Quantum Gravity

K. Braun

An alternative quantum ground state energy model based on a physical H(-1/2) Hilbert space enabling a quantum gravity model

A. Introduction

A mathematical framework for an integrated gravity and quantum model

(Wikipedia) **The ground state** of a quantum mechanical system is its lowest-energy state; the energy of the ground state is known as the zero-point energy of the system. An excited state is any state with energy greater than the ground state. The ground state of a quantum field theory is usually called the vacuum state or the vacuum.

state axiom of quantum mechanics: "*physical states are described by vectors of a Hilbert space, i.e. physical states are mapped injective onto the radiances of a Hilbert space* "**quantum gravity** is a field of theoretical physics that seeks to describe the force of gravity according to the principles of quantum mechanics.

The **Higgs boson** combines the existence of mass together with the action of the weak force. But why it provides especially to the quarks that much mass, **is still a mystery**.

M. Planck: "real *is only that, which is* measurable".
E. Schrödinger: *"Indeed there is no observation concerned with the geometrical shape of a particle or even with an atom."*

Mass is essentially the manifestation of the **vacuum energy**

The Standard Model of Elementary Particles (SMEP), including the Higgs mechanism, is concerned with gauge theory and variational principles (energy and operator norm minimization problems), whereby each of the 4 (observed) Nature forces are related to a specific gauge group. The model does not provide any explanation where the related elementary "particles" are coming from (or have been generated out of "first mover" resp. out of mass-less photons) during the inflation phase of current big bang assumption and why their mass have their specific values.
A. Einstein: "*We can't solve problems by using the same kind of thinking we used when we created them*."

We propose an alternative mathematical framework, which replaces gauge theory and variational principles (with its underlying concept of exterior derivatives and tensor algebra) by (distributional) Hilbert scales (enabling an inner product) and variational principles. As a consequence, the vacuum energy becomes an intrinsic part of the variational principles, i.e. is identical for all considered Lagrange resp. Hamiltonian mechanisms, while the corresponding "force" becomes an observable of the considered minimization problem

The central new technical element is a change in the underlying Hilbert space framework, i.e. the standard L(2)-Hilbert space framework is replaced by the distributional Hilbert space H(-1/2). This has two immediate consequences

- an **alternative Schrödinger momentum operator** can be defined, whereby the complementary closed space H(-1/2)-H(0) enables an alternative way to model "wave functions" of the ground state energy resp. condensates, superfluids & superconductivity (J. F. Annett)

- **an alternative Dirac function model** can be defined, whereby the regularity of the "defining" H(-1/2) Hilbert space is (now) independent from the space dimension "n" and, at the same time, more regular than the Dirac distribution "function" itself even for the space dimension n=1 (!!). In other words, an alternative modelling framework for current "physical" applications of the Dirac "function" is provided with (slightly, but essentially) reduced mathematical model required regularity assumptions to SMEP, which is now (newly) independent from the mathematical model`s space-time dimension.

The conceptual idea

The Berry-Keating conjecture is about an unknown quantization ***H*** of the classical Hamiltonian H=xp, that the Riemann zeros coincide with the spectrum of the operator 1/2+i***H***. This is in contrast to canonical quantization, which leads to the Heisenberg uncertainty principle and the natural numbers as spectrum of the harmonic quantum oscillator. The Hamiltonian needs to be self-adjoint so that the quantization can be a realization of the Hilbert-Polya conjecture.

The central concept is about a proposed alternative harmonic quantum energy model which enables a finite "quantum fluctuation = total energy". The model is based on a fractional (distributional) Hilbert space framework, enabling a self-adjoin Hamiltonian operator. It provides a truly infinitesimal geometry overcoming current handicaps of the manifolds framework of Einstein`s field equations (*differentiable* (?) manifolds resp. exterior (differential) algebra), while, at the same time, the Hilbert space provides a closed subspace of the L(2) test space, which enables continuous spectra.

1. Current gravity model & its handicaps
The main characteristics of current gravity model and its related handicaps regarding the physical model requirements are

a. metric space, affine connexions
handicap: no scalar fields (vector fields, only), no (infinitesimal) geometry
b. differentiable manifolds
handicaps: physical justification is only about continuous manifolds, additional regularity requirements are purely mathematical model driven

c. exterior differential forms, exterior product, exterior algebra
handicaps: no geometry, gravitational collapse and space-time singularities not covered adequately; if physical singularities in space-time are not to be permitted (R. Penrose) inside such a collapsing object at least one of the following holds

- negative local energy occurs
- Einstein's equations are violated
- the space-time manifold is incomplete
- the concept of space-time loses its meaning at very high curvatures, because of quantum phenomena

2. Current quantum model & its handicaps
The main characteristics of current quantum model and its related handicaps regarding the physical model requirements are

a. separable Hilbert space
handicaps: location and momentum operator have different domains (separable Hilbert spaces) leading to non-vanishing related commutator and the Heisenberg uncertainty principle

b. the Dirac function
handicap: Dirac (delta) function regularity depends from the space dimension (due to the Sobolev embedding theorem)

3. An alternative mathematical framework

a. A separable distributional (quantum state) Hilbert space H(-1/2) with slightly better regularity than Delta function Hilbert space (independently from space dimension n and valid for all cases n), where L(2)=H(0) test space is a closed sub-space of it. In other words, Dirac's Location Operator is replaced by the orthogonal projection from H(-1/2) Hilbert space onto test space H(0)

b. The standard derivative definition (momentum operator) is replaced by a Calderon-Zygmund (convolution, singular integral) PDO of order 1. In other words, Schrödinger's momentum Operator is replaced the orthogonal projection of the Hilbert space H(1/2) onto the test space H(0)

c. The Dirac function concept (H(0)-inner product of a "function" and its related Fourier transform) is replaced by the inner product of an element of separable distributional Hilbert space and its dual element of the corresponding domain of the momentum operator.

Rational

The proposed mathematical framework above is supposed to provide a truly infinitesimal geometry (H. Weyl). A physical interpretation could be about "rotating differentials" ("quantum fluctuations"), which corresponds mathematically to Leibniz's concept of monads. Its mathematical counterpart is the ideal points (or hyper-real numbers). This leads to non-standard analysis, whereby the number field has same cardinality than the real numbers. It is "just" the Archimedean principle which is no longer valid. This looks like a cheap prize to be paid, especially as hyper-real numbers might provide at least a proper mathematical language for the "Big Bang" initial value "function" and its related Einstein-Hilbert action functional. Looking on hyper-real numbers from the "real" number perspective one must admit to classify the term "real" as a contraction in itself, if it is understood as *real*. Already the existence of each irrational number (not only the transcendental numbers; and the cardinality of the irrational numbers is different from the rational numbers) is ensured by an axiom, "only", i.e. the "empty space" between two rational numbers is filled with infinite irrational numbers with same cardinality as the field of real numbers itself, i.e. with multiple "universes". The difference of real numbers to hyper-real numbers is "just" the fact that there are additionally infinite small and large numbers "existent", ensured "just" by a second axiom.

Some more details

We propose a fractional (energy) Hilbert space H(1/2), which already plays an elegant role in universal Teichmüller theory. It is also related to the bounded variation functions. Its dual space with respect to the L(2) Lebesgue Hilbert space is the H(-1/2) Hilbert space. The latter one is the proposed quantum state Hilbert space. While the Hermite polynomials (or it Hilbert transforms) build an orthogonal system of the Hilbert space L(2)=H(0) (and related discrete energy spectrum) the basis of the Hilbert space H(-1/2) requires an additional eigenfunction with continuous spectrum. This "eigen-pair" is proposed to be a model for the dark energy model, given by the common (additional) root operator of the ladder "symmetric" operators ("Erzeugungs- und Vernichtungsoperatoren"; "Bosonische und Fermionische Kletteroperatoren"). By this the "symmetry" theory is also anticipated, as the current particles zoo ("materialized" in H(0) Hilbert space) has all the time the same symmetry partner (field), i.e. the "eigen-function" with continuous spectrum, which spans the closed sub-space H(-1/2)-H(0).

The integral of all frequencies of the proposed harmonic quantum oscillator model is finite (which is not true for the current model (!)), while the Heisenberg uncertainty relation is still valid in the distributional Hilbert space, while allowing discrete momentum-location measurements in sub-space H(0). The "measurement" Hilbert space (which is identical to the statistics modelling Hilbert space) can be interpreted as projections from the H(-1/2) (state) space to its sub-space H(0).

At the end, this mathematical model is claimed to enable a new quantum gravity model replacing complex (not sufficient) mathematical models (e.g. differentiable manifolds with dimensions greater than 10, M-Theory, (Super) String theory, loop quantum gravity, all of them w/o any physical interpretations, by a (at the end, 4 dimensional Minkowski-like) Hilbert space providing not only a metric (exterior differentials), but also a geometry (with inner product). It's still beyond human imagination and open for corresponding physical interpretations, but providing a consistent mathematical model (and therefore an appropriate language) combining the "very large" with the "very small" (R. Penrose).

Along with the alternative Hilbert space H(-1/2), as a model for the quantum states two additional conceptual changes are proposed to apply the same Hilbert space alternatively to the current gravity theory framework (differentiable manifolds & affine connexions).

1. Already for the space dimension n=1 the Dirac Delta „function“ is not an element of the (newly proposed quantum state) Hilbert space H(-1/2). This is due to the Sobolev embedding theorem. Therefore, the H(-1/2) Hilbert space concept does not require the Dirac Delta „function“ concept anymore and the Hilbert space extension from H(0) to H(-1/2) enables an alternative wave package concept (with regularity requirements independently from the space dimension). At the same time the model also enables an alternative interpretation of the Neutrinos and their relationship/interaction with Fermions and Baryons. At the same time the concept enables an alternative to current symmetry breaking concept to explain the generation of matter in the early phase of the universe, where energy is required to generate matter w/o violating energy conservation laws out of (massless) photons. Mathematically speaking this is enabled by the compact embedding of the quantum state space H(-1/2) into H(0), which is compactly embedded into the energy space H(1/2).

2. Einstein’s field equations are based on the concept of differentiable manifolds and physical terms are described (indirectly) by vector fields. In case scalar fields would exist this enables direct interpretation/verification of observations and measurements with the mathematical model. The standard scalar field are the real numbers, whereby the term „real“ covers also the irrational numbers and its subset, the transcendental numbers. Ordered fields that have infinitesimal elements are called non-Archimedean. As the distance of two real numbers cannot be infinitely small, G. W. Leibniz argued that the theory of infinitesimals implies the introduction of ideal numbers (monads). This leads to the Non-standard analysis resp. to the concept of hyperreal fields. The differentials (1-forms) can also be brought into relationship to the Hilbert space H(-1/2).

Nota bene

The classical Yang-Mills theory is a generalization of the Maxwell theory of electromagnetism where the *chromo*-electromagnetic field itself carries charges.

As a classical field theory it has solutions which travel at the speed of light so that its quantum version should describe massless particles (gluons). However, the postulated phenomenon of color confinement permits only bound states of gluons, forming massive particles. This is the mass gap. Another aspect of confinement is asymptotic freedom which makes it conceivable that quantum Yang-Mills theory exists without restriction to low energy scales. The problem is to establish rigorously the existence of the quantum Yang-Mills theory and a mass gap.

Identifying "fluids" or "sub-atomic particles" not with real numbers (scalar field, I. Newton), but with hyper-real numbers (G. W. Leibniz) enables a truly infinitesimal (geometric) distributional Hilbert space framework (H. Weyl) which corresponds to the Teichmüller theory, the Bounded Mean Oscillation (BMO) and the Harmonic Analysis theory. The distributional Hilbert scale framework enables the full power of spectral theory, while still keeping the standard L(2)=H(0)-Hilbert space as test space to "measure" particles' locations. At the same time, the Ritz-Galerkin (energy or operator norm minimization) method and its counterpart, the methods of Trefftz/Noble to solve PDE by complementary variational principles (A. M. Arthurs, K. Friedrichs, L. B. Rall, P. D. Robinson, W. Velte) w/o anticipating boundary values) enables an alternative "quantization" method of PDE models (P. Ehrenfest), e.g. being applied to the Wheeler-de-Witt operator.

Braun K., Introduction, A new ground state energy model enabling a quantum gravity model
Some philosophical aspects of proposed mathematical framework

B. Ground state energy

A distributional Hilbert scale framework

We provide a new ground state energy model, which ensures convergent quantum oscillator energy series. This enables the definition of a truly infinitesimal geometry. The corresponding inner product with its induced norm gives the appropriate metric. The domains of related self-adjoint, positive definite operators to build appropriate eigen-pair structures are built on Cartan´s differential forms. By this, H. Weyl´s "truly" infinitesimal (affine connexions, parallel displacements, differentiable manifolds based) geometry is replaced by a truly infinitesimal (rotation group based) geometry.

The today´s well accepted zero energy formula of the quantum oscillator is (just (!)) a divergent series. Nobody seems to be concerned about his. Sophisticated renormalization techniques were developed to overcome this homemade "issue", when trying to build a quantum gravity theory, which failed until today. ("Superstrings" have not yet reached a status to be called a "theory", ... at least, to the author's opinion, from a mathematical perspective; the "loop quantum gravity" builds a Hilbert space framework, but puts the whole complexity in a sum of a series of Hamiltonian operators with corresponding (Hilbert space) domains). The free energy of a system of interacting oscillators to model the Planck blackbody radiation law contains same divergent series as the quantum oscillator (Feynman R., P. Hibbs A. R., "Quantum Mechanics and Path Integrals", (10.85)).

The underlying still unsolved mathematical conceptual problem is similar to the non-vanishing constant Fourier coefficient of the Theta function for the RH duality problem. The above solution of the RH in combination with remarkable properties of the Hilbert/Riesz transforms enables an alternative mathematical ground state energy model.

The new concept has a direct relationship and impact to the idea of H. Weyl of a (to-be-built) truly infinitesimal geometry: we claim, that in case of space dimension n=1 the proposed inner (!) product for 1-forms is the enabler of this. It also provides an alternative to the concept of S. Lie about "contact transformation", which was developed to allow an analysis (enabling the concept of co-variant derivatives) on manifolds. A contact transform is a point transform, which also transforms the Pfaff problem dz-pdx-qdy=0 into itself, i.e. two contacting areas are transformed into again two contacting areas (only in an infinitesimal small area, of cause). This concept is required to "bridge" the gap between the mathematical concept of affine connexion on manifolds with a (still missing) truly infinitely geometry of the continuum, which overcomes the today's particle-field dualism paradox.

Our proposed new mathematical framework replaces the manifold concept by a Hilbert space concept:

the manifold concept requires additionally the concept of gauge theory to ensure group properties in relation to the "affine built" vector spaces, just because those properties are not provided by the manifold itself.

Our proposed alternative Hilbert space framework H(-a) provides “geometrical space” properties per definition, i.e. a Hilbert space anticipates appropriate “geometry structure” properties per definition by its inner product with corresponding norm, which ultimately builds a metric. The tool to build this inner product is based on an appropriately defined alternative "contact" transformation for infinitesimal small entities. In case of space dimension n=1 it is built on ("nomen est omen") the Hilbert transform (which also plays a key role in conformal mapping theory), with its remarkable properties, especially in the context of H(0) Hilbert space theory. It is applied to 1-forms, which is basically a Riemann-Stieltjes (singular) convolution integral, which enables a linkage to Hilbert scale theory.

The Hilbert transformation is a PDO of order "0" with in our case chosen domain of 1-forms. It can be reformulated as singular Calderon-Zygmund (convolution) PDO of order "1".

With respect to F. Klein's algebraic approach classifying a geometry we note: „the entirety of all properties, which do not change by the transformations of a group, defines the geometry".

By this principle there is the relationship between:

- the Euclidean geometry and the group of movements (not truly covering infinitesimal displacements)

- the affine geometry and the affine group (not truly covering all kinds of infinitesimal displacements)

- the projective geometry and the projective group (modeling the "infinity" by managing straight lines, which clip at infinity (but not truly covering all kinds of infinitesimal displacements)).

With respect to our proposition above we propose and claim the following relationship:

"A truly infinitesimal geometry can only be defined by an infinitesimal rotation group":

a "truly" infinitesimal affine (i.e. parallel) only geometry requires uniquely to-be-defined (measurement) directions of the required displacements (which relates one-to-one to the underlying space (-time) dimension). "All" remaining potentially other "out-of-scope "displacements" are of same cardinality as the unit interval, i.e. same cardinality as the field of the real numbers (i.e. the same cardinality as the field of the Non-standard numbers)!!

Therefore, the affine geometry should not be accepted as a "truly" infinitesimal model.

Hilbert scales and differentials
and extended Plemelj Green formula

Let grad(u) denotes the gradient operator applied to a function u and S(u) be the Calderon-Zygmund operator according to (BrK). In variational theory the Dirichlet integral D(u,v)=(grad(u),grad(v)) defines the energy inner product with (Sobolev) domain H(1) x H(1).

The key idea is to replace the gradient (energy) operator by the Calderon-Zygmund operator and the Dirichlet integral by the corresponding inner product (S(u),S(v)) with corresponding energy inner product with the domain H(1/2) x H(1/2). This then implies a physical state Hilbert space H(-1/2), enabling continuous spectrum in H(-1/2)-H(0), while still governed by the Heisenberg principle with respect to the sub-space H(0).

The (singular) Calderon-Zygmund integro-differential operator enables less regularity assumption to its domain than (H(1/2)) than standard theory (H(1)). The Dirichlet integral goes along with modeling energy and momentum (H(1)), which requires the concept of space and extended bodies within this space (WeH, III, 22, d). The primary physical concepts and physical laws are the laws of conservation of energy and momentum.
see also

http://www.navier-stokes-equations.com/

Eskin G. I., "*Boundary Value Problems for Elliptic Pseudodifferential Equations*", AMS, Providence, Rhode Island, Trnaslations of Mathematical Monographs Vol. 52, 1981
Lifanov I.K., Poltavskii L.N., Vainokko G.M., "*Hypersingular Integral Equations And Their Applications*", Chapman & Hall/CRC, Boca Raton, London, New York, Washington D.C., 2004
Translating the equation

$$E = E(kin) + E(pot)$$

into quantum operator language, means, that the two domains of the operators on the right side of the equation above are orthogonal. In our proposed less regular Hilbert space Environment this will be no longer the case, i.e. there is an intersection between both domains, which is not equal to zero. This means there is no discrete "jump" from kinetic to potential energy anymore. But by orthogonal projection of the corresponding variational equation in H(-a) into the higher regular Hilbert space the sum above becomes the corresponding approximation solution.

The same argument is valid for Einstein's thermo-dynamic theorem, built as the sum of quadrates of energy variances, based on classical particle and wave theory ((HeW) V.7 (110)).

With respect to an alternative definition of a mass element "dm", we refer to the great book of

(PlJ) Plemelj J., "*Potentialtheoretische Untersuchungen*“, B. G. Teubner Verlag Leipzig, 1911

It provides a physical interpretation of a mass element "dm" which defines a new concept of a "mass element" creating a potential not only by a density of the mass, but by the element "dm" itself.

In case the mass element "dm" does have also a density it roughly holds:

$$((dm,dm))=(m,m), \text{ as it holds } (Hm,Hm)=(m,m) \ .$$

This means, that the quantity of a quantum "dm" in the sense of quantum mechanics (as an element of the Hilbert space L(2)) corresponds to the norm of the mass element "dm" in our new ground state energy model, which is "just" and only the physical state of its energy (nothing more, but also nothing less!).

With respect to (complementary) variational methods (Friedrichs, Noble) we refer to

Arthurs A. M., "*Complementary Variational Principles*", Clarendon Press, Oxford, 1970

Velte W., "*Direkte Methoden der Variationsrechnung*", Teubner Studienbücher, 1976

We emphasis that the dual operator of T:=grad is given by T(*)=-div, while the dual operator of T:=curl is given by T(*)=curl.

The above enables a quantum gravity model, which supports

- the definition of a manifold, providing a purely inner geometry (1st fundamental forms only), building the Hamilton9an formalism on a H(-1)-based quantum mechanics model

- a purely field based theory enabling a purely infinitesimal geometry with proper linkages to differential forms. Those build the foundation of nearly all relevant physical models.The less regular Hilbert space framework H(-a) than the standard L(2)=H(0)-Hilbert space enables a differentiation between the Hamiltonean and the Lagrange formalism. The Legendre transformation proves the equivalence of both formalisms, in case the Legendre transformation is well defined. If this would be no longer the case, the Hamiltonean formalism (action minimization in H(-a), which is about purpose) keeps valid, but the Lagrange formalism (work minimization in H(0), which is about causality) is only be defined, if experiments gives results (to be modelled by probability theory), which needs to be reflected and validated by an appropriate physical model. The first one is beyond the trancendence border, while the causality model is part of the physics world.
In the context of "*Emmy Noether's Wonderful Theorem*" (D. E. Neuenschwander, The John Hopkins University Press, Baltimore, 2011) we quote:

Noether E. (Invariante Variationsprobleme): "*The problems in variation here concerned are such as to admit a continuous group (in Lie's sense); .... what is to follow, therefore, represents a combination of the methods of the formal calculus of variations with those of Lie's group theory*"Ramond P. (Field Theory: A Modern Primer (1981). "*It is a most beautiful and awe-inspiring fact that all the fundamental laws of Classical Physics can be understood in terms of one mathematical construct called the* ***Action****. It yields the classical equations of motion, and analysis of its invariances leads to quantities conserved in the course of the classical motion. In addition, as Dirac and Feynman have shown, the* ***Action*** *acquires ist full importance in Quantum Physics*."

The Calderon-Zygmund Pseudo Differential Operator

The singular (Calderon-Zygmund Pseudo Differential) operator with domain of (Cartan's) differential forms is proposed to be the non-standard alternative to the "standard", non-bounded (momentum) differential operator. It is basically an isomorphism from H(a+1) --> H(a) with a real. The requirements from physics determines the setting of the scale factor a: it is proposed to put a:=-1 in order to ensure that the range of S is isomorph to L(2)=H(0).

@ http://www.navier-stokes-equations.com/Author-s-papers

The proposed mathematical Hilbert space framework in combination with singular integral operators has been successfully applied also for aerodynamics and electrodynamics:

Lifanov, I. K., Nenashev A. S., Generalized functions on Hilbert spaces, singular integral equations, and problems of aerodynamics and electrodynamics.pdf

We emphasis that the (Poisson formula) series representations of the cot(x) and the Dirac functions are convergent in H(a) with a<-1/2.

C. Quantum Gravity

enabling a truly infinitesimal geometry replacing Weyl's affine geormetry

which is about a Hilbert space of differential forms with appropriately defined inner product replacing an exterior algebra (with exterior derivatives) *over* differential forms.

Prolog

(RoC) xi, "*The problem of what happens to classical general relativity at the extreme short-distance Planck scale of 10*exp(-33) cm is clearly one of the most pressing in all of physics. It seems abundantly clear that profound modifications of existing theoretical structures will be mandatory by the time one reaches that distance scale. There exists several serious responses to this challenge. These include effective field theory, string theory, loop quantum gravity, thermo-gravity, holography, and emergent gravity. .....*

.... it is probably that all these ideologies, including my own (which is distinct from the above listing), are dead wrong. The evidence is history: from the Greeks to Kepler to Einstein there has been no shortage of grand ideas regarding the basic questions."

(RoC) Rovelli C., "*Quantum Gravity*", Cambridge University Press, 2004

(VeW) Velte W., "*Direkte Methoden der Variationsrechnung*", Teubner Studienbücher, 1976

Rational

The characteristic of an affine geometry is the fact, that only parallel distances can be measured against each other, i.e. other kinds of infinitesimal small "actions" between not parallel "objects" are not considered in this kind of "continuum". Vectors are the mathematical model of such translations (resp. parallel displacements) and the underlying (affine) geometry is mathematically described by the group properties of vectors (WeH). An affine geometry with space dimension n is the "same" as its related (n-1)-dimensional projective group. The "enrichment" of the today's n-dimensional space-time affine geometry (manifolds and affine connexions, (ScE), (WeH1) and quotes § 18 below) goes along with the concept of **exterior derivatives** to allow "measurements" and the definition of appropriate metrics resp. to link to the Riemannian metric and the concept of curvature.
The Sobolev H(1/2) space on the circle plays a key role of universal period mapping universal Teichmüller parameter space for all Riemann surfaces via quantum calculus:
Biswas I., Nag S., Jacobians of Riemann Surfaces and the Sobolev Space H(1 2) on the Circle
Nag S., Sullivan D., Teichmüller theory and the universal period mapping via quantum calculus and the H(1 2) space on the circle.

We propose a quantum gravity model

- building on **Hilbert space**, alternatively to ***manifolds*** (metric space, only)

- enabling an infinitesimal small geometry model with an **inner product** defined by "**rotating differential forms**", alternatively to ***exterior derivatives*** based on ***differentiable (!) manifolds***

- enabling a **truly infinitesimal geometry**, alternatively to the **affine connexions** (affine, parallel infinitesimal displacements, only)

- not changing the way, "how to measure distances" (**Archimedean** axiom), but changing the "what to be measured", i.e. the structure of the underlying field from an ordered to a **non-ordered field**

- not increasing the "degree" of transcendence "complexity" (knowing that this is a question of yes/no, of course), if this is measured by Cantor's definition of cardinality (as the field of Non-Standard numbers *R does have the same cardinality than the field of real numbers R)

- applying the **Riesz and Caldéron-Zygmund Pseudo Differential Operators** (PDO) with domains in **Hilbert spaces H(-a), a>0** enabling convergent (!) quantum oscillator energy series in a Hilbert space H(-a), for appropriate a>0.

Here we are:
Braun K., A quantum gravity and ground state energy Hilbert space model

Mass and vacuum

"Atoms" contain basically no mass nearly all of the mass is "built" of the quantum fluctuation of the vacuum energy. This vacuum energy fluctuates, but is finite. It presents itself in form of gluons, which are the interconnection particles, which hold together the quarks. The mass of a proton consists nearly exclusively of the energy of the gluons:

*"**Mass is essentially the manifestation of the vacuum energy**"*

*.Light consists of particles, as the current of electrons increases with the increase of the frequency, but not with the increase of the intensity (the "force" of the light). This phenomenon leads Einstein to the concept of photons with minimal quantum energy. But photons have no mass, nevertheless it holds the Einstein equation: E=m*c*c. In addition light is an electro-magnetic wave in the sense of the Maxwell equations.*

The energy of the Einstein gravitation field is all time negative. The energy in the universe is constant.

Not everything what happens does have a root cause. But is the result of the human "pattern thinking". How in this context can the phenomenon of "time" be explained?

The Higgs boson combines the existence of mass together with the action of the weak force. But why it provides especially to the quarks that much mass, is still a mystery.

We emphasize that our "ground state energy" model, which can "define" a quantum "object" as an element of a H(-1) Hilbert space models a spontaneous (Higgs-) breakdown just by applying the projection operator from H(-1) into H(0)=L(2) Hilbert space.

From the original famous paper of Higgs (see below) we recall the following statements:

...."*the idea, that the apparently approximate nature of the internal symmetries of elementary-particle physics is the result of asymmetries in the stable solution of exactly symmetric dynamical equations .... is an attractive one. .... Within the framework of quantum field theory such a "spontaneous" breakdown of symmetry occurs if a Lagrangian, fully invariant under the internal symmetry group, has a structure that the physical vacuum is a member of a set of (physically equivalent) states which transform according the a nontrivial representation of the group. .... That vacuum expectation values of scalar fields, ... might play such a role in the breaking of symmetries.... in a theory of this type the breakdown of symmetry occurs already at the level of classical field theory...."*

We emphasize that our model fits to this statement, while being valid at the same time for the Maxwell equations without any further requirement for additional space-time dimensions to keep consistency between the models. We note that the Dirac delta function is an element of H(-s) for s>n/2, whereby n denotes the dimension of the field.

SMEP & NMEP

SMEP

The today's Standard Model of Elementary Particles (SMEP) is "just" modeled as the "orthogonal" group stick together by the three "force specific" (gauge) groups. The Standard (field) Model of Elementary Particles (SMEP) is given by SU(3) x SU(2) x U(1). Its components are the following interaction dynamics fields:

1. Electromagnetic Interaction Dynamics (EID): U(1)

2. Weak Interaction Dynamics (WID): SU(2) x U(1)

3. Strong Interaction Dynamics (SID): SU(3) .

NMEP

A Non-Standard Model of Elementary Particles (NMEP) is proposed. It is based on the proposed new ground state energy model, as described in

(BrK) Braun K., "*A new ground state energy model*".

It is a rotation symmetry group, which is built on a negatively scaled Hilbert space, which builds the framework of Pseudo Differential Operators (PDO) with domains of differentials. It leads to a geometry based field theory, which is independent and therefore does not need to build on EID, WID and SID:

It is about a truly and purely (intrinsic) infinitesimal geometry, which enables "Differentials (monads) Interaction Dynamics (DID)" and which is built on the 4 dimensional space-time Minkowski space.

The negatively scaled Hilbert space enables a quantum state statistics (expectations value and variance) for bosons in H(-a), a>0 framework. The consequence is that the related “expectation value and variance” measures are now decoupled from the corresponding probability theory measures. By this the statement “God does not throw dice” is true, but, at the same time, one has to add, that “God doesn’t count, measure and gauge”, and therefore god doesn’t need finite “length” units to measure distances, especially god does not need the Archimedean axiom.

We note that the alternative to the ""Superstring" "theory"", the “Loop Quantum Gravity”, is built on a Hilbert space **K(diff)**, modeling 3D diffeomorphism invariance and transformation properties of spin network states under diffeomorphism ((RoC) 6.4). The Hamiltonian for the fields is built in a standard analysis framework and defined by ((RoC) 6.4.2)

H := H(Einstein) +H(Yang-Mills) + H(Dirac) + H(Higgs) , ((RoC) 7.3.

(RoC) 1.2.1: “*The LQG is characterized by the choice of a different algebra of basis field functions, as in Quantum Field Theory (QFT). In conventional QFT this is generally the canonical algebra formed by the positive and negative frequency components of the filed modes. The quantization of this algebra leads to the creation and annihilation operators a and a(+). The characterization of the positive and negative frequencies requires a background space-time. In contrast to this, what characterizes LQG is the choice of a different algebra of basis field functions: a non-canonical algebra based on the holonomies of the gravitational connection. The holonomy (or “Wilson loop”) is the matrix of the* ***parallel transport along a closed curve***.”

Therefore the LQG struggles with the same handicaps as H. Weyl’s affine geometry:
(WeH3), p. 18: „*Ich bin fest davon überzeugt, dass die Substanz heute ihre Rolle in der Physik ausgespielt hat. Der Anspruch dieses von Aristoteles als einer metaphysischen konzipierten Idee, , das Wesen der realen Materie auszudrücken – der Anspruch der Materie, die fleischgewordene Substanz zu sein, ist unberechtigt. Die Physik muss sich ebenso der ausgedehnten Substanz entledigen, wie die Psychologie schon längst aufgehört hat, die Gegebenheiten des Bewusstseins als „Modifikationen“ aufzufassen, die einer einheitlichen Seelensubstanz inhärieren.*

Weyl affine connexions

affine connexions → “rotating differentials”

We propose to replace H. Weyl's infinitesimal small affine geometry by an infinitesimal small rotation geometry. At the same time this validates Riemann's conjecture about an Euclidean rotation geometry. The rotating "objects/substances" are differentials, which links back to Leibniz's concepts of monads. At the end the concept of a hyper-real universe beyond (Kant's) physical reality (i.e. physics) becomes (Kant's and Plato's) transcendental "reality", which "*is beyond the borders of sensuous experience, where no other theoretical knowledge is possible. In order to lend the term “objectivity”, it needs to be supported in any way by intuition*".

(KaI): "*Ich behaupte aber, dass in jeder besonderen Naturlehre nur so viel eigentliche Wissenschaft angetroffen werden könne, als darin Mathematik anzutreffen ist*".

(KaI) Kant I., "*Metaphysische Anfangsgründe der Naturwissenschaften*"

(PoP) Poluyan P. V., "*Non-Standard Analysis of Non-classical Motion; do the hyperreal numbers exist in the Quantum-relative universe?"*

http://www.oocities.org/quantum_math_poluyan/hy_nu/hy-nu.htm

The proposed mathematical gravity model builds on the definition of the inner product of the "new ground state energy" model. The key mathematical tool is the (Pseudo Differential) Riesz operators being applied to differentials.

The concept to apply Riesz operators to differentials goes in line with J. Plemelj’s alternative definition of a potential building on a mass element "dm", alternatively to a mass density, only. The corresponding Klein´s group, which characterizes the geometry, is the infinitesimal ***rotation group***. This also goes along with Riemann's conjecture of an **infinitesimal small Euclidean geometry**. The Hilbert space is also related to the L(2) Hilbert space, which is the as-is framework of today's quantum mechanics and quantum field theory. Consequently the Hilbert scale (approximation) theory is the proper quantum gravity modeling framework.

As an alternative to the today´s Hermite polynomial orthogonal system we propose the modified Lommel polynomials (D. Dickinson, "On Lommel and Bessel polynomials", Proc. Amer. Soc. 5 (1954) 946-956).

The proposed model overcomes the still unsolved particle-wave paradox providing a purely geometrical rationalized "continuum" (H. Weyl). The model overcomes the "contacting body" interaction challenge of "quants without extension, but equipped with flavor and spin". The latter constraint generates a paradox; this handicap is "solved" by H. Weyl´s affine (only!) geometry, whereby the affine geometry model only focuses on parallelized “quants” (i.e. is restricted to affine vectors only). The related mathematical concept to handle to "contacting body" issue is about the concept of continuous transformations, built on S. Lie's concept of contact transforms.

Einstein universe

Differential manifolds → distributional Hilbert scale

The famous Einstein field equations give the relationship between space-time tensor G=G(i,k) and the corresponding energy-matter tensor T=T(I,k). One of the challenges of this great system of partial differential equations is the fact, that the space-time tensor describes the universe structure and the energy-matter tensor describes the "dynamics" within this universe, i.e. the space-time tensor is a modelling element of the "stage", while the energy-matter tensor is a modelling element of "events resp. actors" happening/acting on this stage.

The equations say that one of the elements determines the other. From a model design perspective this is the root cause, while the field equations do not have, and cannot have, initial value functions or boundary value functions, which is a mathematical problem per se. In the corresponding Hilbert-Einstein functional minimizing (the action) description this issue is reflected by not mathematical adequately defined domain of the underlying operator equation. With respect to a quantum gravity theory this is seen by the author as first opportunity for a more generalized, but then appropriate Hilbert-Einstein characterization of the Einstein field equations. As all those kinds of mathematical models are anyway describing transcendental areas, there is no loss of "truth", but the chance to get a consistent model, which fits also for quantum theory.

The conceptual mathematical elements of quantum theory are functional analysis, Hilbert space framework with INNER product and spectral theory.

The conceptual mathematical elements of gravity theory are PDE, manifolds framework with metric and **exterior derivatives**, and affine connexions.

From a properly designed (mathematical) model of quantum gravity all elements from the above need to be deduced, if the current (gravity and quantum) models should be kept valid for their specific areas. How this can be achieved, when there is no possibility

- to derive a Hilbert space framework from a purely metric space framework (the other way around would be possible, as any Hilbert space is also a metric space)

- to derive a manifolds/affine connexions concept based on exterior differential forms from a (quantum theory) Hilbert space framework ?

The only way out, based on the constraint to keep the Hilbert space framework, is, to build an alternative Hilbert space (which is basically about the definition of an appropriate inner product), which is able to define a geometry for infinitesimal small differentials. This is the linkage to the section "ground state energy".

The above mentioned issues with initial value and boundary functions of the Einstein field equations then turn over to adequate definition of the domains of operators, acting on those domains. Of course, the regularity of such a Hilbert space needs to be less regular than current quantum theory framework. To derive current quantum theory from the new quantum gravity Hilbert space framework can then be achieved by standard orthogonal projection, enabling also the full power of spectral theory; Hilbert space approximation theory then can even quantify the "approximation error": the "truly" quantum gravity model is given by operator norm minimization formulation in the less regular HS framework (which is equivalent to corresponding variational equation in appropriate Hilbert space energy inner product), while the "approximating model" gives the today's "observation model", which is basically the probability Lebesgue L(2) Hilbert space with its orthogonal (Hermite) polynomials.The practical utility of the field equations is pure. There are only a few metrics/solutions derived out of it. The most prominent and applied one is the Schwarzschild metric. The additional mathematical assumptions to define a well posed problem to enable its calculation, are very strong (Trefftz E., "*Das statische Gravitationsfeld zweier Massenpunkte in der Einsteinschen Theorie*"). To the author's best knowledge this metric is in most of the time the preferred metric, when analyzing black holes, big bang and related singularities scenarios. The outcome/consequence of the model seems to "generate" necessarily singularities, which then becomes the starting point for philosophical discussion about space-time structure (expanding universe, the very first moments of the universe, etc.). Why not challenging the mathematical assumption of the model itself, which is basically the metric (affine connexions enabling) space, with its missing capability to capture non-affine manifolds

relationships/derivatives. An alternative Hilbert space framework would very likely provide alternative interpretations of "time arrow" and "entropy".

Einstein action minimization

Hilbert Einstein action minimization in Hilbert scale

*which is a **teleological principle**, e.g. similar as e.g. the*

1. Real number definition, e.g. by Dedekind cut or Cauchy criteria

2. Non-standard number definition by maximal ideals (whereby the field of Non-Standard numbers has the same cardinality as the field of real numbers; the only differentiator is by an additional valid Axiom for the real numbers, which is the Archimedean axiom.
Today's gravity model is based on the mathematical concept of **exterior differential forms**, based on the concepts of differentiable (!),(ScE1)) manifolds, affine connexion and variational principles.

We propose the build a modified gravity model based on the mathematical concept of **"interior" differential "elements"**, as intrinsic part of a (distributional, negative-scaled) Hilbert space.

As a Hilbert space is the truly framework to model "geometry", this provides the proper framework for the ***General Relativity*** *(i.e. the Hilbert-Einstein action minimization principle can formulated as operator norm minimization problem, which is equivalent to a corresponding energy inner product variational equation, (VeW)). A*t the same time, it's already the appropriate framework for ***Quantum Field Theory***. Therefore it defines the proper framework for a

Quantum Gravity.

Braun K., A quantum gravity and ground state energy Hilbert space model
Braun K., An alternative quantization of H=xp

Alternative concepts

A Hilbert space framework for a quantum gravity model

- differentiable manifolds being replaced by Hilbert space

- exterior derivatives being replaced by inner products

- affine connexions being replaced by rotating differentials

- tensor analysis being replaced by Pseudo-Differential Operator theory

- affine (only) infinitesimal small geometry (affine connexions) being replaced by truly infinitesimal small (Hilbert space) geometry

- Lie's contact transform becoming obsolete

Manifolds/Tensor Analysis & Hilbert space/ PDO

Generalized functions on Hilbert spaces

The proposed mathematical Hilbert space framework in combination with singular integral operators has been successfully applied also for aerodynamics and electrodynamics:

Lifanov, I. K., Nenashev A. S., Generalized functions on Hilbert spaces, singular integral equations, and problems of aerodynamics and electrodynamics

We emphasis that the (Poisson formula) series representations of the cot(x) and the Dirac functions are convergent in H(a) with a<.-1/2.

Calderon-Zygmund Pseudo Differential Operator

The singular (Calderon-Zygmund Pseudo Differential) operator with domain of (Cartan's) differential forms is proposed to be the non-standard alternative to the "standard", non-bounded (momentum) differential operator. It is basically an isomorphism from H(a+1) --> H(a) with a real. The requirements from physics determines the setting of the scale factor a: it is proposed to put a:=-1 in order to ensure that the range of S is isomorph to L(2)=H(0).

@ http://www.navier-stokes-equations.com/Author-s-papers

Riesz Operators

The Riesz operators fulfill the following crucially property with respect to the rotation group SO(n), ((StE), (BrK) p.13):

Let m be the Mikhlin multipliers of the Riesz operators and r an element of SO(n), then m(r(x))=r(m(x)).

As a consequence there is a corresponding change from a Riemann manifold (with the related concept of “extension quantities” (Grassmann)) to a Hilbert space framework for differentials (see also (ScE) 1.1.3, to model "extended quantities" in a "continuum", whereby differentiable manifolds are required in case of a Riemannian manifold). We note some other properties of the Riesz resp. the Hilbert operators: The Hilbert transform (as well as the Riesz operators) are “symplectic-like” in the sense, that it holds (Hu,v)=-(u,Hv), H*H=-I. The Riesz operators commutes with translations and homothesis (PeB) example 9.9.-9.11).

The constant Fourier coefficient of a Hilbert transformed function vanishes. This property plays a key role in the two proofs of the Riemann Hypothesis. At the same time there is a similarity to the "cusp form" with its vanishing zero mode in the context of spectral theory in hyperbolic surfaces. This fact indicates a relationship to the proposed vector (domain) field (see e. g. Borthwick D., Introduction to Spectral theory in Hyperbolic Surfaces).

Riemann's continuous manifolds (which ends up to be necessarily differentiable (!!))

it is proposed to replaced manifolds by distributional Hilbert space, which allows a truly modelling of geometry.

The terminology of "multiple extended quantities" was used by B. Riemann synonym to a "continuous manifold", conceptually based on two essential attributes: "continuity" and "multiple extensions". Since Helmholtz, Riemann, Poincare and Lie the history of manifolds are the attempt to build a mathematical structure to model the whole (the continuum) and the particular (the part) to put its combination then into relationship to describe motion, action etc. From the paper from E. Scholz below we recall the two conceptual design strategies:

Strategy I: design of an "atomistic" theory of the continuum: to H. Weyls's opinion this contradicts to the essence of the continuum by itself

Strategy II: develop a mathematical framework, which symbolically explores the "relationship between the part and the whole" for the case of the continuum.

The later one leads to the concept of affine connexion, based on the concept of a manifold, which were developed during a time period of about 100 years.

The concept of manifolds leads to the concept of co-variant derivatives, affine connexion and Lie algebra to enable analysis and differential geometry, but (according to H. Weyl in E. Scholz 1) ...a .." *truly infinitesimal geometry ... should know a transfer principle for length measurements between infinitely close points only*."

Inner products and first fundamental form

We note, that the **1st fundamental form** is related to **(inner) geometry** concepts like lengths, angles**,** Christoffel symbols & the Levi-Civita derivative. The corresponding mathematical model concepts are inner products and (dual) Hilbert spaces. The **2nd fundamental** form addresses the (parallel/affine) displacement of tangential (vector) spaces, i.e. it leaves the (inner geometry) Hilbert space framework. The additionally required mathematical concepts are about "hyper areas" and related distance functions. Therefore, not only the terminology changes to "**exterior geometry**". The gauge theory framework is a consequence to re-build again necessary vector space properties.

Lie transforms

We refer to

Lie S., Ueber die Grundlagen der Geometrie (1890), Wissenschaftliche Buchgesellschaft, Band CXX, Darmstadt, Sonderausgabe MCMLXVII

Quote (p.2): "Für den dreifach ausgedehnten Raum können die betreffenden Eigenschaften folgendermassen zusammengefasst werden:

die Bewegungen des dreifach ausgedehnten Raumes bilden eine Gruppe von reellen Transformationen, welche die folgende Eigenschaft besitzt: Wird ein reeller Punkt und ein reelles hindurchgehendes Linienelement festgehalten, so ist immer noch continuierliche Bewegung möglich; wird jedoch ausserdem ein durch das Linienelement gehendes reelles Flächenstück festgehalten, so bleiben alle Punkte des Raumes in Ruhe.

Diese Eigenschaft kommt der Gruppe der Euclidischen und der Gruppe der Nichteuclidischen Bewegungen, aber keiner anderen Gruppe zu. .....

In einem Raum mit mehr als drei Dimensionen lassen sich die beiden betreffenden Gruppen in ganz entsprechender Weise charakterisieren. Dagegen stellt sich die Sache wesentlich anders in einem zweifach ausgedehnten Raume; in der Ebene giebt es noch weitere Gruppen, welche die genannten Eigenschaften besitzen."

Lie's theory of "contact transformation", which builds the foundation of the Lie theory in the context of the manifolds:

http://openlibrary.org/books/OL7045943M/Geometrie_der_berührungstransformationen

Lie S., "*Vorlesungen über continuierliche Gruppen mit geometrischen und anderen Anwendungen*". Bearb. und hrsg. von Georg Scheffers (1893)

Lie S., "*Ueber die Grundlagen der Geometrie*", Wissenschaftliche Buchgesellschaft Darmstadt, MCMLXVII

Bell J. L., "*Hermann Weyl on intuition and the continuum*"

Scholz E., "*H. Weyl's and E. Cartan's proposals for infinitesimal geometry in the early 1920s"*

Legendre transform

We note that also the Legendre transform is a contact transformation. If the Legendre transform is applicable (ensured by (!) sufficiently high regularity assumptions) it is applied to prove the equivalence of the Lagrange and the Hamiltonian formalisms. We emphasis, that it would be sufficient to have a Hamilton (energy minimization functional) formalism, only, to define existing physical laws in the framework of variational theory. In case of PDO of negative order (in opposite to PDO of postive order, as model of PDE with corresponding regularityx assumption to its domain) the induced Hilbert space with respect to the energy norm is a compactly embedded sub set of the induced Hilbert space with respect to the operator (graph) norm.

Therefore, minimization representations with respect to operator norm are defined, w/o the need that the "standard" minimzation representation with respect to the energy norm (which defined classical resp. weal PDE representation) are not neccessarily defined.

Therefore, it's not a necessary (from a mathematical modelling perspective) that today's PDEs (representing physical models, e.g. the Maxwell equations) need to be valid for both representations, the integral form and in the differential form.

Cartan's differential forms

Probably interesting to mention that today physicists calculate with differentials as "number" objects, but they neglect its physical existence as "particle" objects, while mathematicians calculate with differentials only as "functionals" or within the Cartan differential form calculus, but accept those "objects" as well defined existing "objects" of an e.g. Hilbert space (which is the today´s mathematical standard framework for quantum mechanics modeling quantum "objects", ending up with quotes like the following one from N. Bohr: "*If people are not scared about the quantum theory, they haven´t understood it*").

Berkeley described Leibniz' differentials as "*ghosts of departed quantities*":

Dray T., Manogue C. A., Putting Differentials Back into Calculus

The alternative normal derivative definition of J. Plemelj

A new mathematical concept to define the normal derivative on the boundary with only "continuous" regularity assumption (only using interior domain values) was given by

J. Plemelj, Potentialtheoretische Untersuchungen, B. G. Teubner Verlag Leipzig, 1911

(pdf-copy: see www.navier-stokes-equations.com)

In J. Plemelj´s mathematical concept there exists a massless particle in the form of a differential connected to potentials defined by Stieltjes integrals“; in section I, §8 he states: "*bisher war es ueblich fuer das Potential V(p) die Form (...) vorauszusetzen, wobei dann (...) die Massendichtigkeit der Belegung genannt wurde.*

Eine solche Annahme erweist sich aber als eine derart folgenschwere Einschraenkung, dass dadurch dem Potentials V(p) der groesste Teil seiner Leistungsfaehigkeit hinweg genommen wird."

Archimedian non-ordered fields

Preliminary Notes

(StJ), p.27: "...*the set of **real numbers** is seen as a model for the **number line**. In today's world this number line is perceived as a simple term. But this is not the case. A **"point"** on the number line is **a whole universe**, if one realizes that such **a** "**point**" is a whole **universe**, which **is** about **a Dedekind cut of the infinite number of rational numbers***".

(WeH), p. 1, "Preface", 1917: "*At the center of my reflections stands the conceptual problem posed by the continuum - a problem which ought to bear the name of Pythagoras and which we currently attempt to solve by means of the arithmetical theory of irrational numbers*".

(WeH1), p. 86: "*While topology has succeeded fairly well in mastering continuity, we do not yet understand the **inner meaning** of the restriction to **differential manifolds**. Perhaps one day physics will be able to discard it. At present it seems indispensable since the **laws of transformation of most physical quantities are intimately connected with that of the differentials** dx(i)."* ...

... "*As the true lawfulness of nature, according to Leibniz's continuity principle, finds ist expression in laws of nearby action, connecting only the values of physical quantities at space-time points in the immediate vicinity of one another, so the basic relations of geometry should concern only infinitely closely adjacent points ('near-geometry' as opposed to far-geometry'). **Only in the infinitely small may we expect to encounter the elementary and uniform laws, hence the world must be comprehended through its behavior in the infinitely small".***

(WiL), Preface: *"....The book deals with the problems of philosophy and shows, as I believe, that the method of formulating these problems rests on the miss understanding of the logic of our language. Its whole meaning could be summed up somewhat as follows: What can be said at all can be said clearly; and whereof one cannot speak thereof one must be silent. .... The book will, therefore draw a limit to thinking, or rather - not to thinking, but to the expression of thoughts; for, in order to draw a limit to thinking we should have to be able to think both sides of this limit (we should therefore have to be able to think what cannot be thought). ... The limit can, therefore, only be drawn in language and what lies on the other side of the limit will be simply nonsense*."

(ScE), p. 90".... *beide Paradoxa lösen wird (...), indem man dem Bau unsrer westlichen Naturwissenschaft, die östliche Identitätslehre einverleibt. ... Ich wage, den **Geist unzerstörbar** zu nennen, denn er hat sein **eigenes und besonderes Zeitmaß**; nämlich er ist **jederzeit j e t z t**. Für ihn gibt es in Wahrheit weder früher noch später, sondern nur Jetzt, in das die Erinnerungen und die Erwartungen einbeschlossen sind."*

(ToA), chapter I: "*Time and numbers are different terms, which seem to be independent. This is valid as long as one do not wants to measure. The question, if "time" is measurable or not can be answered positively or negatively. ... Bergson defined the duration as the "real" time.*"(ScE): *"Bohr's standpoint, that a space-time description is impossible, I reject a limine. Physics does not consist only of atomic research, science does not consist only of physics, and life does not consist only of science. The aim of atomic research is to fit our empirical knowledge concerning it into our other thinking. All of this other thinking, so far as it concerns the outer world, is active in space and time. If it cannot be fitted into space and time, then it fails in its whole aim and one does not know what purpose it really serves."*

The **Archimedean axiom** (axiom of Eudoxos, (WeH) p. 41, 45):

From every positive number a one can obtain a number greater than 1 by repeating addition

$$a+a+a....+a \text{ (n times)} > 1$$

where n is a positive integer number.

The field of Non-Standard numbers ***R** is an Archimedean **non-ordered field**, while the field of real numbers **R** is an Archimedean **ordered field**.

This is curious information, whereby in gravitation theory a black whole is seems and accepted as a

"real" "objects" by human's mind (still sophisticated phenomena, but consistently described in mathematical language).

A. Robinson The metaphysics of calculus II.pdf

We note that the "real number" field without the **axiom of Eudoxus** expands to the "Non Standard number" field, with same cardinality (A. Robinson) and same all other properties. Needless to mention, that experimental physics is anyway only requiring rational numbers, while theoretical physics models are calculating with differentials in same manner as with irrational numbers.

The Archimedean axiom is "just" about "distance" measurements of the real x-axis by an integer multiple of a given length standards. Now the delta of non-standard to standard is not about the way, how to measure, but about "ordered field" versus "non-ordered field". **Human beings might need this specific type of "order", but does Nature need this as well?**

The most probably strongest principles in Nature is "entropy", which is the opposite of "ordering".

How our current understanding and interpretation of the physical/ measurable world would look like, if our children would learn right from the beginning mathematical analysis as described in the language of "ideal" points? Analysis, as teached in school, become "standard", because it's part of the standard education program; if it would be replaced by "Non-Standard Analysis" this would be perceived as "standard".The current "Non-Standard-Analysis" would be a standard one and the other way around. This would mean that our universe would be realized and interpreted by mind as "Non-Standard", as we all were learned at school, but "standard" in the way, how Leibniz would have been defined/interpreted the term "differential" and ist actions in the universe. Singularities would become "natural" and consistent to the corresponding physical-mathematical models; big bang would require no t=0perception of R. Penrose versus S. Hawking (mathematicians vs physicists) of what matter are finally goes back to Newton´s miss understanding/ interpretation of Leibniz´s concept of the infinitesimals (monads). Imagine that the development of the infinitesimal calculus would have been built on the original thoughts of Leibniz instead? This would have been lead to the fact, that Robinson´s Non-Standard Analysis would be teach at school as "standard" and an "ideal" point would be a natural "object", as it is a "real" objects/numbers (even if it is a sophisticated transcendental number) today.

Fundamental theorem of set theory (K. Gödel, P. Cohen):

The Cantor continuum hypothesis is neither provably, nor refutably.

(RiM): The system *R of hyper-real numbers (nonstandard reals) is a way of treating infinite and infinitesimal small quantities. The cardinality of the real and hyper-real number fields is the same. The Archimedean axiom, which is related to measure distances on the real number axis with a finite measuring stick of finite length, is valid only for the real number field:

***R is an Archimedean *non-ordered field*, while the field R of the real numbers is an Archimedean *ordered field*.**

Some early thought about the topics above:
Nov 2011, Braun K., Thoughts about a Quantum Gravity Theory
June 2013, Quantum gravity model related mathematical Areas

Hyper-real Universe

Is just a term, like a “Mickey Mouse Universe”

built on purely mathematical terms, in this case based on the "non-standard" analysis framework, alternatively to the "standard" analysis framework (whereby the term "standard" is "just" due to a suitable conditioning in early childhood); the field of non-standard (or Mickey Mouse) numbers (where the field of real numbers is a subset of it) has same cardinality (as defined by Cantor), fulfills same (mathematical) Archimedean principle, but is just non-ordered (according to its mathematical definition), as the field of real numbers. If a (mathematical) Mickey Mouse universe enables a consistent quantum and gravitation theory it's an adequate model, which explains phenomena from both areas, simultaneously. They are images in our mind.

"Something, which is named "*hyper-real*", is perceived as an utopic status, which is only applicable in science fiction stories. But it's just a mathematical definition of something, which is very similar to "real". A "*hyper-real*" universe corresponds to an Archimedean, non-ordered field, while a "*real*" universe corresponds to a Archimedean ordered field. Both fields have same cardinality, which is Cantor's (mathematical) definition to qualify and quantify the different kinds of infinity, e.g. integers, rational numbers, real numbers.

Schrödinger E., "Science and Humanism": 7. The intricacy of the continuum: *... "It seems simple to us, because the idea of the continuum seems simple to us. We have somehow lost sight of the difficulties it implies. That is due to a suitable conditioning in early childhood. Such an idea as 'all the numbers between 0 and 1' or 'all the numbers between 1 and 2' has become quite familiar to us. We just think of them geometrically as the distance of any point like P and Q from 0. ... Among the points P and Q there is also the square(2). We are told that such a number as square(2) worries Pythagoras and his school almost to exhaustion. ...There worry was highly creditable. ... The idea of a continuous range, so familiar to mathematicians in our days, is something quite exorbitant, an enormous extrapolation of what is really accessible to us." ...*
Basically people take as a real "particle entity", what's defined as real number, instead of what's alternatively possible as hyper-real number. This is just due to the fact, that Leibniz (monads, differentials) lost the marketing fight against Newton (particle) concerning the branding and related perception of "differential calculus". This led to so-called *Standard Analysis* and to the perception, that a "real / physical" particle (required as test particle in mathematical physics) is "identical" with a real number: then, finally perception became "reality" in human (western) mind.

(PoP) Poluyan P. V., "*Non-Standard Analysis of Non-classical Motion; do the hyperreal numbers exist in the Quantum-relative universe?"*

1. already in standard models "particles" are "transcendental objects", which are mathematically modeled by real numbers. As an option to this we propose non-standard numbers alternatively, i.e. the monads (= ideal points). The common denominator between both arithmetic models is the Archimedean principle and the same cardinality. The Archimedean principle enables a "measurement" of the distance / "lenght" between zero and any real number on the x-axis by a multiple (integer number) of a given finite measure unit ("!).
Let a=r+i a finite non-standard number with r real and i infinitely small. Then i can be the differential of "something"
2. There was an initial hyper-real „particle“ in the neighborhood of the big bang, the „inflation“.

3. The Weyl curvature of that inflaton was infinitely small, but not zero in the neighborhood of the big bang, i.e. Weyl(i)=i

4. The Ricci tensor, measuring the size of the volume reduction at that point in space-time (i.e. at the point i) was infinitely large, i.e. Ricci(i)=1/i.
5. Taking 3. and 4. as initial value conditions for Einstein's gravity PDE, where the PDE systems are defined as variational equations system, i.e. in weak form only, and formulated in the k-form calculus (coordination system independent)

a. to match to current quantum theory mathematical concepts
b. are embedded in a hyperbolic geometry following R. Penrose´s arguments
c. to link non-standard analysis with Dirac function/Distribution theory.

If there is a chance for a well posted problem a duality/symmetry between Weyl and Ricci could be constructed by the additional condition, that the k-form analogue of Weyl(1/i)=1/i and the k-form analogue of Ricci(1/i)=i. This would define a periodically swinging back and forth (between the infinitely small and the infinitely large) quantum gravity model, where the Weyl and the Ricci tensors are changing their roles.

Some related quotes

1. Well-ordering theorem from E. Zermelo (1904): every set can be well-ordered
2. Theorem of G. Cantor: "*for every set L the cardinal number of its power set is richer*"
Theorem: the cardinal number of the power set of the natural numbers is the same as the cardinality number of the real numbers
3. Cantor's continuum hypothesis: "*every sub set of the real numbers has either the cardinality of the natural numbers or the cardinality of the real numbers*"
The Zermelo "**axiom** of choise" (i.e. every set of non-empty sets has a function of choice) was key/necessarily required to answer the CH of G. Cantor positively.
4. L. Kronecker, "*God made integers, all else is the work of man*"
5. Hoffmann D. W., Die Gödel'schen Unvollständigkeitssätze, Springer Spektrum
For the related Gödel's incompleteness theorems we refer to
Ebbinghaus H.-D., Flum J, Thomas, *Mathematical Logic*
6. Weyl H., "*The Continuum, a critical examination of the foundation of analysis*"

H. Weyl: "***Preface**: It is not the purpose of this work to cover the "firm rock" on which the house of analysis is founded with a fake wooden structure of formalism - a structure which can fool the reader and, ultimately, the author into believing that it is the true foundation. Rather, I shall Show that this house is to a large degree built on sand. I believe that I can replace this shifting foundation with pillars of enduring strength. They will not, however, support everything which today is generally considered to be securely grounded. I give up the rest, since I see no other possibility.*
*At the center of my reflections stands the conceptual problem posed by the continuum - a problem which ought to bear the name of Pythagoras and which we currently attempt to solve by means of **the arithmetical theory of irrational numbers**. ..... Concerning the epistemological side of logic, I agree with the conceptions which underlie Husserl. .... Our examination of the continuum problem contributes to critical epistemology's investigations into relations between what is immediately (intuitively) given and the formal (mathematical) concepts through which we seek to construct the given in geometry and physics*. ...(chapter I, concluding remarks): *The concept of function has two historical roots. first, this concept was suggested by the "natural dependencies" which prevail in the material world - the dependencies which consist, on the one Hand side, in the fact that conditions and states of real things are variable over **time**, the paradigmatic independent variable, on the other hand, in the **causal** connections between **action and consequences**. ....With the help of a tradition bound up with that complex of notions which even today enjoys absolute primacy in mathematics and which is connected above all with the names of Dedekind and Cantor, I have discovered, traversed, and here set forth my own way out of this circle. Only after having done so did I become acquainted with the ideas of Frege and Russell which point out in exactly the same direction*. ....chapter II, §6, *... If the time-points with their relations of "earlier" and "equal" can really furnish the foundation of a pure theory of time, then the intuition of time must suffice to determine whether this correspondence between time-points and real numbers holds or not. If it does not hold, then we should attempt to expand or modifiy our principles of definition in such a way that the desired agreement comes about. ....In confronting these questions we cannot avoid the concept of set (or sequence), no matter how we twist and turn; **and the scope of this concept depends on the principles of definition!** Now, I think that everything we are demanding here is obvious nonsense: to these questions, the intuition of time provides no answer - just as a man makes no reply to questions which clearly are addressed to him by mistake and, therefore, when addressed to him are unintelligible*. ....

Chapter I, §4, *No one can describe an infinite set other than by indicating properties which are characteristic of the elements of the set. And no one can establish a correspondence among infinitely many things without indicating a **rule**, i.e. a relation, which connects the corresponding objects with one another. The Notion that infinite set is a "gathering" brought together by infinitely many individual arbitrary acts of selection, assembled and then surveyed as a whole by consciousness, is nonsensical; "inexhaustibility" is essential to the infinite...But as things now stand we must point out that, in spite of Dedekind, Cantor, and Weierstrass, the great task which has been facing us since the Pythagorean discovery of the irrationals remains today as unfinished as ever; that is, the continuity given to us*

*immediately by intuition (**in the flow of time and motion**) has yet to be grasped mathematically as a totality of discrete "stages" in accordance with that part of its content which can be conceptualized in an "exact" way. More or less arbitrarily axiomatized systems (be they ever so "elegant" and "fruitful") cannot help us here. we must try to attain a solution which is based on objective insight. At this point, we would do well to explore somewhat further the consequences for the foundations of analysis and set theory of our view concerning the concepts of set and function. .....*
chapter II, §6, *The system which, for the moment, we shall call "hyperanalysis" arises if, starting from the level attained in §3 of this chapter, we lay a new foundation for pure number theory, a foundation in which we admit the real numbers as a new basic category alongside the naturals. ... This new system certainly does not coincide with our version of analysis. On the contrary, in hyperanalysis there are, e.g. more sets of real numbers than in analysis. For hyperanalysis admits sets in whose definition "there is" appears in Connection with "a real number". **Thus, hyperanalysis contains neither Cauchy's convergence principle nor, in General, our theorems about continuous functions**. ..."*

7. A common basis to synchronize Kant´s philosophy with mathematics was given by
Riemann B., "On the Hypothesis which lie at the Bases of Geometry"

8. In his work, "Principles of Nature and of Grace Founded on Reason", G. W. Leibniz summed up the problem "why there is somethning and not nothing?" (cited from the book of S. Blackborn, "Philosophy"):
"*Nothing takes place without sufficient reason, that is to say that nothing happens without it being possible for one who has enough knowledge of things to give a reason sufficient to determine why it is thus and not otherwise. This principle having been laid down, the forst question we are entitled to ask will be: why is there something rather than nothing? For "nothing" is simpler and easier than "something". Further, supposing that things must exist, it must be possible to give a reason why they must exist just as they do and not otherwise.*
Now this sufficient reason of the existence of the universe cannot be found in the series of contingent things, that is to say, of bodies and of their representation in souls ... Thus the sufficient reason, which needs no further reason, must be outside this series of contingent things, and must lie in a substance which is the cause of this series, or which is a necessary being, bearing the reason of ist existence within itself; we should still not have a sufficient reason, with which we could stop.And this final reason of things is called God."

9. Some comments from A. Einstein, *Grundzüge der Relativitätstheorie, WTB, Bd. 58, 1956*:
*... (48a) ...Die MAXWELLschen Gleichungen bestimmen das elektrische Feld, wenn die Verteilung der elektrischen Ladungen und Ströme und Ladungen bekannt ist. De Gesetzte aber, nach denen sich Ströme und Ladungen verhalten, sind uns nicht bekannt. Wir wissen wohl, dass die **Elektrizitäten in Elementarkörperchen** (Elektronen, positiven Kernen) bestehen, aber **wir begreifen es nicht vom theoretischen Standpunkte** aus. Wir kennen die energetischen Faktoren nicht, welche die Anordnung der Elektrizität in Körperchen von bestimmter Grösse und Ladung bewirken, und alle Versuche, die Theorie nach dieser Seite hin und zu vervollständigen, sind bisher gescheitert. Wir kennen daher, falls wir überhaupt die MAXWELLschen Gleichungen zugrunde legen dürfen, den Energietensor für elektromagnetische Felder nur ausserhalb der Elementarteilchen.*

*... 49) ... Wir wissen heute, dass die Materie aus elektrischen Elementarteilchen aufgebaut ist, sind aber **nicht im Besitze der Feldgesetze, auf welchen die Konstitution jener Elementarteilchen beruht**. Wir sind daher genötigt, uns bei der Behandlung der mechanischen Probleme einer **ungenauen Beschreibung** der Materie zu bedienen, welche der von der klassischen Mechanik verwendeten entspricht. Die **Dichte der ponderablen Substanz und der hydrodynamischen Druckkräfte** (Flächenkräfte) sind die Grundbegriffe, auf die eine derartige Beschreibung sich stützt.*
Die Gleichung (90) beschreibt die Bewegung (motion) des materiellen Punktes unter der alleinigen Einwirkung der Trägheit (inertia) und Gravitation. ...(90a) ... Die ***Christoffel-Symbole*** (in dieser Gleichung) spielen die Rolle der ***Feldstärke des Gravitationsfeldes***. Diese Grössen haben ***nicht Tensorcharakter***. ...Die Einheit von Trägheit und Gravitation wirkt sich formal dadurch aus, dass wohl der gesamte (Summen-) Term von (90) Tensorcharakter hat, nicht aber die ***beiden Glieder*** einzeln genommen, von denen man in Analogie zu den NEWTON-schen Gleichungen das erste als Ausdruck der ***Trägheit***, das zweite als Ausdruck der ***Gravitationskraft*** zu betrachten hätte.
...unter dem Gesichtspunkte einer tieferen Analyse ist der ***Energietensor der Materie nur als ein vorläufiges, wenig tiefgreifendes Darstellungsmittel für die Materie*** anzusehen. In Wahrheit besteht ja die Materie aus elektrischen Elementarteilchen und ist selbst Teil, ja als der Hauptteil des elektromagnetischen Feldes anzusehen. Nur der Umstand, dass die wahren Gesetze des elektromagnetischen Feldes für sehr intensive Felder noch nicht hinreichend bekannt sind, zwingt uns

vorläufig dazu, die wahre Struktur dieses Tensors bei der Darstellung der Theorie unbestimmt zu lassen.
... (121) .... Die Materie besteht aus elektrischen Elementarteilchen. Diese können auf der Basis der MAXWELLschen Theorie nicht singularitätsfrei als elektromagnetische Felder aufgefasst werden; man braucht in MAXWELLs Theorie nicht enthaltene energetische Terme, um der Tatsache gerecht zu werden, dass das einzelne Elementarteilchen trotz der abstossenden Wirkung seiner gleichnamig geladenen Teile aufeinander Bestand hat. ***Poincare hat daher, um dieser Tatsache gerecht zu irgendwie und vorläufig gerecht zu werden, im Innern dieser Teilchen einen Unterdruck angenommen, welcher die elektrostatische Abstossung kompensieren soll****. Es kann nun nicht behauptet werden, dass dieser Druck ausserhalb der Elementarteilchen verschwinde. Diesem Umstand werden wir in unserer phänomenologischen Darstellung dadurch gerecht, dass wir der Materie ein Druckglied beifügen. Dieses ist aber nicht mit dem Druck der Hydrodynamik zu verwechseln, der ja nur zur energetischen Darstellung dynamischer Verhältnisse innerhalb der Materie dienen soll.*

10. Additional comments on those topics are given e.g. given by E.Schrödinger, "Space-Time Structure", (12.29) ff.
"*Physical interpretation of the GRT cannot be done in a general coordinate system. It requires locally a SRT coordinate system (10.13) ff), which does not necessarily require (3,1)-spacelike/timelike space-time structure: K. Gödel gave an example of a new type of cosmological solutions of Einstein´s field equations of gravitation*."

11. H. Bergson, "*Materie und Gedächtnis*", *Dieses Buch bejaht die Realität des Geistes und die Realität der Materie und versucht die Beziehung zwischen beiden klarzulegen an dem speziellen Beispiel des Gedächtnisses. Es ist also ausgesprochen dualistisch. Aber andererseits betrachtet es Körper und Geist auf eine solche Art, dass es viel zur Milderung wenn nicht Hebung der theoretischen Schwierigkeiten beizutragen hofft, die immer aus dem Dualismus erwachsen sind und die daran schuld sind, dass er, den durch das unmittelbare Bewusstsein nahelegt und der gesunde Menschenverstand annimmt, bei den Philosophen in sehr geringem Ansehen steht.*"

Affected areas

The vision of an all-inlcusive „form, fit and function“ framework

covering the areas of mathematics, natural science and philosophy

TERMINOLOGIES

Form
not finally defined, yet: the below puts together some input information and great ideas to achieve this as a future output; the building of the form is essentially about a tbd common language based on adequately defined terms to enable the

Fit
based on the foundation of the ideas of Kant, Schopenhauer, Schrödinger in combination with the corresponding mathematical concepts of Weyl, Leibniz and with the propositions of this homepage

Function
is claimed to be achieved by
- integrating compatible concepts (e.g. teleology, onenes of mind, continuum, monads, infinitesimal "matter contact transforms" w/o only affine (momentum) directions, non experienced "be-ing" from the above (form) areas
- targeting for a consistent (language) framework, "only", not for a "true" ideology / religion, which is anyway always only built on human judgements.

FORM & FUNCTION OF LEIBNIZ' LEAST ACTION PRINCIPLE
From (KnA) p. 2-4, we summaries to following essence with respect to our new concept to build a quantum gravity model:

Leibniz’s basic conception is about the fact, that natural processes can be derived from (science specific) integral principles

A general teleology is about the fact that for every perception view (caused by a physical event) there can be detected a corresponding effort principle.

The principle of least action is the (purely) **form** of the (Leibniz) integral principle. It is characterized by the fact that the present is determined by the past and the future, while the corresponding natural principle defintion of Newton determines the future by the past and the present.

Related to the purely *form* there are multiple perception area /sciences specific integral principles.

(KnA) p. 43, "*Den tiefsten Zusammenhang der Teleologie oder sagen wir geradezu des Prinzips der kleinsten Wirkung mit der Kantischen Gedankenwelt gewinnen wir erst, wenn wir uns der Kritik der Urteilskraft zuwenden* ..."

(KnA) p. 55, " ... *so dürfen wir endgültig als Beziehung unseres Prinzips zur Kantischen Urteilskraft feststellen: D a s P r i n z i p d e r k l e i n s t e n W i r k u n g i n s e i n e r m o d e r n s t e n A l l g e m e i n h e i t i s t e i n e M a x i m e d e r r e f l e k t i e r e n d e n U r t e i l s k r a f t*."

PURPOSE

The purpose of this section is not to provide a foundation to define a new or modified existing philosophical concept, which incorporates the proposed quantum gravity in the one or the other philosophical concept dealing with the (still unsolved) dualism (mind & matter) challenges. It puts the spot to those philosophical concepts, which are seen as having the potential to be leveraged to one single integrated philosophical concept, while at the same time would be consistent with the axioms of the proposed quantum gravity concerning the infinitesimal small & large and the matter & mind challenges.

CONTINUUM, MATTER, FORM, SUBSTANCE

We claim, that the mathematical concept of a “point particle”, which is required to test the presence of (continuously "acting") forces, is the root cause for current conceptual miss matches between quantum and gravitation theory. For the GRT, as well as for the quantum theory point wise convergence of functions is of no interest. A to-be-developed mathematical GUT model needs to overcome the corresponding inherited constraints, basically caused by the concept of "particles", which goes along with the requirement to formulate a ("continuous") contact transformation (S. Lie) between "objects" w/o extensions. "Particles" are e.g. required to describe (directly) its movement (which requires the conception of "continuity", leading e.g. to the famous paradox of Zenon) or (indirectly, as test particle) to define "forces" as a consequence of a potential, which is only then "reality", if there is a test particle". This then ends up to the paradox of continuous forces in combination with felt "continuous" actions, but with "discrete" energy quantum.

If our proposed (truly inner (!) infinitesimal geometry based) quantum gravity model, based on the proposed "new ground state energy model", is resp. becomes a valid model, this would end up in some of the following conclusions:
"*God does not throw dice, God do not measure "displacements / distances / extensions" by counting (Peano axiom system) the number of normalized (finite) gauges and God does not need integers and rational numbers (ratios of integers) to measure "subsets / ratio of distances" of such normalized gauges. Both concepts are required to define the axiom of Archimedes / Eudoxus, which ensures a distance measurement between zero and any real number x not equal to zero"."*

For example the physical concept of "force" (through which physics represents reality) is an observable (source) of physical measurable attributes of matter, only, ... enabling "continuous" action transmission between "truly substances / monads (Leibniz)".
As the Legendre transformation is no longer valid in a strong sense in the infinitesimal small the Lagrange formalism is not applicable in a strong sense (differentiable functions) in the infinitesimal small."
The 30 seconds (...:)) elevator speech scope
Weyl continuum *not a really truly infinitesimal geometry* *Weyl* affine connexions *differential manifolds & exterior derivatives* Schrödinger oneness mind/psyche *effective operations / differential change* Schrödinger oneness form/shape *no substance & no observations geom. shape* Kant teleology *proposed Nature* e*ffectiveness principle* Kant judgments *verification of judgment&reality=(again judgm.* Schrödinger organism/metabolism *insistence: form/shape vs. not minutiue: material* Schopenhauer anorganic matter *insistence: material vs. not minutiue: form/shape* Schopenhauer opposition *both insistence/not minutiue: judgment of subject* Schrödinger fractale/discrete *judgm. change=discrete, even i. perfect continuum* Heidegger The(re-) being *new ontological difference principle* Heidegger "The turn" *mind & form: two sides of the same coin*.
Schrödinger emphasis e.g. on the question, when starting from a particle, then atome, molecule the chain up to an organism the entity starts to exist: a cell division generates two entities, just this and this is a discrete phenomenon. The counterpart in the plant life (flora) is the photosynthesis, which is perceived as a distant effect phenomenon, when solar energy is transformed to chemical energy. Both "development" processes generate a kind of *There-being*.

E. Schrödinger, "Mind and Matter":
"The objective world has only been constructed at the prize of taking the self, that is, mind, out of it remaking it; mind is not part of it; obviously, therefore, it can neither act on it nor be acted on by any of its parts. If this problem of the action of mind on matter cannot be solved within the framework of our scientific representation of the objective world, where and how can it be solved?"

"No single man can make a distinction between the realm of his perceptions and the realm of things that cause it, since however detailed the knowledge he may have acquired about the whole world, the story is occurring only once and not twice. The duplication is an allegory suggested mainly by communication with other beings."

THOUGHTS FIT FOR PURPOSE

Philosophical and mathematical ideas/thoughts

In order to make first some steps back in the way to consider and approach the several related topics of this area, we recommend to start with the philosophical thoughts of E. Schrödinger (mind & matter), which refer basically back to Kant and Schopenhauer.

E. Schrödinger: *"Vielleicht ist es sogar zulässig zu sagen: Metaphysik v e r w a n d e l t sich im Laufe der Entwicklung in Physik - freilich nicht in dem Sinne, wie es v o r Kant den Anschein haben mochte. Nämlich n i e durch allmähliche Sicherstellung vorerst noch unsicherer Meinung, sondern durch Klärung und W e c h s e l des philosophischen Standpunktes."*

(Sch3) german text, p. 31 "*Die Substanz hat ihre Rolle ausgespielt. Wir haben es nur mit Gestalten zu tun, die teils wechseln, aber doch auch verharren. .... Dabei müssen wir freilich Gestalt in viel weiterem sinn verstehen denn geometrische Form. Es gibt überhaupt keine Beobachtung, die auf die räumliche Gestalt eines Partikel oder eines Atoms abzielte*."

english text, p. 125, "But when you come to the ultimate particles constituting matter, there seems to be no point in thinking of them again as constituting of some material. There are, as it were, purely shape, nothing but shape; what turns up again and again in successive observations is this shape, not an individual speck of material.
In this we must, of course, take shape (or Gestalt) in a much wider sense than as geometrical shape. ***Indeed there is no observation concerned with the geometrical shape of a particle or even with an atom****."*

Schrödinger's major critique is about the common handicap of all western philosophy baseline assumptions, which is about *spacial and temporal multiplicity of examining and thinking individuals*. He is just rejecting this multiplicity and proposes instead a purely monoism of psyche ("cogitat - est") with its two parts, consciousness and subconsciousness, refering to Buddhistic philosophy (philosophy of Vedanta): "*multiplicity is only a p p e a r a n c e s, i t d o e s n o t e x i s t i n r e a l i t y*". The later one (subconsciousness) ensure functional / proper operations of this world's organisms (relating to the *"that which has being*"), while the first one is triggered, when differential changes happen to "effective operations" (of organisms); this is related to the philosophical term "das *Werdende*".

While Schrödinger refers back to philosophy history proposing new ideas with respect to the "*mind & matter*" question, Weyl refers back to philosophy with respect to the concept of "*infinity*". (The concept of a "zero" was introduced when changing from roman numbering system to decimal system to enable calculus dealing only with 10 integers, instead of infinitely integers).

(Sch1) p. 119, "*The second (antinomy) is our fruitless quest fot he place where mind acts on matter or vice-versa, so well known from Sir Charles Sherrington's honest search, …in "Man on his Nature". The material world has only been constructed at the prize of taking the self, that is, mind, out if it, removing it; mind is not part of it; obviously, therefore, it can neither act on it be acted on by any of its parts.* " ..

p. 121,"*In my own words I would express this by saying: Mind has erected the objective outside world of the natural philosopher out of its own stuff. Mind could not cope with this gigantic tasks otherwise than by the simplifying device of excluding itself – withdrawing from its conceptual creation. Hence the latter does not contain its creator. …*

Physical science … faces us with the impasse that mind per se cannot play the piano – mind per se cannot move a finger of a hand. Then the impasse meets us. The blank of the "how" of mind's leverage on matter. The inconsequence staggers us. Is it a misunderstanding?" ..

p. 122, "*Neither can the body determine the mind to think, nor the mind determine the body to motion or rest or anything else (if such there be)."*

p. 128, "The reason why our sentient, percipient and thinking ego is met nowhere within our scientific world picture can easily be indicated in seven words: ***because it is itself that world picture****."*

(Sch3), p. 115, "*Radical change in our ideas of matter*"
p. 122 "*Form, not substance, the fundamental concept"*
p. 125, " *The nature of our 'models'* "
p. 130, "*Continuous description and causality*"
p. 133, "*The intricacy of the continuum (das Kontinuumsproblem)"*
p. 151, "*The alleged break-down of the barrier between subject and object*"

Heidegger's concept of "*ontologische difference*" might provide the appropriate concept to link the philosophical terms of the-being / there-being with mathematical terms with respect to weak and strong PDO equations embedded in a distributional Hilbert scale framework.

(ReW1) p. 519, „*Die ontologische Differenz ist … ein Grundbegriff. … Grundbegriffe …. Sind Begriffe, mit denen „der Grund" begriffen, gegriffen, gefaßt, ja überhaupt erst erreicht, ja zuvor erst auch nur erahnt werden soll". … selbstwidersprüchliche Moment bestimmt auch den Grundbegriff der ontologischen Differenz."*

p. 520, „*Der Unterschied kann jetzt nur so gedacht werden, dass er gleichzeitig als ein Unterschied ‚in' und ‚ausserhalb' des Daseins gedacht wird; wobei das ‚in' und ‚ausserhalb' gleichermassen als räumlich und zeitlich und als nicht-räumlich und nicht-zeitlich zu verstehen ist. Die logisch unzulässige Kombination von ‚innerhalb' und ‚ausserhalb" zeigt an, dass die Dimension der Alltagspraxis, für die das Gesetzt der Kausalität und des Widerspruchsverbots gilt, transzendiert wird*."

p. 522, „*Wissenschaftliche Erkenntnis scheint zunächst etwas zu sein, was …. in der empirischen Welt vorkommt – es gibt sie einfach: Sie ist ein Seiendes unter Seienden. Damit aber, dass dieses Seiende, weil es ja nicht nur ‚ist', sondern anderes Seiende objektivieren, zum Gegenstand machen kann und somit einen Unterschied zwischen sich und dem Objekt legen kann, zeigt sich, dass es etwas gibt, was diese Ebene des Seienden übersteigt – nennen wir es ‚Sein'."*

p. 523, „*Wir sollten Denken nicht nur als Feststellen von Qualitäten und Ursachen/Folgen verstehen, sondern vor allem als Fragen danach, wie das möglich ist." … Dieses Fragen ist ein Fragen nach der besonderen Art, in der die Wesen, die so fragen, ‚sind'. Es ist ein Fragen nach dem Sein des Seienden – also nach dem Unterschied von Sein und Seiendem. … Das heisst also, dass es das Sein nicht mehr gab, als es vergessen wurde.* ***Das Sein gibt es erst dann wieder, wenn es gedacht wird****.*

Wissenschaftliche Erkenntnisse werden in Urteilen gefasst. Die Übereinstimmung zwischen dem Urteil und der Wirklichkeit aber bleibt unsicher. Jede sogenannte Verifikation ist ja auch nichts weiter als wieder ein Urteil. …Jedes Urteil ist Interpretation, oder, wie Heidegger es nennt ‚Verstehen'."

p. 526, „*Wahrheit", so meint Heidegger, ist bei Plato und Aristoteles definitiv als Richtigkeit der Aussage (miss-) verstanden worden. … Das richtige Urteil beruht dabei auf einem angemessenen Vorstellen, das sich entweder – idealistisch – auf das vorgestellte (perceptum oder idea) oder auf den Gegenstand (Realismus) bezieht*."

p. 527, „*Heidegger fasst nun seinerseits die Beziehung zwischen Denken und Seiendem als ‚Offenheit'. Nur Dank der Offenheit ist Richtigkeit möglich - nicht umgekehrt*."

p. 529, „*Die Grundfrage richtet sich nun auf das ‚Kehrige' von Seyn und Seiendem, das sich im Dasein ereignet. Seyn und Seiendes werden nun als Gegensätze, die sich zu einander kehren, in einer Einheit gefasst. Sie sind dabei nicht als Teile eines Ganzes zu begreifen, denn sie sind zugleich Teil und ganzes, indem sie jeweils füreinander – und für sich – das Andere und das Selbe sind. .... Heidegger verabschiedet sich hier von seinem Ansatz in Sein und Zeit".*

p. 531, „*Es gibt also eine klar fassbare Korrespondenz zwischen Seyn und Da-sein*."

p. 537, "*Mit dem Denken der kehre setzte in Heideggers Denken eine radikale und konsequente Änderung ein. Mit ihm löst er erst die radikalen Versprechen aus Sein und Zeit ein."*

p. 538, „*Dieser Versuch artikuliert sich beispielhaft in der Auflösung der gängigen Vorstellungen von Identität und Differenz*."

Weyl / Schrödinger: Matter, mind, mathematics and natural science

((Sch) p. 49, "*Vielleicht ist es sogar zulässig zu sagen: Metaphysik v e r w a n d e l t sich im Laufe der Entwicklung in Physik - freilich nicht in dem Sinne, wie es v o r Kant den Anschein haben mochte. Nämlich n i e durch allmähliche Sicherstellung vorerst noch unsicherer Meinung, sondern durch Klärung und W e c h s e l des philosophischen Standpunktes."*

NaT),p. 13: "*Ausgangspunkt für meine Argumentation ist das Scheitern des psychophysischen Reduktionismus, eine Position in der Philosophie des Geistes, die weitgehend von der Erwartung motiviert ist, zeigen zu können, dass die physikalischen Wissenschaften im Prinzip eine Theorie von allem liefern könnten*. .."
(TaR) p. 31, "*Gödel bewies: Cantor, der sich .... bemühte, die Stufe der Unendlichkeit des Kontinuums zu fixieren, musste scheitern. Steckt man die Mengenlehre in ein formales Korsett, kann niemand, ..., die Unendlichkeit des Kontinuums orten, und dies wird auch in aller Zukunft niemandem ?"*
(TaR) p. 32: *".. dass aber gerade die naheliegende Frage nach dem Wesen der Unendlichkeit einer Geraden unentscheidbar sein sollte, empfand er (*Gödel*) trotzdem als kläglichen Mangel der Mengentheorie"*

(ReW) p 12, "nur Philosophie kann zeigen, warum die Erkenntnisse er Wissenschaften wirklich wahr sind, sie liefert die Kriterien mit Hilfe derer wir zwischen Wahrheit und Unwahrheit unserer Urteile unterscheiden können."

(ReW) p. 17, "*Sein ist das transcendens schlechthin"*

(ReW) p. 19, "Heidegger versucht mit dem Denken des Daseins, die Trennung von res extensa und res cogitans ... zu unterlaufen. ..... bekämpft Heidegger die nach ihm auf Descartes zurückgehende Vorstellung, dass das Subjekt als die Grundlage und gar das Zentrum der Philosophie gedachtbwerden sollte. Wer so denkt, verfehle unvermeidlich die Frage nach dem Sein und dem Dasein, weil er beide ontisch denkt..... Das, was Heidegger ontologische Differenz nennt, ist also der Unterschied von Sein und Seiendem, zugleich aber ihre dynamisch gedachte Identität. ...."
(ReW) p. 109, "*Heidegger: Sprache ist das Haus des Seins*"
(WeH2) p. 18: *"I am convinced that the substance has lost its role in physics"*

(WeH2) p. 19: ""*the concept of "momentum" appears to be primarily to the concept of "mass/matter""*

(WeH2) p. 20. *"the mass of a body is determined by its state"*

(WeH2) p 31: *"when using a test particle to test/model the action of a field one already disturbs the field"*

(WeH2) p. 44: *"a strictly intuitive rational of a mathematical theory of the continuum (as drafted by Brouwer and Weyl) were required to build the continuum as a medium, where single particles can be identified, but where the set of particles can be resolved"*

(WeH2) p. 49: *"the today's relationship between matter and field is dynamical: the matter builds the field, the field acts on the matter*"

(WeH2) p. *"For Leibniz the "reality" of movement is not built on movements (change of the position), but on the causing force; "La substance est un etre capable d'action - une force primitive""*

(WeH2) p. 58: *"....the Leibniz agens theory of matter can be executed by the GRT. Based on this a matter particle is even not a point in the field space, even not any kind of something related to "space" (extensive)"*

(WeH2) p. 59: *"what is matter? After the perception of the concept of substance has been quashed, the today's beam vacillates between a dynamic and a field theory of matter"*

(BlS)… Kant thought, that if we can only know objects because of their potential effects on others, their powers, then it seems that we are only responsive to what they do but not responsive, necessarily, to what they are. He thought that there have to be „other intrinsic properties, without which the relational properties would not exist because there would be no subject in which they inhered“. But it is not clear how we can know about this „subject“…. Are we therefore cut off from the world as that? Then we would be caught in a „false imaginary world“ (Bishop Berkeley).
Michael Faraday thought, that we could just do without Kant´s „other intrinsic properties“. Suppose we try to distinguish a particle x from the powers or forces m whereby it makes its influence known. Then, Faraday writes,

„"*to my mind … the x or nucleus vanishes, and the substance consists of the power, or m, and indeed what notion can we form of the nucleus independent of its power: what thought remains on which to*

hang the imagination of an x independent of the acknowledged forces? Why then assume the existence of that of which we are ignorant, which we cannot conceive, and for which there is no philosophical necessity?“"

The problem which this is whether we can be satisfied with the idea that „the substance consists of the powers“, or whether contrary to Faraday there is some kind of philosophical necessity to posit a substance as well, a nucleus or thing that actually possesses the powers.

But there is an argument that we need Kant´s further category of intrinsic properties. We might call it the not-just-washing argument, after Bertrand Russell, who talks in his book „The Analysis of Matter“ of how „there are many possible ways of turning things hitherto regarded as „real“ into mere laws concerning the other things,“ and remarks, „Obviously there must be a limit to this process, or else all things in the world will merely be each other´s washing.“ The conclusion is that even if we have trouble understanding things apart from their powers, nevertheless we seem to need them. We seem to need them because otherwise we have no conception at all the actual world.

(ScE) p. 1594): "***... a truly infinitesimal geometry ...should know a transfer principle for length measurements between infinitely close points only***".

(TaR), p. 17, „*Das Problem, von dem die Rede ist, lautet knapp formuliert so: Wie hängen Arithmetik und Geometrie, die beiden Grundpfeiler der Mathematik, zusammen? …*

***Geometrie**, …, fußt unmittelbar auf sinnlichen visuellen Eindrücken, raubt ihnen jedoch Buntheit, Körperlichkeit, Vergänglichkeit, Verletzbarkeit, kurz: alle opaken und barocken Reize. Geometrie verkürzt die optische Wahrnehmung so lange, bis nur mehr einzelne Punkte und Linien übrig bleiben. …*

***Arithmetik**, die Theorie der Zahlen, …, hat hingegen nur mittelbar mit sinnlichen Eindrücken zu tun: zwar sehen und fühlen wir Geldstücke, die wir zählen, aber wir sehen und fühlen nicht die Zahl Dreißig selbst, sondern nur die Silberlinge, die wir mental mit Dreißig verbinden. ….Weder optisch, noch taktil, noch akustisch, noch sonst wie sinnlich empfinden wir eine Zahl sui generis, kein Sinnesorgan vermag sie direkt zu empfangen.*

Jedoch: Man kann Zahlen geometrisch veranschaulichen. Jeder Maßstab beleg es: Er bringt Zahlen auf einer Gerade unter. Arithmetik erweist sich folglich als geometrische Disziplin. Wie man hingegen alle, ausnahmslos alle Punkte einer Geraden umgekehrt als Zahlen zu deuten vermag, blieb seit den Tagen des Pythagoras … ein Rätsel.

… Ist die Geometrie sogar so exakt, dass sich die sinnlichen Anschauungen, auf der sie zu beruhen scheint, in Wahrheit als überflüssig entpuppt? Würden wir, ohne Rückgriff auf Sehen und Tasten, alleine aufgrund arithmetischer Gesetze alle Einsichten und Erkenntnisse der Geometer gewinnen?“

(ScE) p. 77: " *Der Grund dafür, daß unser fühlendes, wahrnehmendes und denkendes Ich in unserem naturwissenschaftlichen Weltbild nirgends auftritt, kann leicht in fünf Worten ausgedrückt werden: Es ist selbst das Weltbild. ... Aus diesem (*arithmetischen*) Paradoxon gibt es zwei Auswege, die beide vom Standpunkt unsres heutigen naturwissenschaftlichen Denkens aus .... reichlich unsinnig aussehen. Der eine ist die Vervielfachung der Welt in Leibniz' schrecklicher Monadenlehre, in der jede Monade eine Welt für sich ist, es ist keine Verbindung zwischen ihnen. .... Offenbar gibt es nur e i n e n andren Ausweg: die Vereinigung aller Bewußtseine in eines. Die Vielheit ist bloßer Schein; in Wahrheit gibt es nur ein Bewußtsein.*

(ScE) p. 109: "*Indessen liegt die überragende Bedeutung von Kants Behauptung gar nicht in einer richtigen Verteilung der Rollen auf den Geist und auf sein Objekt -die Welt- in dem Prozeß, in dem "sich der Geist eine Vorstellung von der Welt bildet". Das Große war, den Gedanken zu fassen, daß dieses e i n e D i n g - Geist oder Welt - sehr wohl andrer Erscheinungsformen fähig sein kann, die wir nicht zu erfassen vermögen und die die Begriffe Raum und Zeit nicht enthalten. Das bedeutet eine eindrucksvolle Befreiung von einem eingewurzelten Vorurteil. Wahrscheinlich gibt es andre Arten, die Erscheinungswelt zu ordnen als die raum-zeitliche. Ich glaube, es war* Schopenhauer*, der Kant zuerst so verstanden hat."*

(ScE) p. 117: "*Einstein hat nicht - ... - Kants tiefe Gedanken über die Idealisierung von Raum und Zeit*

widerlegt. Er hat im Gegenteil einen großen Schritt in Richtung auf ihre Vollendung gemacht.

(ScA) Bd 2, §29, 193: *"Daß diese Welt, in der wir leben und sind, ihrem ganzen Wesen nach, durch und durch Wille und zugleich durch und durch Vorstellung ist; daß diese Vorstellung schon als solche eine Form voraussetzt, nämlich Objekt und Subjekt, mithin relativ ist; und wenn wir fragen, was nach Aufhebung dieser Form und aller ihr untergeordneten, die der Satz vom zureichenden Grund ausdrückt, noch übrig bleibt; dieses als ein von der Vorstellung* toto genere *Verschiedenes, nichts Anderes seyn kann, als Wille, der sonach das eigentliche Ding an sich ist. Jeder findet sich selbst als diesen Willen, in welchem das innere Wesen der Welt besteht, so wie er sich auch als das erkennende Subjekt findet, dessen Vorstellung die ganze Welt ist, welche insofern nur in Bezug auf sein Bewußtseyn, als ihrem nothwendigen Träger, ein Daseyn hat. Jeder ist also in diesem doppelten Betracht die ganze Welt selbst, findet beide Seiten derselben ganz und vollständig in sich selbst. Und was er so als sein eigenes Wesen erkennt, das Selbe erschöpft auch das Wesen der ganzen Welt, des Makrokosmos: auch sie also ist, wie er selbst, durch und durch Wille, und durch und durch Vorstellung, und nichts bleibt übrig. So sehn wir hier die Philosophie des Thales, die den Makrokosmos, und die des Sokrates, die den Mikrokosmos betrachtete, zusammenfallen, indem das Objekt beider sich als das Selbe aufweist."*

(ReW) p. 32, "*Wissenschaftliche Erkenntnisse werden in Urteilen gefasst. Die Übereinstimmung zwischen dem Urteil und der Wirklichkeit aber bleibt unsicher. Jede so genannte Verifikation ist ja auch nichts weiter als wieder ein Urteil. Deswegen ist es ratsam, von vornherein zu akzeptieren, dass Urteile keinen absoluten Wahrheitsanspruch geltend machen können. Jedes Urteil ist Interpretation oder, wie Heidegger es nennt, "*Verstehen*"."*

(KaM) p. 12, "*because general relativity and quantum mechanics can be derived from a small set of postulates, one or more of these postulates must be wrong. The key must be to drop one of our commonsense assumptions about Nature on which we have constructed general relativity and quantum mechanics. Over the years, several proposals have been made to drop some of our common sense notions about the universe:*

1. *continuity*
2. *causality*
3. *unitarity*
4. *locality*
5. *point particles*".

We propose to redefine this list starting with the concept of ordered fields. It is obvious, that this mathematical principle is essential for all of the above 5 notions above, which is a mix of mathematical and physical notions anyway. In other words, we recommend re-defining the list above with purely mathematical (transcendental) axioms (with first attempt to replace ordered field by non-ordered field and causality by purpose).

We claim that the following notions are sufficient for a quantum gravity theory:

- distributional Hilbert space H(-a) to model physical laws as (teleological) action minimization problem

- Archimedean non-ordered field as appropriate model for transcendental "energy" "particles".

(ToA) p. 21, H. Berson, "*Damit erhält man zwei Ausdehnungen: einmal eine Ausdehnung, in der sich die endlichen Dinge bewegen, während die andere Ausdehnung als Unendliches, Homogenes betrachtet wird. .... Die Bewegung ist wie die wirkliche Zeit nicht teilbar und auch nicht auf den Raum reduzierbar. Sie ist im wahrsten Sinne des Wortes nur mit der Dauer möglich. In diesem Sinne hat die Bewegung mit der teilbaren, messbaren Zeit wenig zu tun."*

In order to link practical reason (*subject* related with will and freedom of human beings) with theoretical reason (object related with natural recognition), Kant built "the critique of judgement" with the (newly introduced) conception of ***effectiveness*** (purpose related) to the two areas, aesthetics (as part of the supernatural human being world: subjective effectiveness) and organisms (as part of Nature: objective effectiveness). Kant's proposal to put beside the (mechanical determined) ***causality*** conception of Nature the additional principle (="axiom", which cannot be proved, as it is transcendental per definition) of ***effectiveness*** has been forgotten due to the success of natural science in the last

century, but the conception of this homepage is again referring to it, which goes in line with (NaT).

"Wave-particle dualism" related statements

1. "*Nature must* (now, can) *be also thought teleological*", Kant I., Critique of judgement

2. "*God does not play dice*" seems to be right (if he/she/it exists, as teleology is a human built concept, same as causality): the framework of probability theory is the Hilbert space L(2)=H(0), which is a compact embedded subset of any Hilbert space H(-a), a>0

3. Planck's comments to his black body radiation observations are still valid, as any experimental results/measurements can only be validated in at least a H(0) framework

4. Bohr's interpretation of experimental results is building on wrong assumption, that the H(0)=L(2) Hilbert space, which provides the proper mathematical quantum mechanics (Hilbert space) model, is also the adequate one to model quantum dynamics and underlying action principles.

Kant: "Critique of Judgment"
(KnA) p. 43, "*Den tiefsten Zusammenhang der Teleologie oder sagen wir geradezu des Prinzips der kleinsten Wirkung mit der Kantischen Gedankenwelt gewinnen wir erst, wenn wir uns der Kritik der Urteilskraft zuwenden, und zwar unter der Führung des hervorragenden Neukantianers Stadler,* ..."
(KnA) p. 55, " ... *so dürfen wir endgültig als Beziehung unseres Prinzips zur Kantischen Urteilskraft feststellen: D a s P r i n z i p d e r k l e i n s t e n W i r k u n g i n s e i n e r m o d e r n s t e n A l l g e m e i n h e i t i s t e i n e M a x i m e d e r r e f l e k t i e r e n d e n U r t e i l s k r a f t*."
A big miss-understanding concerning mathematical model of acknowledged physical laws is, that 'real numbers' are all real, in fact nearly all of them are transcendental; the set of rational numbers is a zero set in the sense of the Lebesgue integral, which is the standard inner product for probability theory and quantum theory. As a consequence every mathematical ((weak or strong) partial differential equation(s)) model, based on which a physical law is described, is per definition transcendental.

Current inconsistency between the mathematical model of quantum theory and gravity theory are therefore inconsistent from a mathematical model point of view in the transcendental area. The root causes of this are either over-determined axioms/principles (put in place by human reason/mind) or inconsistent axioms, if one rejects the (stupid?) option, that there can be two or more transcendental areas, which are "inconsistent" from a mathematical point of view.

Each mathematical model built on real numbers is per definition transcendental; if the model is declared as a physical law, this is building of human reason/*Vernunft*, which goes beyond human intellect/mind/understanding/*Verstand*. According to ***Critique of Pure Reason*** (*the area, which human develops by sensuous perception; Nature / recognition / notion / sensualism; causality*) the intellect a priori disposes about notions which prove right but which cannot a posteriori be verified. The same approach is applied by Kant in his ***Critique of Practical Reason*** (*the area, which human develops only in human thoughts/mind; human /will / freedom / transcendence / supernatural; purpose*) to "explain", why there is obviously a room of freedom for human beings to decide, while everything else in the Nature seems to determine by causality only. Not the intellect defines a priori laws/axioms, but the reason. The area of the *Critique of Practical Reason* is per definition the transcendence, which is the area, which can be acknowledged by the human being only in his thoughts. The same is true for real numbers, as mentioned above.

We emphasis, that variational equations are equivalent (if certain regularity requirements are fulfilled) to corresponding "operator norm (action) minimization" problems. This is per definition a model following a principle of "purpose/effectiveness", not of causality.

The concept of "effectiveness" (teleology) to build the bridge between *Critique of Pure Reason* and *Critique of Practical Reason*, answering the "problem of the concrete", was introduced by Kant in his "***Critique of Judgment*** (CoJ)", (LuR), p. 121 "*Die teleologische Ureteilkraft*".
Kant used the following definition of „judgment": It is the capability to think the specialty/specific thing as part of the general thing.

The CoJ is built on two conceptual elements:

1. the subjective effectiveness: senses of perception and aesthetic sensitivity & sublime related; the later one with the interior/mathematical part, covering the infinite small and large(!), and the exterior/dynamic part, covering power and violence

2. the objective effectiveness: matter/Nature and formal/mathematics related.

We focus on the formal/mathematics area with Kant's exterior/relative and interior/absolute effectiveness principles. We propose to introduce back Kant's teleological concept into the Nature (additionally to the causality principle), i.e. to put to Nature (per reason decision) the effectiveness principle beside the "causality" principle. We claim, that our distributional Hilbert space framework in combination with the mathematical "action minimization" model (equivalent to variational equation) is an appropriate (per definition transcendental (!) model for Kant's exterior and interior effectiveness:

exterior/relative effectiveness: physical laws based on variational theory and Lagrange formalism, force (always related to matter/real numbers), probability theory, H(0) Hilbert space

interior/absolute effectiveness: mathematical models based variational theory and on Hamiltonian formalism, energy (related to hyper-real numbers), distributional theory, H(-a) Hilbert space.

The "organisms" world (as part of the Nature) with its acknowledged concept of matter following causality principles is built based on the standard L(2)=H(0) Hilbert space (HS). This HS is by accident, by chance or by purpose the "natural" HS to enable probability theory.

L(2) is compact embedded into any HS H(-a), a>0, i.e. the measure theory enables the building of measures that events in H(0) become zero sets, when they are created by projection (realization in organism world) from H(-a) "world".

The HS H(0) is the framework of today's acknowledged causality determined world, following HS energy or operator norm minimization principles to formulate physical laws. The mathematical concept for "organisms" is the Lagrange principle, which is basically about "work" minimization, based on the concept of force.

In case the Legendre transformation is valid, this is mathematically equivalent to the Hamiltonian formalism, based on the concept of "energy".

The proposal is, that there is a "mind" related "energy" definition in the larger HS H(-a), based on the concept of Leibniz' monads (which are differentials), modelling no longer "dead" matter, but "active" (of course, transcendental entities, as any real ,transcendental number) "entities".

The corresponding Hamiltonian minimization principle is no longer equivalent to the "real" world Lagrange formalism, as the Legendre transformation is no longer valid.

The Penrose-Hawking dispute about "*The Nature of Space and Time*"

Penrose rep. Hawking are supporter of platonism resp. positivism. This goes along (more or less a kind of re-branding, building on the first Solvay conference) with the dualism of idealism resp. empiricism (F. Bacon). Both concepts one can belief or not, but it needs a choise. If one has made a choice, there were/are antonomies in both concepts. Kant was the first and last of the philosophers, who developped a concept to overcome this dualism. There is no chance to show/prove evidence to this concept, which is an intrinsic consequence of the concept of transcendence; and that's where we are and where we will be!

Penrose's "3-world-model" (PeR1), which adds a "mind world" to "Plato's world" and "physical world", just increases the complexity of "Plato's world", while not adding any additional value to Kant's conception. Needless to say that Hawking's concept of integrating "mind world" into the "physical world" (i.e. finally mind can be "explained" experientially!) has also been overcome by Kant resp. is the source of antinomies.

Sorry for the kind of blasphemous sounding statement, but this comes across (just!) like a kind of reincarnation of Leibniz (mathematician) versus Newton (physicist), both "platonists", with their concepts of E(pot)= m * v*v/2 and (transcendental) monads versus E(pot) = m* v * v (!!), F = m * a and

(trancendental (!)) particles.

(WiL); "*what we cannot speak about it, we muss pass over in silence*"

The probably more fruitful approach to this "dualism" challenge could be an analysis of the question, which of the following alternatives are the more appropriate axiom in order to conceptual capture the "*problem of the concrete*" of the connection between the physical "object related/natural" relevance of physical (natural) laws and its description by mathematical ("transcendental") (PDE) models:

axiom option 1: mathematics is a construct of human mind, i.e. mathematics is an invention of mind (with its consequences to Schrödinger's arithmetic paradox (ScE)

axiom option 2: mathematics exists independently from human existence, i.e. mathematics is a discovery of mind.

At a first glance, the first option might sound somehow disappointingly, especially perhaps for mathematicians, but in the light of Gödel's result, it might turn around to be in fact good news, at a second glance! It is related to Kant's *Critique of Pure Reason* and model theory is a follow-up invention driven by rationality, not by mind (according to Kant), of course, with the risk of antinomies.

For axiom option 2 in his *Critique of Judgement* Kant puts mathematics as an own category to deduce from the general to the specific with respect to two aspects: an esthetical and teleological perspective. In other words, mathematics is a characteristic of an esthetical and fitness for purpose principle of Nature.

Quotes from (Kal)

"*Als* (innerer Natur-) *Zweck ist ein Ding nur möglich, wenn es nicht ursprünglich von einem Mechanismus der Natur verursacht ist, sondern von einer Wirkung, die durch Begriffe bestimmt ist. Auch darf seine Form nicht vom Verstand alleine erkannt werden können. ..*

Ich würde vorläufig sagen: das Ding existiert als Naturzweck, wenn es von sich selbst (obgleich in zwiefachem Sinne) Ursache und Wirkung ist.

Ein organisiertes Wesen ist also nicht bloß Maschine; denn die hat lediglich bewegende Kraft; sondern sie besitzt in sich bildende Kraft ..."

Kant's CoJ is about the subjective (aesthetical) and objective (teleological) concept of judgement is about the following structure:

Die teleologische Hypothese lautet, dass die Dinge vielleicht nicht alleine von wertfreier Chemie und Physik festgelegt werden, sondern außerdem noch von etwas anderem, nämlich einer kosmischen Prädisposition für die Schaffung von Leben, Bewusstsein und Wert, der von ihnen nicht ablösbar ist.

Der Hypothese einer natürlichen Teleologie zufolge besäße die Welt der Natur einen Hang, Wesen von der Art entstehen zu lassen, die ein Wohl haben -Wesen, für welche die Dinge gut oder schlecht sein können. Dies sind alle tatsächlichen und möglichen Lebensformen (Organismen).

The **mathematical model of the "teleology**" part of the Nature is given by the **Hamiltonian formalism** in distributional Hilbert space. The Lagrange formalism is valid in the higher regular Hilbert space H(0), which is by chance, by purpose or by accident the appropriate framework for probability theory.

With respect to the above we would like to draft the following analogue:

1. material related effectiveness: -->Nature, matter-mind, physical-transcendental related, i.e. an exterior effectiveness according to Kant --> physical world with the concept of force (Lagrange)

2. formal/mathematical effectiveness: force - energy, physical-mathematical related, i.e. an interior effectiveness according to Kant --> mathematical world with the concept of energy (Hamilton)

We emphasis, that the Lagrange and the Hamiltonian formalisms are only equivalent, in case the

Legendre transformation is valid.

The H(-a) into H(0) projection can be interpreted as zero sets (same as the rational number as part of the "standard" real number or as part of the "non-Standard" numbers). From this there might be a further extension from Kant's judgement critique to Heideggers "being and time" with respect to

--> "dem Hineingeworfen-sein".
(ReW), "*Für Heidegger ist es die Zeit, die das Sein und damit den Sinn von Sein konstituiert. Wesentlich für das Verständnis von Sein ist die Einsicht, dass das Sein unzertrennbar mit dem Nichts verknüpft, ja davon durchzogen ist. Unser Dasein ist wesentlich durch die Möglichkeit und die Realität des Nicht-seins bestimmt. Der Tod ist die jede Sekunde präsente Möglichkeit des Nicht-seins. Realität ist das Nicht-sein selbstverständlich in dem Sinn, dass wir wissen, dass es uns lange Zeit vor unserer Geburt nicht gegeben hat und dass es eine lange Zeit geben wird, in der wir nicht mehr da sein werden.*

(ScA), p. 730: "*However, the dying needs to be qualified as the very rationale purpose of life*".

"*Materie ist "Wirken", ist "Kausalität"; Veränderung von Körpern stellt sich in Raum und Zeit ein, aber realisiert sich doch an der Materie". ....Materie ist Wahrnehmbarkeit von Raum und Zeit*",... "

Nun aber erhält das Gesetz der Kausalität seine Bedeutung und Notwendigkeit allein dadurch, daß das Wesen der Veränderung nicht im bloßen Wechsel der Zustände an sich, sondern vielmehr darin besteht, daß an dem selben Ort im Raum jetzt EIN Zustand ist und darauf an ANDERER, und zu EINER und derselben bestimmten Zeit HIER dieser Zustand und DORT jener; nur diese gegenseitige Beschränkung der Zeit und des Raumes durch einander giebt einer Regel, nach der die Veränderung vorgehen muß, Bedeutung und zugleich Nothwendigkeit. Was durch das Gesetz der Kausalität bestimmt wird, ist also nicht die Succession (Folge) der Zustände in der bloßen Zeit, sondern diese Succession in Hinsicht auf einen bestimmten Raum, und nicht das Daseyn der Zustände an einem bestimmten Ort, sondern an diesem Ort zu einer bestimmten Zeit ... Demzufolge vereinigt die Kausalität den Raum mit der Zeit (§4, S. 39)"

Wirken im Sinne von Kausalität --> Veränderung zeigt sich an der Materie
Schopenhauer*: The World as Will and Representation*":

http://en.wikipedia.org/wiki/Critique_of_Judgment

Heidegger: "Science" and "Ontological Difference"
"*The "logical" concept of **science** understands science with respect to its results and defines it to be a "nexus of explanatory and true — i.e., valid, correctly formed — propositions". The existential concept of science understands it, in contrast, to be a mode of existence and therefore to be a mode of being-in-the-world, a mode that uncovers or discloses either what-is or being. A fully adequate existential interpretation of science can only be carried out once the sense of being and the relation between being and truth have been understood on the basis of the temporality of existence…It is only within this understanding that the ideal of phenomenology can be developed — in contradistinction to the pre-conception that has already been indicated.*

The essence of what today is called science is research. It is essential to research that cognition, as practice, orient itself toward a realm of beings — of nature or of history. Here, procedure does not mean just method or procedure; every practice has need of an open region within which it already moves. However, the opening up of such a region is the basic practice of research. The opening-up is carried out here by projecting within some realm of beings, e.g., within nature, a definite outline of nature-processes. The project predelineates in what way cognitive practice has to commit itself to the region which is opened up. It is this commitment or stricture which is the rigor of research. Through its projection of the outline and through the definition of rigor, the practice of research assures itself of its region of objects within the relevant realm of being.
*Only on the way toward solving the basic **ontological** problem of the **difference** and **relationship between being and beings** can the Kantian thesis "being is not a real predicate" be at once both grounded and complimented by a radical interpretation of being at large as extantness (actuality, being there, existence)."*

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