)}{m} \right] \quad \text{with} \quad m = \frac{4a \sin(\theta)}{a^2 + r^2 + 2a r \sin(\theta)}$$

Here are $K(m)$ and $E(m)$ elliptic integrals of first and second order, which are calculated numerically by a program.

This formula is applied for two circuits which are shifted by z_0 upward and downwards. In both circuits the magnetic current flows in opposite directions. From the geometry of the setup the appropriate radii and angles of each circuit can be determined, see fig. 13 . The system of equations from fig. 13 are solved

$$\begin{aligned} \theta_1 &= \text{arccot} \left(\frac{r \cdot \cos\theta - z_0}{r \cdot \sin\theta} \right) \rightarrow r_1 = r \cdot \sin\theta / \sin\theta_1 \\ \theta_2 &= \text{arccot} \left(\frac{r \cdot \cos\theta + z_0}{r \cdot \sin\theta} \right) \rightarrow r_2 = r \cdot \sin\theta / \sin\theta_2 \end{aligned}$$

Then it is possible to write down the electric vortex potential Γ which has only one component in Φ -direction which is perpendicular to the plane of the paper, see fig.11

$$\Gamma_{\Phi}(r,\theta)=\Gamma_{\Phi}(r_1,\theta_1)+\Gamma_{\Phi}(r_2,\theta_2)$$

For the purpose of a simple calculation the equally distributed magnetic charge on the surface was approximated by 11 charges at equidistant positions on the surfaces each. The calculated intensity of the Φ -component of the electric vortex field Γ is already shown in fig.10. Then, the \mathbf{E} -field is calculated according to $\mathbf{E} = \text{rot } \Gamma$, fig. 11 a)+b). So the field has been estimated having a maximum 80 kV/cm near the edges at the surface of the moving magnet. This seems to coincide with the observations of Godin&Roschin [72, 73]. They observed a luminescence and therefore a ionisation of the air near the surface of moving magnets. However, the prediction of electrodynamics is contrary to the other reported claims like the self-acceleration and the weight loss of the setup at higher angular velocities. Electrodynamics predicts a braking down due to the Lenz-rule. Any currents generated by the \mathbf{E} -field leads to braking and dissipation as well. Whether electric cycles can be built up with a gain hysteresis or not has to be proved separately.

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