Music melodies signals
between red (Brownian) and white noise

Internet information

Brownian (red) noise is the kind of signal noise produced by Brownian motion. A Brownian motion (i.e. a Wiener process) is a continuous stationary stochastic process having independent increments, i.e. $B(t)-B(0)$ is a normal random variable with mean μt and variance $\sigma^2 t$ with μ,σ^2 constant real numbers. The density function of a Brownian motion is given by

$$f_{B(t)}(x)=\frac{1}{\sqrt{2\pi\sigma^2 t}}e^{-\frac{(x-\mu t)^2}{2\sigma^2 t}}$$

The sample paths of Brownian motion are not differentiable, a mathematical fact explaining the highly irregular motions of small particles. The total variation of Brownian motion over a finite interval $[0,T]$ is infinite. It holds

$$Var\left[\frac{B(t)}{t}\right]=\frac{\sigma^2}{t}.$$

If $W(t)$ is a Wiener process on the interval $[0,\infty)$, then, as well the process

$$W^*(t):=\begin{cases} tW(1/t), & t>0 \\ 0 & ,t=0 \end{cases}.$$

White noise can be defined as the derivative of a Brownian motion (i.e. a Wiener process) in the framework of infinite dimensional distribution theory, as the derivative $B'(t)$ of $B(t)$, does not exist in the ordinary sense. Not only $B'(t)$, but also all derivatives of Brownian motion are generalized functions on the same space. For each t, the white noise $B'(t)$ is defined as a generalized function (distribution) on an infinite dimensional space. A Brownian motion is obtained as the integral of a white noise signal $dB(t)$, i.e.

$$B(t)=\int_0^t dB(\tau)$$

meaning that Brownian motion is the integral of the white noise $dB(t)$ whose power spectral density is flat

$$S_0=\left|Fourier[B'](\omega)\right|^2=const.$$

This means, that the spectral density S_0 for white noise is flat , i.e. $S_0/\omega^0=c$ i.e. it is inversely proportional to ω^0. It holds $Fourier[B'](\omega)=i\omega Fourier[B](\omega)$. Therefore the power spectrum of Brownian noise is given by

$$S(\omega)=\left|Fourier[B](\omega)\right|^2=S_0/\omega^2.$$

i.e. it is inversely proportional to ω^2, meaning it has more energy at lower frequencies, even more so than pink noise.

The proposed framework for proof P2 indicates a spectral density $S^*(\omega)$ for harmonic music “noise” signals given by

$$S^*(\omega)=\left|Fourier[B^*](\omega)\right|^2=S_0/\omega\ .$$