„Plasma Physics“
Consolidated pages

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Plasma

The key differentiator between plasma to neutral gas or neutral fluid is the fact that its electrically positively and negatively charged particles are strongly influenced by electric and magnetic fields, while neutral gas is not. An ideal plasma is a non-dissipative flow of the incompressible charged particles, (CaF).

> (CaP) p. 1: "*Plasma is sometimes called the "fourth state of matter". This kind of "matter" needs to fulfill the following two pre-requisites:*
>
> *- there must be electromagnetic interactions between charged particles*
>
> *- the number of positively and negatively charged particles per considered volume element may be arbitrarily small oder arbitrarily large, but both numbers need to be approximately identical. The number of neutral particles (atomes or molecules) is irrelevant for the definition of a plasma.*
>
> *Therefore, plasma is a conductor. As electric current generates magnetic fields, and as electric charged particles are influenced by electric and magnetic fields a plasma is influenced by external electrical and magnetical fields, creating itself such fields. This means that plasma can interact with itself. In electrodynamics plasma is a anisotropic non-linear dispersive conductor.*
>
> *As interstellar gas and all stars consist of ionized gases 99% of the whole matter of the universe is in plasma state with values of state variables far apart*".

Question 1: when a star "is born", what is the first trigger initiating electromagnetic interactions?

Question 2: when a star "is born", why the number of positively and negatively charged particles per "considered volume element" is approximately identical?

From general relativity and quantum theory it is known that all of them are fakes resp. interim specific mathematical model items. An adequate model needs to take into account the *axiom of (quantum) state* (physical states are described by vectors of a separable Hilbert space ***H***) and the *axiom of observables* (each physical observable ***A*** is represented as a linear Hermitian operator of the state Hilbert space). The corresponding mathematical model and its solutions are governed by the Heisenberg uncertainty inequality. As the observable space needs to support statistical analysis the Hilbert space, this Hilbert space needs to be at least a subspace of ***H***. At the same point in time, if plasma is considered as sufficiently collisional, then it can be well-described by fluid-mechanical equations. There is a hierarchy of such hydrodynamic models, where the magnetic field lines (or magneto-vortex lines) at the limit of infinite conductivity is “frozen-in” to the plasma. The “mother of all hydrodynamic models is the *continuity equation* treating observations with macroscopic character, where fluids and gases are considered as continua. The corresponding infinitesimal volume “element” is a volume, which is small compared to the considered overall (volume) space, and large compared to the distances of the molecules. The displacement of such a volume (a fluid particle) then is a not a displacement of a molecule, but the whole volume element containing multiple molecules, whereby in hydrodynamics this fluid particle is interpreted as a mathematical point.

Regarding the concept "entropy" we note that plasma is characterized by a constant entropy and that the entropy is related to the so-called dissipative function. It describes the amount of heat per volume unit and per time unit generated by the friction forces.

An ideal plasma is a frictionless plasma without Joule heat and without heat conduction, and therefore with constant entropy, i.e., it is a non-dissipative flow.

Basically, there are three mathematical modelling classes based on two fundamental baseline theories, anticipating the several classes of considered physical modelling cases, (CaF) p. 20,

1. Magnetohydrodynamics models
2. Hydrodynamics model (single or multiple fluid media) considering low resp. high density plasma, where
    - "plasma particles" collisions are ignored (e.g. Vlasov equation)
    - "plasma particles" collisions cannot be ignored (e.g. Fokker-Planck equations)

Magnetohydrodynamics (MHD)

MHD is concerned with the motion of electrically conducting fluids in the presence of electric or magnetic fields. In MHD one does not consider velocity distributions. It is about notions like number density, flow velocity and pressure. The MHD equations are derived from continuum theory of non-polar fluids with three kinds of balance laws:

1. conservation of mass
2. balance of linear momentum
3. balance of angular momentum (Ampere law and Faraday law).

The MHD equations consists of 10 equations with 10 parameters accompanied with appropriate boundary conditions from the underlying Maxwell equations (CaF).

In (EyG) it is proven that smooth solutions of non-ideal (viscous and resistive) incompressible magneto-hydrodynamic (plasma fluid) equations satisfy a stochastic (conservation) law of flux. It is shown that the magnetic flux through the fixed Plasma is an ionized gas consisting of approximately equal numbers of positively charged ions and negatively charged electrons.

Hydrodynamics

The kinetic theory of gases w/o and with “molecular chaos“

The “mother" of all hydrodynamic models is the *continuity equation* treating observations with macroscopic character, where fluids and gases are considered as continua. The corresponding infinitesimal volume “element” is a volume, which is small compared to the considered overall (volume) space, and large compared to the distances of the molecules. The displacement of such a volume (a fluid particle) then is a not a displacement of a molecule, but the whole volume element containing multiple molecules, whereby in hydrodynamics this fluid is interpreted as a mathematical point.

The Boltzmann equation (*), is a (non-linear) integro-differential equation which forms the basis for the kinetic theory of gases. This not only covers classical gases, but also electron /neutron /photon transport in solids & plasmas / in nuclear reactors / in super-fluids and radiative transfer in planetary and stellar atmospheres.

The Boltzmann equation for polyatomic gases, mixtures, neutrons, radiative transfer is derived from the Liouville equation for a gas of rigid spheres, without the assumption of “molecular chaos”. The basic properties of the Boltzmann equation are then expounded and the idea of model equations introduced, e.g. the Vlasov equation (w/o plasma particle collisions) and the Fokker-Planck (Landau) equations (with plasma particle collisions)

The treatment of corresponding boundary conditions leads to the discussion of the phenomena of gas-surface interactions and the related role played by proof of the Boltzmann H-theorem.

(*) (LiP1): "*The Boltzmann and Landau equations provide a mathematical model for the statistical evolution of a large number of particles interacting through "collisions". The unknown function f corresponds at each time t to the density of particles at the point x with velocity v. If the collision operator were zero, the equations would mean that the particles do not interact and f would be constant along particle path. If collisions occur, in which case the rate of changes of f has to be specified such a description was introduced by Maxwell and Boltzmann and involves an integral operator. This model is derived under the assumption of stochastic independence of pairs of particles at (x,t) with different velocities (molecular collision assumption)*", (LiP), (LiP1).

The Vlasov equation type case w/o "plasma particles" collisions

In case there are no plasma particles interactions the Liouville distribution function becomes for each considered particle a specific distribution function (separation approach), which is characterized by the Vlasov equation. The following is basically taken from (ShF) p. 390 ff:

Much of the complication of plasma physics stems from the dominance of internal electromagnetic fields in affecting the motions of charged particles. These fields have a dynamics self-consistent with the dynamics of the particles. An example of such a fully dynamic phenomeon is the high-frequency oscillations that can take place if charge separation occurs with periodic structure, in particular the problem of longitudinal plasma oscillations when electrons possess non-zero random motions. For cold plasma in which only the electrons are mobile, spacial displacement of the electrons from the ions introduces restoring electric forces that set up oscillations at a certain frequency. In case of hot plasma the question arises, how does finite temperature for the electrons modify this specific frequency.

The corresponding perturbational Vlasov equation treatment continues to adapt the model of a two-component plasma in which the ions provide only a positively charged background of uniform density, while the electrons have a phase space description given by a distribution function f(x,v,t). In the absense of magnetic fields or collisions, f(x,v,t) satisfies the Vlasov kinetic equation, where the self-consistent electric field arises from the charge distribution of electrons and ions, and where the equilibrium state of the electric field is assumed to be unifirm and electrically neutral. With the ions immobile, and only considering perturbances that generate no magnetic fields the purely electric oscillations are purely longitudinal.

The related Landau damping decrement depends only on the derivative of the electron distribution function evaluated at the exactly resonant value, (ShF) p. 401. The resonant wave-particle interaction is claimed to underline the physical mechanism behind the Landau damping. Electrons with random velocities substantially different from the phase speed of the wave, drfit in and out oft he crests and troughs of the wave. Sometime they are accelerated by the collective electric field, sometime decelerated; but integrated over time, no net interaction results (in the linear approximation!) because the time spent in crests and troughs averages out for a sinusoidal disturbance.

In its purest form, Landau damping represents a phase-space behavior peculiar to collisionsless systems. Analogs to Landau damping exist, for example, in the interactions of stars in a galaxy at the Lindblad resonances of a spiral density wave. Such resonance in an inhomogeneous medium can produce wave absorptions (in space rather than in time), which does not usually happen in fluid systems in the absense of dissipative forces.

Longitudinal plasma oscillations and Landau damping

The Landau equation type case with "plasma particles" collisons

In case of high density plasma corresponding particle collisions cannot be ignored. The corresponding equations also need to be derived from the underling Liouville equation.

In case there are different types of plasma particles (typical case: plasma heating, which is about a two media plasma fluid of negatively charged electrons and positively charged ions without "neutralization" over time) each of the considered particle class needs to be governed by different Vlasov equations (CaF) p. 66.

There are two classes of plasma collision processes:

(1) binary collisions with smaller densities and interacting forces and short range (like intermolecular forces) This is the situation in a diluted, neutral gas as modelled by the Boltzmann equation, which has been adapted resp. improved by Landau in case of plasma particles

(2) because with a long range Coulomb forces play an essential role and it will occur multiple collisions. The long range of Coulomb forces causes collisions with small scattering angle.

There are three methods available to derive appropriate equations adressing (1) and (2), (CaP) p. 67:

a) the BGKBY method resulting into the Boltzmann-Landau equation

b) the method of Klimontovitsch and Deupree resulting into the Boltzmann-Landau equation and the Fokker-Planck equation

c) the Balescu method (solving the Liouville equation applying the Green function concept) resulting into the Fokker-Planck equation and the Lenard-Balescu equations.

The main objective of all methods is to derive the Boltzmann collision equation and the corresponding Boltzmann collision integral from the Liouville equation. The common idea of all methods is to replace the "one-particle distribution function" first by a "two-particle distribution function" in the framework of the Liouville equation. This corresponds to a twofold (pair) correlation function in the framework of the Boltzmann equation. This conceptual approach results into a hierarchical ordered PDE system, which is equivalent to the Liouville equation, but only solvable for a finite number of those PDEs.

The reduction to only binary collision in small density gases and neglecting all other than 2-particle short distance interactions results into the Vlasov equation.

The case of neglected 3- (and higher) particle interactions allows the definition of a third distribution function (stochastic independent from the already existing 2-particle distribution functions) resulting into the Vlasov-Liouville equation.

For plasma gases Landau has modified the collision integral of the Boltzmann equation in case of collisions with small scattering angles. His collision integral can be derived from the Fokker-Landau equations, as well as from the Lenard-Balescu equations. This shows that in the context of the several possible approximation equations above all those equations are in a certain sense equivalent. The Fokker-Planck equation is mainly used one in plasma physics.

We emphasis that from a physical modelling perspective the 1-plasma particle Vlasov equation is inappropriate for collision processes of class (2) above, i.e., it is an inappropriate physical model for the non-linear Landau phenomenon in the context of long range Coulomb forces.

Plasma heating

and the proposed $H_{1/2}$ Hilbert space based integrated gravity and quantum field theory

The conclusion from the below is that the proposed Hilbert scale model replaces the model for "gas-surface interactions", where the Boltzmann equation in the corresponding variational framework form the adequate kinetic and potential theory of gases, and where the corresponding linearized Boltzmann collision operator forms the quanta kinematical operator. In other words, the proposed variational framework avoids the expounded model equations above, while providing a well defined variational PDE system accompanied with corresponding finite energy norm estimates.

A characteristics of the plasma "matter" state is the fact that the number of positively charged and negatively charged "particles" are approximately equal over time. In the related standard (cold plasma) PDE model (Vlasov equation) this is about a two media plasma fluid of negatively charged electrons and positively charged ions without "neutralization" over time, where each of the considered particle class needs to be governed by different Vlasov equations (CaF) p. 66.

The "plasma particles collisions" related mathematical model is about a Landau type PDE. This model is basically about friction forces ocurring when the molecules of the fluid collide with the considered particle. In case of the plasma heating phenomenon this requires a two media plasma fluid PDE system with interactions of same type and different type particles. The latter interaction type then is the model for the hot plasma specific plasma heating phenomenon.

The claims are

1. that for several reasons all three classical plasma theory types, the fluid PDE models, the statistical PDE theory and MHD, provide inappropriate models for the (cold and hot) plasma heating and Landau damping phenomena, and

2. that the proposed H_1 kinematical Hilbert space decompostion into two positively and negatively charged kinematical energy sub-spaces (with same cardinality governed by an indefinite inner product) in combination with a multiple (at least a binary, but proposed tertiary ("positron", "electron", "neutron")) particle type fluid/statistical/MHD variational theory overcomes the several handicaps of the current models. The "+/- fluid" media are governed by the complex Lorentz group with its underlying two connected components L^+ and L^-, (StR).

Claim 2 is also related to the theory of equilibrium critical phenomena:

"*The main aim of the theory of critical phenomena is about to be the (implicitly or explicitly) calculation of the observable properties of a system from first principles using the full microscopic quantum-mechanical description of the constituent electrons, protons and neutrons. Such a calculation, however, even if feasible for a many-particle system which undergoes a phase transition need not and, in all probability, would not increase one's understanding of the observed behaviour of the system. Rather, the aim of the theory of a complex phenomenon should be to elucidate which general features of the Hamiltonian of the system lead to the most characteristic and typical observed properties. Initially one should aim at a broad qualitative understanding, successively refining one's quantitative grasp of the problem when it becomes clear that the main features have been found*", (FiM) p. 619.

In this context, (FiM) p. 639:

"*binary fluids undergo,*

i) phase separation, when AA and BB contacts are favoured energetically over AB contacts

ii) ordering, when AB contacts are most favourable.

In case i) the mole fraction, of, say, the A component is analogous to the density in a one-component fluid system. Below the critical point T(c) the mixture will separate into a A-rich and a B-rich phase.

Case ii) of an ordering crystalline binary alloy such as beta-brass is most directly analogous to an antiferromagnet since the phase transition is signalled by the appearance below T(c) of a coherent super lattice scattering line".

The most striking handicap is the fact that obviously the two "plasma particles" of the two +/- charged media are on an equal footing concerning their physical properties and their "parallel existences" over time induce the heating phenomenon. At the same time their existences are "a priori" given before the related fields can "act". The "collisions-less" (Vlasov) model is only about Coloumb forces. The "collisions" (Landau type) models are about "heat energy generation" by collisions without "neutralization".

The proposed theory can be supported by the theory of "vortex dynamics", which is (in the classical PDE case) about coherent structures in turbulence. It is a natural paradigm for the field of chaotic motion and modern dynamical system theory, e.g., accompanied with singular distributions of vorticity, vortex momentum, related creation processes, and the dynamics of line vortices, (SaP).

The instability criteria of MHD are based on the two methods, the *normal modes method* and the (kinematical) *energy principle method*. Regarding the first method we note that there are also existing non-normal modes, (FiM). Regarding the latter method we recall the issue of generated virtual plasma waves "out of the blue" in the context of "explaining" the plasma damping phenomeon. Regarding the micro (quanta) and macro (galaxy) "*instability*" phenomena scope of plasma, anticipating that the macro plasma world makes 99% of the whole matter universe in the context of a common related field based theory we note the paper (RoK). It is about a plasma slab of infinite extent in the z-direction, with finite resistivity and finite shear, accompanied with twisted slicing modes, i.e. there are modes which are neither localized near a particular horizontal surface, nor dependent on a boundary layer.

The theories of thermodynamics and critical phenomena and the proposed Hilbert scale based gravity and quantum field model

„*The standard theory of thermodynamics tells us that knowledge of a thermodynamic potential as a function of its natural variables completely specifies the thermodynamics of a system. Several potentials have this property of encoding complete thermodynamic systems, e.g., the Helmholtz free energy F and the Gibbs free energy G. F's natural variables are the temperature and whatever macroscopic parameters determine the system's energy levels. … The Gibbs free energy ist he Legendre transform of the Helmholtz free energy with respect to its second variable. Thus we have*

$$G(T,p) = F + pV \text{ and } G(T,M) = F + BM$$

for fluid and magnetic systems, respectively", (BiJ) p. 21.

„*The Ginzburg-Landau model is a kind of „meta model" in the theory of critical phenomena*", (BiJ) p. 179.

„*The great beauty of the Ginzburg-Landau model is that it allows one to solve many difficult problems in superconductivity (e.g. surfaces of superconductors, the GL theory of inhomogeneous systems, or the GL theory in a magnetic field)*", (AnJ) p. 83.

„*The Landau theory is an approximation of the partition function in the Ginzburg-Landau model*", (BiJ) p. 188.

Regarding the proposed $H_{1/2}$ energy model we note that only the Hamiltonian formalism is valid, as the Legendre transform is not defined due to the reduced regularity assumptions; therefore

1. the Lagrange formalism
2. the Ehrenfest theorem
3. the GL (meta-) model for critcial („turbulence") phenomena

are only valid in the compactly embedded kinematical H_1 framework.

Legendre transforms

With respect to the application of probabilistic considerations to the study of turbulence we refer to (SwH) 2.3:

"*there is one feature of the initial distribution which is generally agreed to be universal: Any specific set of initial states of zero state space volume (i.e., zero Lebesgue measure) will also have probability zero with respect to the initial distribution. In other words, a phenomenon which occurs only for a set of initial states of zero volume will never be seen. .... For what sorts of differential equations are all but a neglegible set of solution curves statistical regular? One answer provides by the Birkhoff pointwise ergodic theorem, is that Hamiltonian equations of motion have this property provided that the microcanonical measure is finite on each energy surface*."

In this context we note that the compactly embedded kinematical sub-Hilbert space H_1 of $H_{1/2}$ is a "zero set" with respect to the $H_{1/2}$ inner product.

Regarding the notion „*spontaneous symmetry break down*" in the context of a Hamiltonian, we recall from (BiJ) p. 48:

„*When an exact symmetry of the laws governing a system is not manifest in the state of the system the symmetry is said to be spontaneously broken. Since the symmetry of the laws is not actually broken it would perhaps be better described as „hidden", but the term „spontaneously broken symmetry" has stuck*."

The Landau damping phenomenon and
the proposed Hilbert scale based gravity and quantum field model

(CaF) p. 390 ff: "*The turbulence of plasma differs from the hydrodynamic turbulence by the action of the magnetic field. A more relevant difference is due to the hydrodynamic interaction between the plasma particles, the interaction with the magnetic fields, and the interaction between the electromagnetic waves. ... All of them are the root cause of electromagnetic plasma turbulence. ...*

The case of interactions between quasi-stationary electromagnetic waves is called weak turbulence. ...

The case of non-linear Landau damping (strong plasma turbulence) leads to the generation of virtual waves, which transfer their energy to the affected particles asymptotically with 1/t; the plasma is heated (turbulence heating) faster than this may happen by purely particles collisions", (TsV).

In the context of the statistical theory the Landau damping phenomenon is about "*wave damping w/o energy dissipation by collisions in plasma*", because electrons are faster or slower than the wave and a Maxwellian distribution has a higher number of slower than faster electrons as the wave. As a consequence, there are more particles taking energy from the wave than vice versa, while the wave is damped over time.

Regarding the above mentioned two phenoemona area, the micro-plasma-particles world and the macro-galaxy-stars-world, we note that in the latter case no SRT validated relevant physical parameters are taken into account, while in the latter case additional purely mathematical assumptions are required (micro turbulance relevant Penrose stability condition) to govern the considered electric (Coloumb) force.

Mathematical speaking the Landau damping is a specific behavior of linear waves in plasma governed by the non-linear term of the considered PDE system. In this context we note that in (RoK) it is shown that there exist modes influencing plasma, which are not of the form e^{iwt}.

In case of the statistical theory the core defining element of a plasma (i.e. a system of nearly equal numbers of positive and negative charged plasma particles staying over time with constant entropy) requires a two-component plasma fluid media. The Vlasov equation is about a positively charged ions background of uniform density, and negatively charged electrons having a phase space description given by a distribution function $f(x, v, t)$.

The electromagnetic turbulence of plasma is caused by

1. the action of the magnetic field
2. the hydrodynamic interaction between the plasma particles
3. the interaction of the plasma particles with the magnetic fields
4. the (quasi-stationary resp. non-linear) interactions between the electromagnetic waves (weak resp. strong plasma (heating) turbulence).

The proposed modelling framework governs 1.-3., while the quasi-linear or non-linear term of the considered PDE governs 4.

In the proposed $H_{1/2}$ energy Hilbert space framework the strong plasma heating energy is provided from the total of potential differences (~ "pressure", ~ potential operator) of the colliding plasma quanta particles resp. from the related energy norms.

The Landau damping is a characteristic of collisionsless plasma. In the context of the proposed $H_{1/2}$ Hilbert space framework it is a characteristic of this framework, i.e. it is related to the complementary sub-space of the compactly embedded (kinematical energy) Hilbert sub-space H_1 of $H_{1/2}$.

(ChF) p. 245: „*Landau damping may also have applications in other fields. For instance, in the kinetic treatment of galaxy formation, stars can be considered as atoms of a plasma interacting via gravitational rather than electromagnetic forces. Instabilities of the gas of stars cause spiral arms to form, but this process is limited by Landau damping*".

Wavelets: a mathematical microscope tool

Wavelets, a mathematical microscope tool

This following is about the mathematical "théoreme vivant" (MoC), which is about a "proof" of the physical observed (Plasma physics) Landau damping phenomenon based on the classical (PDE) Vlasov equation.

To the author´s humble opinion, ...

the situation: with (MoC) there exists now a complex and sophisticated mathematical proof of the Landau phenomenon based on the classical Vlasov equation, which is inappropriate from a physical modelling perspective with respect to the plasma heating phenomenon (as there is only one class of distribution functions governing the electron fluid, while the approximately identical number of positively charfed particles is considered as a background field). At the same time the proof requires strong mathematical assumptions, like analytical regularity assumptions of the PDE solution function to enable the defintion of so-called "hybrid" and "gliding" analytical norms, while at the same time the Landau-Penrose stability criterion to govern the singularity of the Coulomb potential governs micro turbulences of the plasma particles.

the comment: regarding the "two-class distribution functions" modelling requirement we note that the counterpart of the Heisenberg uncertainty inequality in probability theory is the co-variance concept of two independent random variables. The special physical effect of the Landau damping phenomenon context is about plasma heating caused by strong turbulence behavior of nearly the same number of negatively and positively charged ions and electrons w/o "neutralization" over time. Any classical thermostatistical considerations subsuming those two phenomenon enabling media into a single "fluid" governed by a single class of probability distributions at least looks inappropriate, especially in a quantum field framework governed by the Heisenberg inequality.

the conclusion: the existence of a non-linear Landau damping proof based on the statistical "turbulence" classical Vlasov PDE provides evidence that this classical PDE is an inappropriate physical model for the observed physical phenomena.

Below we sketch appropriate $H_{1/2}$ energy norm estimates in the context of a variational representation of the Vlasov equation, where the analytical norms in (MoC) are replaced by an "exponential decay" Hilbert space norm, which is even weaker than any polynomial distributional Hilbert space norm.

Just to anticipate the kinetic treatment of galaxy formation, where stars can be considered as atoms of a plasma interacting via gravitational rather than electromagnetic forces with the "plasma matter" of the universe the considered "one-particle-type" variational representation of the Vlasov equation still remains to be an inappropriate physical model. Clearly some type of Maxwell/Lie-type boundary value conditions with the plasma heating particles are missing. Additionally a global/universal time variable is conflicting already with the SRT.

An integrated kinematical discrete & continuous entropy concept

1. The three laws of thermodynamics

- The first law distinguishes two kinds of transfers of energy, (kinematical) heat and thermodynamic „work“ (called „internalenergy“), governed by the principle of „conservation of law“.

- The second „law“ only describes an observed phenomenon. It is about the concept of „entropy“ predicting the direction of spontaneous irreversible processes, despite obeying the principle of „conservation of law“; the corresponding (continuous) Boltzmann entropy cannot derived from the model parameters.

- The third (Nernst distribution) law governs the distribution of a solute between two non miscible solvents.

2. Schrödinger’s statistical (classical & quanta) thermodynamics

A thermodynamic state of a system is not a sharply defined state of the system, because it corresponds to a large number of dynamical states. This consideration led to the Boltzmann entropy relation S = k*log(p), where p is the (infinite) number ofd ynamical states that correspond to the given thermodynamic state. The value of p, and therefore the value of the entropy also, depends on the arbitrarily chosen size of the cells by which the phase space is divided of which having the same hyper-volume s. If the volume of the cells is made vanishing small, both p and S become infinite. It can be shown, however, that if one changes s,p is altered by a factor. But from the Boltzmann relation it follows that an undetermined factor in p gives rise to an undetermined additive constant in S. Therefore, the classical statistical mechanics cannot lead to a determination of the entropy constant. This arbitrariness associated with p can be removed by making use of the principles of quantum theory (providing discrete quantum state without making use of the arbitrary division of the phase space into cells). According to the Boltzmann relation, the value of p which corresponds to S=0 is p=1.

3. The proposed quantum field model enabling a truly second *law* of thermodynamics

The third (Nernst distribution) law stays untouched governed by the kinematical Hilbert space H_1.

The first law governs the energy transfer between a kinematical (heat) energy and an „internal energy“. The two energy concepts ("heat" and "internal energy") are now reflected by the decomposition of the Hilbert space $H_{1/2}$ into H_1 and its complementary sub-space in $H_{1/2}$.

The kinematical energy Hilbert space H_1 is now governed by the (discrete) Shannon entropy.

The second law is now about the two probabilities for such an energy transfer.
This is determined by the ratio of the cardinalities of both spaces, where the H_1 Hilbert space is compactly embedded into the overall Hilbert space $H_{1/2}$, i.e., the sub-space H_1 in $H_{1/2}$ is a zero set only.

Considering hermitian operators with either domain H_1 or its complementary space this results into either discrete spectra or purely continuous spectra. In other words, the cardinalities ratio determines the probability of energy transfers between both spaces. This probability is „zero“ for a transfer from the internal energy space into the heat space; it can be interpreted as the probability to generate a matter particle out of the „internal (ether) energy“ space. The probability into the other direction is measured by an exponential decay norm (in line with the Boltzmann probability distribution), which governs all polynomial decay norms of the considered Hilbert scales defined by eigenpair solutions of hermitian operators.

Two types of thermo-kinematical entropies
discrete L_2 -based & complementary („continuous") $L_2^{\perp}$ -based entropy

The discrete Shannon entropy in information theory is analogous to the entropy in thermostatistics. The analogy results when the values of the random variable designate energies of microstates. For a continuous random variable, differential entropy is analogous to the „continuous" Boltzmann entropy. However, the continuous (Boltzmann) entropy cannot be derived from the Shannon (discrete) entropy in the limit of n, which is the number of symbols in distribution $P(X)$ of a discrete random variable X, (MaC1). In other words, the Boltzmann entropy cannot be derived from the underlying model parameters. Therefore, the second theorem of thermodynamics states only an observation, which cannot be derived from the underlying model parameters.

The central notion in Schrödinger's thermostatistics which makes the difference between the classical and the quanta world is the „vapour-pressure formula of an ideal gas" for computing the so-called entropy constant or chemical constant. The crucial „auxiliary" term to build the vapour-pressure formula is the „thermodynamical *potential*", from which then the entropy itself is derived. The essential physical law it the third theorem of thermodynamics (Nernst), which states, that the ground state energy level is always a constant in any considered system, i.e. there is a part of the entropy, which does not vanish at $T = 0$, and which is independent from all system parameters. The only mathematical relevant assumption is that the considered particles are energy quanta without individuality (ScE) pp. 16, 43.

In the physics of plasma the entropy is constant, information is conserved and the initial state data is always known, caused by the so-called Landau damping.

Regarding the notion„*vapour-pressure*" we note that „pressure" is nothing else than a potential difference. Therefore, the proposed coarse-grained kinematical H_1 energy Hilbertspace model and its complementary closed („*potential*") the subspace in $H_{1/2}$ accompanied with model intrinsic concepts of a potential function and a potential barrior enablean alternative model for the „vapour-pressure", resulting in a corresponding entropy concept between both spaces. We note that H_1 is compactly embedded into $H_{1/2}$, i.e. from a probability theory perspective it is a zero set withd iscrete spectrum of the corresponding „energy operator".

From a mathematical perspective we note that the distributional Hilbert scales (accompanied with polynomial degree norms) are governed by a weaker norm with exponential degree. This enables norm weighted estimates with two parts, a statistical $H_0 = L_2$ based part and a corresponding „exponential degree" part. For a corresponding apprximation theory we refer to (NiJ), (NiJ1).

Appendix

The Boltzmann equation

The Boltzmann equation isa (non-linear) integro-differential equation which forms the basis for the kinetic theory of gases. This not only covers classical gases, but also electron /neutron /photon transport in solids & plasmas / in nuclear reactors / in super-fluids and radiative transfer in planetary and stellar atmospheres. The Boltzmann equation is derived from the Liouville equation for a gas of rigid spheres, without the assumption of "molecular chaos"; the basic properties of the Boltzmann equation are then expounded and the idea of model equations introduced. Related equations are, e.g. the Boltzmann equations for polyatomic gases, mixtures, neutrons, radiative transfer as well as the Fokker-Planck (or Landau) and Vlasov equations. The treatment of corresponding boundary conditions leads to the discussion of the phenomena of gas-surface interactions and the related role played by proof of the Boltzmann H-theorem.

The Boltzmann equation is a nonlinear integro-differential equation with a linear first-order operator. The nonlinearity comes from the quadratic integral (collision) operator that is decomposed into two parts (usually called the gain and the loss terms). In (LiP) it is proven that the gain term enjoys striking compactness properties. The Boltzmann equation and the Fokker-Planck (Landau) equation are concerned with the Kullback information, which is about a differential entropy. It plays a key role in the mathematical expression of the entropy principle. The existence of global solutions of the Boltzmann and Landau equations depends heavily on the structure of the collision operators (LiP1). The corresponding variational representation of $B = A + K$ with a H_α-coercive operator A and a compact disturbance K fulfills a Garding type coerciveness condition (KaY).

In (ViI) the existence and uniqueness of nonnegative eigenfunction is analyzed.

In (MoB) the eigenvalue spectrum of the linear neutron transport (Boltzmann) operator has been studied.The spectrum turns out to be quite different from that obtained according to the classical theory. The two theories about related physical aspects have one aspect in common: namely that there exists a region of the spectral plane which filled up by the spectrum.

The Fokker-Planck equation

The Fokker-Planck equation is also denoted as Landau equation. It is a model describing time evolution of the distribution function of Plasma consisting of charged particles with long-range interaction) is about the Boltzmann equation with a corresponding Boltzmann collision operator where almost all collisions are grazing. The mathematical tool set is about Fourier multiplier representations with Oseen kernels (LiP), Laplace and Fourier analysis techniques (e.g. LeN) and scattering problem analysis techniques based on Garding type (energy norm) inequalities (like the Korn inequality). Its solutions enjoy a rather striking compactness property, which is main result of P. Lions ((LiP) (LiP1)).

The following is a collection from (RiH):

The Fokker-Planck equation was first used by Fokker and Planck to describe the Brownian motion of particles. If a small particle of mass is immersed in a fluid a friction force will act on the particle. The physics behind the friction is that the molecules of the fluid collide with the particle. The momentum of the particle is transferred to the molecules of the fluid and the velocity of the momentum of the particle therefore decreases to zero. If the mass of the particle is large so that its velocity due to thermal fluctuations is neglible, then the related PDE is a deterministic equation governed by the mean energy of the particle. If the mass of a small particle is still large compared to the mass of the molecules the equation needs to be modified so that it leads to a correspondingly adapted thermal energy equation based on the corresponding thermal velocity observable by adding a fluctuation force. In other words, the total forces of the molecules acting on the small particle is decomposed into a continuous damping force and a fluctuating force. The latter one is a stochastic or random force, the properties of which are given only in the average (the fluctuation force per unit mass is called the Langevin force). The mathematical concept is about a stochastic differential equation.

Asking for the probability to find the velocity in the interval (v,v+dv), or in other words, asking for the number of systems of the ensemble whose velocities are in the interval (v,v+dv) divided by the total number of systems in the ensemble results into Fokker-Planck type equations. Its solution is a probability density function W(v), also called probability distribution. The probability density times the length of the interval dv is then the probability of finding the particle in the interval (v,v+dv). This distribution function depends on time t and the initial distribution. Mathematically, the Fokker-Planck equation is a linear second-order PDE of parabolic type.

Roughly speaking, it is a diffusion equation with an additional first-order derivative with respect to the x variable. In mathematical literature the Fokker-Planck equation is also called a forward Kolmogorov equation.

A complete solution of a macroscopic system would consist in solving all microscopic equations of the system. Because one cannot generally do this one uses instead a stochastics description, i.e., one describes the system by macroscopic variables which fluctuate in a stochastic way. The Fokker-Planck equation is just an equation of motion for the distribution function of fluctuating macroscopic variables. For a deterministic treatment one neglects the fluctuations of the macroscopic variables. Therefore, for the Fokker-Planck equations one neglects the diffusion term. Mathematically speaking, one neglects the defining term for the kinematical energy norm (one dimensional case).

The N stochastic variables case requires the concepts of a probability current and joint probability functions (Markov processes). The latter one can be expressed by a transition probability function for small times and a distribution function at the time t. The diffusion coefficients become a (positive definite) diffusion matrix. Roughly speaking, the drift and the diffusion coefficients become the Nabla and the Laplace operators, allowing a reformulation of the Fokker-Planck equation in covariant form.

The Leray-Hopf operator and the linearized Landau collision operator

In a weak $H_{-1/2}$ Hilbert space framework in the context of the Landau damping phenomenon the linerarized Landau collision operator can be interpreted as a compactly disturbed Leray-Hopf operator.

The Leray-Hopf operator plays a key role inexistence and uniqueness proofs of weak solutions of the Navier-Stokes equations, obtaining weak and strong energy inequalities.

Both operators, the Leray-Hopf (or Helmholtz-Weyl) operator and the linearized Landau collision operator are not classical Pseudo-Differential Operators, but Fourier multipliers with same continuity properties as those of the Riesz operators (LiP1).

For the related Oseen operators Fourier multiplier we refer to (LeN).

The related hypersingular integral equation theory, including the Prandtl operator, is provided in (LiI).

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