Quantum Gravity

Dr. K. Braun

homepage text 2019
www.fuchs-braun.com

1. Einstein's geometrodynamics

(CiI) 2.8: *Einstein's "general relativity" or ""geometric geometry of gravitation" or "geometrodynamics", has two central ideas: (1) Space-time geometry "tells" mass-energy how to move, (2) mass-energy "tells" space-time geometry how to curve. The* ***concept (1)*** *is automatically obtained by the Einstein field equations,* (CiI) (2.3.14), *basically as the* ***covariant divergence of the Einstein tensor is zero****. At the same point in time there are multiple tests of the geometrical structure and of the geodesic equation of motion, e.g. gravitational deflection and delay of electromagnetic waves, de Sitter and Lense-Thirring effect, perihelion advance of Mercury, Lunar Laser Ranging with its relativistic parameters: time dilation or gravitational redshift, periastron advance, time delay in propagation of pulse, and rate of change of orbital period,* (CiI) 3.4.

(CiI) 3.5: "*Hilbert used a variational principle and Einstein the requirement that the conservation laws for momentum and energy for both, gravitational field and mass-energy, be satisfied as a direct consequence of the field equations. ... Einstein geometrodynamics, ..., has the important and beautiful property the the equations of motion are a direct mathematical consequence of the Bianchi identities.*"

With respect to the overall conceptual idea of this homepage a Hilbert space based geometrodynamics is built on "space-time states", which are represented by elements of H(-1/2), while their corresponding "space-time energy" elements are represented by the corresponding "dual" (wavelets) elements in H(1/2). We emphasis that if ((u,v)) denotes the inner product of H(-1/2) the following relationships hold true: ((div(u),v)) ~ (u,v) ~ ((u,grad(v))).

2. Loop Quantum Theory (LQT)

The LQT is a modern version of the **Wheeler/deWitt theory** ((CiI), (WhJ)). The H(-1/2) Hilbert space of this page is proposed as a model for the "LQT spin network state/quanta of gravity" Hilbert space, enabling an (**unified Dirac+Yang-Mills+Higgs+Einstein**) **hamiltonian operator** ((RoC) 7.2.3), including the **4th matter (plasma) state**. It is known from general relativity and quantum theory that all of them are fakes resp. interim specific mathematical model items.

The ***loop quantum theory (LQT)*** (**C. Rovelli**) is the choice of a different algebra of basic field functions: a noncanonical algebra based on the holonomics of the gravitational connections ((RoC) 1.2.1). The holonomy (or the "Wilson loop") is the matrix of the parallel transport along a closed curve and **spacetime** itself is **formed** by **loop-like states**. Therefore the position of a loop state is relevant only with respect to other loops, and not with respect to the background. The **state space** of the theory is a **separable Hilbert space** spanned by loop states, admitting an orthogonal basis of spin network states, which are formed by **finite linear combinations of loop states**, and are defined precisely as the spin network states of a lattice **Yang-Mills theory**."

We briefly sketch the central **conceptual differentiators of LQT** to other GUT theory attempts ((RoC) p. 10, p. 14, p. 140) with its relationship to the topic of this page (related conceptual elements of the Wheeler theory are sketched below). "*The theory of LQT combines general relativity with quantum mechanics in a rather conservative way, because it does not employ any other hypothesis apart from those of the two theories themselves, suitable rewritten to render them compatible*" ((RoC2) p. 144).

(RoC1): "*the **key differentiator** to **Einstein's field theory** is the **absence of** the familar **"space-time" stage** (**background independence**), which is technically realized by the gauge invariance of the action under (active) diffeomorphisms (or **diffeomorphism invariance**). It is the combination of two properties of the action: its invariance under arbitrary changes of coordinates and the fact that there is **no nondynamical "background" field**. ... The notions of space and time is given up. The **space** continuum "on which" things are located and the **time** "along which" evolution happens are **semiclassical approximation** notions; the LQT makes only use of the general tools of quantum theory: a Hilbert space of states, operators related to the measurements of physical quantities, and transition amplitudes that determine the probability outcome of measurements of these quantities. ... In the macroscopic world, the physical variable t measured by a clock has peculiar properties. The fact, that time is not a special variable at the fundamental level needs to be reconciled, leading to the thermal time hypothesis* ((Roc) 3.4, "the thermal hypothesis", (RoC1)): "*In Nature, there is no preferred physical time variable t. There are no equilibrium state "r" preferred a priori. Rather, all variables are equivalent: we can find the system in an arbitrary state "r"; if the system is in a state "r", then a preferred variable is singled out by the state of the system. This variable is what we call time*."

The three conceptual elements of the quantum mechanics (remaining in LQT) are (RoC1): "*(1) **granularity** (2) **indeterminism** (3) **fluctuation** ((RoC2), p. 116):*

*(1) **Granularity**: the information in the state of a system is finite, and limited by Planck's constant*
*(2) **Indeterminacy**: the future is not determined unequivocally by the past. Even the more rigid regularities we see are, ultimately, statistical*
*(3) **Relationality**: the events of nature are always interactions. All events of a system occur in relation to another system (i.e. it is about relations of physical variables resp. phenomena).*

With respect to the physical phenomenon "time" this means that for all physical phenoma there is (1) granularity: a smallest "time" unit, the Planck time (2) indeterminism: quantum super position of time (3) fluctuation (Heisenberg), when trying to determine the position of an electron today and tomorrow".

*In (RoC) 5.5.2, the relationship of the naturally defined "physical" scalar product of a Hilbert space **H** (defined based on the solutions of the Wheeler-DeWitt equations) and the related "kinematical" inner product of the (**kinematical state**) Hilbert space **K** is considered ((Roc) 5.1.2). This relation depends on the hamiltonian H. The space **H** is the **eigenspace** of the **hamiltonian** H corresponding to the **eigenvalue zero** ("**zero point energy**").*

*By definition, an **indefinite inner product space** is a real or complex vector space together with a symmetric (in a complex case: hermitian) **bilinear form** prescribed on it so that the corresponding quadratic form assumes both positive and negative values. The most important special case arises when a Hilbert space is considered as an **orthogonal direct sum of two subspaces**, one equipped with the original inner product, and the other with -1 times the original inner product* (BoJ).

In quantum mechanics "**time**" and "**energy**" are conjugated variables linked by the concept of "**action**" ((HeW) II, 2c). Therefore, the conceptual elements above find its counterpart with respect to the proposed quantum state Hilbert space H(-1/2) by the facts, that the Hilbert sub-space **L(2)=H(0)** is **compactly embedded into H(-1/2),** where physical quantum mechanics phenomena are "measured" by corresponding **hermitian (projection) operators onto L(2)**. This property is proposed as the mathematical model for quantum mechanics "granularity" states in H(0). At the same point in time this embedded "granular" Hilbert space (with respect to the norm of H(-1/2)) is the standard **L(2)** framework of **probability theory**, statistical analysis and quantum mechanics. Some related aspects related to **Schrödinger's view** on **statistical thermodynamics** are given in (BrK7) p. 25. In other words, the physical (H(0), state resp. H(1), energy Hilbert) spaces are made of quanta (H(-1/2), state resp. H(1/2) energy Hilbert) spaces.

An analogue situation regarding the compactly embedded Hilbert space H(a) into H(b) for a>b, is given by the rational numbers (~H(0), H(1)) as subset of the real or hyperreal numbers (~H(-1/2), H(1/2)): the rational numbers are embedded into the **ordered field** of **real numbers**, **which is a subset of** the **ordered field** of **hyper-real (ideal) numbers**. The field of hyper-real numbers (or ideal points) contains infinitely great and small numbers. It is constructed abstractly using Zorn's lemma. The key differentiator of both fields is, that the **Archimedian axiom** (which is valid for the real numbers) is no longer valid for the ordered hyper-real numbers.

From a purely mathematical point of view the baseline of all mathematical models are "axioms"; the very first one to be mentioned in the context of the above is the "**well-ordering theorem**" (which is NOT a "theorem" as such). It is equivalent to the "**axiom of choice**" and "**Zorn's lemma**" (which is NOT a "lemma", as such). At the same time all "physical" gravity and quantum theory models are a purely mathematical models building on those kind of axioms. With respect to an appropriate definition of a "mathematical time" beyond the ""physical"/ thermodynamical time" ((RoC) III.9) one could decide for a hyper-real number (which is nothing else than a Leibniz monad), where the corresponding standard part of it (in case the hyper-real number is finite) is the "thermodynamical time" variable. If this option is beeing seen as too sophisticated, please note that already each **irrational number** is its own mystery/universe, as EACH irrational number is only "existing" (i.e. purely mathematically defined) as the limit of a sequence of INFINITE rational numbers.

The last section of (RoC1) is related to philosophical aspects (including the words of Anaximander, (HeM) "Der Spruch des Anaximander"). From (HeM), Die Zeit des Weltbildes, 72) we recall the following: "*Die neuzeitliche Physik heisst mathematische, weil sie in einem vorzüglichen Sinne eine ganz bestimmte Mathematik anwendet. Allein, sie kann in solcher Weise nur mathematisch verfahren, weil sie in einem tieferen Sinne bereits mathematisch ist. .... Keineswegs wird aber das Wesen des Mathematischen durch das Zahlenhafte bestimmt. ... Wenn nun die Physik sich ausdrücklich zu einer mathematischen gestaltet, dann heisst das: Durch sie und für sie wird in einer betonten Weise etwas als das Schon-Bekannte im vorhinein ausgemacht. Dieses Ausmachen betrifft nichts Geringeres als den Entwurf dessen, was für das gesuchte Erkennen der Natur künftig Natur sein soll: der in sich geschlossene* ***Bewegungszusammenhang raum-zeitlicher Massenpunkte***."

From the below we quote: The **least action principle** can be also seen as THE fundamental principle to develop laws of nature in strong alignment with Kant's philosophy: ((KnA), p. 55, p. 56): (translated*) "the least action principle in his most modern general public is a* ***maxime of Kant's reflective judgment***. ... *Offenbar haben wir beim Energieprinzip eine typische Entwicklung vor uns: wenn das Prinzip der reflektierenden Urteilskraft mit einer seiner Maximen vollen Erfolg gehabt hat, rückt sein Ergebnis aus dem Reich der Vernunft im Kantischen Sinne, zu welchem die reflektierende Urteilskraft gehört, in die Sphäre des Verstandes herab und wird zum allgemeinen Naturgesetz (****law of nature****)*".

The ***loop quantum theory** (**LQT**)* (**C. Rovelli**) is the choice of a different algebra of basic field functions: a noncanonical algebra based on the holonomics of the gravitational connections ((RoC) 1.2.1). The holonomy (or the "Wilson loop") is the matrix of the parallel transport along a closed curve and **spacetime** itself is **formed** by **loop-like states**. Therefore the position of a loop state is relevant only with respect to other loops, and not with respect to the background. The **state space** of the theory is a **separable Hilbert space** spanned by loop states, admitting an orthogonal basis of spin network states, which are formed by **finite linear combinations of loop states**, and are defined precisely as the spin network states of a lattice **Yang-Mills theory**."

3. The Einstein-Hilbert action functional and its related (differentiable) manifold framework

The central mathematical concepts of the GRT are differentiable manifolds, affine connexions with the underlying covariant derivative definition on corrresponding tangential (linear) vector spaces. Already the "differentiability" condition is w/o any physical justification. The only "affine" connexion concept and its corresponding locally defined metrical space framework jeopardizes a truly infinitesimal geometry, which is compatible with the Hilbert space framework of the quantum theory and the proposed distributional Hilbert space concept in (BrK). In sync with the above we propose a generalized Gateaux differential operator:

let H(1/2) = H(1) + H(*) denote the orthogonal decomposition of the alternatively proposed "energy/momentum/velocity" Hilbert space, whereby H(1) denotes the (compactly embedded) standard energy space with its inner product, the Dirichlet integral; "lim" denotes the limes for t --> 0 for real t. Then for x,y ex H(1/2) the operator VF(x,y) is defined by VF(x,y):=lim((F(x+t*y)-F(x))/t), whereby the limes is understood in a weak H(-1/2) sense. The operator is homogeneous in y; however, it is not always a linear operator in y ((VaM) 3.1).

The main tools used in geometrical theory of gravitation are tensor fields defined on a Lorentzian manifold representing space-time. A Lorentz manifold L is likewise equipped with a metric tensor g, which is a nondegerated symmetric bilinear form on the tangential space at each point p of L. The Minkowski metric is the metric tensor of the (flat space-time) Minkowski space.

The least action principle can refer to the family of variational principles. The most popular among these is Hamilton's principle of least action. It states that the action is stationary under all path variations q(t) that vanishes at the end points of the path. It does not not strictly imply a minimization of the action.

The **least action principle** can be also seen as THE fundamental principle to develop laws of nature in strong alignment with Kant's philosophy: ((KnA), p. 55, p. 56): (translated*) "the least action principle in his most modern general public is a* ***maxime of Kant's reflective judgment***. ... *Offenbar haben wir beim Energieprinzip eine typische Entwicklung vor uns: wenn das Prinzip der reflektierenden Urteilskraft mit einer seiner Maximen vollen Erfolg gehabt hat, rückt sein Ergebnis aus dem Reich der Vernunft im Kantischen Sinne, zu welchem die reflektierende Urteilskraft gehört, in die Sphäre des Verstandes herab und wird zum allgemeinen Naturgesetz (****law of nature****)*".

The **Einstein-Hilbert action functional** W(g) is about the **scalar curvature** S=scal (which is the Ricci scalar of the Ricci tensor "Ric") applied to the metric tensor g. It is the simplest curvature invariant of a Riemannian manifold. The scalar curvature is the Lagrangian density for the Einstein-Hilbert action. The stationary metrics are known as Einstein metrics. The scalar curvature is defined as the trace of the Ricci tensor. We note that the trace-free Ricci tensor for space-time dimension n=4 is given by Z(g):=Ric(g)-(1/4)*S(g)*g, and that Z vanishes identically if and only if Ric = l*g for some constant l. In physics, this equation states that the manifold (M,g) is a solution of Einstein's vacuum field equations with cosmological constant. We further note, that the Ricci tensor corresponds to the Laplacian operator multiplied by the factor (-1/2) plus lower order terms.

The Einstein-Hilbert action functional leads to the Einstein field equations with the Einstein tensor G := Ric(g)-(1/2)*S(g)*g, whereby the negative Einstein tensor -G is the L(2) gradient of the Einstein-Hilbert functional W(g). Its counterpart in elasticity theory is given by the **principle of Castigliano** (for elastic bodies), which is about the minimization of the potential energy. It is a weak form representation of the corresponding classical boundary value problem -grad(S(**u**))=f and boundary conditions (!), whereby, in this case, S denotes the stress tensor ((VeW) (4.127), (4.128)), i.e. what's missing in the Einstein-Hilbert action representation are appropriate "boundary" conditions. This links back to the "origin of inertia in the Einstein geometrodynamics" to develop a modified well defined Einstein-Hilbert functional in sync with appropriate Hilbert scale (CiI). In (HoA) a generalized concept of Minkowski space is provided embedded in a semi-indefinite-inner-product space using the concept of a new product, that contains the classical cases as special ones. It is proposed as alternative (integration) concept for the 3 geometries of the (special relativity) universe, which are the Minkowski, the de Sitter and the anti-de Sitter space with corresponding zero, positive and negative curvature.

The Einstein-Hilbert functional is an *invariant* integral, which is a must to describe the field-action of graviation ((WeH), §28). From a physical perspective a field-action term should be based on a scalar density G, which is composed of the potentials g(i,k) and of the field-components of the gravitation field (which are the first derivatives of the g(i,k,), i.e., g(i,k);r): "*it would seem to us that only under such circumstances do we obtain differential equations of order not higher than the second for our gravitation laws .... Unfortunately a **scalar density G**, of the type we wish, does not exist at all; for we can make all g(i,k);r vanish at any given point choosing the appropriate co-ordinate system. Yet, the scalar R, the curvature defined by Riemann, has made us familiar with an invariant which involves the second derivatives of the g(i,k)'s only lineary. ... In consequence of this linearity we may use the invariant integral (the Einstein-Hilbert functional) to get the derivatives of the second order by partial integration. ... We then get a sum of a truly field-action functional (*with a scalar density G*) plus a divergence integral, that is an integral whose integrand is of the form div(**w**). Hence for the corresponding variations of theh potential functions g(i,k) the variations of both funtionals are identical; therefore the replacement of the physically required scalar density G by the integrand of the W(g) is justified (as the essential feature of the Hamilton's principle is fulfilled with W(g)).*" This is where a alternative field-action functional of gravitation in a alternative framework (as proposed above) can be defined, based on a **"*scalar density*" function** in a "**Plemelj**" (Stieltjes integral) sense.

The electromagnetic field is built up from the co-efficients of an invariant *linear* differential form. The potential of the gravitational field is made up of the co-efficients of an invariant quadratic differential form. Replacing the Newtonian law of attraction by the Einstein theory is about discovering the invariant law of gravitation, according to which matter determines the components of the graviation fields. The topic of the chapter above is about the **substance-action** and the **field-action** of electricity and gravitation in the context of the least action principle. The substance-action is based on the mathematical concept "density", while the field action is based on the mathematical concept "potential (function)". The substance-action related "**tensor density**" of electricity can be easily extented to the substance-action related "tensor density" of gravitation ((WeH) §28). A corresponding field-action of gravitation based on an invariant integral *and* an approporate potential "scalar density" is not possible from a mathematical perspective, as by choosing the appropriate co-ordinates the field components of the gravitational field vanish. The alternatively proposed aproach of this page can be summarized as follows:

- replacing of the mathematical "*density*" concept by Plemelj's "mass element" concept, which goes along with an alternative (more general) "*potential*" function concept

- replacing the manifold concept by a (semi) Hilbert space-based concept, where a non-linear invariant integral functional F(V(g)) is defined by a distributional (semi-) inner product, which is equivalent to a corresponding functional F(R(g)) of a related inner product (where R denotes the Riesz operators (which commute with translations & homothesis having nice properties relative to rotations)) plus a (non-linear) compact disturbance term; the concept enables variational methods of nonlinear operators based on Stieltjes and curvilinear integrals (VaM).

The Yang-Mills functional is of similar structure than the Maxwell functional regarding the underlying constant fundamental tensor. The field has the property of being self-interacting and equations of motions that one obtains are said to be semilinear, as nonlinearities are both with and without derivatives. The **YME mass gap problem** is about the **energy gap for the vacuum state**. Therefore, the above proposed model alignments for the "electricity & gravity forces" phenomena covers also the cases of the "weak & strong nuclear forces" phenomena.

To merge two inconstent theories requires changes on both sides. In the above case this is about a newly proposed common "mass/substance element" concept, alternatively to the today's "mass density" concept, while, at the same time, the linear algebra tensor tool (e.g. a "density" tensor) describing classical PDE systems is replaced by non-linear operator equations defined by weak (variational) functional systems. Those (weak) equations provides the mathematical model of physical phenomena, while its correspondin classical PDE systems (requiring purely mathematical additional regularity assumptions) are interpreted as approximation solutions, only.

4. Geometrodynamics, distortion-free, progressing Maxwell and Einstein waves and space-time matter

As a shortcut reference to geometrodynamics is given by (WhJ). For a review of discoveries in the nonlinear dynamics of curved spacetime, we refer to ((ScM). An introduction to the foundations and tests of gravitation and geometrodynamics or the meaning and origin of inertia in Einstein theory is provided in (CiI).

In ((CiI) 4.6) the Gödel model universe is discussed, which is a four-dimensional model universe, homogeneous both in space and time, which admits the whole four-dimensional simply transitive group of isometries, in other words, a space-time that admits all four "simple translations" as independent Killing vectors. As the Gödel model universe is homogeneous both in space and time it is stationary. In other words, in this model the cosmological fluid is characterized by zero expansion and zero shear. Thus the Gödel model runs into difficulty with the expansion of the universe.

The initial-value problem and the interpretation of the origin of inertia in geometrodynamics is considered in ((CiI) 5.1, 5.2):

"*The specification of the relevant features of a three-geometry and its time rate change on a closed (compact and without boundary manifolds), initial value, space-like hypersurface, together with the energy density and density of energy flow (conformal) on that hypersurface and together with the expansion of the equation of state of mass-energy, determines the entire space-time geometry, the local inertial frames, and hence the inertial properties of every test particle and every field everywhere and for all time.*"

The related clarifcations regarding the distortion tensor or gravitomagnetic field is provided in ((CiI) §5.2.6, § 5.2.7).

The Laplacian equation for the gravitomagnetic vector potential W, in terms of the current J of mass-energy is discussed in ((CiI) 5.3). The Neumann problem and its related integral equations with double layer potential leads to the Prandtl operator, defining a well posed integral equation in case of domain H(1/2) with range H(-1/2) ((LiI) theorem 4.3.2).

The Prandtl operator with appropriate domain ((LiL) theorem 4.3.2) is proposed to be applied defining an adequate (distributional) Hilbert space framework for the geometrodynamics (GMD) (gravitation & inertia and 3 manifolds geometries (ScP), requiring an appropriate definition of a corresponding inner product. This proposed inner product (in opposite to the standard "*exterior*" product) is in line with the idea of (BrK), defining an alternative Hilbert space framework for an alternative new ground state energy model (for the harmonic quantum oscillator model). In essence it is about a inner product (and corresponding norm = metric) of "Plemelj's mass elements" (represented as 1-forms (i.e. differentials)):

((((du,dv)))):=(((u,v))):=((P(u),v))

whereby (((*,*))) defines the H(-1) inner product and ((*,*)) defines the H(-1/2) inner product of the corresponding Hilbert scales building on the eigen-pair solutions of the Prandtl operator equation with domain H(1/2).

The proposed alternative Hilbert space based framework provides also a "variational wave equation/ function" based approach of the "evolution of geometric structures on 3-manifolds" in the context of Thurston's "**geometrization conjecture**" and its underlying **Poincare conjecture** (which have been established by **Perelman**), where the **Ricci**

flows play a central conceptual solution element to build "nice behavior" metrics in manifolds.

"T*he hypothesis that the* ***universe is infinite*** *and* ***Euclidean at infinity****, is, from a relativistic point of view, a complicated hypothesis. In the language of the general theory of relativity it demands that the* ***Riemann tensor*** *of the fourth rank* ***shall vanish at infinity****, which furnishes twenty independent conditions, while* ***only ten curvarture components*** *enter the laws of the* ***gravitational field****. It is certainly unsatisfactory to postulate such far-reaching limitation without any physical basis for it.*

If we think these ideas consistently through to the end we must expect ***the whole inertia****, that is, the whole* ***g(i,k)-field****,* ***to be determined by the matter of the universe****, and* ***not*** *mainly by the* ***boundary conditions at infinity****.*

The possibility seems to be particularly satisfying that the universe is spatially bounded and thus, in accordance with our assumption of the constancy of the mass-energy density, is of constant curvature, being either spherical or elliptical; for then the boundary conditions at infinity which are so inconvenient from the standpoint of the general theory of relativity, may be replaced by the much more natural conditions for a closed surface" ((CiI) 5.2.1)

The wave equation can be derived from the Maxwell equations by applying the rot-operator. It results into the "light" phenomenon. A similar tranformation is not possible for Einstein equations, which results into the "gravitation" phnomenon. The "approximation" approach is about the split g(i,k)=m(i,k)+h(i,k), where m(i,k) denotes the flat Minkowski metric. The perturbance term h(i,k) admits a retarded (only) potential representation, representing a gravitational perturbance propagating at the speed of light ((CiI) 2.10). An alternative splitting with defined distortion tensor enabling an analogue approach with electrodynamics is provided in ((CiI) 5.2.7).

In ((CiI) (2.7.10)) an „*energy-momentum pseudotensor for the gravity field*" is introduced representing the energy and momentum of the gravitation field. Then, using the corresponding "***effective energy-momentum pseudotensor for matter, fields and gravity field***", in analogy with special relativity and electromagnetism, the conserved quantities on an asymptotically flat spacelike hypersurface are defined by the sum of four-momentum, energy and angular momentum operators (2.7.19-21). Following an analogue approach, which lead to the modified Maxwell equation (as proposed in the above paper), leads to an alternative *effective energy-momentum tensor for matter, fields and gravity field*". As the Einstein (gravity) tensor is derived from the condition of a divergence-free energy-momentum tensor, this results to an alternative Einstein tensor. The additional term of this alternative Einstein tensor could be interpreted as "cosmologic term", not to ensure a static state of the universe (which is not the case due to Hubbles observations), but to model the "vacuum energy" properly. This then would also be in sync with the physical interpretation of the corresponding term in the modified Maxwell equations with its underlying split of divergence-free and rotation-free tensors. At the same point in time the approach avoids the *affine connexion* concept and the "*differentiable*" manifolds regularity requirement, which is w/o any physical justification.

There are eight 3-dimensional geometries in the context of "nice" metrics. The nicest metrics are those with a constant curvature, but there are other ones. Their classification in dimension three is due to Thurston (ScP).

In (GrJ) philosophical aspects of the geometrodynamics are considered. We quote from the cover letter summary:

"*The central conceptual idea of the* ***contemporary theory of general relativity*** *– or geometrodynamics – is the* ***identification of matter with the structure of space-time****. No identities foreign to space-time, like masses, charges, or independent fields are needed, and physics thus becomes identical with the geometry of space-time. This idea implies a philosophical description of the universe that is monistic and organic, characterized by an all-encompassing interdependence of events. …. The Newtonian independence and distinctness of objects is at the polar extreme from their Einsteinian interdependence and continuity. …. He (the author) then presents the remarkable recent developments in geometrodynamics which allow the program of identifying matter with space-time to be carried further than even Einstein suspected possible. The surprising discovery that electromagnetism can be incorporated into geometrodynamics without modifying Einstein's original equations appears to be formally correct, but reliance on multiply connected topologies ("****wormholes****") to represent charge raises various unresolved questions. Graves concludes that the present language of physics, like that of every-day life, is based on concepts of independence and separation, and that a wholly new language may needed to describe the world in terms of geometrodynamics, in which space-time appears as the only substance, with curvatures as its attributes, and in which objects have no absolute individuality, distinctness, or location.*"

The above *questions* concerning singularities and non-geometric manifolds can be revisited based on the above alternative conceptual framework; the corresponding physical interpretation of the geometrodynamics are in line with Schrödinger's vision (resp. critique about the common handicap of all western philosophy baseline assumptions, propagating instead a purely monoism) of a truly quantum field theory (see also www.quantum-gravitation.de).

In (CoR) there is a **conjecture** formulated, that **distortion-free families of progressing, spherical waves of higher order** exist if and only if the **Huyghens' principle** is valid, and that families of spherical, progressing waves only exist for space-time dimension n=2 and n=4 ((CoR) VI, §10.2, 10.3). In combination with Hadamard conjecture (that the wave equations for even space-time dimension are the only partial differential equations, where the Huyghens' principle is valid) this would lead to an essential characterization of the four-dimension space-time space with its underlying Maxwell field theory.

With respect to the geometrodynamics (gravitation and inertia) we note that ..

1. .. Huyghens' principle is valid under the same conditions for both, the initial value problem of the wave equation and the corresponding radiation problem. For each t>0 the latter one is defined by a certain sphere integral limit regarding of the normal derivative defining the intensity of the radiation as function of the time variable. Spherical waves are defined in that way, that the family of its corresponding characteristic surfaces builds characteristics conoids, those tips lie all on a time-like curve ((CoR) VI, §10.1).

2. *.. in order to avoid the problem of existence of closed time-like curves and the problem of special non-compactness, Gödel proposed a rotating cosmological model that have no closed time-like curves and that expand but are spatially homogeneous and compact* (CiI), 4.6, 4.7)

3. *.. there is an alternative postulate that space geometry shall be asymptotically flat with two problems of principle, (1) it imposes "flatness from on high", (2) "the quantum fluctuations rule the geometry of space in the small". No natural escape has ever presented itself from these two difficulties of principle excepts to say that space in the large must be compact. No one will deny that space-time approaches flatness well out from many a localized center of attraction. However, nothing, anywhere, in any finding of*

astrophysics of our day makes it unattractive to treat every such nearly flat region, or even totally flat region, "not as infinite, but as part of a closed universe". … The role of spatial closure in the context of finding the magnetic field associated with a stationary system of electric currents lead to an additionally to be added magnetic field, that is free of curl and divergence (and which therefore goes on and on to infinite with its twisty, wavy lines of force) to the well-known obvious solution to obtain another (unique) solution. Transferring this concept to geometrodynamics is tame, when the S(3) topology is supplemented by one or more wormholes. Then the solution is not unique until one restores uniqueness by specifying the flux through each wormhole (CiI) (5.2.1)).

Related to topic 1 above we note that spherical waves are only relative distortion-free and progressing due to the special role of the time-like curves.

The singularities of wormholes are the main challenges of current status of geometrodynamic (topic 3 above). We propose Plemelj's alternative "mass element", "flux" and "flux strength" concept to specify the inertia condition for the corresponding radiation conditions (in analogy to the wave equation radiation condition (CoR). It is based on an alternative "normal derivative" concept. Its definition requires only information from the surface. The corresponding field equations are defined in a (weak) variational representation based on a H(-1/2) "space-time matter fluid/particle". It avoid the Dirac "function" concept, which is a "singularity governing function". It avoid the concept of "continuity" resp. "differentiable continuity", requiring regularity conditions to enable the Sobolev embedding theorem (H(k) sub-space of C(0), if k>n/2).

We mention that the existing electromagnetic phenomena on earth are the result of plasma physics phenomena underneath the earth crust. Those "activities" are all triggered by gravitation "forces".

The above (distributional) Hilbert space based alternative geometrodynamic modelling framework provides an alternative approach to **Penrose**'s "**cycles of time**" concept of a "**conformal cyclic cosmology**", addressing e.g. the "collapsing of matter" of an over-massive star to a black hole problem (PeR) and "the problem of time" (AnE).

"*What characterizes the **loop quantum theory** (LQT) is the choice of a different algebra of basic field functions: a noncanonical algebra based on the holonomics of the gravitational connections ((RoC) 1.2.1). The holonomy (or the "Wilson loop") is the matrix of the parallel transport along a closed curve. ... In LQT, the holonomy becomes a quantum operator that creates "loop states" (to overcome the issue of current dynamics model of coupled gravity + matter system, simply defined by adding the terms defining the matter dynamics to the gravitational relativistic hamiltonian ((RoC) 7.3)). ... **Space-time** itself is **formed** by **loop-like states**. Therefore the position of a loop state is relevant only with respect to other loops, and not with respect to the background. ... The state space of the theory is a separable Hilbert space spanned by loop states, admitting an orthogonal basis of spin network states, which are formed by finite linear combinations of loop states, and are defined precisely as the spin network states of a lattice **Yang-Mills theory**.*" The proposed distributional quantum state H(-1/2) above admits and requires infinite linear combinations of those "loop states" (which we call "quantum fluid" state), i.e. overcomes the **current challenge of LQT** defining the scalar product of the spin network state Hilbert space ((RoC) 7.2.3). The physical space is a quantum superposition of "***spin networks***" in LQT corresponds to an orthogonal projection of H(-1/2) onto H(0). This othogonal projection can be interpreted as a general model for a "**spontaneous symmetry break down**".

References

(AbM) Abramowitz M., Stegun A., Handbook of mathematical functions, Dover Publications Inc., New York, 1970

(AnE) Anderson E., The Problem of Time, Springer, Cambridge, UK, 2017

(AnM) Anderson M. T., Geometrization of 3-manifolds via the Ricci flow, Notices Amer. Math. Sco. 51, (2004) 184-193

(AzT) Azizov T. Y., Ginsburg Y. P., Langer H., On Krein's papers in the theory of spaces with an indefinite metric, Ukrainian Mathematical Journal, Vol. 46, No 1-2, 1994, 3-14

(BeB) Berndt B. C., Ramanujan's Notebooks, Part I, Springer Verlag, New York, Berlin, Heidelberg, Tokyo, 1985

(BiP) Biane P., Pitman J., Yor M., Probability laws related to the Jacobi theta and Riemann Zeta functions, and Brownian excursion, Amer. Math. soc., Vol 38, No 4, 435-465, 2001

(BoJ) Bognar J., Indefinite Inner Product Spaces, Springer-Verlag, Berlin, Heidelberg, New York, 1974

(BrK) Braun K., A new ground state energy model, www.quantum-gravitation.de

(BrK1) Braun K., An alternative Schroedinger (Calderon) momentum operator enabling a quantum gravity model

(BrK2) Braun K., Global existence and uniqueness of 3D Navier-Stokes equations

(BrK3) Braun K., Some remarkable Pseudo-Differential Operators of order -1, 0, 1

(BrK4) Braun K., A Kummer function based Zeta function theory to prove the Riemann Hypothesis and the Goldbach conjecture

(BrK5) An alternative trigonometric integral representation of the Zeta function on the critical line

(BrK6) Braun K., A distributional Hilbert space framework to prove the Landau damping phenomenon

(BrK7) Braun K., An alternative Schroedinger (Calderón) momentum operator enabling a quantum gravity model

(BrK related papers) www.navier-stokes-equations.com/author-s-papers

(CaD) Cardon D., Convolution operators and zeros of entire functions, Proc. Amer. Math. Soc., 130, 6 (2002) 1725-1734

(CaJ) Cao J., DeTurck D., The Ricci Curvature with Rotational Symmetry, American Journal of Mathematics 116, (1994), 219-241

(ChK) Chandrasekharan K., Elliptic Functions, Springer-Verlag, Berlin, Heidelberg, New York, Tokyo, 1985

(CiI) Ciufolini I., Wheeler J. A., Gravitation and Inertia, Princeton University Press , Princeton, New Jersey, 1995

(CoR) Courant R., Hilbert D., Methoden der Mathematischen Physik II, Springer-Verlag, Berlin, Heidelberg, New York, 1968

(DrM) Dritschel M. A., Rovnyak, J., Operators on indefinite inner product spaces

(EbP) Ebenfelt P., Khavinson D., Shapiro H. S., An inverse problem for the double layer potential, Computational Methods and Function Theory, Vol. 1, No. 2, 387-401, 2001

(EdH) Edwards Riemann's Zeta Function, Dover Publications, Inc., Mineola, New York, 1974

(ElL) Elaissaoui L., Guennoun Z. El-Abidine, Relating log-tangent integrals with the Riemann zeta function, arXiv, May 2018

(ElL1) Elaissaoui L., Guennoun Z. El-Abidine, Evaluation of log-tangent integrals by series involving zeta(2n+1), arXiv, May 2017

(EsO) Esinosa O., Moll V., On some definite integrals involving the Hurwitz zeta function, Part 2, The Ramanujan Journal, 6, p. 449-468, 2002

(EsR) Estrada R., Kanwal R. P., Asymptotic Analysis: A Distributional Approach, Birkhäuser, Boston, Basel, Berlin, 1994

(FaK) Fan K., Invariant subspaces of certain linear operators, Bull. Amer. Math. Soc. 69 (1963), no. 6, 773-777

(FeR) Feynman R. P., Quantum Electrodynamics, Benjamin/Cummings Publishing Company, Menlo Park, California, 1961

(GaW) Gautschi W., Waldvogel J., Computing the Hilbert Transform of the Generalized Laguerre and Hermite Weight Functions, BIT Numerical Mathematics, Vol 41, Issue 3, pp. 490-503, 2001

(GeM) Geissert M., Heck H., Hieber M., On the equation div(u)=g and Bogovskii's operator in Sobolev spaces of negative order, in Operator Theory: Advances and Applications, Birkhäuser Verlag Basel, 2006, Vol. 168, 113–121

(GrI) Gradshteyn I. S., Ryzhik I. M., Table of integrals series and products, Academic Press, New York, San Franscisco, London, 1965

(GrJ) Graves J. C., The conceptual foundations of contemporary relativity theory, MIT Press, Cambridge, Massachusetts, 1971

(HaG) Hardy G. H., Riesz M., The general theory of Dirichlet's series, Cambridge University Press, Cambridge, 1915

(HeM) Heidegger M., Holzwege, Vittorio Klostermann, Frankfurt a. M., 2003

(HeW) Heisenberg W., Physikalische Prinzipien der Quantentheorie, Wissenschaftsverlag, Mannhein, Wien, Zürich, 1991

(HoM) Hohlschneider M., Wavelets, An Analysis Tool, Clarendon Press, Oxford, 1995

(HoA) Horvath A. G., Semi-indefinite-inner-product and generalized Minkowski spaces, arXiv

(IvV) Ivakhnenko, V. I., Smirnow Yu. G., Tyrtyshnikov E. E., The electric field integral equation: theory and algorithms, Inst. Numer. Math. Russian of Academy Sciences, Moscow, Russia

(KaM) Kaku M., Introduction to Superstrings and M-Theory, Springer-Verlag, New York, Inc., 1988

(KeL) Keiper J. B., Power series expansions of Riemann's Zeta function, Math. Comp. Vol 58, No 198, (1992) 765-773

(KlB) Kleiner B., Lott J., Notes on Perelman s papers, Mathematics ArXiv

(KnA) Kneser A., Das Prinzip der kleinsten Wirkung von Leibniz bis zur Gegenwart, B. G. Teubner, Leipzig, Berlin, 1928

(KoH) Koch H., Tataru D., Well-poseness for the Navier-Stokes equations, Adv. Math., Vol 157, No 1 22-35 2001

(KoJ) Korevaar J., Distributional Wiener-Ikehara theorem and twin primes, Indag. Mathem. N. S., 16, 37-49, 2005

(LaE) Landau E., Die Lehre von der Verteilung der Primzahlen I, II, Teubner Verlag, Leipzig Berlin, 1909

(LaE1) Landau E., Ueber die zahlentheoretische Function phi(n) und ihre Beziehung zum Goldbachschen Satz, Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, Vol 1900, p. 177-186, 1900

(LeN) Lebedev N. N., Special Functions and their Applications, translated by R. A. Silverman, Prentice-Hall, Inc., Englewood Cliffs, New York, 1965

(LeN1) Lerner N., A note on the Oseen kernels, in Advances in phase space analysis of partial differential equations, Siena, pp. 161-170, 2007

(LiI) Lifanov I. K., Poltavskii L. N., Vainikko G. M., Hypersingular Integral Equations and their Applications, Chapman & Hall/CRC, Boca Raton, London, New York, Washington, D. C., 2004

(LiP) Lions P. L., On Boltzmann and Landau equations, Phil. Trans. R. Soc. Lond. A, 346, 191-204, 1994

(LiP1) Lions P. L., Compactness in Boltzmann’s equation via Fourier integral operators and applications. III, J. Math. Kyoto Univ., 34-3, 539-584, 1994

(LiX) Li Xian-Jin, The Positivity of a Sequence of Numbers and the Riemann Hypothesis, Journal of Number Theory, 65, 325-333 (1997)

(LoA) Lifanov I. K., Poltavskii L. N., Vainikko G. M., Hypersingular Integral Equations and Their Applications, Chapman & Hall/CRC, Boca Raton, London, New York, Washington, D. C. 2004

(LuL) Lusternik L. A., Sobolev V. J., Elements of Functional Analysis, A Halsted Press book, Hindustan Publishing Corp. Delhi, 1961

(MaJ) Mashreghi, J., Hilbert transform of log(abs(f)), Proc. Amer. Math. Soc., Vol 130, No 3, p. 683-688, 2001

(MoJ) Morgan J. W., Tian G., Ricci Flow and the Poincare Conjecture, Mathematics ArXiv

(NaS) Nag S., Sullivan D., Teichmüller theory and the universal period mapping via quantum calculus and the H space on the circle, Osaka J. Math., 32, 1-34, 1995

(OlF) Olver F. W. J., Asymptotics and special functions, Academic Press, Inc., Boston, San Diego, New York, London, Sydney, Tokyo, Toronto, 1974

(OlF1) Olver F. W. J., Lozier D. W., Boisvert R. F., Clark C. W., NIST Handbook of Mathematical Functions

(OsH) Ostmann H.-H., Additive Zahlentheorie, erster Teil, Springer-Verlag, Berlin, Göttingen, Heidelberg, 1956

(PeR) Penrose R., Cycles of Time, Vintage, London, 2011

(PoG) Polya G., Über Nullstellen gewisser ganzer Funktionen, Math. Z. 2 (1918) 352-383

(RoC) Rovelli C., Quantum Gravity, Cambridge University Press, Cambridge, 2004

(RoC1) Rovelli C., The Order of Time, Penguin Random House, 2018

(RoC2) Rovelli C., Reality is not what it seems, Penguin books, 2017

(RoC3) Rovelli C., Seven brief lessons on physics, Penguin Books, 2016

(ScP) Scott P., The Geometries of 3-Manifolds, Bull. London Math. Soc., 15 (1983), 401-487

(SeA) Sedletskii A. M., Asymptotics of the Zeros of Degenerated Hypergeometric Functions, Mathematical Notes, Vol. 82, No. 2, 229-237, 2007

(SeJ) Serrin J., Mathematical Principles of Classical Fluid Mechanics

(ShM) Scheel M. A., Thorne K. S., Geodynamics, The Nonlinear Dynamics of Curved Spacetime

(ShM1) Shimoji M., Complementary variational formulation of Maxwell s equations in power series form

(StE) Stein E. M., Conjugate harmonic functions in several variables

(ThW) Thurston W. P., Three Dimensional Manifolds, Kleinian Groups and Hyperbolic Geometry, Bulletin American Mathmematical society, Vol 6, No 3, 1982

(TiE) Titchmarsh E. C., The theory of the Riemann Zeta-function, Clarendon Press, London, Oxford, 1986

(VaM) Vainberg M. M., Variational Methods for the Study of Nonlinear Operators, Holden-Day, Inc., San Francisco, London, Amsterdam, 1964

(VeW) Velte W., Direkte Methoden der Variationsrechnung, B. G. Teubner, Stuttgart, 1976

(VlV) Vladimirow V. S., Drozzinov Yu. N., Zavialov B. I., Tauberian Theorems for Generalized Functions, Kluwer Academic Publishers, Dordrecht, Boston, London, 1988

(WhJ) Wheeler J. A., On the Nature of Quantum Geometrodynamics

(WhJ1) Whittaker J. M., Interpolatory Function Theory, Cambridge University Press, Cambridge, 1935

(WeH) Weyl H., Space, Time, Matter, Cosimo Classics, New York, 2010

(WeH1) Weyl H., Matter, structure of the world, principle of action, ..., in (WeH) §34 ff.

(WeH2) Weyl H., Was ist Materie? Verlag Julius Springer, Berlin, 1924

(ZhB) Zhechev B., Hilbert Transform Relations

(ZyA) Zygmund A., Trigonometric series, Volume I & II, Cambridge University Press, 1959